Radiative Energy Loss in the absorptive QGP

Excited QCD 2012 Peniche-Portugal

P.B. Gossiaux
SUBATECH, UMR 6457
Université de Nantes, Ecole des Mines de Nantes, IN2P3/CNRS

with

M. Bluhm, J. Aichelin, Th. Gousset

The context: Probing QGP in URHIC with heavy flavors and jets at large p_t

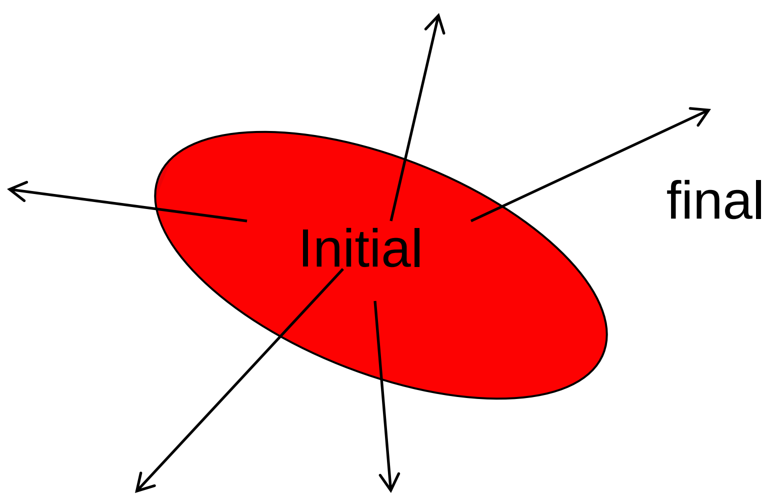
The method: tomography

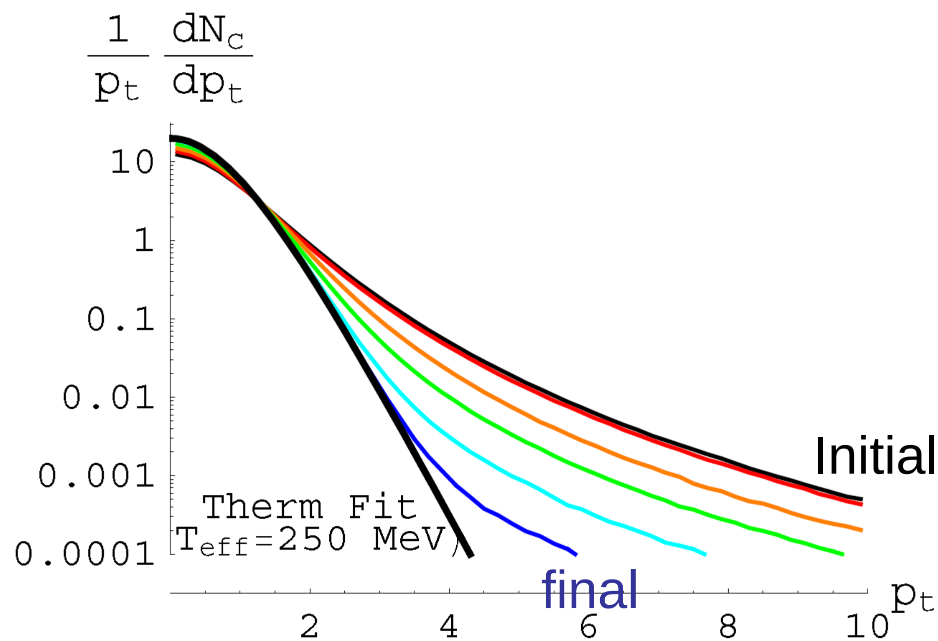
Ideally:

$$\frac{dN_{\rm in}}{dp_t} + \rho(t,\vec{x}) \otimes \frac{dE}{dx} \rightarrow \frac{dN_{\rm fin}}{dp_t} \quad \Rightarrow \quad \rho(t,\vec{x}) \left(\frac{dN_{\rm in}}{dp_t}, \frac{dN_{\rm fin}}{dp_t}, \frac{dE}{dx} \right)$$

deconvolution

$\frac{dN_{\rm in}}{dp_t} \approx \frac{dN_{\rm pp}}{dp_t}$; $\frac{dE}{dx}$: known





