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Excited-QCD 2012

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OZI - violating eight quark interactions
and its effects
on the physics of the P-NJL model

B. Hiller, J. Moreira, A. A. Osipov, A. H. Blin

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- **$SU(3)_L \times SU(3)_R$ chiral symmetry and instability of the vacuum for $4q + 6q$ NJL Lagrangian.**
  - A. A. Osipov, B. Hiller, V. Bernard, A. H. Blin, Annals Phys. (NY), 321:2504-2534,2006.
- **A solution for stability: addition of $8q$ terms.**
  - A. A. Osipov, B. Hiller, J. da Providencia, Phys. Lett. B 634 (2006) 48-54.
- **A mechanism of dynamical χSB**
- **Consequences on the low lying spin 0 meson spectra.**
  - A. A. Osipov, B. Hiller, A. H. Blin, J. da Providencia, Annals of Phys. (NY), 322:2021-2054,2007
- **Conseq. for magnetic catalysys on χSB .**
  - A. A. Osipov, B. Hiller, A. H. Blin, J. da Providencia, Phys. Lett. B 650 (2007) 262-267

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• Consequences for chiral transitions at finite temperature and phase diagram.

- A. A. Osipov, B. Hiller, J. Moreira, A. H. Blin, J. da Providencia, Phys. Lett. B 646:91-94,2007.
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General arguments

• We consider the minimal chiral extension needed to render the vacuum stable, based on the assumption of hierarchy in multi-quark interactions in large N_c counting.
• We suppose that these interactions are localized in the interval $\Lambda_{conf} < \Lambda < \Lambda_{\chi SB}$.

• QCD inspired model of instanton vacuum $\Rightarrow$ evidence in favour of $2N_f$-quark interactions in low-energy region (in zero mode approximation). Leading $1/N_c$ order $\Rightarrow$ 't Hooft determinant, which breaks the axial $U_A(1)$ symmetry and is a source of OZI-violating effects.

• G.'t Hooft, Phys. Rev. D 14 (1976) 3432; Phys. Rev. D 18 (1978) 2199.

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• The effective quark Lagrangian derived from the instanton gas model, considered beyond the zero mode approximation, predicts the existence of $4q, 6q, \ldots, 2nq, \ldots$ quark interactions, all equally weighted at large N_c. The 't Hooft type Ansatz emerges if only zero modes contribute.

• Yu. A. Simonov, Phys. Lett. B 412 (1997) 371; Phys. Rev. D 65 (2002) 094018.

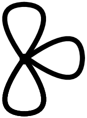
• Lattice results for gluon field correlators show hierarchy with dominance of lowest one.

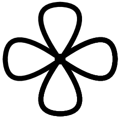
• G. S. Bali, Phys. Reports 343 (2001) 1.

• Expect similar hierarchy for multiquark interactions after averaging over gluon fields.

Scale arguments

$$G\,(\bar{\Psi}\Psi)^2 \;\rightarrow\; \frac{\bar{G}}{\Lambda^2}$$  $$\propto\; \bar{G}\Lambda^2$$

$$K\,(\bar{\Psi}\Psi)^3 \;\rightarrow\; \frac{\bar{K}}{\Lambda^5}$$  $$\propto\; \bar{K}\Lambda^1$$

$$g\,(\bar{\Psi}\Psi)^4 \;\rightarrow\; \frac{\bar{g}}{\Lambda^8}$$  $$\propto\; \bar{g}\,\Lambda^0$$

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Effective multi-quark Lagrangian: $1/N_c$ arguments

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m)q + \mathcal{L}_{4q} + \mathcal{L}_{6q} + \mathcal{L}_{8q} + \dots \tag{1}$$

• NJL four-quark interactions of the scalar and pseudoscalar types with the $U(3)_L \times U(3)_R$ chiral symmetry are given by

$$\mathcal{L}_{4q} = \frac{G}{2}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right]. \tag{2}$$

After bosonization they lead to effective meson vertices of order N_c ($G \sim 1/N_c$).

• Six-quark OZI violating 't Hooft interaction

$$\mathcal{L}_{6q} = \kappa(\det \bar{q}P_L q + \det \bar{q}P_R q), \tag{3}$$

leads after bosonization to effective meson vertices of order 1 ($\kappa \sim 1/N_c^3$).

