The phase diagram in T-μ-N_c space

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Excited QCD — Peniche (Portugal), May 9 2012

Outline

1. Introduction

2. The percolation model

3. The deconfinement line in (ρ, N_c)

4. Final remarks

QCD phase diagram

- QCD in the low-temperature, high-density ($\mu_B \sim \Lambda_{QCD}$) region is poorly understood:
  - lattice QCD of little help there;
  - perturbation theory not yet available.

- Crucial ingredient for: high-density matter, HIC, neutron stars, ...

- Valuable insight from the large-N_c limit !

Large-N_c approach

Idea ['t Hooft; Witten, '70s]: assume there is a confining limit

$$N_c \to \infty \quad , \quad g \to 0 \quad \text{such that} \quad g^2 N_c = \lambda \quad (\Lambda_{\text{QCD}} \sim N_c^0)$$

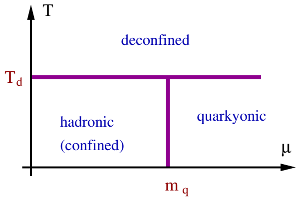
Simpler calculations (planar diagrams only):

mesons $\to$ stable and noninteracting;

baryons $\to$ states of $M_B \sim N_c$ and size $\sim \Lambda_{\text{QCD}}^{-1}$

Corrections appear as $1/N_c$, $1/N_c^2$ (in real life: $1/3$, $1/9 \ldots$)

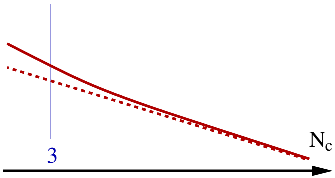
Large-N_c phase diagram [McLerran, Pisarski '07]:

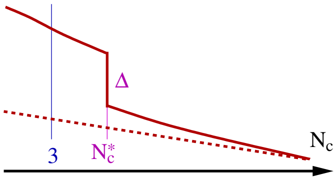


Possible objections

$N_c = 3$ is not "large" ...

{"Large N_c" is such that deconfinement is *first order* for $N_f = 2$ [P. Petreczky] }

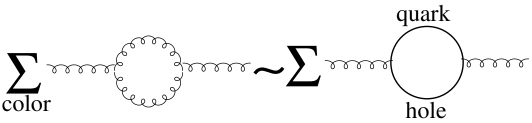




Most important: if there is a discontinuity, how reliable are results extrapolated from $N_c = \infty$ to our world?

Compare: the "Skyrme crystal" of large-N_c dense baryons $\Leftrightarrow$ the $N_c = 3$ "nuclear liquid" ...

We expect deconfinement when
quark-hole (screening) is $\sim$ gluon loop (anti-screening):



$$N_c^2 \quad \sim \quad \mu_q^2 N_c N_f \quad \Rightarrow \quad \mu_q = \sqrt{N_c/N_f}\Lambda_{\text{QCD}}$$

On the other hand, quarks are as close as $\sim N_c^{-1/3}$:
at large N_c, they feel "asymptotically free" to reach each other.

$\Rightarrow$ Apparent paradox starting at $\mu_q \sim \Lambda_{\text{QCD}}$ ...

Quark DOFs inside, baryonic DOFs on the surface ...

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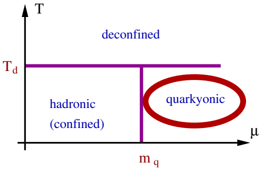
"Quarkyonic phase"

"Quarkyonic phase" suggested by [McLerran, Pisarski '07] for $T < T_d$ and μ high enough, that is, dense matter



- Quarks below the Fermi surface are quasi-free, while on the surface we have baryonic excitations.
  $\Rightarrow$ Pressure and entropy density scale as $\sim N_c$
  (unlike confined: N_c^0, or deconfined: N_c^2).
- "Still confined but already chirally restored".
- It is an *educated guess*, quite hard to test rigorously
  (e.g. standard AdS/CFT has $N_c = \infty \ldots$).

