Intriguin solutions of Bethe-Salpeter equation for radially excited pseudoscalar charmonia

Vladimír Šauli
NIP, Rez near Prague
sauli@ujf.cas.cz

Excited QCD
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Outline

- Motivation and introduction
- 4d BSE in Euclidean space
- phenomena- doubling of excited states
- way out ? "confining propagator" (BSE in Minkowski space)

Main motivation

- main aim: simple relativistic -fully Lorentz covariant- model for excited mesons (heavy quarkonia)

- BSE has never been used for excited quarkonia in its full 4d form!

  Are Heavy quarkonia heavy enough? that QM or RQM can be used

- What are the retardation effects and how much is relativity (quantum field theory description) important and urgent for Charmonia, recall $< v^2 >= 0.25c$ Quigg-Rosner (1977)

- What is the full Lorentz form of the interaction? $V_s + V_v$

Motivation and Introduction

- theory:

  QFT BSE —nonrelativistic limit —> S.R. for quarkonia

  QFT BSE — instantaneous approximation—> 3d "semirelativistic" Salpeter equation (retardation effect neglected)

- practice:

  S.R., Salpeter, Wilson approaches are extensively used —> EMG +hadronic transitions + spectroscopy —> energy eigenvalues are mixture of orthogonal states due to the relativity

  overview:

N. Brambilla,..., Heavy quarkonium, puzzles, and opportunities, arXiv:1010.5827

E. Eichten, arXiv:0701208, Quarkonia and their transitions

most observations- QM +(some knowledge from QCD) is enough spectroscopy is simple , nonrelativic QM with phenomenological V

Schrodinger equation with quark-antiquark static potential reproduce plethora of observed meson

QM to be generalized - Linear quark-antiquark static potential from lattice Wilson loops => Phenomenology and practice

Cornell potential $V = c_1/r + c_2 * r + c_3$

Eichten (1975, 1976, 1978...)

Quigg-Rosner (1977) $V = clog(r/r_0)$

or more recently: (string breaking effect+non-Abelian Schwinger mechanism)

$V = c_1 * exp(-c_3 r) + c_2/r + c_4$

Bai-Qing Li,... ,arXiv:0903.5506 V. Vento, ... arXiv:1108.2347

suggestion: backward check of ideas:

- 1. Choose modern and succesful model of Quarkonia available on a market....
- 2. find Poincare invariant generalization of the interaction V ...
- 3. and solveQuarkonium BSE with V in its full form for given J^{PC}...
- 4. and their transitions

Peniche 2012: 1+2+3 for $\eta(nS)$ for arbitrary n.

Conventional BSE

QM model to be generalized :

SR with P $V = c_1 * exp(-c_3 r) + c_2/r + c_4$ or

$V = c_1 * exp(-c_3 r) + c_2 * exp(-c_5 r)/r + c_4$

$$\Gamma(q,P)_{\mathsf{k,l}} = -i \int \frac{d^4k}{(2\pi)^4} \left[S(q-P/2)\Gamma(p,q)S(q+P/2)\right]_{\mathsf{j,i}} V(k,q,P)_{\mathsf{i,k,l,j}}$$

where Latin letters $i, j..$ represent Dirac indices. Explicitly for pseudoscalar

$$\Gamma_P(q,P) = \gamma_5 \left(A(q,P) + \not{P} C(q,P) + \not{q} B(q,P) + [\not{q}, \not{P}] D(q,P)\right)$$

Alternatively the BSE (more singular) BS wave function χ

$$S^{-1}(q-P/2)\chi(p,q)S^{-1}(q+P/2) \; S^{-1}(p) = \not{p} - m_c$$

V_s dominant Lorentz scalar (spin degeneracy in heavy meson sector) V_v Vectorial interaction is naturally assumed in QCD

$$V(k,q,P)_{iklj} = 1V_s + \gamma_{\mu,ik}\gamma^{\mu}_{lj}V_v$$

$$V_s = \frac{C}{((k-q)^2-\mu_s^2)^2} \; V_v = \frac{g^2}{(k-q)^2-\mu_v^2}$$

Numerical search

Projection+Wick rotation in relative momentum only

Integration over the spacelike part (angles)