• The most general eight-quark $U(3)_L \times U(3)_R$ symmetric Lagrangian for spin zero interactions (without derivatives)

$$\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}$$

$\mathcal{L}_{8q}^{(1)}$: OZI violating

$$\begin{aligned}
\mathcal{L}_{8q}^{(1)} &= 8\, g_1 \left[(\bar{q}_i P_R q_m)(\bar{q}_m P_L q_i)\right]^2 = \frac{g_1}{32}\left[\mathrm{tr}(S - iP)(S + iP)\right]^2 \\
&= \frac{g_1}{8}\left(S_a^2 + P_a^2\right)^2, \qquad (4) \\
\mathcal{L}_{8q}^{(2)} &= \frac{g_2}{16}\mathrm{tr}\left[(S - iP)(S + iP)(S - iP)(S + iP)\right] \\
&= \frac{g_2}{8}\left[d_{abe}d_{cde}\left(S_aS_bS_cS_d + P_aP_bP_cP_d + 2S_aS_bP_cP_d\right)\right. \\
&+ \left. 4 f_{ace}f_{bde}S_aS_bP_cP_d\right]. \qquad (5)
\end{aligned}$$

Trace over flavour indices $i, j = 1, 2, 3$;
$S_{ij} = S_a(\lambda_a)_{ij} = 2\bar{q}_j q_i, \quad P_{ij} = P_a(\lambda_a)_{ij} = 2\bar{q}_j(i\gamma_5)q_i.$

Bosonization

$$Z = \int \mathcal{D}q\mathcal{D}\bar{q} \exp(i\int \mathrm{d}^4x \mathcal{L}(\bar{q}, q)) \tag{6}$$

$$\begin{aligned} Z = & \int \mathcal{D}q\mathcal{D}\bar{q} \prod_a \mathcal{D}\sigma_a \prod_a \mathcal{D}\phi_a \exp\left(i\int \mathrm{d}^4x \mathcal{L}_q(\bar{q}, q, \sigma, \phi)\right) \\ & \times \int_{-\infty}^{+\infty} \prod_a \mathcal{D}s_a \prod_a \mathcal{D}p_a \exp\left(i\int \mathrm{d}^4x \mathcal{L}_r(\sigma, \phi, \Delta; s, p)\right), \end{aligned} \tag{7}$$

Effective lagrangian

$$\mathcal{L}_{\rm eff} = W_{\rm ql}(\sigma,\phi) + \mathcal{L}_{\rm st}$$

- Quark loop integrals

$$W_{\rm ql}(\sigma,\phi) = \frac{1}{2}\ln|\det D_E^\dagger D_E| = -\int \frac{\mathrm{d}^4 x_E}{32\pi^2}\sum_{i=0}^\infty I_i \mathrm{tr}(b_i),$$

$$I_i = \frac{1}{3}\sum_{j=u,d,s} J_i(M_j^2), \quad J_i(M^2) = 16\pi^2\Gamma(i+1)\int \frac{\mathrm{d}^4 p_E}{(2\pi)^4}\frac{1}{(p_E^2+M^2)^{i+1}}$$

$$b_0 = 1, \quad b_1 = -Y, \quad b_2 = \frac{Y^2}{2} + \frac{\Delta_{us}}{\sqrt{3}}\lambda_8 Y, \quad \ldots,$$

$$Y = i\gamma_\alpha(\partial_\alpha\sigma + i\gamma_5\partial_\alpha\phi) + \sigma^2 + \{\mathcal{M},\sigma\} + \phi^2 + i\gamma_5[\sigma+\mathcal{M},\phi]$$

- Stationary contribution

$$\begin{cases} s_{st}^a &= h_a + h_{ab}^{(1)}\sigma_b + h_{abc}^{(1)}\sigma_b\sigma_c + h_{abc}^{(2)}\phi_b\phi_c + \ldots \\ p_{st}^a &= h_{ab}^{(2)}\phi_b + h_{abc}^{(3)}\phi_b\sigma_c + \ldots \end{cases}$$

$$\mathcal{L}_{\rm st} = h_a\sigma_a + \frac{1}{2}h_{ab}^{(1)}\sigma_a\sigma_b + \frac{1}{2}h_{ab}^{(2)}\phi_a\phi_b + \mathcal{O}(\text{field}^3)$$

Stationary Phase Equations and Gap Equations

Three coupled equations to determine the quark condensates or $h_a\lambda_a = \mathrm{diag}(h_u, h_d, h_s)$

$$\begin{cases} Gh_u + \Delta_u + \frac{\kappa}{16}\, h_d h_s + \frac{g_1}{4}\, h_u(h_u^2 + h_d^2 + h_s^2) + \frac{g_2}{2}\, h_u^3 = 0, \\ Gh_d + \Delta_d + \frac{\kappa}{16}\, h_u h_s + \frac{g_1}{4}\, h_d(h_u^2 + h_d^2 + h_s^2) + \frac{g_2}{2}\, h_d^3 = 0, \\ Gh_s + \Delta_s + \frac{\kappa}{16}\, h_u h_d + \frac{g_1}{4}\, h_s(h_u^2 + h_d^2 + h_s^2) + \frac{g_2}{2}\, h_s^3 = 0. \end{cases} \tag{8}$$

$\Delta_i = M_i - \hat{m}_i$

to be solved selconsistently with

$$h_i(M_i) + \frac{N_c M_i}{2\pi^2} J_0(M_i^2) = 0, \tag{9}$$

$\Uparrow$ Quark one loop tadpole.