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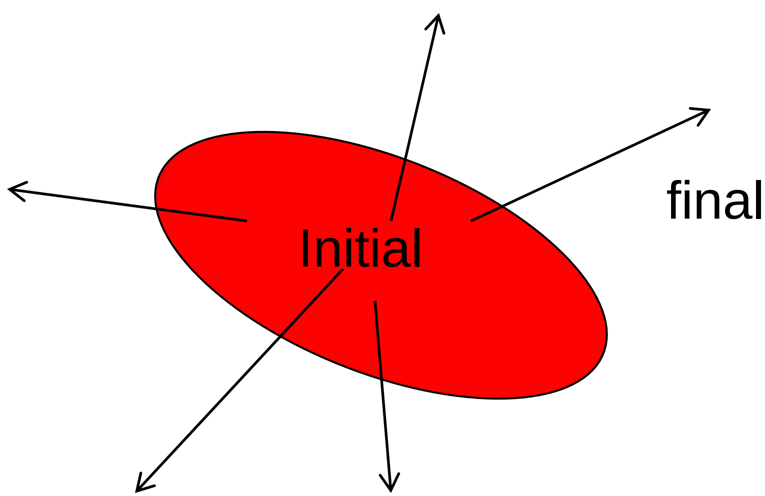
$$\frac{dN_{\rm in}}{dp_t} + \rho(t,\vec{x}) \otimes \mathcal{P}(\Delta E) \rightarrow \frac{dN_{\rm fin}}{dp_t} \quad \Rightarrow \quad \rho(t,\vec{x})\left(\frac{dN_{\rm in}}{dp_t}, \frac{dN_{\rm fin}}{dp_t}, \mathcal{P}(\Delta E)\right)$$

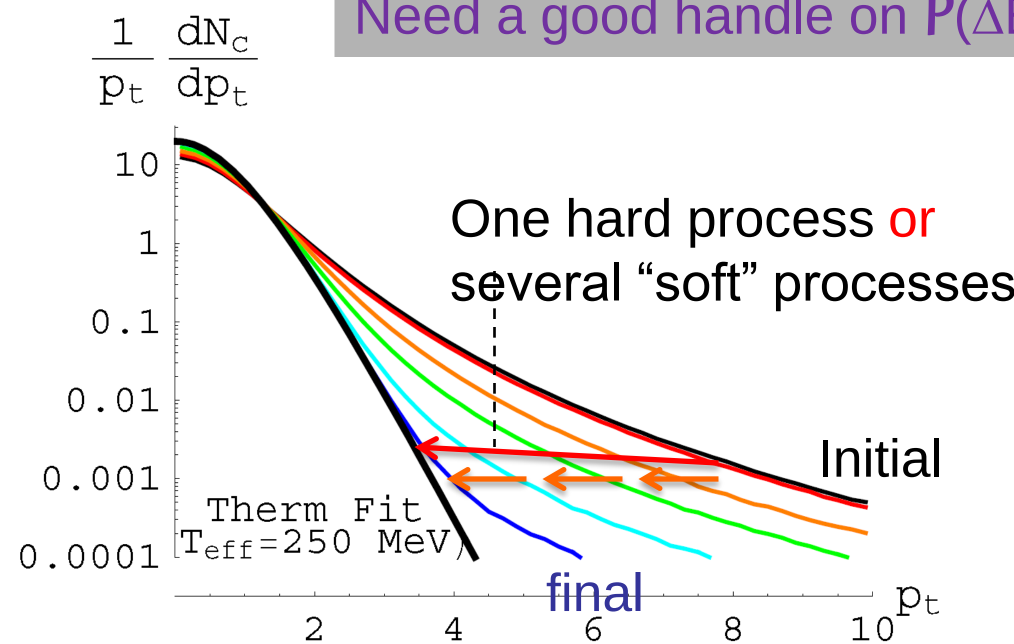
deconvolution

$$\frac{dN_{\rm in}}{dp_t} \approx \frac{dN_{\rm pp}}{dp_t}$$

$\mathcal{P}(\Delta E)$: Known ?