–> 2dim coupled set of four real scalar Eqs. for $A, \beta = iB, C, D$

Solving 2dim integral equations by brute force iteration

$$
\begin{aligned}
A(p,P) &= \lambda(P)\int_{-\infty}^{\infty} dk_4 \int_0^{\infty} \frac{d\mathbf{k}\mathbf{k}^2}{(2\pi)^3} U_A \left(\kappa I_S^{[1]} - 4g^2 I_V^{[1]}\right) G_2 \\
U_A &= A(k,P)\left(k_E^2 + \frac{P^2}{4} + m_c^2\right) + 2D(k,P)\left(-k_E^2 P^2 + (k_4 M)^2\right) + \\
\beta, C, D, (p,P) &= ....
\end{aligned}
$$

Stability achieved by suitable "normalization"

$$
\lambda^{-1}(P) = \frac{1}{2}\int dk_4 \int d\mathbf{k} \frac{\left[A_i(k,P) + A_{i+1}(k,P)\right]^2}{k_4^2 + \mathbf{k}^2 + m_c^2}
$$

i- iteration step

$$\sigma^{-1}(P) = \frac{1}{2}\int dk_4 \int d\mathbf{k} \frac{\left[A_i(k,P) - A_{i+1}(k,P)\right]^2}{k_4^2 + \mathbf{k}^2 + m_c^2}$$

$64 * 128$ integration points , $\simeq$ few weeks with single processor machine solutions $\lambda = 1, \sigma = 0$.

