GLUON MASS THROUGH MASSLESS BOUND-STATE EXCITATIONS

David Ibáñez Gil de Ramales
Department of Theoretical Physics and IFIC,
University of Valencia - CSIC, Spain

in collaboration with A.C. Aguilar, V. Mathieu and J. Papavassiliou
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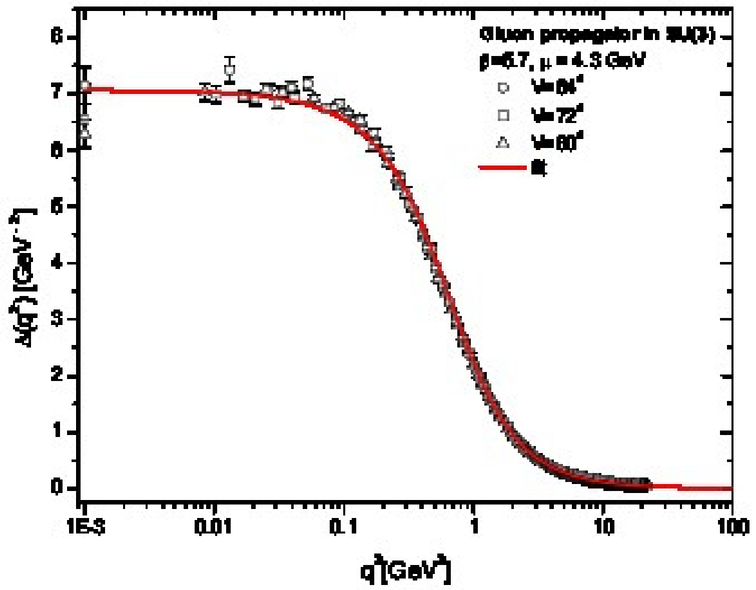
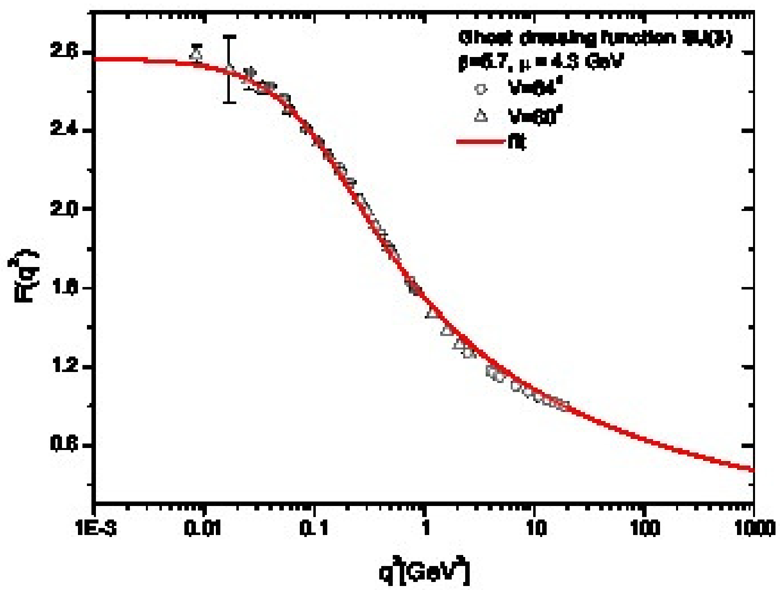
Outline

- Motivation
- Schwinger Mechanism and the need of massless bound-state excitations
- Structure and properties of the bound state vertex
- Dynamical implementation – The Bethe-Salpeter equation
- Numerical Results
- Conclusions

Lattice results

- Gluon propagator and ghost dressing function from lattice data (Landau gauge).





- This behavior can be explained by means of an effective gluon mass.

Gluon mass generation

- The dynamical gluon mass should be generated without modify the QCD lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_a^{\mu\nu}G_{\mu\nu}^a + \frac{1}{2\xi}(\partial^\mu A_\mu^a)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g f^{abc}(\partial^\mu \bar{c}^a) A_\mu^b c^c$$

where the gluonic field strength tensor $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$

- A mass term ($m^2 A_\mu^2$) is forbidden by gauge invariance

- **How to obtain massive solutions gauge invariantly?**

Schwinger Mechanism

J.S. Schwinger, Phys. Rev.125, 397 (1962); Phys.Rev.128, 2425 (1962).