Need a good handle on $\mathcal{P}(\Delta E)$

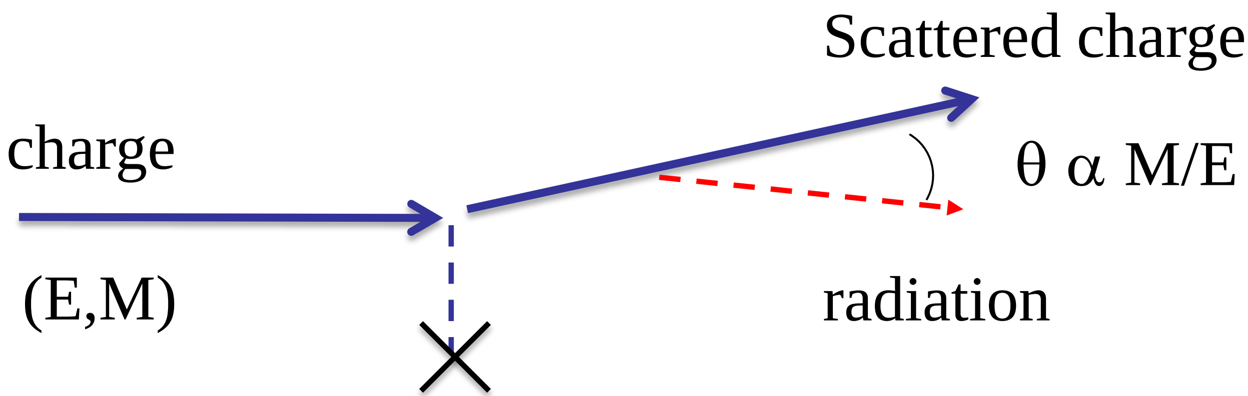




Basic & simple idea

I. Radiation:

- dominant mechanism in parton energy loss…
- But it takes time !!!



Lorentz boost delays decoherence =>

Formation time t_f

II. If anything on the way to t_f: radiation pattern will be affected

Obvious: rescatterings: celebrated LPM effect in QED $\rightarrow$ BDMPS-Z in QCD.

Mostly neglected: damping of radiation in hot/dense medium… “competes” with usual LPM

V. M. Galitsky and I. I. Gurevich, Il Nuovo Cimento 32 (1964) 396.

Based on

- *Plasma damping effects on the radiative energy loss of relativistic particles*, M. Bluhm, P. B. Gossiaux, & J. Aichelin, Phys. Rev. Lett. 107 (2011) 265004 [arXiv:1106.2856]
- Radiative and Collisional Energy Loss of Heavy Quarks in Deconfined Matter *Radiative*, J. Aichelin, P.B. Gossiaux, T. Gousset, J.Phys.G38 (2011) 124119 [arXiv:1201.4192v1]
- On the formation of bremsstrahlung in an absorptive QED/QCD medium, M. Bluhm, P. B. Gossiaux, T. Gousset & J. Aichelin, submitted to PRC [arXiv:1204.2469v1]

Plan

Novel issue of this talk: influence of the damping of time-like radiated photons / gluon in an absorptive hot plasma on energy loss of relativistic particles ?