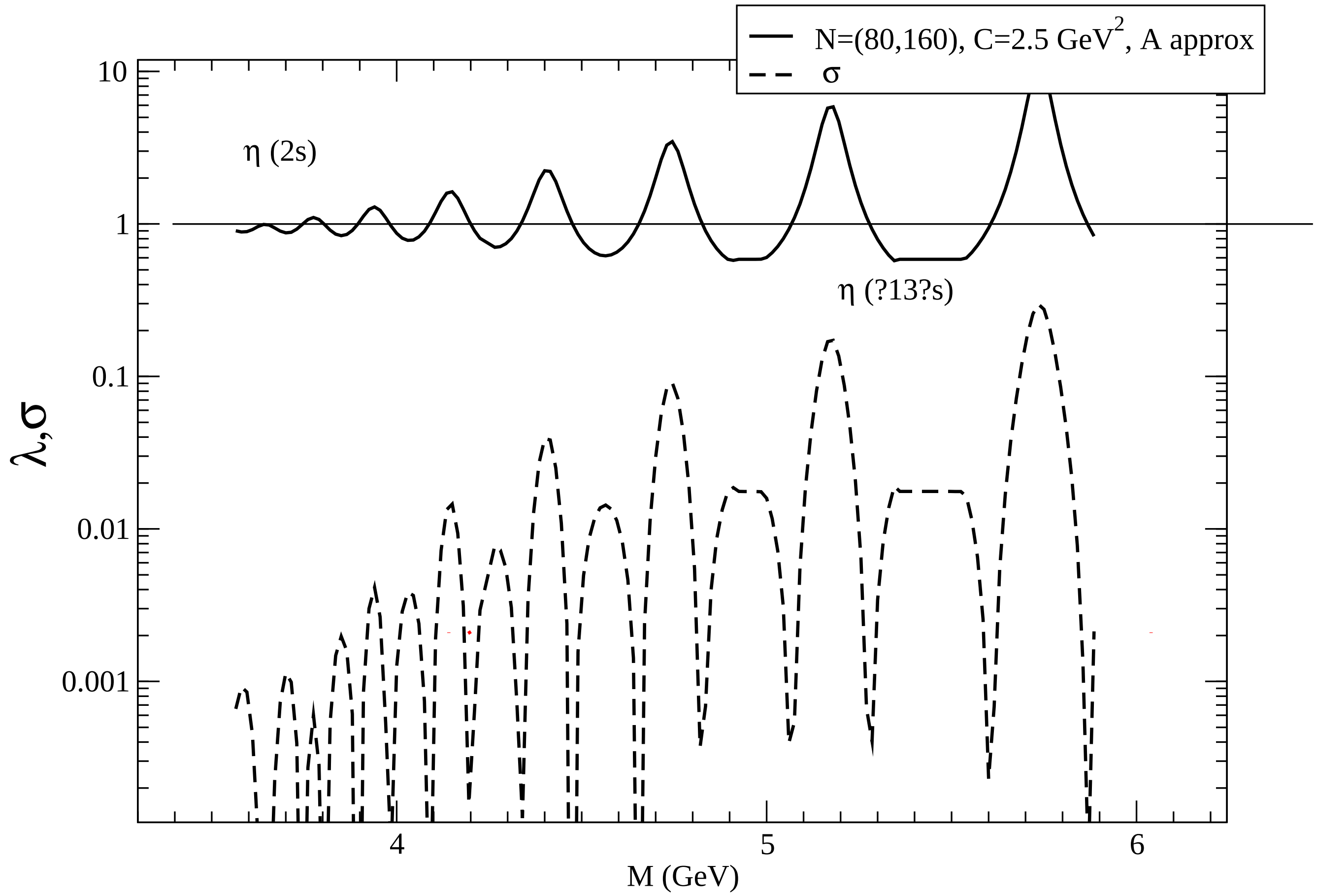

Figure 1: Numerical search for $m_c = 1.5GeV$, $\alpha_s = 0.063$, $\mu_v = \mu_s = 350MeV$

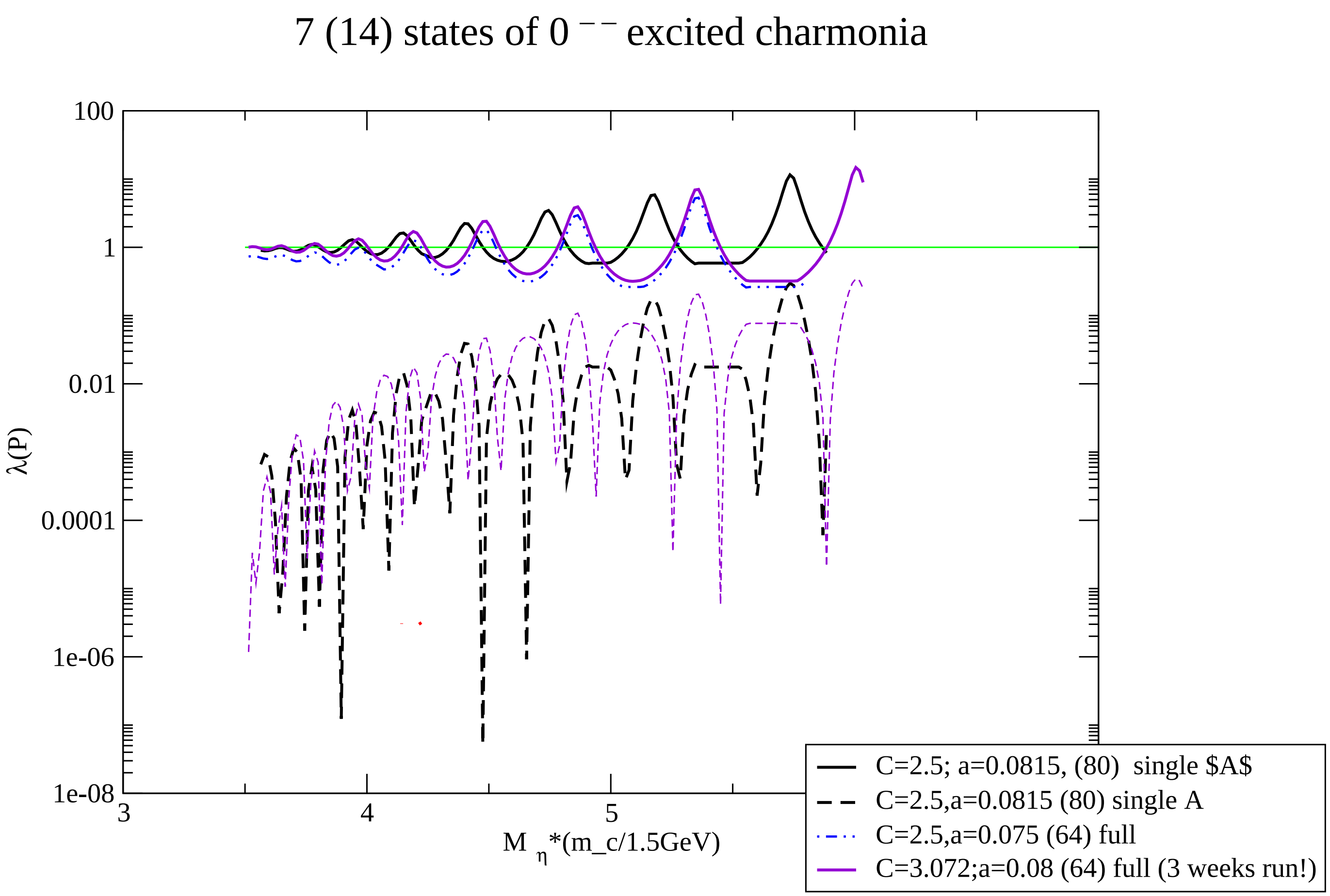

Figure 2: Example of numerics for various parameter space

Interpretation

observation:

A is dominant and reproduces spectrum within few $\%$ for excited states as well

Doubling of excited states!