- Dyson resummation:

$$\Delta(q^2) = \frac{1}{q^2[1+\Pi(q^2)]}$$

- If the vaccum polarization $\Pi(q^2)$ has a pole with positive residue m^2 i.e.

$$\Pi(q^2) = m^2/q^2$$

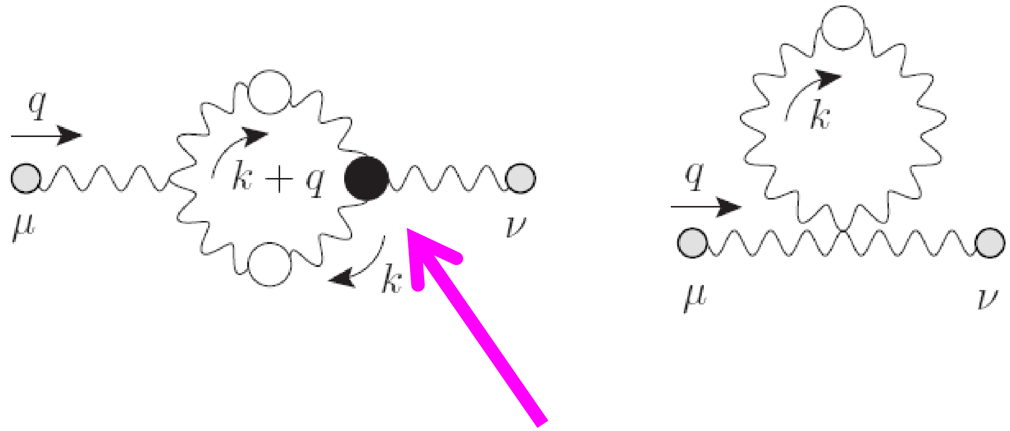
- Then

$$\Delta^{-1}(q^2) = q^2 + m^2 \quad \Rightarrow \quad \Delta^{-1}(0) = m^2$$

The vector meson becomes massive even though it is massless at the level of the fundamental Lagrangian

Triggering the Schwinger Mechanism in QCD

- The PT-BFM SDE truncation scheme



- The way the Schwinger mechanism is integrated into the SDE is through the form of the **three-gluon vertex.**

J.M. Cornwall, Phys.Rev.D**26**, 1453 (1982)

A.C. Aguilar and J. Papavassiliou, JHEP 0612, 012 (2006)

A.C.Aguilar, D.Binosi and J.Papavassiliou, Phys. Rev. D **78**, 025010 (2008)

D. Binosi and J.Papavassiliou, Phys.Rept. **479**, 1 (2009)

A.C.Aguilar, D.Binosi and J.Papavassiliou arXiv:1107.3968 [hep-ph]

□ Necessary ingredient:

The existence of a nonperturbative vertex V that contains massless, longitudinally coupled, bound-state excitations.

1. Composite Nambu-Goldstone-like massless poles (~1/q2).
2. Such **poles** must ***occur dynamically***, even in the ***absence*** of canonical ***scalar fields***.
3. **Composite excitation:** a pole in a off-shell Green's function representing a field that does not exist in the classical action.
4. It is purely longitudinal $$P^{\alpha'\alpha}(q)P^{\mu'\mu}(r)P^{\nu'\nu}(p)V_{\alpha\mu\nu}(q,r,p) = 0$$
5. It decouples from on shell amplitudes.

R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)
J. M. Cornwall and R.E. Norton, Phys. Rev. D8, 3338 (1973)
E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)
E.C.Poggio, E.Tomboulis and S.H.Tye, Phys.Rev.D 11, 2839 (1975)
R. Jackiw, In *Erice 1973, New York 1975, 225-251

V enforces **gauge invariance** **in the presence of a gluon mass**

⇕

The STIs **remain the same before** and **after mass generation**

$\Delta^{-1}(q^2) = q^2 J(q^2),$ → $\Delta_m^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$

$\mathrm{I\Gamma}$ → $\mathrm{I\Gamma}' = \mathrm{I\Gamma} + V$

- $\mathrm{I\Gamma}$ satisfies

$$\begin{aligned} q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) &= p^2 J(p^2) P_{\mu\nu}(p) - r^2 J(r^2) P_{\mu\nu}(r), \\ &= \Delta^{-1}(p^2) P_{\mu\nu}(p) - \Delta^{-1}(r^2) P_{\mu\nu}(r) \end{aligned}$$

- whereas V

$$q^\alpha V_{\alpha\mu\nu}(q,r,p) = m^2(r^2) P_{\mu\nu}(r) - m^2(p^2) P_{\mu\nu}(p),$$

