NLO corrections for large multiplicity processes

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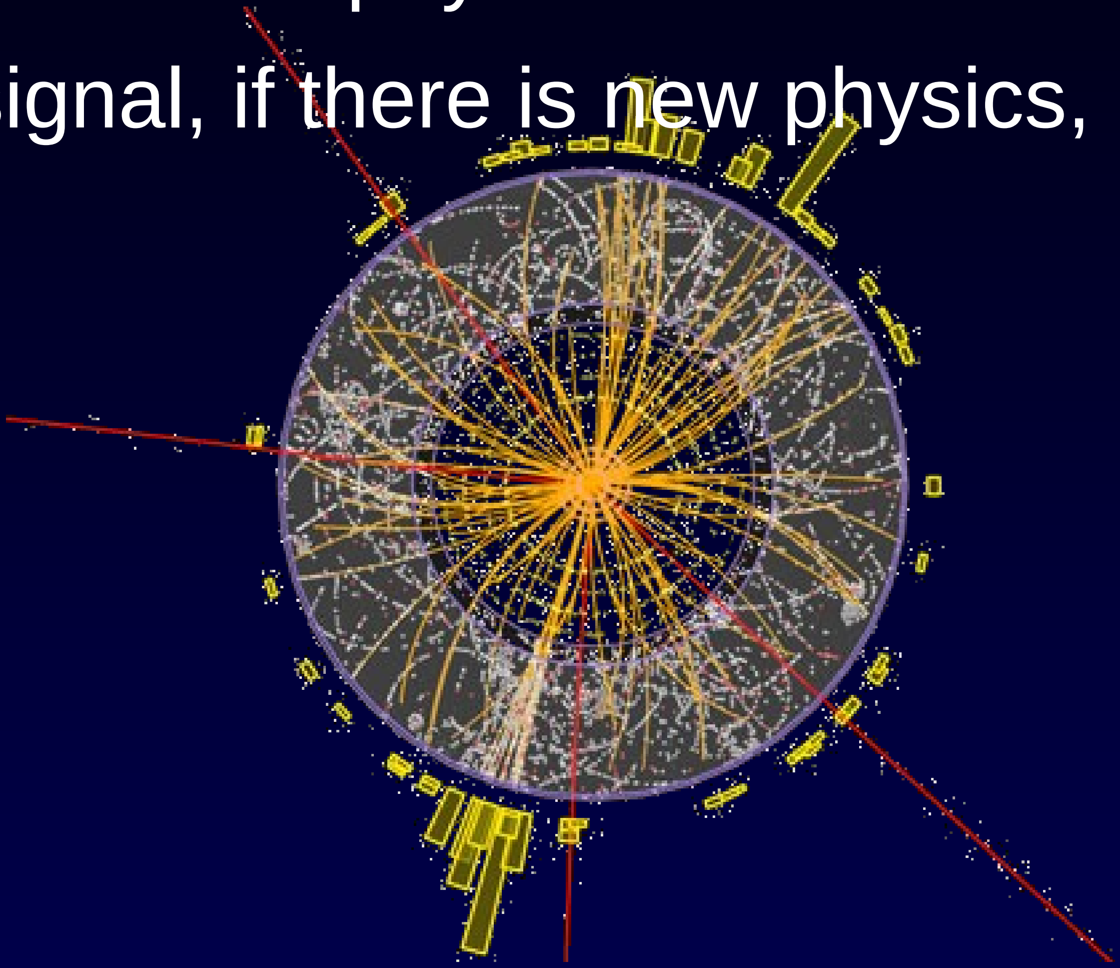
LHC

- High energy frontier
- Hope to find signals of new physics
- So far no striking signal, if there is new physics, it is hiding

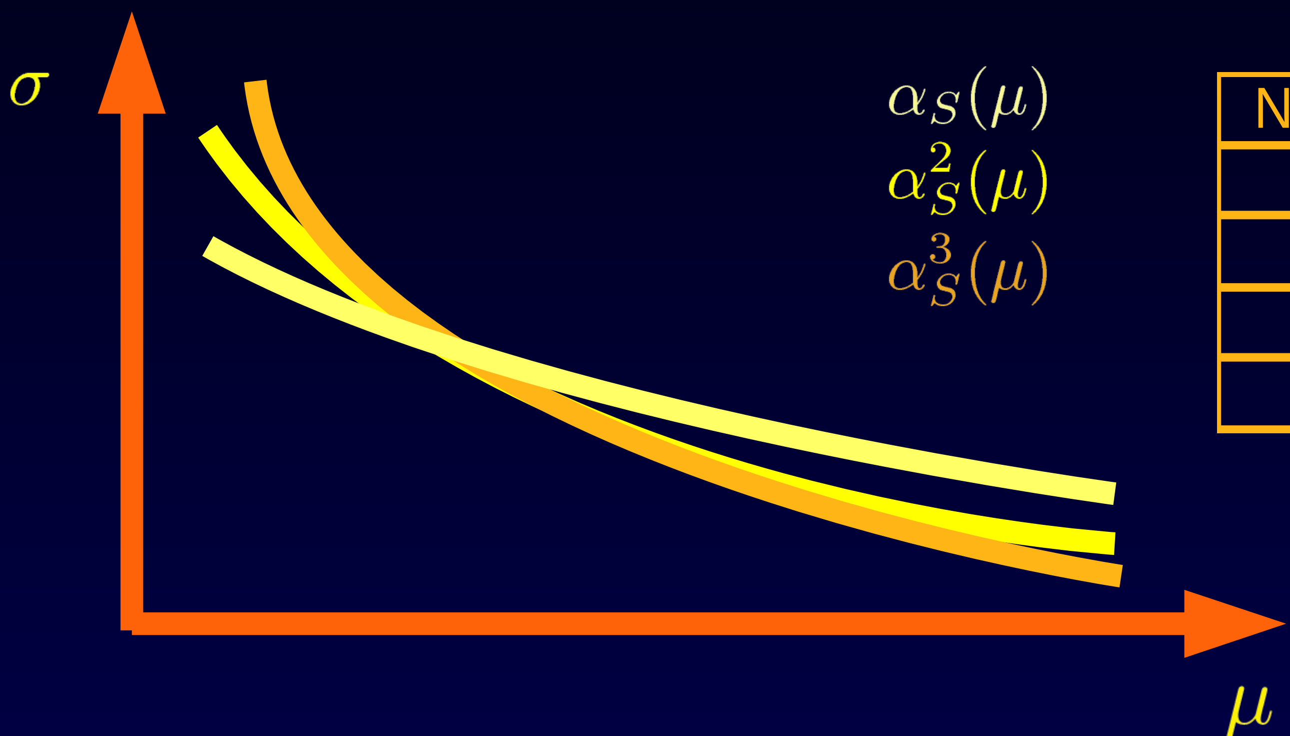


Motivation

- Very good understanding of the SM backgrounds are mandatory for all searches at the LHC
- NLO accuracy is needed for a reliable pQCD prediction
- NLO improvement
  - Absolute normalisation
  - Shapes of distributions
  - Scale dependence