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Stability conditions

$$g_1 > 0, \quad g_1 + 3g_2 > 0, \quad G > \frac{1}{g_1}\left(\frac{\kappa}{16}\right)^2 . \qquad (10)$$

$$\Downarrow$$

and counting rules $G \sim 1/N_c$, $\kappa \sim 1/N_c^3 \Rightarrow$
g_1 cannot scale as $1/N_c^6$ or smaller.

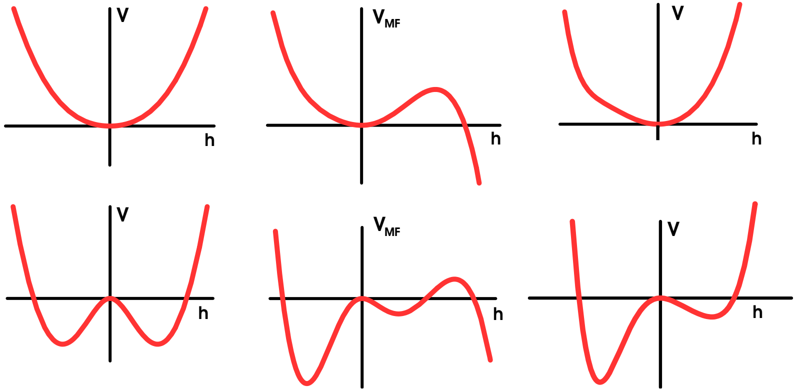
On the other hand $\mathcal{L}_{8q}^{(1)}$ is an additional (to the 't Hooft determinant) source of OZI-violating effects and thus it cannot be stronger than the 't Hooft interactions, i.e., $g_1 \sim 1/N_c^4$ or less.

$\Rightarrow$

$$1/N_c^5 \leq g_1 \leq 1/N_c^4 . \qquad (11)$$

8q and stability; Effective scalar potential V, $SU(3)$ chiral limit.

$\tau < 1$ $\boxed{4q}$ $\boxed{4q + 6q}$ $\boxed{4q + 6q + 8q}$

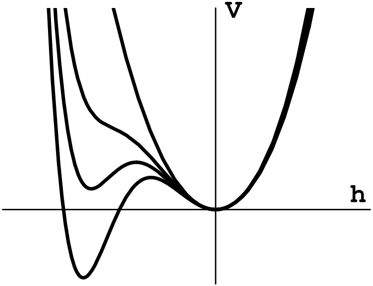


$\tau > 1$ χSB by $4q \Uparrow$

$h \sim$ quark condensate. $\tau = \frac{N_c G \Lambda^2}{2\pi^2} \sim$ curvature of V at origin.

Effective potential V (closer look)

4q+6q+8q $\tau < 1$



$\Uparrow$

χSB by $6q$

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$8q$-interactions may strongly affect *magnetic catalysis* **and** *thermodynamic observables*, **without changing the spectra at $T = \mu = 0, H = 0$.**

- G and g_1 dependence of SPA and masses of light 0^{-+} and 0^{++} mesons:

$$\xi = G + g_1(h_u^2 + h_d^2 + h_s^2)/4,$$

except 00, 08 and 88 states of scalar nonet.

$\rightarrow$ almost identical spectra can be obtained by changing G, g_1 and freezing all other parameters.

- But at finite T, μ or H:

$h_i(T, \mu, H)$ via gap equations $\longrightarrow$ ξ steered by g_1. .

Pseudoscalar masses

- The topological susceptibility is not affected by $8q$ in leading order N_c.

- The $\eta - \eta'$ splitting gets an additional small correction $\sim 2\%$ to the Witten-Veneziano term.

7 Parameters: $\hat{m}_u, \hat{m}_s, G, \kappa, \Lambda, g_1, g_2$

Fit 6 couplings by fixing $m_\pi, m_K, f_\pi, f_K, m'_\eta, m_\eta$. Vary g_1 from set to set. .

Scalar masses

• Definite hierarchy in scalar masses: $m_{f_0^-} < m_{a_0} < m_{K_0^*} < m_{f_0^+}$

• 8q terms do no alter the hierarchy, i.e. it is the same for the conventional NJL + 't Hooft Lagrangian.