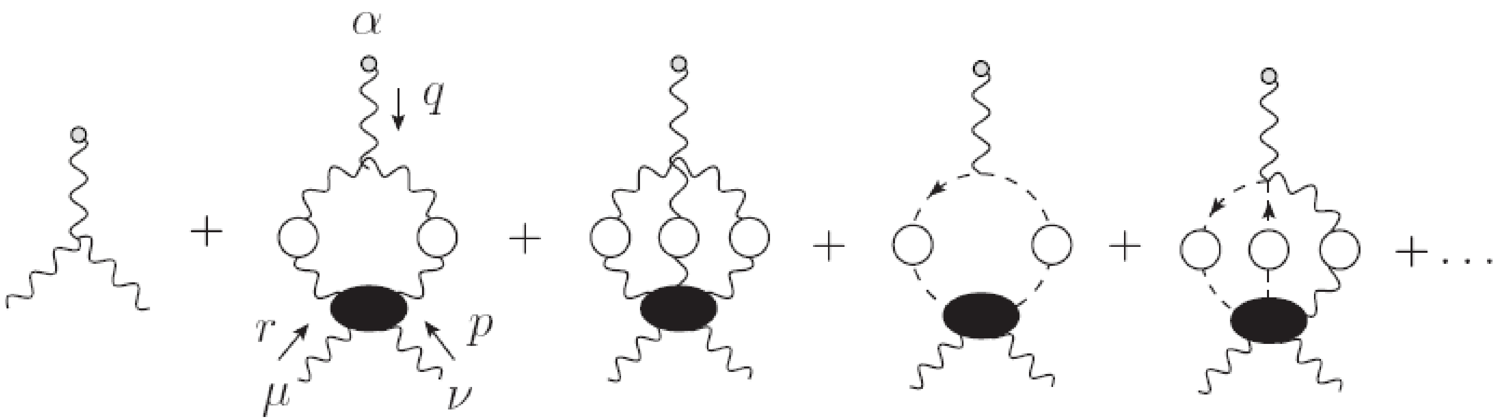
- It folllows that

$$\begin{aligned} q^\alpha \mathrm{I\Gamma}'_{\alpha\mu\nu}(q,r,p) &= q^\alpha \left[\mathrm{I\Gamma}(q,r,p) + V(q,r,p)\right]_{\alpha\mu\nu} \\ &= [p^2 J_m(p^2) - m^2(p^2)] P_{\mu\nu}(p) - [r^2 J_m(r^2) - m^2(r^2)] P_{\mu\nu}(r), \\ &= \Delta_m^{-1}(p^2) P_{\mu\nu}(p) - \Delta_m^{-1}(r^2) P_{\mu\nu}(r) \end{aligned}$$

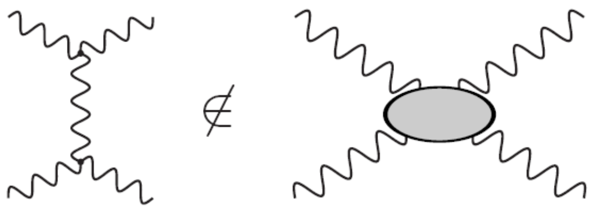
D.Binosi,J.Papavassiliou,JHEP1103,121(2011)

Dynamical implementation

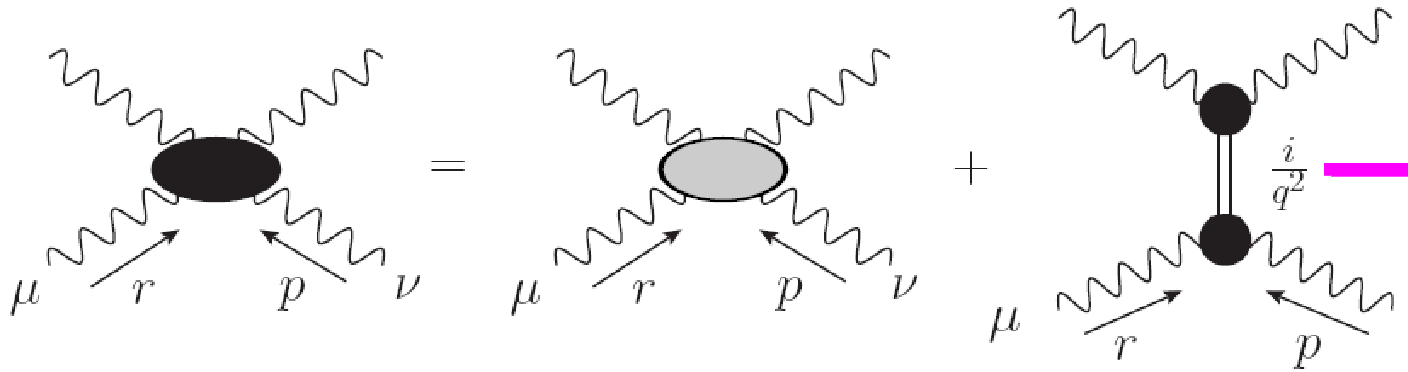
- With the Schwinger mechanism **turned off**



- **1PI diagrams** with respect to cuts in the direction of the momentum q **are excluded** from the **four-gluon kernel**