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Goal of the model

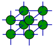
- Suppose close-by baryons "exchange" (energy, momentum)
- *Confined* system, but net effect may be *large-scale* exchanges

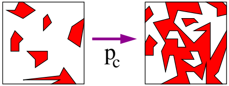
  (on a short timescale $\sim 1/\Lambda_{\text{QCD}}$).

$\Rightarrow$ Identify this to **quarkyonic phase**?

- Formal framework is *(bond-) percolation theory*: call p the probability of neighbours "speaking": then

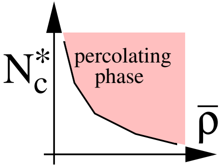
  $p = p_c$ is a second-order point:

  ($p_c \simeq 0.2488$ for )



- Build a model for confined baryons $\rightarrow$ find a $p(N_c, \rho, \ldots)$

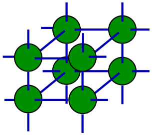
  $\rightarrow$ get a curve for critical $N_c^*(\rho)$:



$\Longrightarrow$ compare to *deconfinement* ...

Model for baryonic matter

Matter is a cubic lattice of baryons.



A *baryon* is a hard-sphere with N_c randomly-distributed quarks:

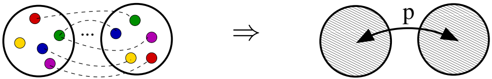
$$f(\mathbf{x}) \sim \theta(1 - \Lambda_{\mathrm{QCD}}|\mathbf{x} - \mathbf{x}_{\mathrm{centre}}|) \quad .$$

Density $\leftrightarrow$ lattice spacing $2\epsilon\Lambda_{\mathrm{QCD}}^{-1}$:

$$\overline{\rho} = \overline{\rho}_0/\epsilon^3 = \left(\frac{\Lambda_{\mathrm{QCD}}^3}{8}\right)\frac{1}{\epsilon^3} \quad .$$

From “interquark” to “interbaryon”

quark-quark *squared propagator* $\Rightarrow$ b-b “exchange” probability



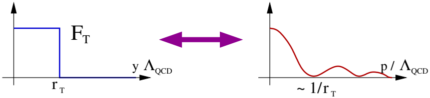
This is done using

$$p(N_c) = 1 - \left[\int f_A(\mathbf{x}_A) \mathrm{d}^3\mathbf{x}_A \int f_B(\mathbf{x}_B) \mathrm{d}^3\mathbf{x}_B \Big(1 - F(|\mathbf{x}_A - \mathbf{x}_B|) \Big) \right]^{N_c^2}$$

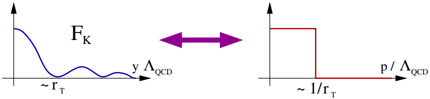
Choices for “squared propagator”

strength λ/N_c, range $\propto r_T/\Lambda_{\text{QCD}}$.

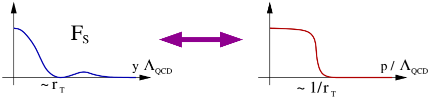
- Coordinate-space step function



- Momentum-space step function



- Momentum-space rounded step



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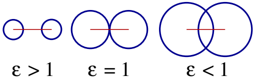
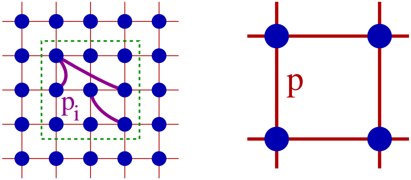
RG-inspired blocking out

High density: next-to-nearest neighbours important!



- Define $\{p_i\}$ for various b-b distances, $(1,0,0),(1,1,0),\ldots(2,2,2)$.
- With a blocking-out of step $b = 3,\ldots 7$,
  a b^3 cell becomes 1^3 in a *superlattice*:



- Also, $\{p_i\} \to p'$, to compare with p_c (the RG fixed point).

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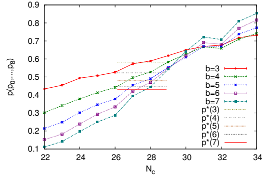
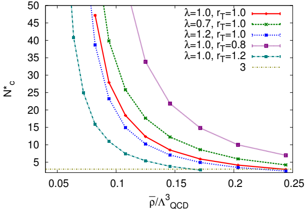
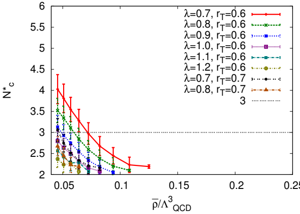
Numerically get curves $p(N_c; \quad F, \lambda, r_T, \epsilon)$.