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• Eight-quark interactions may contribute to the sum rule of Dmitrasinovic: 1996

already at leading $1/N_c$ order, if $g_1 \sim 1/N_c^4$

$$m_{\eta'}^2 + m_\eta^2 - 2m_K^2 + m_{f_0^+}^2 + m_{f_0^-}^2 - 2m_{K_0^*}^2 = -6E_1^{LO} + \mathcal{O}\left(\frac{1}{N_c^2}\right). \quad (12)$$

• This term has a negative sign, decreasing the sum $m_{f_0^-}^2 + m_{f_0^+}^2$.

• g_1 lowers the value of $m_{f_0^-}$ and the octet-singlet splitting grows with increasing g_1 in the scalar nonet.

• Sum rule: good illustration of the possible impact of the eight-quark OZI violating terms on the scalar mesons.

Thermal and medium effects

Thermodynamic potential with Polyakov loop[1]

Integrating the gap equations we get:

$$\Omega\left(M_f, T, \mu, \phi, \overline{\phi}\right) =$$

$$= \frac{1}{16}\left(4Gh_f^2 + \kappa h_u h_d h_s + \frac{3g_1}{2}\left(h_f^2\right)^2 + 3g_2 h_f^4\right)\Bigg|_0^{M_f}$$

$$+ \frac{N_c}{8\pi^2}\sum_{f=u,d,s}\Big(J_{-1}(M_f^2, T, \mu, \phi, \overline{\phi}) + C(T,\mu)\Big) + \mathcal{U}\left(\phi, \overline{\phi}, T\right)$$

[1] For details see: Phys. Rev. D 81, 116005 (2010) and IJMPA 27 (2012) 1250060

Pauli-Villars: $\hat{\rho}_\Lambda^{PV} = 1 - \left(1 - \Lambda^2 \partial_{\vec{p}^2}\right) \exp\left(\Lambda^2 \partial_{\vec{p}^2}\right)$

- Regulator: $\hat{\rho}_\Lambda^{PV} f(|\overrightarrow{p}|^2) = f(|\overrightarrow{p}|^2) - f(|\overrightarrow{p}|^2 + \Lambda^2) + \Lambda^2 \frac{\partial}{\partial |\overrightarrow{p}|^2} f(|\overrightarrow{p}|^2 + \Lambda^2)$

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- Vacuum and medium parts:

$$J_{-1}^{vac}(M^2) = \frac{1}{2}\left(\left(M^4 - \Lambda^4\right) \ln(1 + \frac{M^2}{\Lambda^2}) - M^2\left(\Lambda^2 + M^2 \ln \frac{M^2}{\Lambda^2}\right)\right)$$

$$J_{-1}^{med}(M^2) = -\frac{8}{3} \int_0^\infty \mathrm{d}|\overrightarrow{p}_E||\overrightarrow{p}_E|^4 \hat{\rho}_\Lambda^{PV}\left(\frac{n_{q\ M} + n_{\bar{q}\ M}}{E_M} - \frac{n_{q\ 0} + n_{\bar{q}\ 0}}{E_0}\right)$$

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- *M* independent:

$$C^{PV}(T, \mu) = -\frac{8}{3}\int_0^\infty \mathrm{d}pp^4 \left(\frac{n_{q\ 0} + n_{\bar{q}\ 0}}{p}\right)$$

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- M independent:

$$C^{PV}(T,\mu) = -\frac{8}{3}\int_0^\infty \mathrm{d}p p^4 \left(\frac{n_{q\ 0} + n_{\bar{q}\ 0}}{p}\right)$$

Inclusion of the Polyakov loop.

Introduce homogeneous background A_4 gluonic field

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + \imath A^\mu, \quad A^\mu = \delta_0^\mu g A_a^0 \frac{\lambda^a}{2}, \quad L = \mathcal{P} e^{\int_0^\beta \mathrm{d}x_4 \imath A_4}, \quad \phi = \frac{1}{N_c}\mathrm{Tr} L, \quad \overline{\phi} = \frac{1}{N_c}\mathrm{Tr} L^\dagger$$

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- $\sim$ order parameter (exact $\lim_m \to \infty$) for (de)/confinement ($\phi = 0 \leftrightarrow$ confined)
- enters the action as an imaginary μ

$$n_q(M, p, \mu, T) = \left(1 + e^{\left(\sqrt{M^2+p^2} - \mu\right)/T}\right)^{-1}$$

$$n_{\overline{q}}(M, p, \mu, T) = \left(1 + e^{\left(\sqrt{M^2+p^2} + \mu\right)/T}\right)^{-1}$$

$$\tilde{n}_q(M, p, \mu, T, \phi, \overline{\phi}) \equiv \frac{1}{N_c}\sum_{i=1}^{N_c} n_q(\sqrt{M^2 + p^2}, \mu + \imath (A_4)_{ii}, T)$$