I. Some reminder about radiative energy loss in QED

II. Rigorous calculation in (Q)ED

III. Time scales analysis

IV. Extension to genuine QCD

Important facts about radiative *induced* energy loss

QED (also valid for Abelian approximation to QCD; no gluon rescattering):

1. Photon Radiation on a single scatterer is well known (Bethe Heitler result).

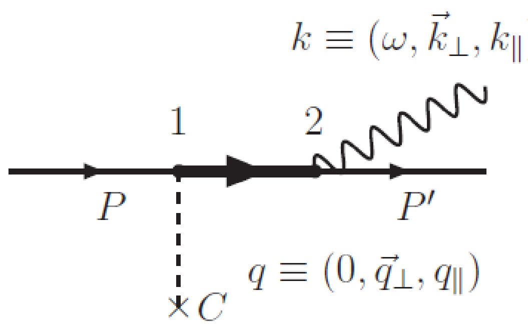
$$\hbar\omega \frac{d\sigma_{\mathrm{BH}}(v \approx c)}{d\hbar\omega} = \frac{16\hbar c^2}{3} \frac{Z^2 z^4 \alpha_{\mathrm{QED}}^3}{(Mc^2)^2} \left(1 - \frac{\hbar\omega}{E} + \frac{3}{4}\left(\frac{\hbar\omega}{E}\right)^2\right) \left[\ln\left(\frac{2E(E-\hbar\omega)}{Mc^2\hbar\omega}\right) - \frac{1}{2}\right]$$

$\omega \frac{d\sigma_{\mathrm{BH}}}{d\omega} \approx \mathrm{cst}(\omega) \times \ln(E)$ ⟹ Per collision: $\Delta E := \frac{\int \omega \frac{d\sigma}{d\omega} \mathrm{d}\omega}{\sigma_{\mathrm{col}}} \sim \alpha_{\mathrm{QED}} \frac{\mu^2}{M^2} \times E$

Reduced for heavier fermions

2. Radiation takes a finite formation time t_f before the photon and the incoming charge can be considered as independent (Heisenberg principle). Although t_f (inverse virtuality of the off-shell fermion) depends on all “microscopic” variables, a good overall estimate in vacuum is

$$t_{\mathrm{f}}^{(s)} \approx \frac{2E^2}{\omega M^2} \approx \frac{2E}{x M^2}$$



(s): single scattering

Important facts about radiative *induced* energy loss

3. Two regimes at high energy:

$$t_f^{(s)} \approx \frac{2E}{xM^2}$$

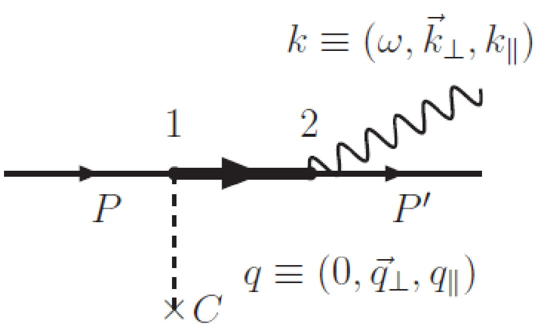
$t_f^{(s)} \leq \lambda$ (mean free path)