Renormalisation scale dependence

- Scale dependence increases with number of jets

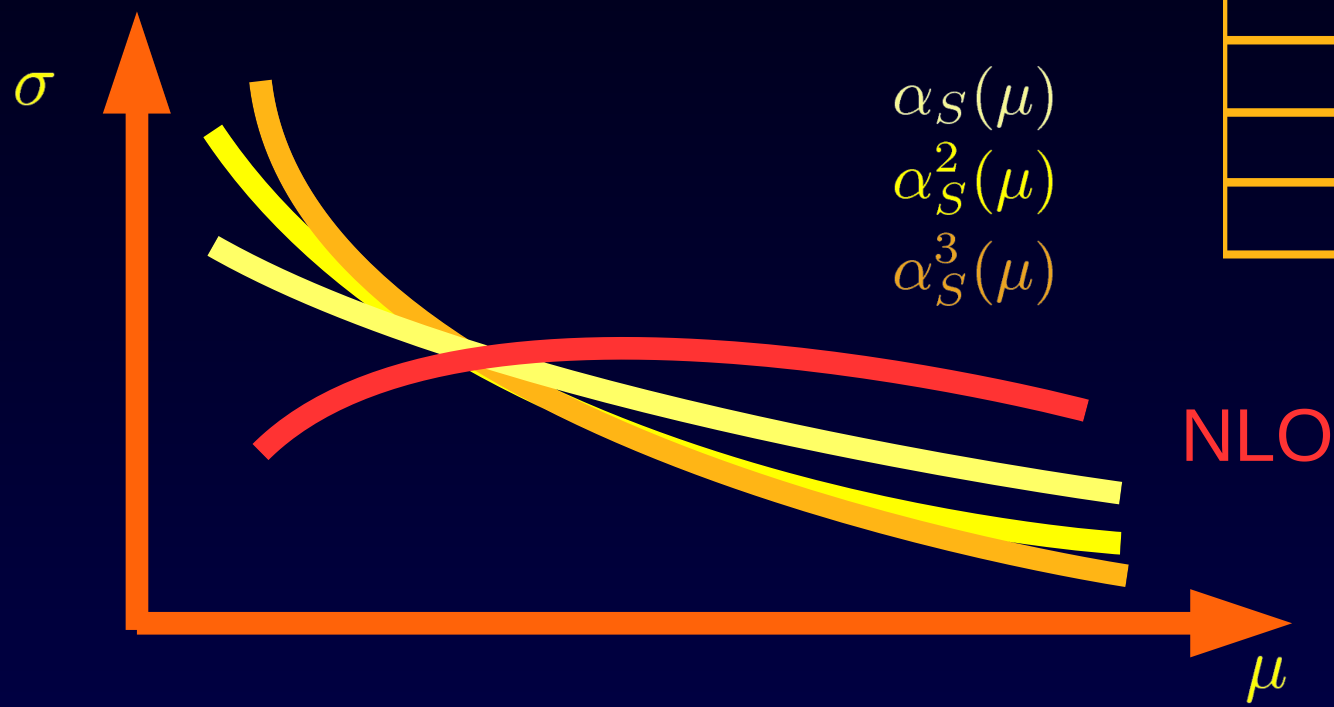


Number of jets	LO
1	9%
2	28%
3	47%
4	64%

[from table I in arXiv:1009.2338]