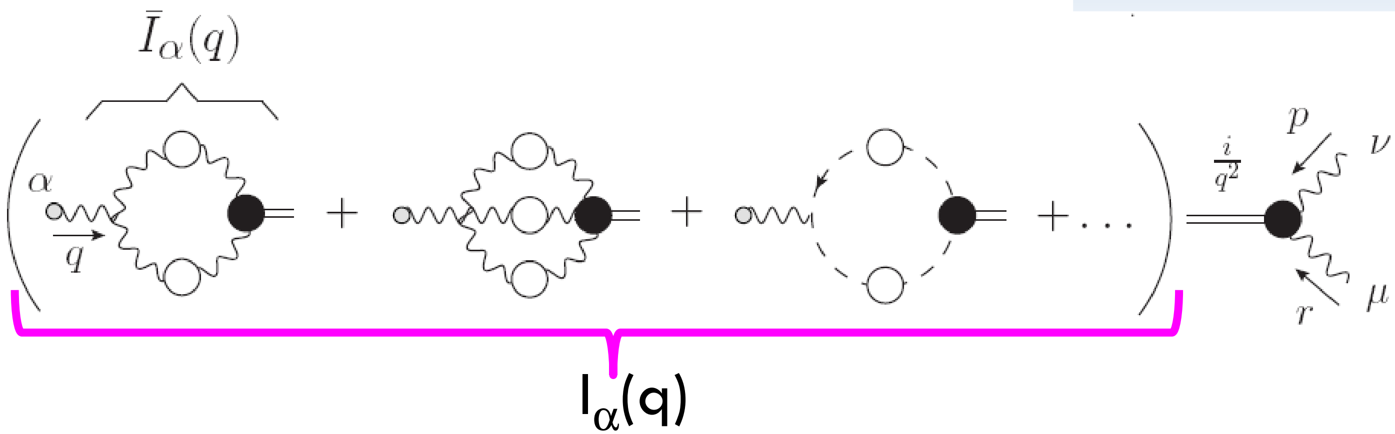


- Schwinger mechanism turned on → modify the structure of the kernel



Presence of composite massless excitation

- The sum of these terms belongs to V

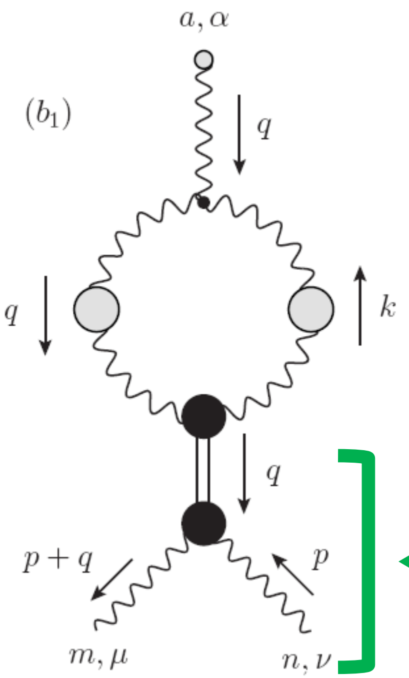


$$\overline{V}_{\alpha\mu\nu}(q,r,p) = I_\alpha(q)\left(\frac{i}{q^2}\right)B_{\mu\nu}(q,r,p)$$

$I_\alpha(q)$ is the transition amplitude mixing gluon and bound-state

$$I_\alpha(q) = q_\alpha I(q)$$

$$I(q) = \frac{q^\alpha I_\alpha(q)}{q^2}$$



$B_{\mu\nu}$ is the gluon-pole interaction vertex

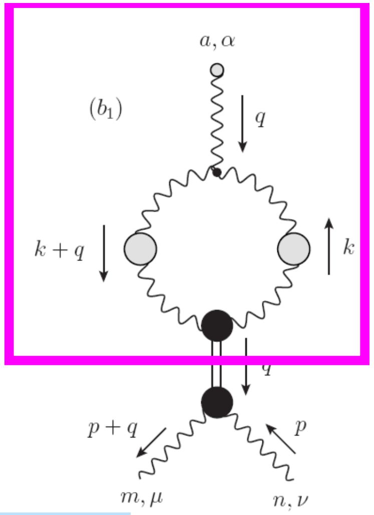
- $B_{\mu\nu}$ can be decomposed as

$$B_{\mu\nu}(q,r,p) = B_1 g_{\mu\nu} + B_2 q_\mu q_\nu + B_3 p_\mu p_\nu + B_4 r_\mu q_\nu + B_5 r_\mu p_\nu$$