States are overpopulated! Fitting lowest two states, one allways gets (approximately) two times more excited states when comparing to non-relativistic limit. From (2s) or (3s) energy level there is "doubling of states" characterized by (approximately) identical wave (vertex) functions.

(important note: one can fit number of bound states to the experiment be hand, but doubling remains!)

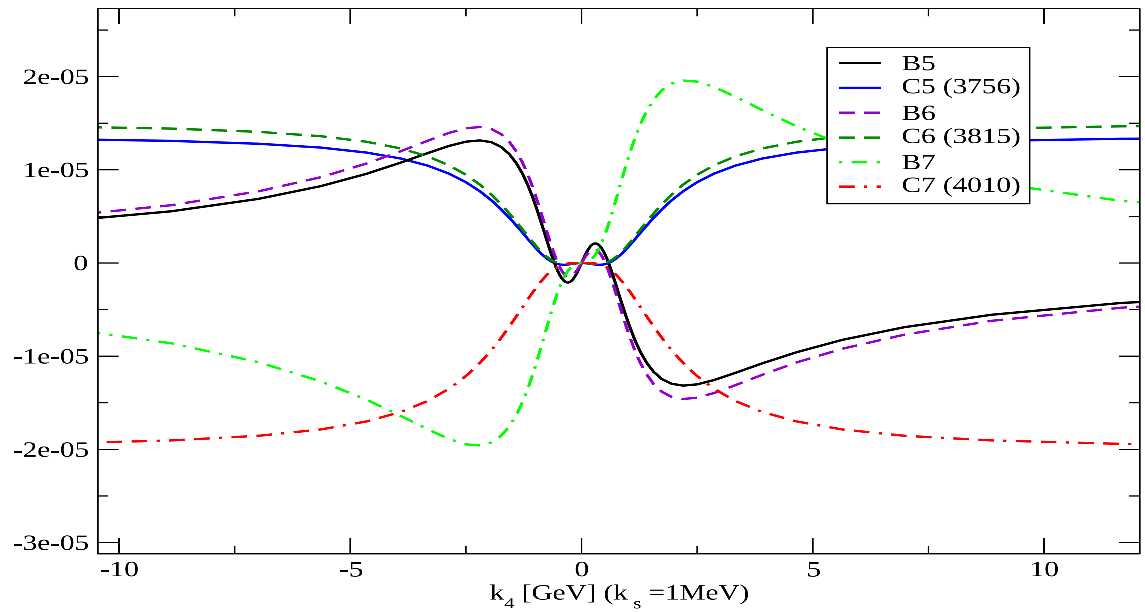

Figure 3: Dependence of B and C components of three excited states shown for fixed three-momenta $\mathbf{k} = 0.2MeV$. Levels labeled by B(C)5 (fifth energy excited state) and B(C)6 (sixth excited state) belong to the neigbouring levels, they have approximately identical vertex functions. They have same nodes (odd function B has three nodes, while C functions have two nodes and zero minimum at beginning, next two higher levels have only one zero, only seventh level is shown). Results shown for the model fit $m_c = 1.5GeV, \mu = 350MeV, C = 3.073GeV^2, \alpha_s = 0.07987$