Renormalisation scale dependence

- Scale dependence increases with number of jets

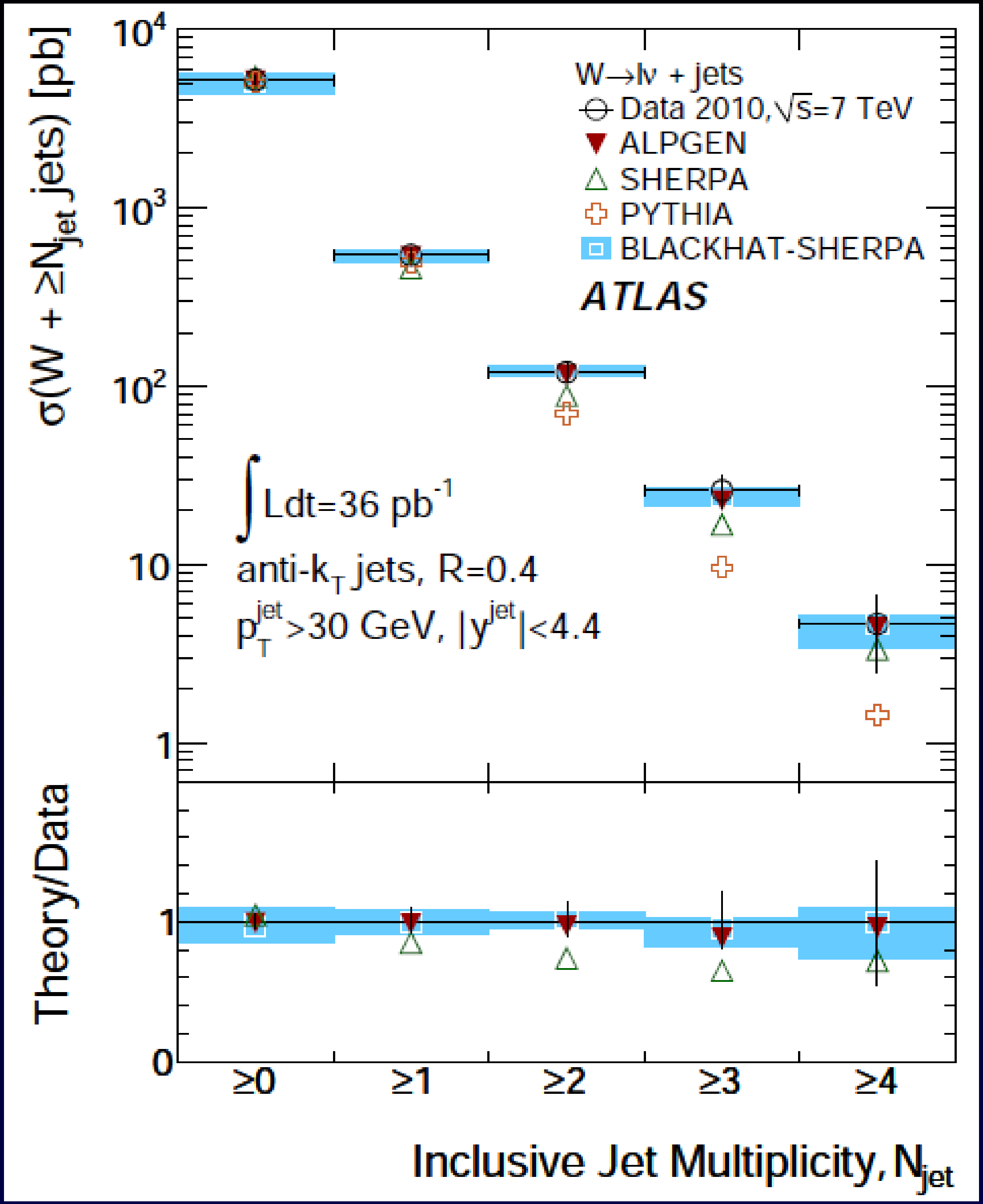


Number of jets	LO	NLO
1	9%	4.5%
2	28%	5.2%
3	47%	7.8%
4	64%	8.4%

[from table I in arXiv:1009.2338]

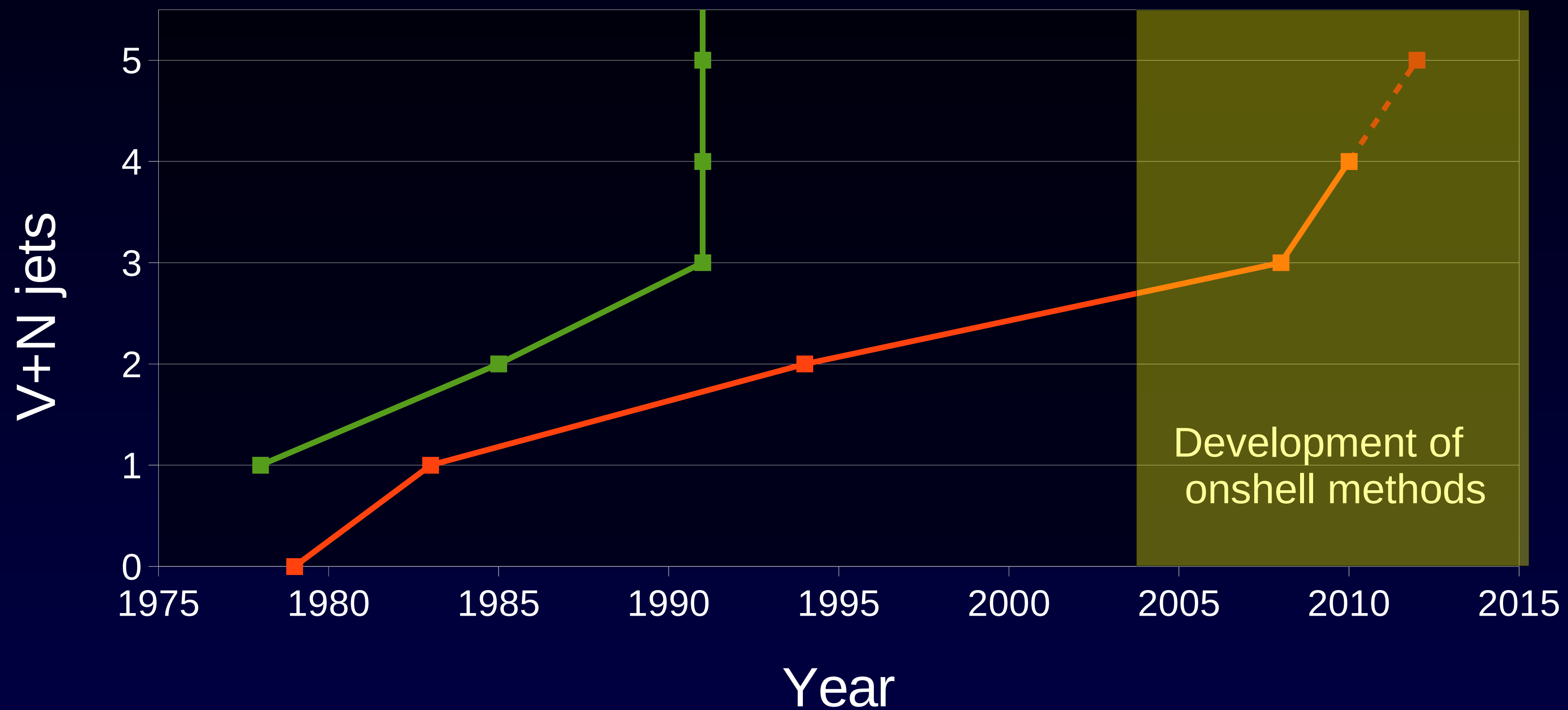
NLO vs Data

- Good agreement for all multiplicities
- Small scale dependence



Recent progress

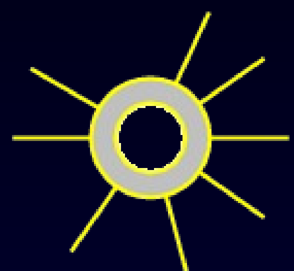
- Number of jets in addition to the vector boson



NLO Corrections

Consider (infrared safe) observable and add contributions that have an higher order in perturbation theory