- Due to Bose symmetry (interchange $\mu \leftrightarrow \nu$ and $p \leftrightarrow r$)

$$B_{1,2}(q,r,p) = -B_{1,2}(q,p,r) \quad \Rightarrow \quad B_{1,2}(0,-p,p) = 0$$

The "one-loop" dressed approximation



- The gluon-pole transition amplitude in the Landau gauge *is given by*

$$I_\alpha(q) = \frac{1}{2} C_A \int_k \Delta(k)\Delta(k+q)\Gamma_{\alpha\beta\lambda} P^{\lambda\mu}(k) P^{\beta\nu}(k+q) B_{\mu\nu}(-q,-k,k+q)$$

- Then

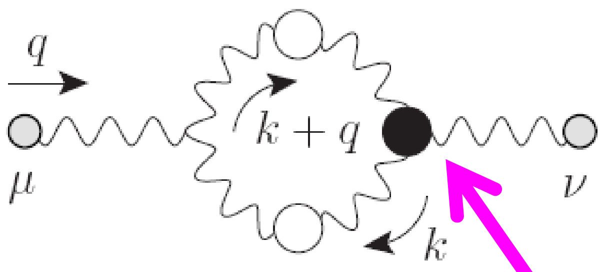
$$I(q) = -\frac{C_A}{q^2} \int_k k^2 \Delta(k)\Delta(k+q) P^{\mu}_{\beta}(k) P^{\beta\nu}(k+q)[B_1 g_{\mu\nu} + B_2 q_\mu q_\nu]$$

- In the limit of q→0, I(q) becomes

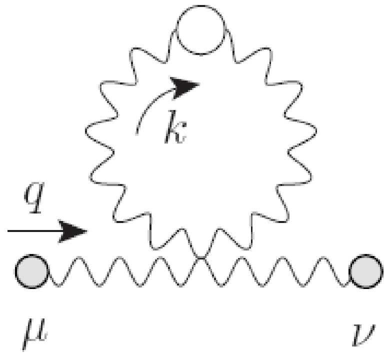
$$I(0) = \frac{3}{2} C_A \int_k k^2 \Delta^2(k) B_1'(k)$$

Gluon mass equation

- The one-loop dressed approximation is given by



Use $\mathrm{I}\!\Gamma + V$

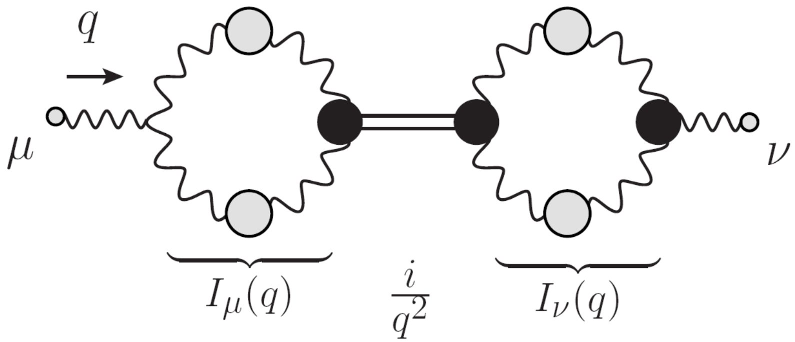


$$\Delta^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\,\Pi_{\mu\nu}(q)}{[1+G(q^2)]^2},$$

- Before and after the gluon mass generation we mantain **the crucial transversality property**