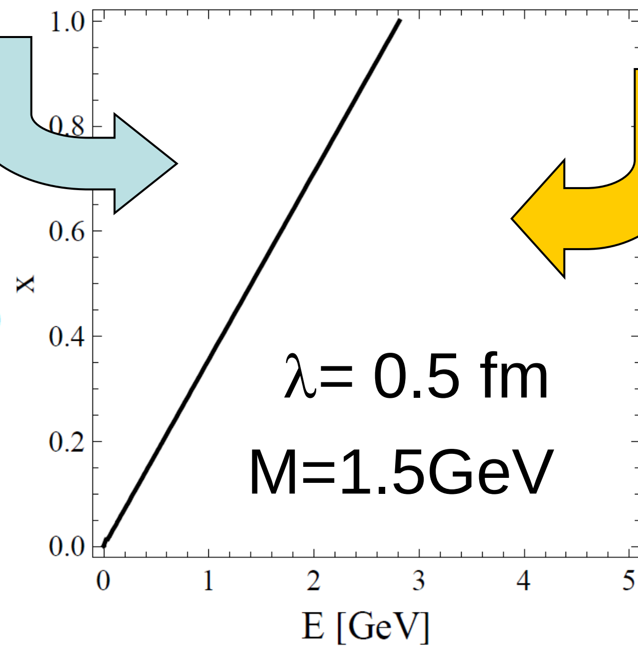
Iterated BH regime

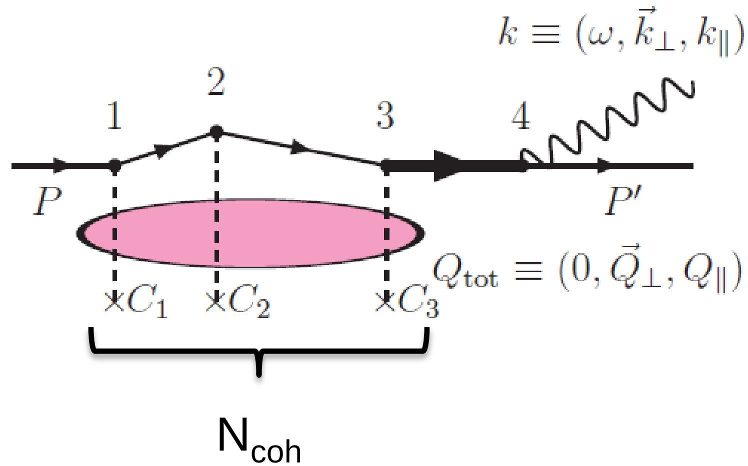
$t_f^{(s)} >> \lambda$

Strong coherence effects (further collisions happen although photon still not formed)

LPM effect (overall reduction of the formation time)