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- Extra term, the Polyakov potential: $\mathcal{U}\left(\phi, \overline{\phi}, T\right)$

Pseudo-critical temperatures

$m \rightarrow 0$: exact chiral symmetry **Vs** $m \rightarrow \infty$: ϕ exact order parameter

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- peaks of susceptibility ($\Omega \to \Omega' = \Omega - T(\eta\phi + \overline{\eta}\overline{\phi})$)

$$\chi^i_{chi} = -\frac{1}{T^2}\left(\left.\frac{\partial^2 \Omega}{\partial^2 m_i}\right|_T - \left.\frac{\partial^2 \Omega}{\partial^2 m_i}\right|_{T=0}\right)$$

$$\chi^i_{num} = -\frac{1}{T^2}\frac{\partial^2 \Omega}{\partial \mu_i^2}$$

$$\chi_\phi = \frac{1}{4}\left(\frac{\partial^2 \Omega'}{\partial \eta^2} + 2\frac{\partial^2 \Omega'}{\partial \eta \partial \overline{\eta}} + \frac{\partial^2 \Omega'}{\partial \overline{\eta}^2}\right)$$

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- dynamical mass, quark condensates, Polyakov loop ($\frac{\partial^2 X}{\partial T^2}=0$, $X(T)=X_{max}/2$)
- peaks of susceptibility ($\Omega \to \Omega' = \Omega - T(\eta\phi + \overline{\eta}\overline{\phi})$)

$$\chi^i_{chi} = -\frac{1}{T^2}\left(\left.\frac{\partial^2\Omega}{\partial^2 m_i}\right|_T - \left.\frac{\partial^2\Omega}{\partial^2 m_i}\right|_{T=0}\right)$$

$$\chi^i_{num} = -\frac{1}{T^2}\frac{\partial^2\Omega}{\partial\mu_i^2}$$

$$\chi_\phi = \frac{1}{4}\left(\frac{\partial^2\Omega'}{\partial\eta^2} + 2\frac{\partial^2\Omega'}{\partial\eta\partial\overline{\eta}} + \frac{\partial^2\Omega'}{\partial\overline{\eta}^2}\right)$$

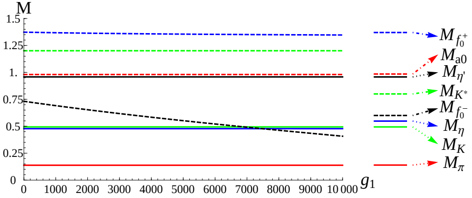
- dual quark condensate ($j = 1$ dressed polyakov loop)

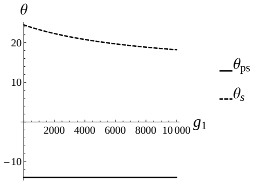
$$\Sigma_i^{(j)} = \int_{-\pi}^{\pi} \frac{e^{-\imath\alpha j}}{2\pi} h_i(\alpha)\mathrm{d}\alpha$$

$h_i(\alpha)$ given by $\frac{\partial\Omega}{\partial h_i} = 0$ with $\mu \to \mu + \imath T\alpha$, (where $-\pi \leq \alpha < \pi$)

Fitting the model parameters

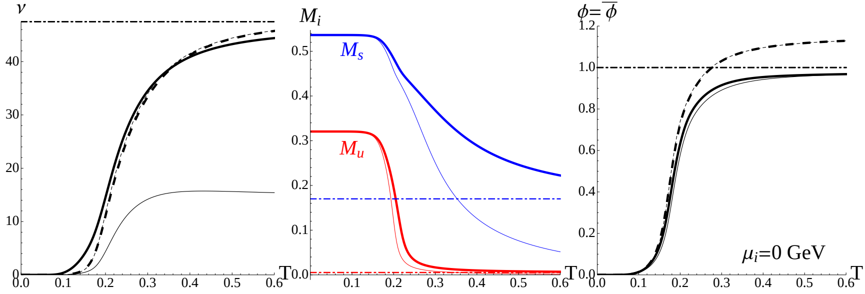
m_u, m_s, G, K, g_1, g_2, Λ can be fit using meson properties (e.g. M_π, f_π, M_K, f_K, M_{a_0}, $M_{\eta'}$)





Regularization effects

- Moreira, BH, Osipov, Blin, IJMPA 27 (2012) 1250060



PNJL, $\mu = 0$, $g_1 = 1000 GeV^{-8}$, polynomial Polyakov Potential $\mathcal{U}^I$ from Ratti, Thaler, Weise, Phys. Rev. D 73, 014019 (2006).