$$q^{\mu}\Pi_{\mu\nu}(q) = 0$$

- When the pole vertex is inserted in the SDE we obtain the "squared" diagram

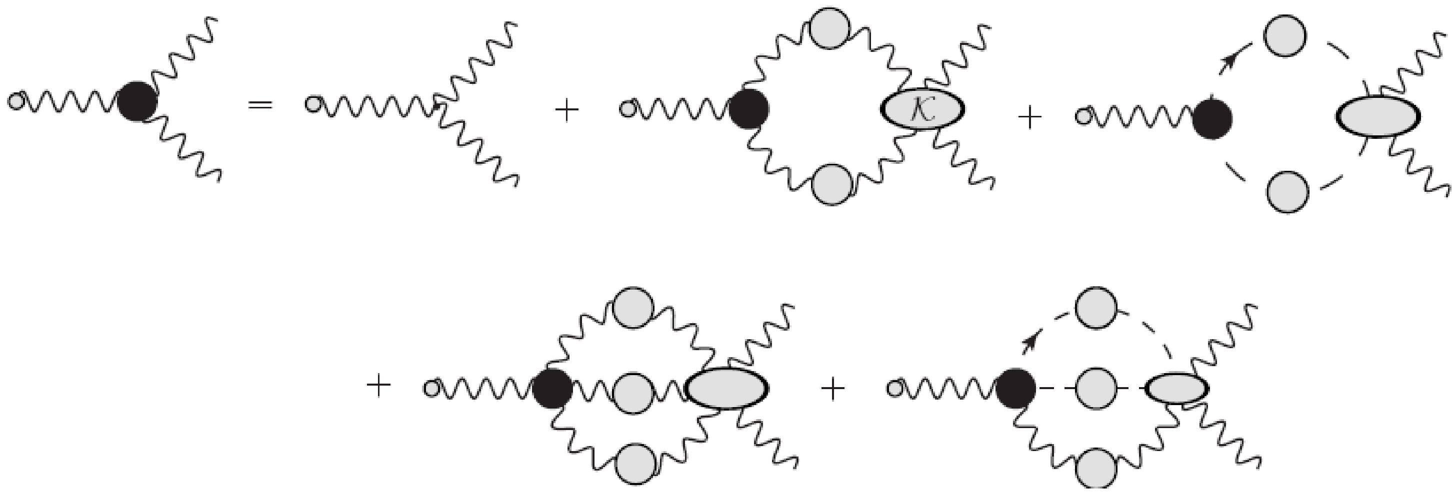


- We arrive at the following relation (in euclidean space)

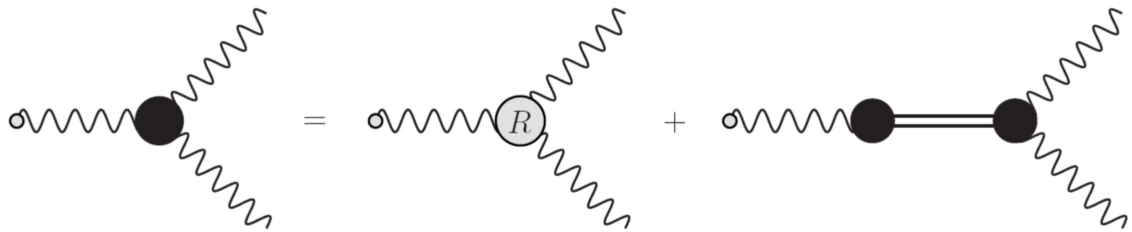
$$m^2(0) = g^2 F^2(0) I^2(0)$$

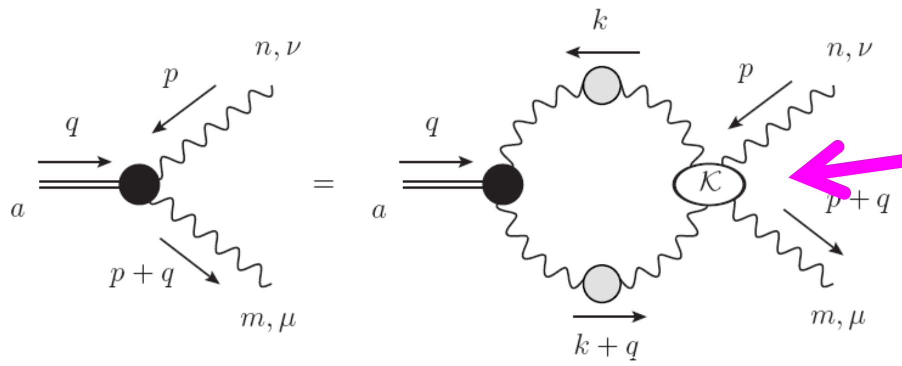
Bethe Salpeter equation

- The BSE for $\mathbb{I}\Gamma'$ is



- Separate **regular** and **pole parts**

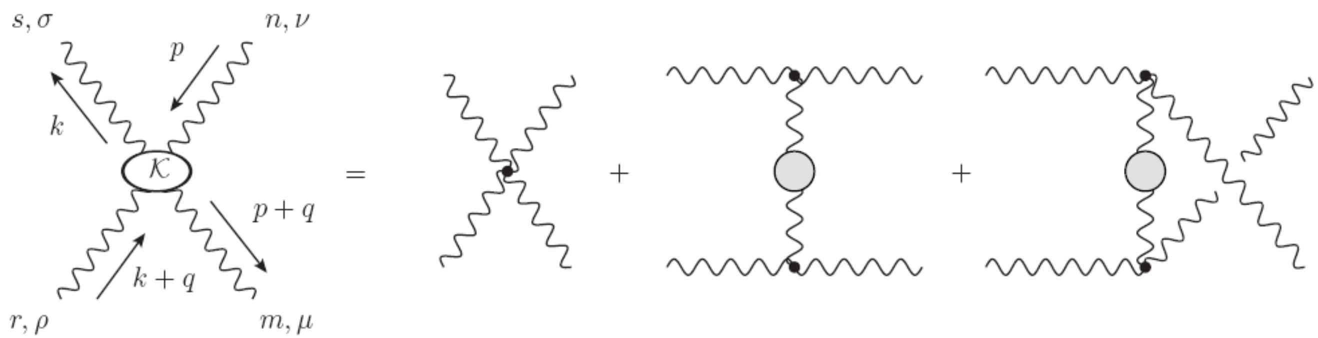




Four gluon BS Kernel

$$B_{\mu\nu} = \int_k B_{\alpha\beta}\Delta^{\alpha\rho}(k+q)\Delta^{\beta\sigma}(k)\mathcal{K}_{\sigma\nu\mu\rho}$$