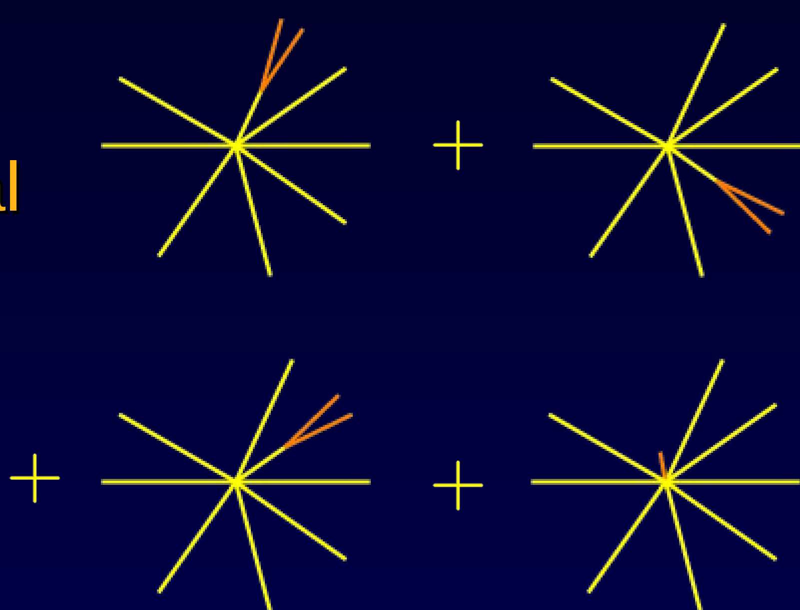
Virtual



Has explicit divergences coming from integration over the loop momentum

cancellation

Real



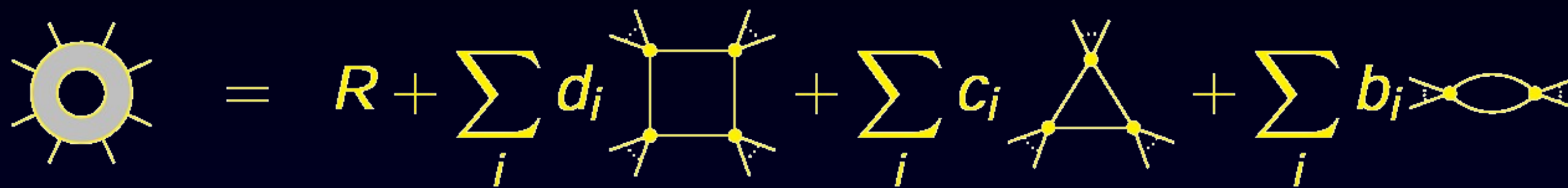
Has divergences when integrating over soft and collinear phase space

On-shell vs Feynman diagrams

- Why are amplitudes obtained through Feynman diagrams so complicated?
  - Many diagrams
  - Gauge information
  - Unphysical particles/degrees of freedom

One-loop decomposition

A one-loop amplitude can be written in terms of scalar integrals



Scalar integrals are known

Coefficients are rational polynomials of spinor products

To compute one-loop integral, it is enough to compute the coefficients of the scalar integrals

Unitarity/on-shell technique

- Reduction at the integrand level [Ossola,Papadopouls,Pittau] + generalised unitarity [Bern,Dixon,Dunbar,Kosower]
- Can obtain the coefficient of the scalar integrals
- Use factorization properties of the amplitude
- Use complex momenta [Britto,Cachazzo,Feng]
- Compute coefficients with “cuts”
- Cut can be seen as a projector onto structures that have a given set of propagators

Unitarity cut

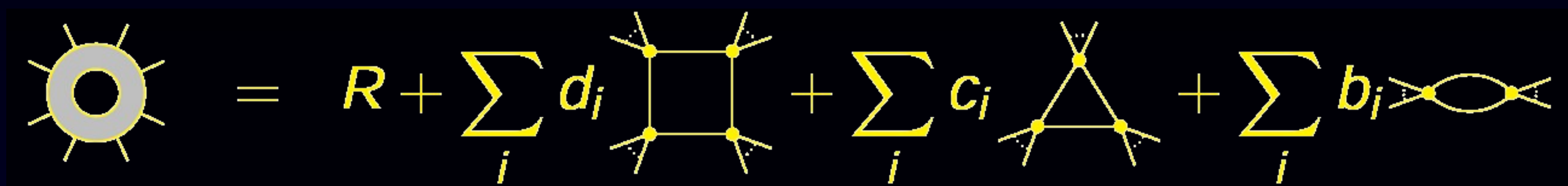
- Replacement under the loop integral propagator → delta function

$$\frac{1}{P^2} \to 2\pi i \delta(P^2)$$

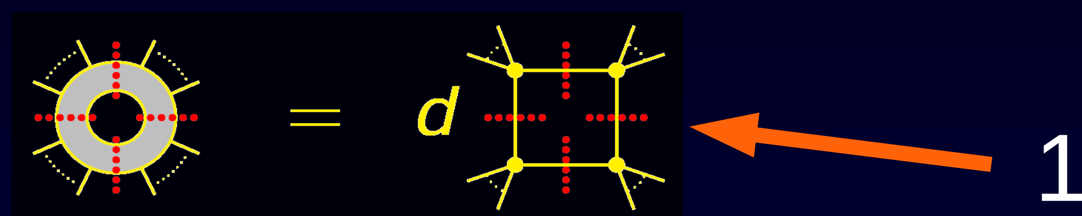
- Can apply more than one cut
  - Double cut
  - Triple cut
  - Quadruple cut
- Only possible in general with complex momenta

Unitarity cut

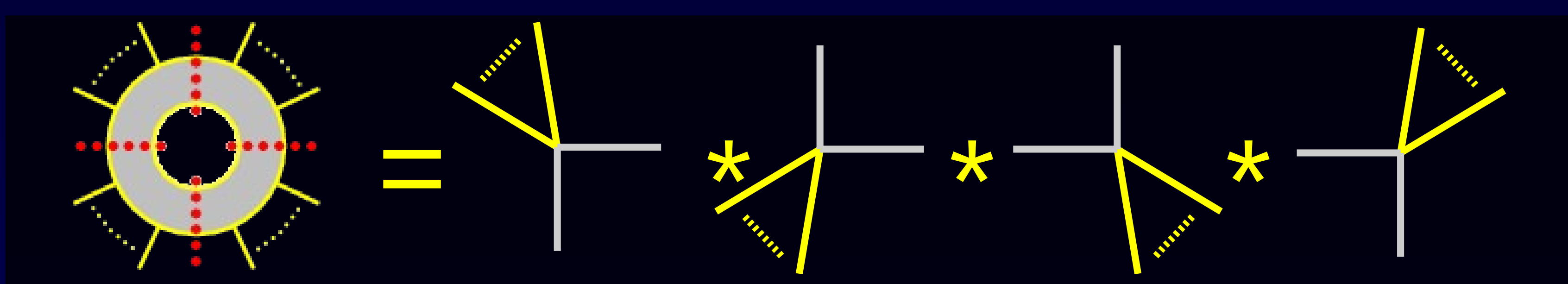
- One-loop decomposition



- Quadruple cut is a projector



- Quadruple Cut breaks the one-loop amplitudes in a product of tree amplitudes



Quadruple cut

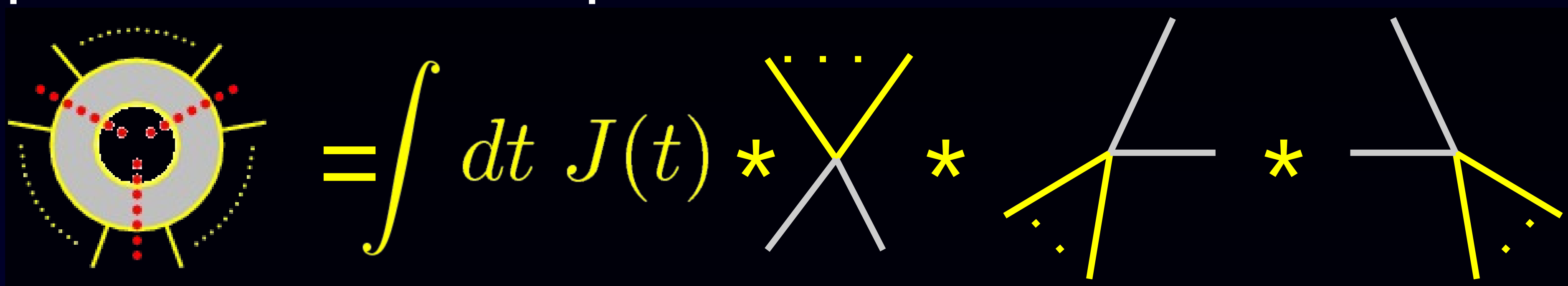
- The box coefficient is

$$d = \sum A_1 A_2 A_3 A_4$$

- Given in terms of on-shell trees
  - No gauge dependence
  - Compact expressions
  - Numerically stable
  - But with two complex momenta