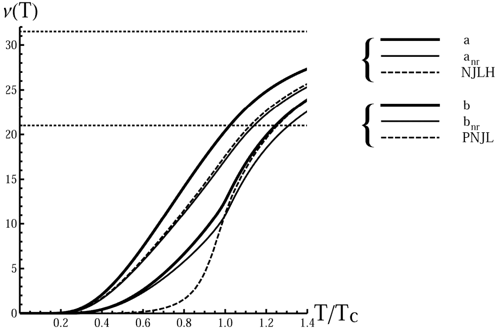
- Left: degrees of freedom; bold PV all, thin: 3D all; dashed: PV and 3D, no regulator on thermal piece.
- Middle: constituent quark masses; bold PV all, thin PV, no regulator on thermal piece. Similar for 3D (P. Costa, M. Ruivo, C. A. de Sousa, Phys. Rev. D 77, 096009 (2008)).
- Right: Traced Polyakov loop; bold PV, thin 3D, dashed PV and 3D no reg on thermal part. .

	3D Vacuum+matter	3D Vacuum	PV Vacuum+matter	PV Vacuum
M	**Current mass**	Zero	**Current mass**	Zero
Condensate	**Zero**	Changes sign	**Zero**	Changes sign
Degrees of freedom	Not the SB limit	**SB limit**	**SB limit**	**SB limit**
Polyakov loop	**Expected value**	Not the expected value	**Expected value**	Not the expected value

.

Number of effective degrees of freedom at $\mu = 0$

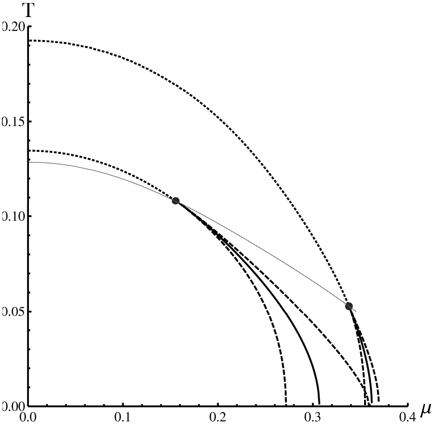
- Bold lines: NJL, Pauli-Villars regulator on vacuum +thermal integrals,
- Thin lines: NJL, PV on vacuum only
- (upper): $g_1 = 1000 GeV^{-8}$ $T_c = 190 MeV$ (PV all), $T_c^\infty = 179 MeV$
- (lower) $g_1 = 8000 GeV^{-8}$ $T_c = 135 MeV$ (PV all), $T_c^\infty = 132 MeV$
- dashed curves: K. Fukushima, Phys. Rev. D77, 114028 (2008); lower curve (PNJL), $T_c^\infty = 204.8$; upper (NJL), $T_c^\infty = 171.6 MeV$, $3D$
- $8q$: **additional source for suppression of degrees of freedom**

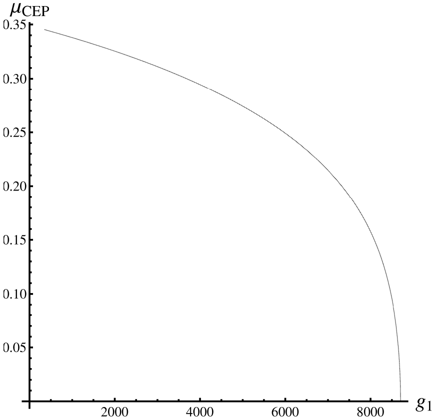


$$\nu(T) = \frac{P(T) - P(0)}{\frac{\pi^2 T^4}{90}}$$

Phase Diagram for NJL

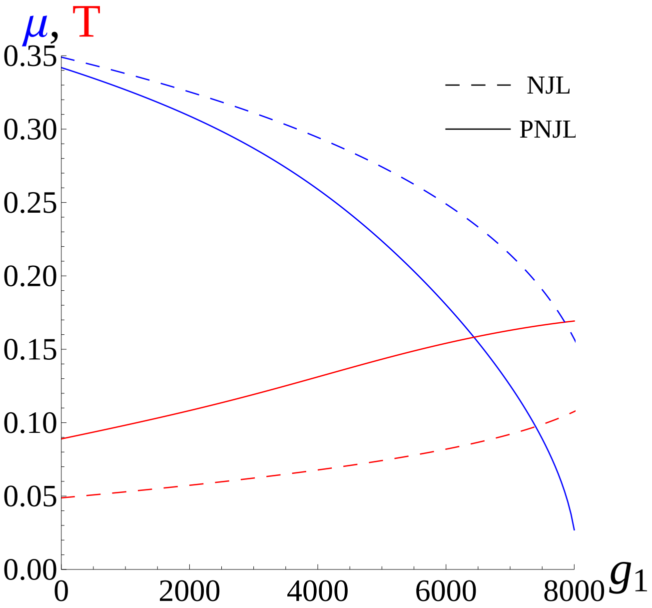
8q decrease T_c and shift CEP to higher T, smaller μ.