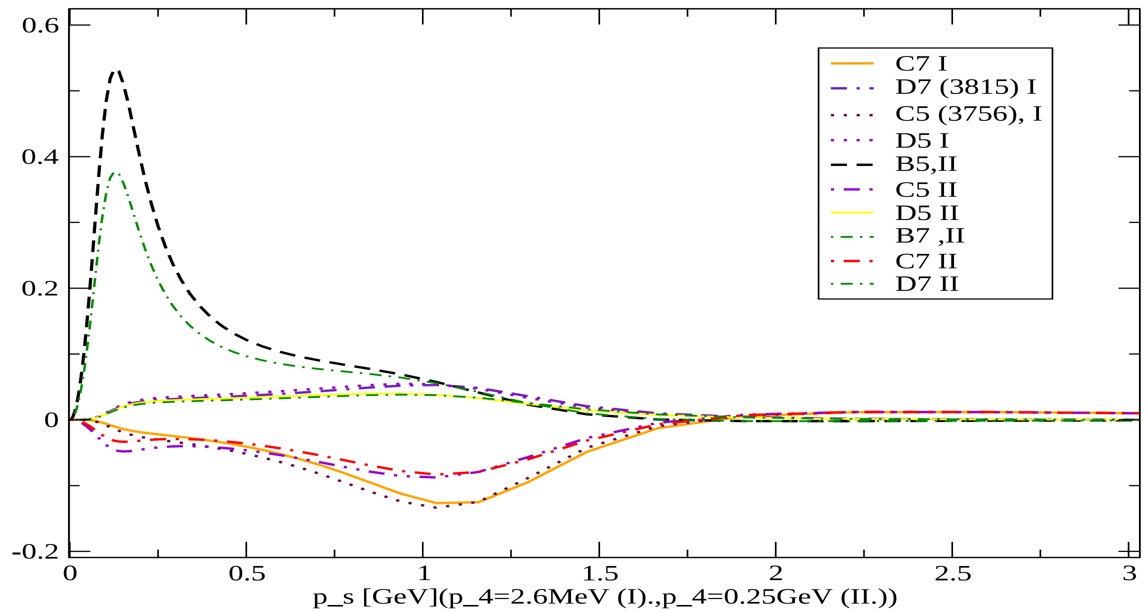

Figure 4: Dependence of components B, C, D on $\mathbf{k}$ of two excited states shown for two slices with given $k_4 = 2.6MeV$ (I) and $k_4 = 0.25GeV$ (II).

$m_c = 1500$	$m_c = 1442$	$<>$	a.
3100	2980	2980	2980 (1s)
3785	3638	3638	3638 (2s)
3940 3990	3787 3835	3811	3940* (3s)
4160 4235	4000 4071	4036	- (4s)
4430 4535	4259 4360	4310	-(5s)
4790 4925	4373 4734	4554	-(6s)
5270 5435	5066 5224	5145	-(7s)

Table 1: Preliminary results from conventional BSE. Up to date the best fit of conventional BSE solutions for $\eta_c(nS)$. We use conventional quantum mechanical assigment nS in order to label states that we expect in nonrelativistc or "instanatneous" approximations. The first column represent actual numerical solution in units where $m_c = 1.5GeV, C = 2.849GeV^2$, in the second column the experimental value of $\eta(1S)$ has been used to scale other levels. The doubling appears for the states $n > 2$, and the energy doublets are identified by comparison of vertex functions (e.g. by number of nodes in B, C, D functions).

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Table 2: ... After rescaling we produce experimentally known $\eta(2S)$. For other states -to make levels meaningfully comparable with quantum mechanical labeling- the masses of energy levels are averaged for given energy doublets. $\alpha_s = 0.07407$. *Belle observed X(3940) in $e^+e^- \rightarrow J/\psi + X$, for the interpretation see [?].

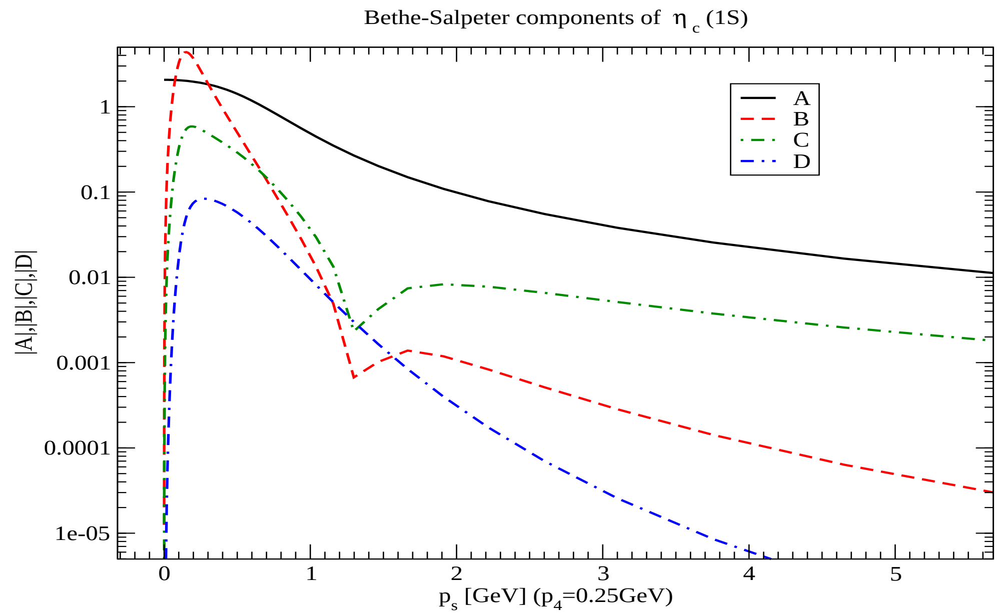

Figure 5: Absolute valuea of the A, β, C, D components of $\eta(1s)$ state. Note the large value of the function β (for the model in the Table)