Triple cut

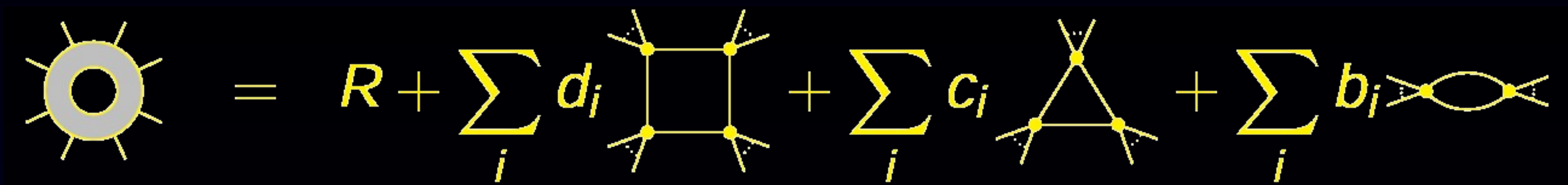
- Triple cut breaks the one-loop amplitudes in a product of tree amplitudes



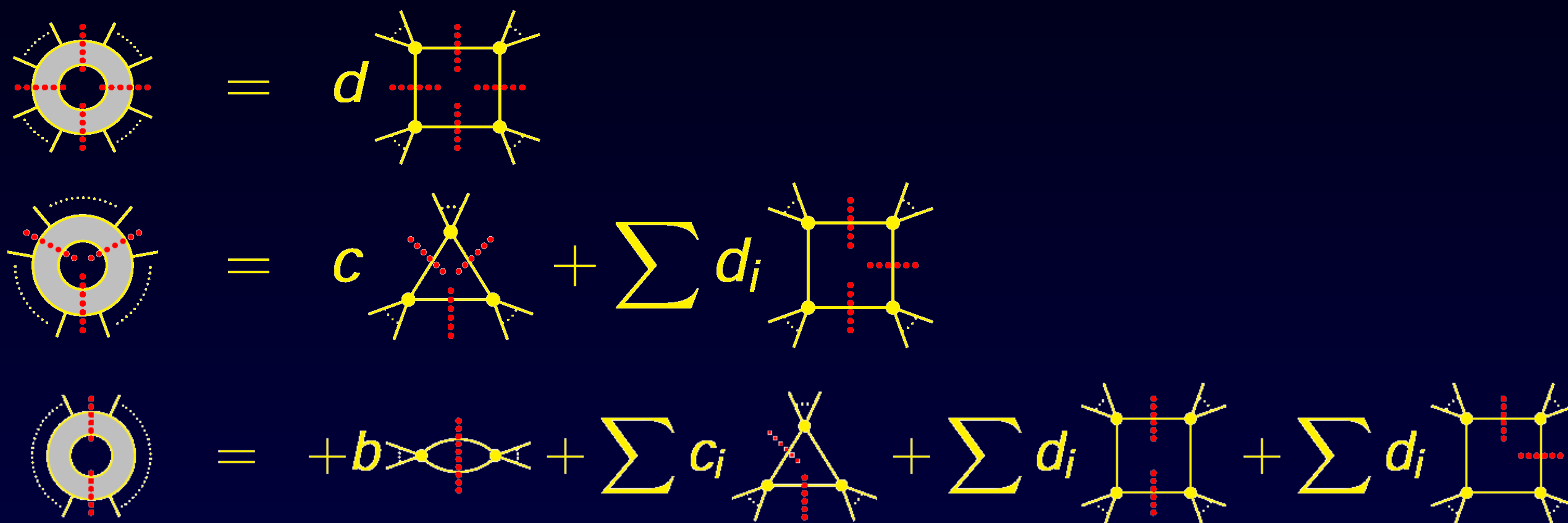
$$\int d^4k\ \delta(P_1)\delta(P_2)\delta(P_3) = \int dt J(t)$$

We know the structure of the integrand
→ can extract the relevant information by sampling different points (choices of *t*)

Generalized Unitarity



- Can obtain the coefficient of the scalar integrals



- Need to compute *R* by other means

Cuts in practice

Given external momenta configuration:

- Generate loop momenta configurations that satisfy the cut conditions (complex momenta)
- For each configuration, compute and multiply the trees at the corner of the cut diagram
- Combine the results appropriately

All the integral coefficients

effectively reduce a loop computation to tree computation

Different types of unitarity

- 4 Dimensional (A = C + R)
  - Recursion relations
  - Special Feynamn diagrams
    - [Draggiotis,Garzelli,Malamos,Papadopoulos,Pittau]
    - [Xiao,Yang,Zhu]
- D-Dimensional
  - Use different dimensions (C(D=D1) , C(D=D2))

    [Ellis,Giele,Kunszt,Melnikov,Zanderighi]
  - Stay in 4 Dimensions and emulate the additional dimensions as an additional mass in the propagators [Badger]

W/Z+4 jets @ LHC

- Very challenging processes
- Important background
  - To SM processes
  - to SUSY and other NP

W+4 jets @ LHC

- Same technology for virtual part
- Real part is very challenging
- Real matrix elements are supplied by BlackHat (BCFW recusion+analytic formulae [arXiv:1010.3991])