Ladder approximation



$$\mathcal{K}_{\sigma\nu\mu\rho} = -ig^2\Gamma^{(0)}_{\sigma\gamma\nu}\Delta^{\gamma\lambda}(k-p)\Gamma^{(0)}_{\mu\lambda\rho}$$

- Using the ladder approximation for 4 gluon kernel , the BSE becomes

$$B_{\mu\nu} = -2\pi i\alpha_s C_A \int_k B_{\alpha\beta}\Delta(k+q)\Delta(k)\Delta(k-p)P^{\alpha\rho}(k+q)P^{\beta\sigma}(k)P^{\gamma\lambda}(k-p)\Gamma^{(0)}_{\sigma\gamma\nu}\Gamma^{(0)}_{\mu\lambda\rho}$$

- After some calculation

$$B_1'(x) = \frac{\alpha_s C_A}{24\pi}\left\{\int_0^x dy B_1'(y)\Delta^2(y)\frac{y^2}{x}\left(3+\frac{25}{4}\frac{y}{x}-\frac{3}{4}\frac{y^2}{x^2}\right)+ \right.$$
$$\left. + \int_x^\infty dy B_1'(y)\Delta^2(y) y\left(3+\frac{25}{4}\frac{x}{y}-\frac{3}{4}\frac{x^2}{y^2}\right)\right\} .$$

$$x \equiv p^2 \quad ; \quad y \equiv k^2 \quad ; \quad z \equiv (p+k)^2$$

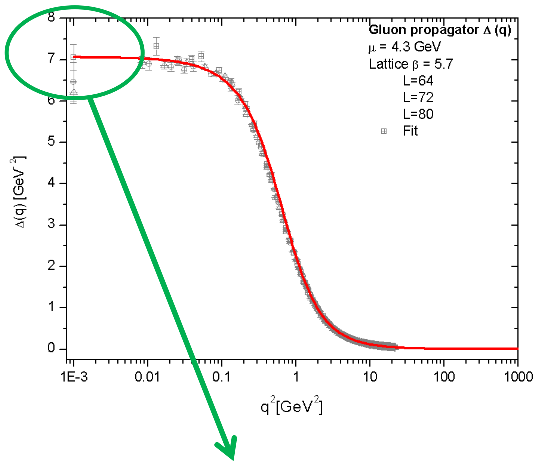
- In the limit of $x \to 0$, we have

$$B_1'(0) = \frac{\alpha_s C_A}{8\pi}\int_0^\infty dy\, y B_1'(y)\Delta^2(y)$$

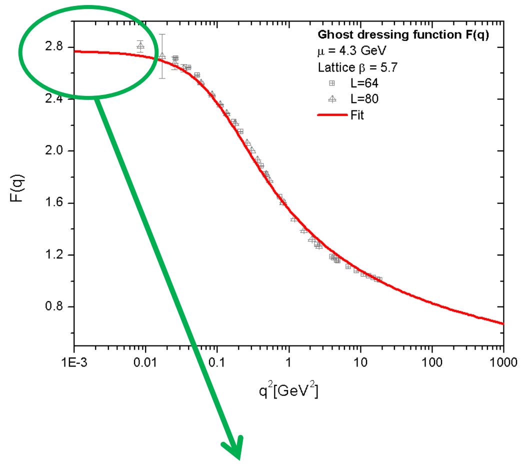
Numerical Solution

- We will solve numerically the integral equation

$$B_1'(x) = \frac{\alpha_s C_A}{24\pi}\left\{ \int_0^x dy B_1'(y)\Delta^2(y)\frac{y^2}{x}\left(3 + \frac{25}{4}\frac{y}{x} - \frac{3}{4}\frac{y^2}{x^2}\right) + \right.$$
$$\left. + \int_x^\infty dy B_1'(y)\Delta^2(y) y \left(3 + \frac{25}{4}\frac{x}{y} - \frac{3}{4}\frac{x^2}{y^2}\right)\right\} \quad .$$