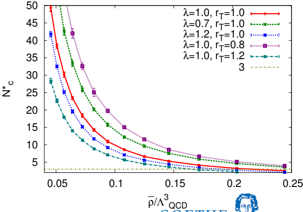
Intersect with p_c, find $N_c^*(\overline{\rho}; \quad F, \lambda, r_T)$:



F_T



F_K



F_S

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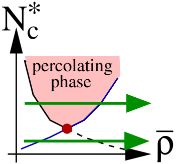
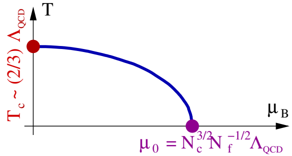
Deconfinement line

Deconfinement line vs. percolation line in (ρ, N_c):

a minimum $N_c^{\heartsuit}$ for percolation !



One such line for each T, i.e. each point on:



Find it with Boltzmann integrals (N_c-scaling, parametrisations . . .)

first at $T = 0$, then at $T > 0$ . . .

$T = 0$ deconfinement line

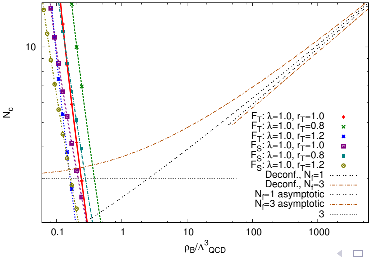
Implement

$$\rho_{(T=0)} = \frac{4\pi g_f g_s}{(2\pi)^3}\left[\int \frac{p^2 \mathrm{d}p}{1+\exp\left(\frac{\sqrt{p^2+m^2}-\mu_0}{T}\right)} - \langle \mu_0 \leftrightarrow -\mu_0 \rangle\right]$$

N_c-friendly change of variables

$$\gamma = \frac{\sqrt{N_c}}{\mu_0} m \;;\quad \alpha = \frac{\sqrt{N_c}}{\mu_0} p \;;\quad \beta = \frac{\Lambda_{\mathrm{QCD}}}{T}\frac{N_c}{\sqrt{N_f}} \;;$$

At zero temperature, can be solved analytically: here for $N_f = 1, 3$:



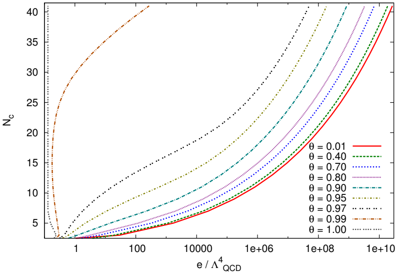
$T > 0$ deconfinement line

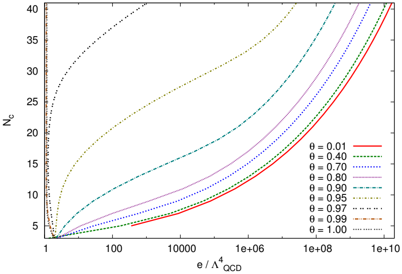
- Higher-spin states (a spin-flip costs $\sim \Lambda_{\mathrm{QCD}}/N_c$)
- Antibaryons start to contribute
- Numerical computation

(some adjustment to convert the physical ρ into the $\overline{\rho}$ of percolation . . .)

Analogous procedure for the *energy density* curve in (e, N_c)
(including mesons. . .)