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

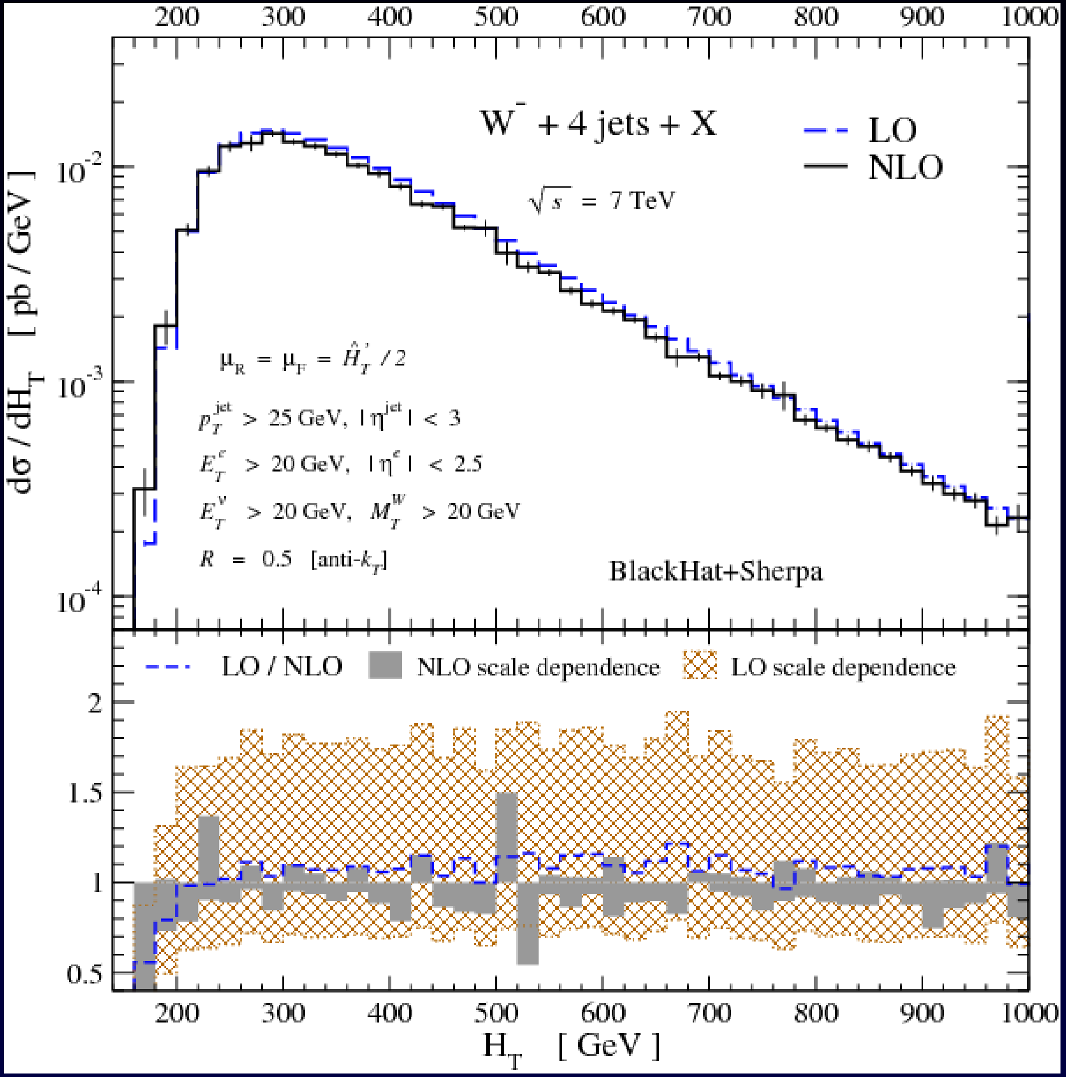
W+4 jets @ LHC

- Scale dependence reduced
- Using scale $\hat{H}'_T/2$

$$\hat{H}'_T = \sum_{\text{partons}} p_T^j + E_T^W$$

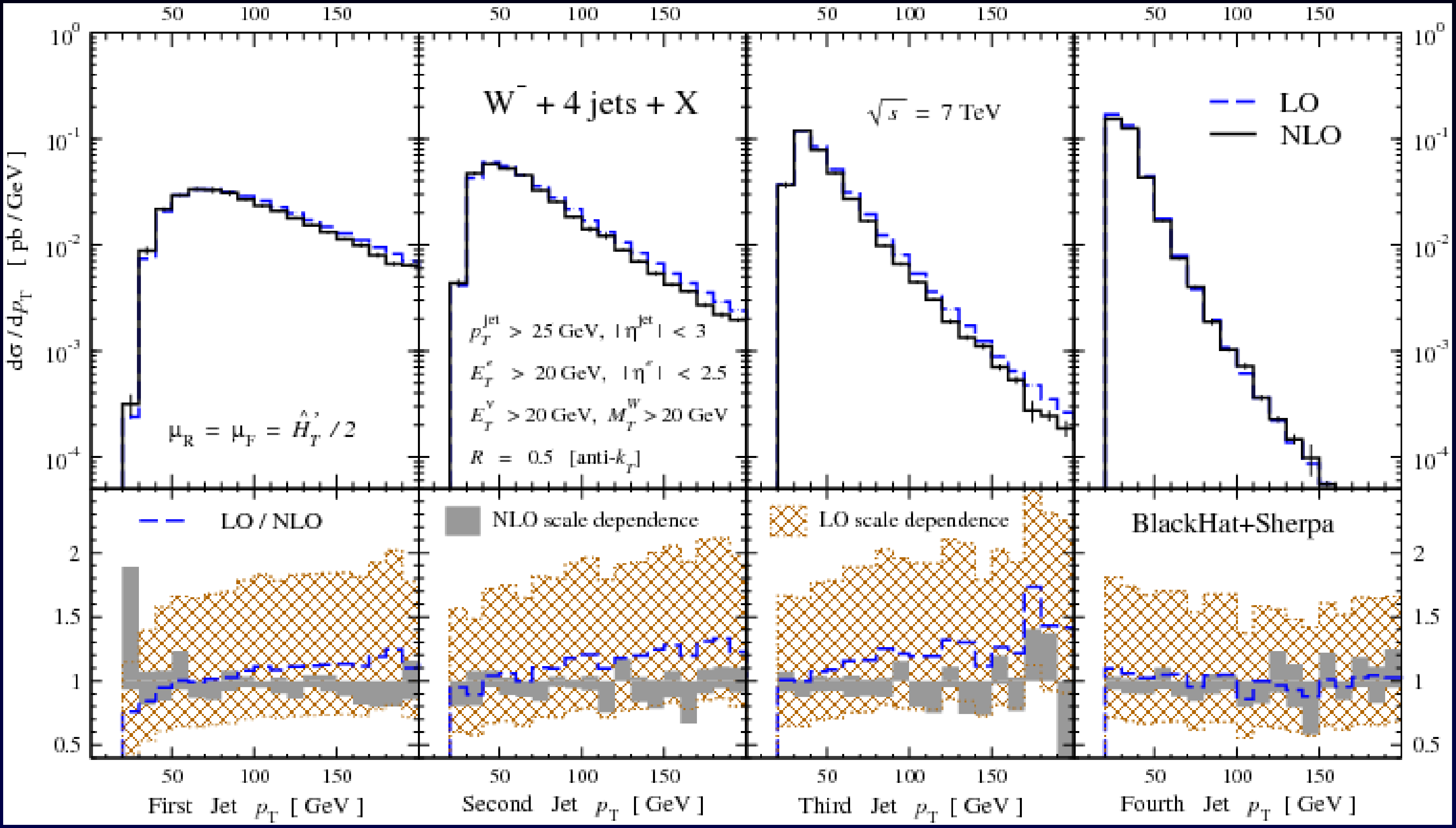
$$E_T^W = \sqrt{M_W^2 + (p_T^W)^2}$$

shape of LO and NLO are similar above 200 GeV



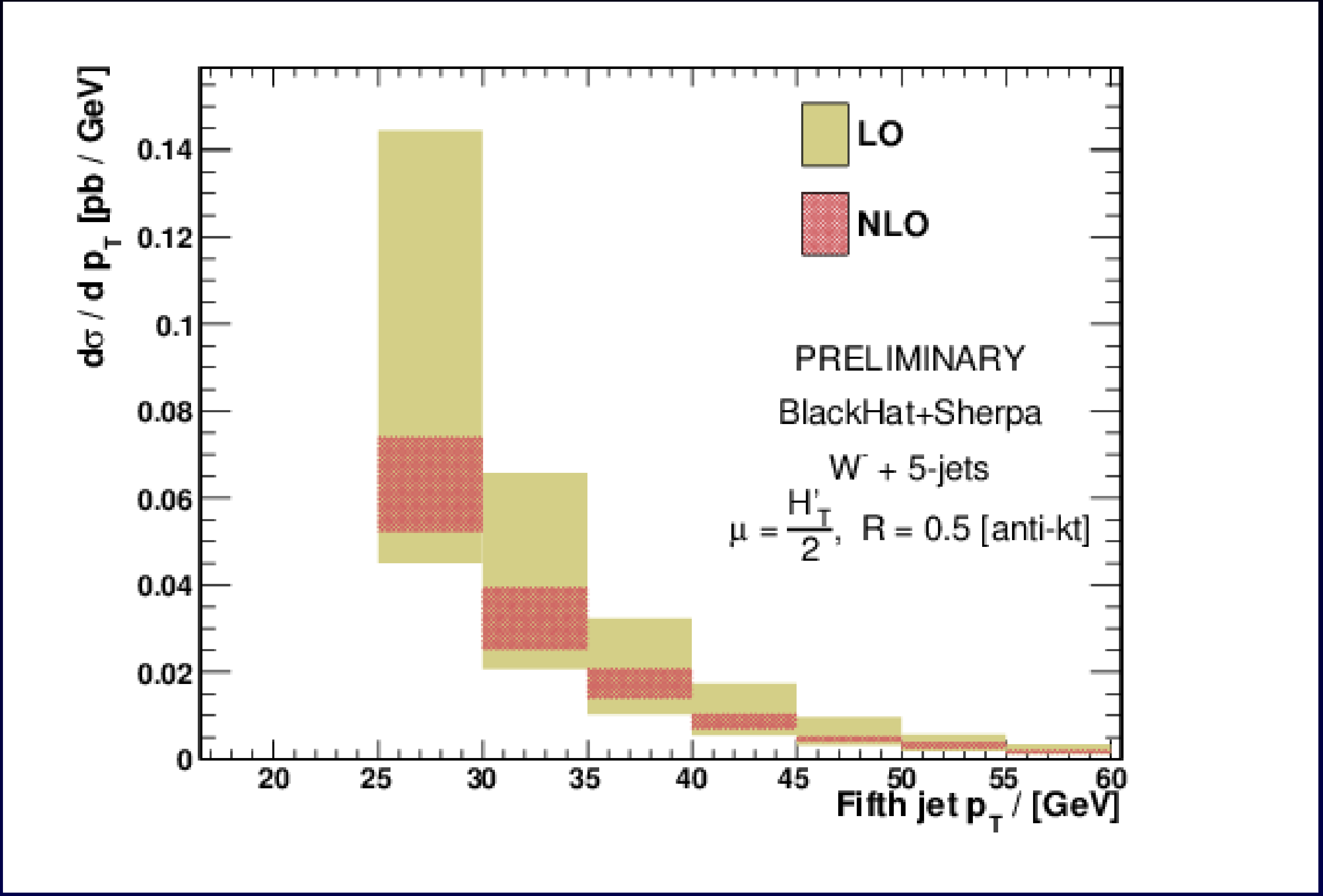
W+4 jets @ LHC

- Jet transverse momentum distribution



Preliminary results for W+5 jets

- First 2 --> 6(7) calculation at NLO for the LHC



Conclusions

- Unitarity-based techniques allow for the calculation of high multiplicity processes
- W + jets are important background for SM processes and BSM physics