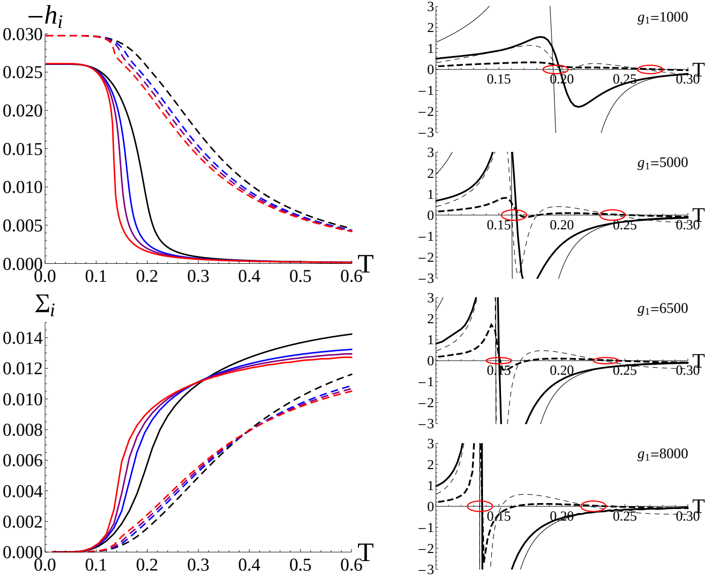


Right upper curve for: $g_1 = 1000 GeV^{-8}$, lower curve for: $g_1 = 8000 GeV^{-8}$;
thin line: CEP in T, μ diagram, depending on g_1;
Left: CEP (other view)

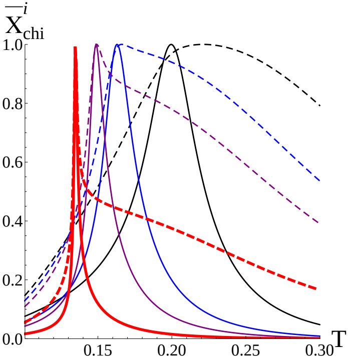
CEP - $\mathcal{U}^1, T_0 = .190 GeV$



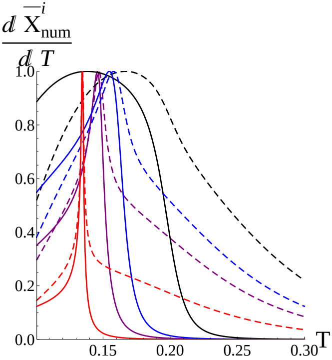
NJL: condensates



NJL: susceptibilities

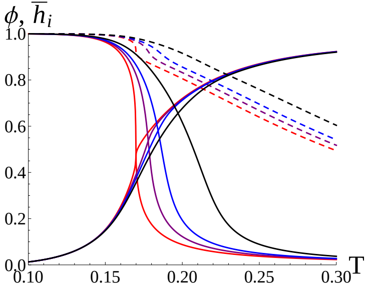


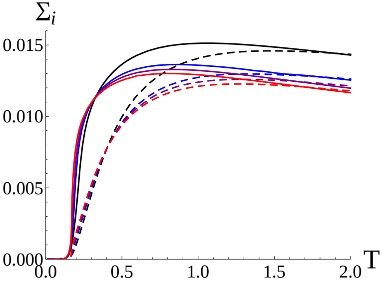
(c)

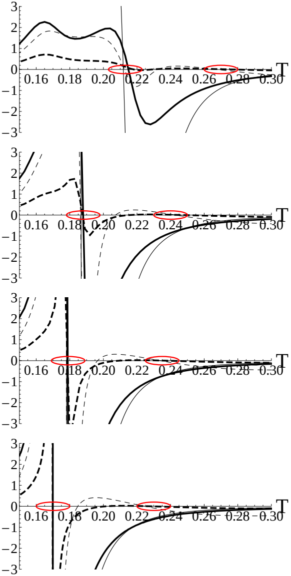


(d)