Energy density e ($N_f = 1, 3$)





$$(\theta = T/T_c)$$

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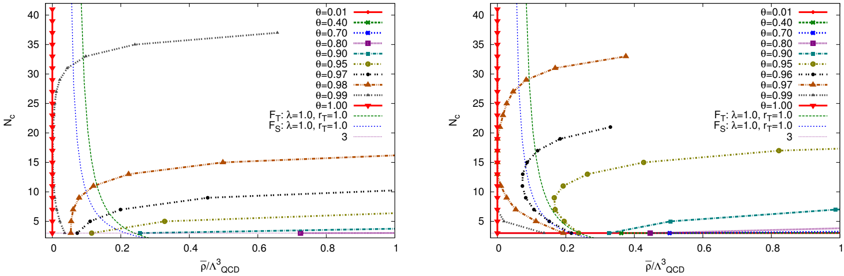


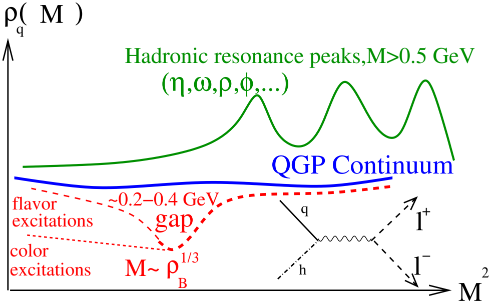
$$(\theta = T/T_c)$$

$\Rightarrow$ Non-trivial behaviour in T: temperature-dependent $N_c^{\heartsuit}$!

An analogy from condensed matter

- In the metal-insulator transition, electrons start to tunnel among the atoms' potential wells.
- Large $N_c \leftrightarrow$ "confined conductor" picture.
- Atoms remain well-defined objects; electrons *delocalise*: Bloch constraints $\rightarrow$ **gaps** in the spectral functions.

Gaps could be detected within dilepton spectral functions !



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Conclusive comments

- This quite rough model should be taken *cum grano salis*: it should mostly suggest a direction for more quantitative future works.
  (Convert the many "$\sim$" appearing here into "$=$")
- If the "percolating" (quarkyonic?) phase is accessible in our $N_c = 3$ world, this happens at very low temperatures and *really high densities*.
  (maybe relevant for e.g. cold compact stars)
- How to test this picture?
  $\Rightarrow$ Our world has fixed N_c, but there is the lattice!
  $\Rightarrow$ And, phenomenologically:
  - The $\sim N_c$ pressure jump should affect supernova explosions.
  - The percolation phase could alter the dilepton spectral function.

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End of the talk.

Clues to a hidden phase transition

Take a Skyrme crystal of baryons at large N_c:

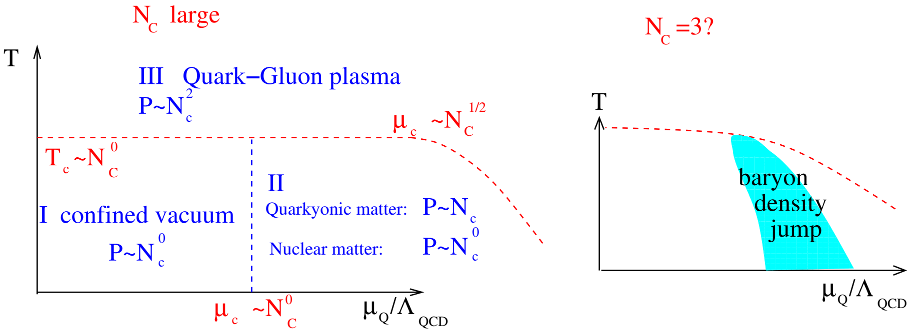
$$\left.\begin{array}{r}\text{Mass} \sim N_c \\ \text{Fermi motion energy} \sim 1\end{array}\right\} \Rightarrow \text{Fermi momentum} \sim \sqrt{N_c}$$

Interbaryonic binding energy in the crystal $\sim N_c$

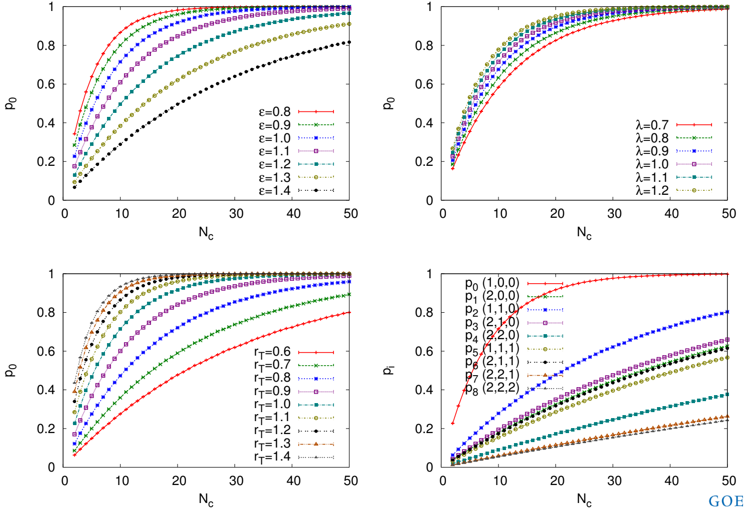
$\Rightarrow$ below some N_c^* the *crystal melts into a liquid*