- NLO predictions for high multiplicity W+jets observables are becoming available


NO BEGINNER
TERRAIN BEYOND
THIS POINT

Cut Illustration

- Toy amplitude

$$\mathcal{A} = \int \frac{\mathcal{N}}{P_1 P_2 P_3} \qquad P_i = (k - p_i)^2$$

- Using tensor reduction we know

$$\mathcal{A} = T I_3 + b_1 I_2(K_1) + b_2 I_2(K_2) + b_3 I_2(K_3)$$

- We want to find *T*
  - We don't like integrals
  - We don't like algebraic manipulation
  - We only like numerical evaluation

Cut Illustration

$$\mathcal{A} = T\int \frac{1}{P_1P_2P_3} + b_1\int \frac{1}{P_2P_3} + b_2\int \frac{1}{P_1P_3} + b_3\int \frac{1}{P_1P_2}$$

$$= \int \frac{T + b_1P_1 + b_2P_2 + b_3P_3}{P_1P_2P_3}$$

- Define cut

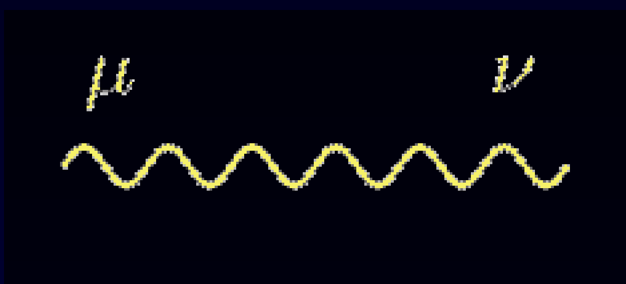
$$\mathcal{C}_i \;:\quad \frac{1}{P_i} \to \delta(P_i)$$