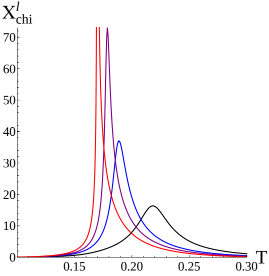
PNJL: condensates ($\mathcal{U}^{I}$, $T_0 = 0.190\mathrm{GeV}$)



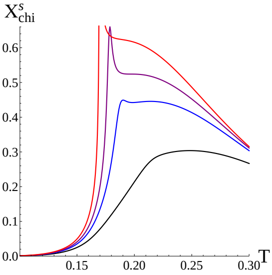




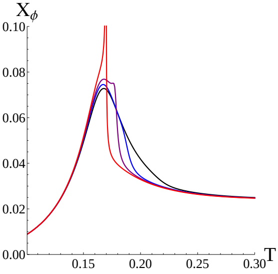
PNJL: susceptibilities ($\mathcal{U}^l$, $T_0 = 0.190\mathrm{GeV}$)



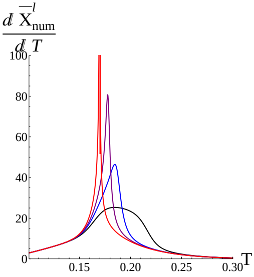
(e)



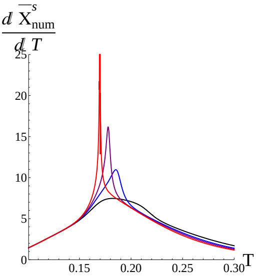
(f)



(g)



(h)

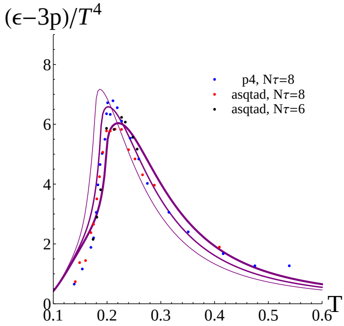


(i)

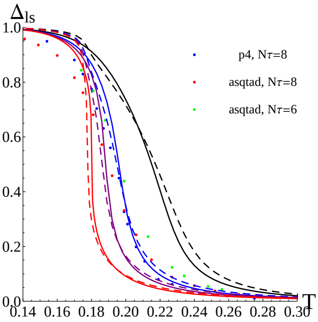
lQCD comparison

(j) $\mathcal{U}^{I}$, $T_0 = .19, .21, .23\,GeV$;

(k) $\mathcal{U}^{I}, \mathcal{U}^{II}$, $T_0 = .21\,GeV$[2]



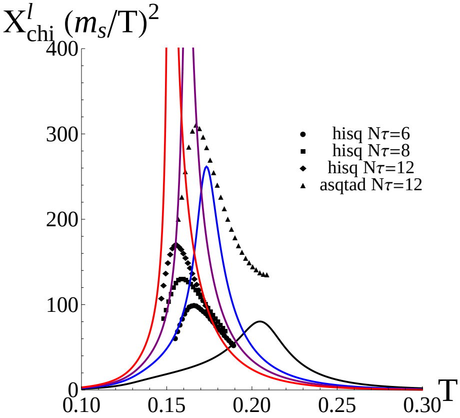
(j)



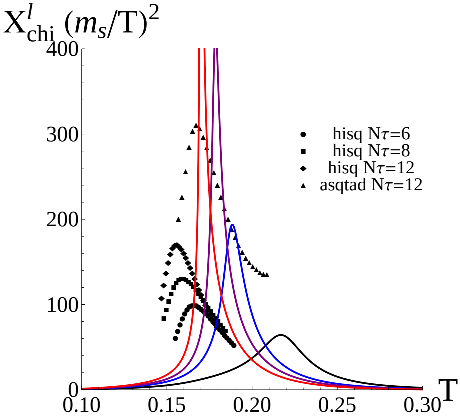
(k)

[2] lQCD data taken from: Bazavov *et al*, Phys. Rev. D80, 014504 (2009); $\mathcal{U}^{II}$ from S. Roessner, C. Ratti, W. Weise, Phys. Rev. D 75, 034007 (2007)

Light quark chiral susceptibility



$\mathcal{U}^l, T_0 = .15GeV$



$\mathcal{U}^l, T_0 = .19GeV$

lQCD data (continuum extrapolation) taken from A.Bazavov et al, Phys. Rev. D85 (2012) 054503

CONCLUSIONS

8q

New vertices with the explicit chiral symmetry breaking.

The impact of 8q forces on meson dynamics.

are needed to stabilize the ground state of the model.

almost do not change the meson mass spectra.

are important in studies of chiral phase transitions in a dense and hot medium and in presence of strong magnetic fields.