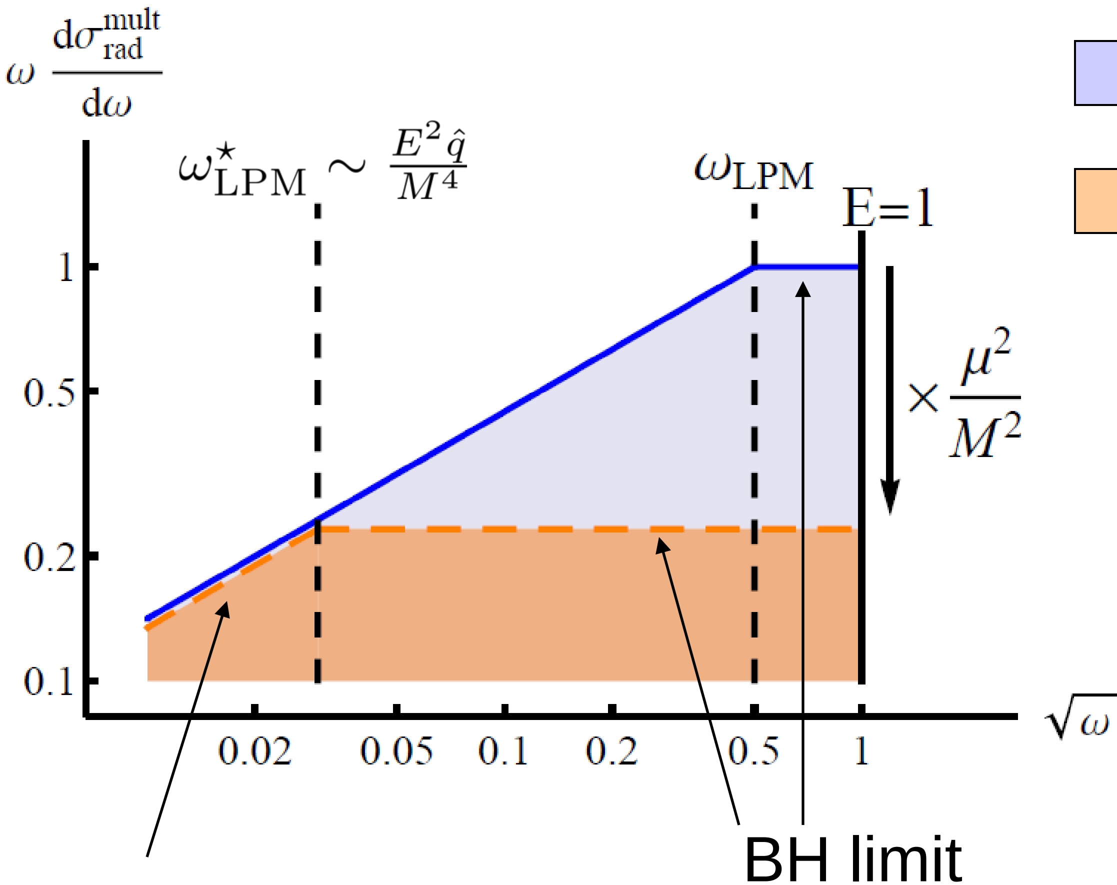






Important facts about radiative *induced* energy loss

Usual LPM spectrum:



Light fermion (M≤μ)

Heavy fermion (M≥μ)

μ: typical q_t in a single scattering

Coherence => suppression according to $1/N_{coh}$ but also to t_f

Effect of medium polarization on radiation

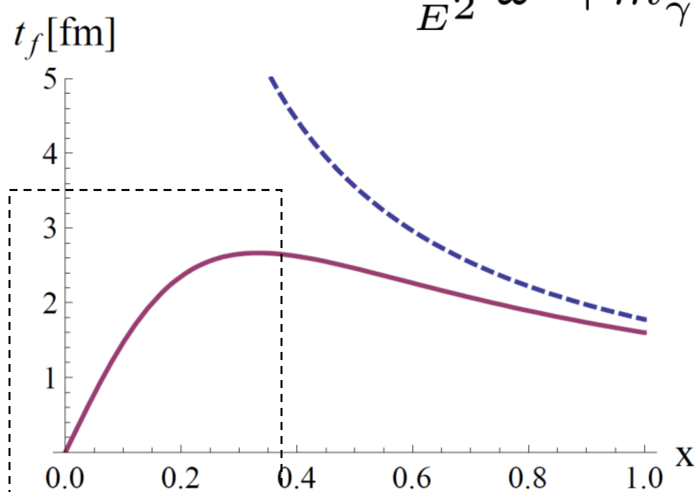
Polarization effect (QED: Ter-Mikaelian 1954)

• formation length modified by medium polarization (effects on radiated quanta)

• loss of coherence, i.e. suppression of emission process, by dielectric polarization of medium

$$t_f^{(s)} \approx \frac{2E}{xM^2} \Rightarrow t_f^{(s)} \approx \frac{2\omega}{\frac{M^2}{E^2}\omega^2 + m_\gamma^2} \approx \frac{2x}{x^2 M^2 + m_\gamma^2} \times E$$