Symmetry is changed: has a phase transition occurred? [suggested by Klebanov already in 1985...]

Large-N_c vs. $N_c = 3$ phase diagram



Baryon-baryon “exchange” probabilities



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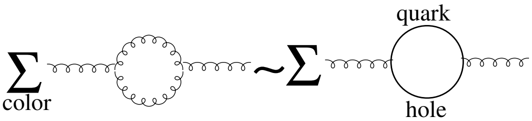
N_c-scaling of $\mu_0(T \simeq 0)$



$$N_c^2 \quad \sim \quad \mu_q^2 N_c N_f$$

$$\Rightarrow \quad \mu_B(\text{deconfinement}) \sim N_c \mu_q \sim \frac{N_c^{3/2}}{\sqrt{N_f}} \Lambda_{\text{QCD}}$$

Density integral

$$\rho_B^{\mathrm{conf}} = \frac{4\pi g_f}{(2\pi)^3}\frac{N_c^3}{N_f^{3/2}}\Lambda_{\mathrm{QCD}}^3 \sum_{\eta=0,1\ldots}^{Q}(2\eta+2)\Bigg\{ \int \frac{\alpha^2 \mathrm{d}\alpha}{1+\exp\left[\frac{3}{2}\frac{N_c}{\sqrt{N_f}}\frac{1}{\theta}\left(\sqrt{\alpha^2+N_f}+\eta\frac{\sqrt{N_f}}{N_c^2}-\sqrt{N_c}\sqrt{1-\theta^2}\right)\right]} - \int \frac{\alpha^2 \mathrm{d}\alpha}{1+\exp\left[\frac{3}{2}\frac{N_c}{\sqrt{N_f}}\frac{1}{\theta}\left(\sqrt{\alpha^2+N_f}+\eta\frac{\sqrt{N_f}}{N_c^2}+\sqrt{N_c}\sqrt{1-\theta^2}\right)\right]}\Bigg\}$$

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Energy density integral

$$e^{\mathrm{conf}} = N_f^2 \frac{\pi^2}{15} T^4 + e_B^{\mathrm{conf}}$$

$$\begin{aligned} e_B^{\mathrm{conf}} \quad = \quad & \frac{4\pi g_f}{(2\pi)^3} \frac{N_c^4}{N_f^2} \Lambda_{\mathrm{QCD}}^4 \sum_\eta (2\eta + 2) \Bigg\{ \\ & \int \frac{\alpha^2 \left[\sqrt{\alpha^2 + N_f} + \eta \frac{\sqrt{N_f}}{N_c^2} \right] \mathrm{d}\alpha}{1 + \exp\left[\frac{3}{2} \frac{N_c}{\sqrt{N_f}} \frac{1}{\theta} \left(\sqrt{\alpha^2 + N_f} + \eta \frac{\sqrt{N_f}}{N_c^2} - \sqrt{N_c}\sqrt{1 - \theta^2} \right) \right]} \\ & + \int \frac{\alpha^2 \left[\sqrt{\alpha^2 + N_f} + \eta \frac{\sqrt{N_f}}{N_c^2} \right] \mathrm{d}\alpha}{1 + \exp\left[\frac{3}{2} \frac{N_c}{\sqrt{N_f}} \frac{1}{\theta} \left(\sqrt{\alpha^2 + N_f} + \eta \frac{\sqrt{N_f}}{N_c^2} + \sqrt{N_c}\sqrt{1 - \theta^2} \right) \right]} \Bigg\} \end{aligned}$$

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