$$\Delta^{-1}(0) = m^2(0) = 0.14$$



$$F(0) = 2.76$$

Under the constraints

$$I(0) = \frac{3}{2}C_A \int_k k^2\Delta^2(k)B_1'(k)$$

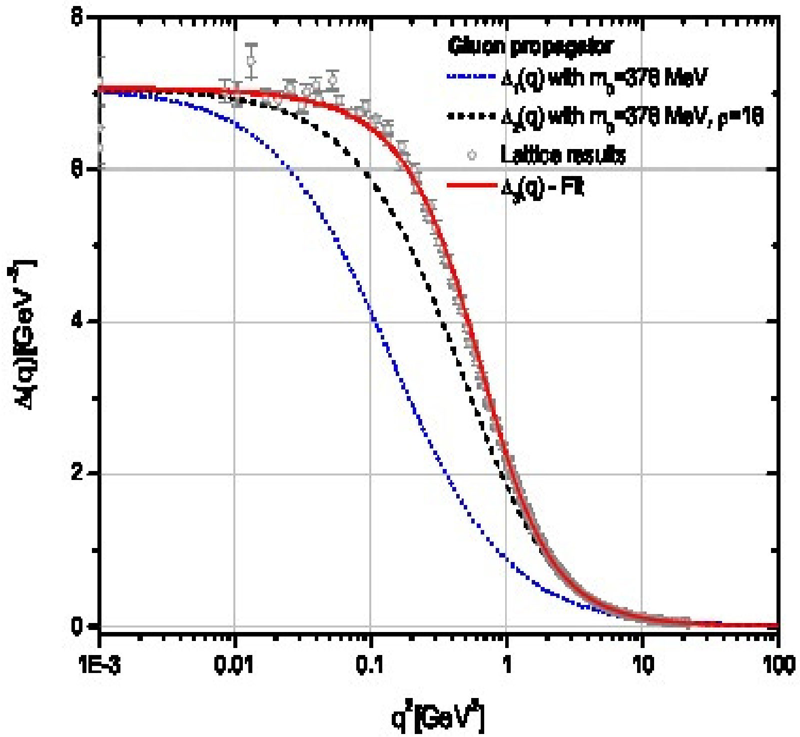
$$m^2(0) = g^2 F^2(0) I^2(0)$$

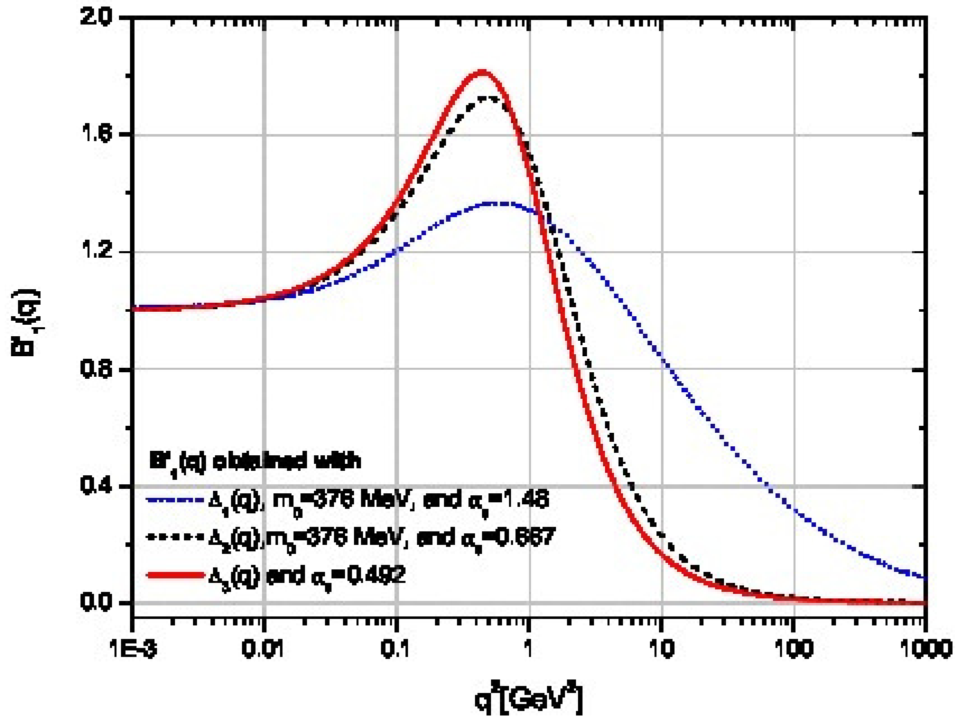
$$I(0) = \frac{m^2(0)}{F^2(0)g^2} = 0.18/g^2$$

I.L.Bogolubsky, et al , PoS **LAT2007**, 290 (2007)

Numerical Solution

- Ansätze for the gluon propagators and solutions for the derivative of the form factor B_1





Conclusions

- The **gauge-invariant** generation of a gluon mass **relies on** the existence of **massless bound-state excitations**, which trigger the **Schwinger mechanism.**

- The study of a simplified **Bethe-Salpeter equation suggests** that the nonperturbative **QCD dynamics lead** indeed **to** the formation of such **massless bound states.**

- Our approximations should be further refined, and the possibility of having **additional similar excitations** (related to the **other vertices**) **must be explored** (coupled system of integral equations).