Strong reduction of coherence for small x photons



λ= 0.5 fm

M=1.5GeV

m_g=0.5GeV

E=10 GeV

Investigations of the induced gluon radiation spectrum

• Kämpfer+Pavlenko (2000): constant thermal mass

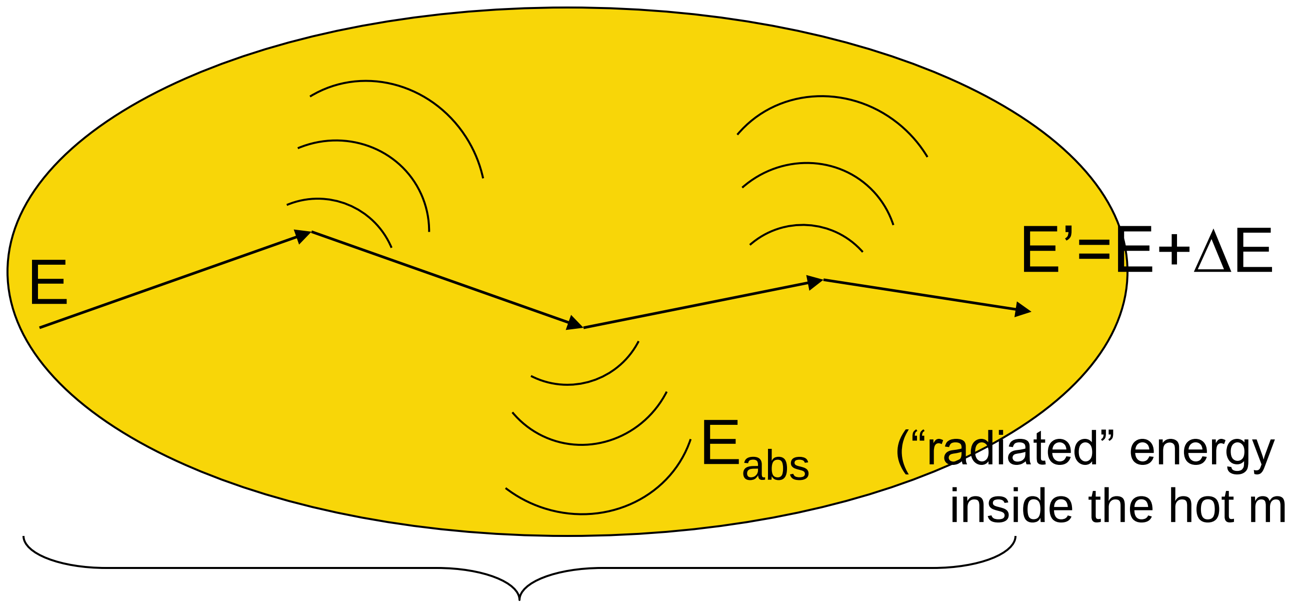
• Djordjevic+Gyulassy (2003): colour-dielectric modification of gluon dispersion relation using HTL self-energy

Modification of LPM effect due to medium effects

Quantity under study:

See PRL 107 for details

E_{rad} (radiated energy outside the hot medium)



We would like <ΔE> or even better $\mathcal{P}(\Delta E, L)$

Energy conservation: $\Delta E = -(E_{rad} + E_{abs})$: complicated

In fact: $\Delta E = W$ (work performed on the charge by the total electric field)

Modification of LPM effect due to damping effects

Naïve thoughts (bets) about the consequences of photon damping

Emitted radiation will be reduced (trivial) but what about DE ?

a) Relaxed attitude: “Nothing special happens to the Work, as photons are absorbed after being emitted” => equal energy loss

b) Vampirish thoughts: as the medium “sucks” the emitted photons (as much as Francesco can eat fish), the charge will have a tendency to emit more of them => increased energy loss

c) Reduction of energy loss

Modification of LPM effect due to medium effects

Evaluation of the work (cf Thoma & Gyulassy 1991):

$$W = 2\mathrm{Re}\left(\int d^3r' \int_0^{+\infty} d\omega \vec{E}(\vec{r}', \omega) \cdot \vec{\jmath}(\vec{r}', \omega)^\star\right)$$

Solving Maxwell's equations with point-like current $\vec{\jmath}(\vec{r}', t) = q\vec{v}(t)\delta^{(3)}(\vec{r}' - \vec{r}(t))$