Way out? -Confinement

Quarks do not live onshell- they do not freely propagate

Minkowski space model with confined form of propagator Stingl 1986

$$S_c(k) = f\,[\not k + m]\,\frac{k^2 - m_c^2}{(k^2 - m_c^2)^2 + \delta^4}$$

δ - the inverse of maximal wavelength S.J. Brodsky, R. Shrock arXiv:0806.1535

Naive guess $m_c = 1.5GeV$ $\delta = 0.5GeV$.

f- complex phase

BSE kernels:

$$
\begin{aligned}
V_s &= \frac{C}{(q^2-\mu^2)^2+\lambda_s^4} \ , \\
V_v &= \frac{g^2}{q^2-\mu^2} \ ,
\end{aligned}
$$

Numerically $\mu = 0.25 GeV$ and $\lambda = 0.5 GeV$

(Recent stage: small δ, λ are numerically impossible, instability)

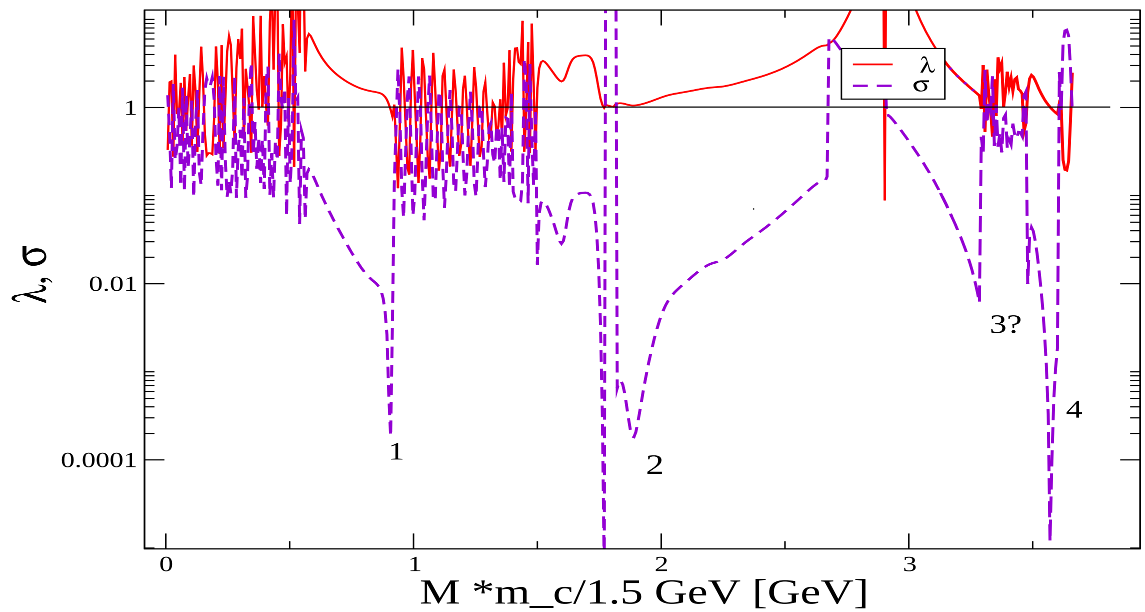

Figure 6: Identification of bound states through the solution of BSE in Minkowski space (one week at 20 procerors). Main Results:

1. BSE is soluble in Minkowski space we get ground states and (some) excited states as well

2. Overpopulation of energy states dissapear

3. Model improvement is necessary and better quantitative agreement is required.

Conclusion

From 1-4

- 1. Choose modern and suscefull model of Quarkonia available on a market....
- 2. find Poincare invariant generalization of the interaction V ...
- 3. and solveQuarkonium BSE with V in its full form for given J^{PC}...
- 4. and their transitions

1.-3. were attemted, problems identified with suggested solution

Analytical forms of Greens functions describing colored objects should reflect confinement, we suggest using Stingl propagators. New methods and proposals on BSE with confinement in Minkowski space. (no doubling)

4. -Transitions ? f_h. EMG T, production (e.g. double charmonium)

future: bc mesons, D and light mesons, etc...

Details not mentioned, but already done:

P^2 dependence of V

running of α_s included