- Apply three cuts

$$\mathcal{C}_1\mathcal{C}_2\mathcal{C}_3\mathcal{A} = \int T\delta(P_1)\delta(P_2)\delta(P_3) = TJ$$

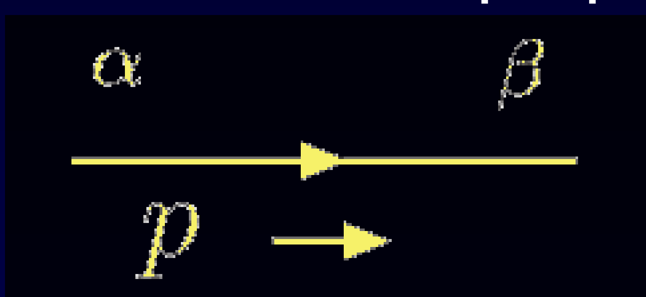
Cut Illustration

- What happens to the amplitude as the cuts are applied?
  - Gluon propagator



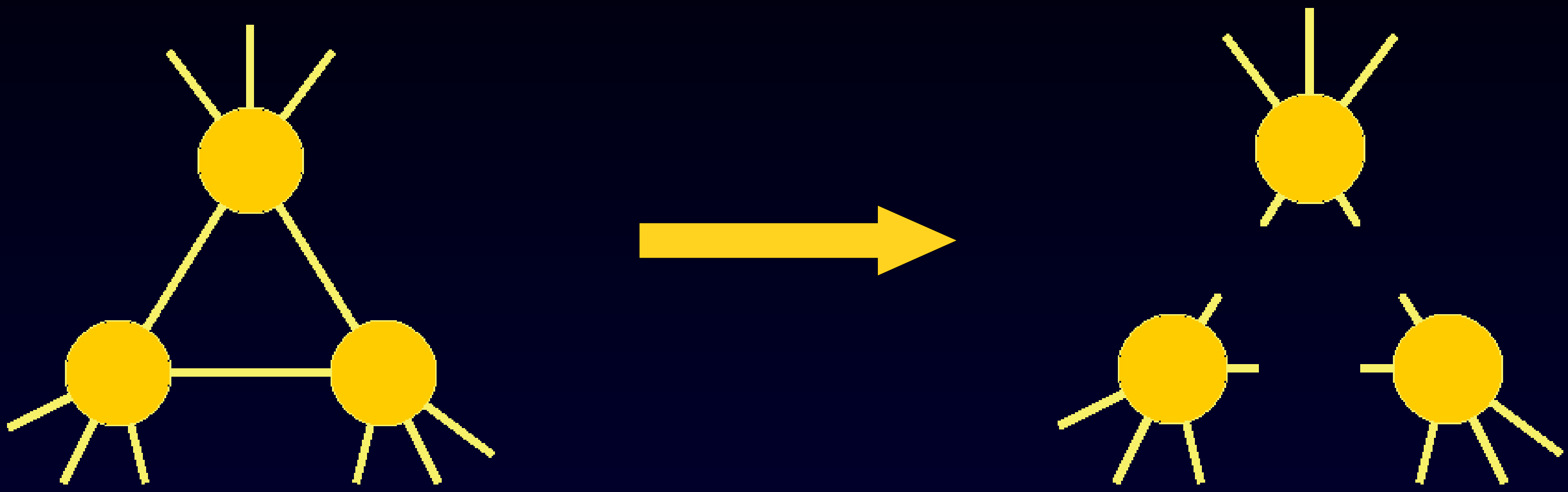
$$\frac{-ig_{\mu\nu}}{p^2} \longrightarrow \sum_{\text{pol}} \epsilon^*_\mu(p)\epsilon_\nu(p)$$

  - Fermion propagator



$$\frac{i\ \not{p}}{p^2} \longrightarrow \sum_{\text{pol}} u(p)\overline{u}(p)$$

Cut Illustration



$$\mathcal{A} = \int dl A(l)$$

$$\mathcal{C}_1\mathcal{C}_2\mathcal{C}_3\mathcal{A} = \int A_1 A_2 A_3 \delta(P_1)\delta(P_2)\delta_(P_3)$$

Cut illustration

$$\mathcal{C}_1\mathcal{C}_2\mathcal{C}_3\mathcal{A} = \int A_1 A_2 A_3 \delta(P_1)\delta(P_2)\delta_(P_3)$$

We can parametrise the loop momentum

$$\int d^4 l \, \delta(P_1)\delta(P_2)\delta(P_3) = \int dt J(t)$$

So we obtain

$$\mathcal{C}_1\mathcal{C}_2\mathcal{C}_3\mathcal{A} = \int dt \, \underbrace{A_1(k(t))A_2(k(t))A_3(k(t))J(t)}_{i(t)}$$

- If the functional form of I(t) is known (and nice enough) one can obtain $\mathcal{C}_1\mathcal{C}_2\mathcal{C}_3\mathcal{A}$ without integration

Cut Illustration

- Suppose $i(t) = c_0 + c_1 t$

  and the integral one has to compute is:

$$I = \int_0^1 i(t)dt = c_0 + \frac{1}{2}c_1$$

$$f(0) = c_0 \ , \qquad f(1) = c_0 + c_1$$

then

$$I = \frac{i(0) + i(1)}{2}$$

Cut Illustration

- Less trivial example

$$i(t) = \frac{1}{t} \underbrace{\left(\frac{c_{-3}}{t^3} + \frac{c_{-2}}{t^2} + \frac{c_{-1}}{t} + c_0 + c_1 t + c_2 t^2 + c_3 t^3 \right)}_{f(t)}$$

- Integration contour = circle of radius 1

$$I = 2\pi i c_0$$

$$I = \frac{2\pi i}{7} \sum_{j=0}^{6} f(e^{\frac{2\pi i j}{7}})$$

Unitarity vs FD

Preferences (not restrictions)

- Feynman diagrams
  - More Masses
  - Less jets
  - More EW
- Unitarity
  - More massless
  - More jets
  - Less EW

Approaches are complimentary

Standard integral reduction

- The One-loop amplitude is the sum of a large number of Feynman diagrams
- Each of these Feynman diagrams is composed of a lot of tensor integrals
- Each tensor integral can be written in terms of scalar integrals
- To find the coefficients a lot of computer algebra has to be performed

Standard integral reduction

- Coefficients of the scalar integral are generally
  - Very large analytical expressions
  - Have numerical instabilities due to so-called Gram determinants
- These problem can be addressed
- $pp \to t\bar{t}b\bar{b}$ [Bredenstein,Denner,Dittmaier,Pozzorini]
- $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth,Greiner,Guffanti,Guillet,Reiter,Reuter]
- ...