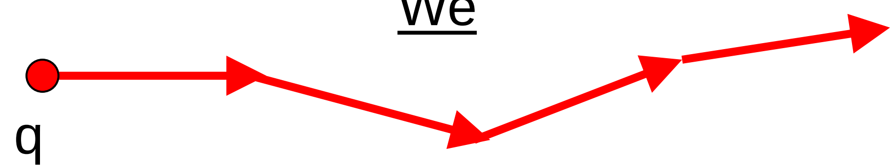
$$\frac{1}{\mu(\omega)}\left[k^2\vec{E}_{\vec{k}}(\omega) - \vec{k}(\vec{k}\cdot\vec{E}_{\vec{k}}(\omega))\right] - \omega\epsilon(\omega)\vec{E}_{\vec{k}}(\omega) = \frac{iq}{(2\pi)^2}\int dt'\vec{v}(t')e^{i\omega t' - i\vec{k}\cdot\vec{r}(t')}$$

Main differences w.r.t. Thoma & Gyulassy:

TG

- constant velocity $\Rightarrow \delta(\omega - \vec{k}\cdot\vec{v})$ Space-like
- Medium polarization along HTL (Landau damping of space-like modes only)
- Collisional E loss

We



- Transverse stochastic kicks (as in Landau work), allowing for time-like components (induced radiation)
- implement damping mechanisms as small corrections by complex $\varepsilon(\omega)$ and $\mu(\omega)$; simplification: ε and μ depend on ω only (only sensitive to time-like poles in the momentum space)

Modification of LPM effect due to medium effects

Sketch of the calculation:

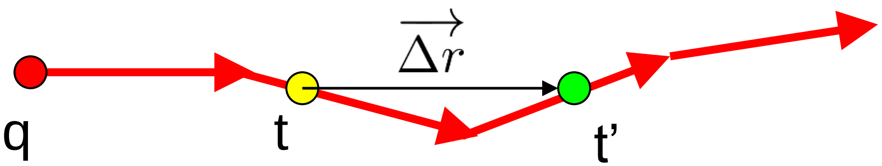
- Mixed representation: $W = \int \frac{dW}{d\omega} d\omega$ with (after momentum integration)

$$\frac{dW}{d\omega} = Re\left(\frac{iq^2}{4\pi^2}\int dt \int dt' \,\frac{\omega^2 n^3(\omega)}{\epsilon(\omega)} e^{-i\omega(t-t')}\mathcal{A}(t,t')\right)$$

Interpreted as the average spectral work, quadratic form of **v**

with

$n^2(\omega)=\varepsilon(\omega)\mu(\omega)$, the complex index of refraction, decomp. as $n(\omega)=n_r(\omega)+in_i(\omega)$



$$\mathcal{A}(t,t') = \left(\vec{v}(t)\vec{v}(t') + (\vec{\nabla}_g \vec{v}(t))(\vec{\nabla}_g \vec{v}(t'))\right)\frac{e^{i\,sgn(n_i)g}}{g}, \quad \vec{g} = \omega n(\omega)\vec{\Delta r}$$

$e^{i\,sgn(n_i)\Delta r \omega n_r} e^{-\Delta r \omega |n_i|}$ ⇒ $e^{i\omega|\vec{\Delta r}|}$ for vacuum

- Similar expression as the radiation spectra in the original work of LP

Modification of LPM effect due to medium effects

Sketch of the calculation:

➢ Stochastic average on transverse kicks + expansion in small deflection angle from t → t':

$$\langle \theta^2 \rangle = \frac{\hat{q}}{E^2} \times \underbrace{(t' - t)}_{\bar{t}}$$

➢ All correlations are functions of $\bar{t}$ only, so that $\frac{dW}{d\omega} \approx \int_0^L \frac{d^2W}{dz\,d\omega} dt$

with

$$\frac{d^2W}{dz\,d\omega} \simeq -Re\left(\frac{2i\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\frac{\omega n^2(\omega)}{\epsilon(\omega)}\exp\left[-\omega|n_i(\omega)|\beta\bar{t}\right]\right.$$
$$\left.\cos(\omega\bar{t})\exp\left[isgn(n_i(\omega))\omega n_r(\omega)\beta\bar{t}\left(1-\frac{\hat{q}}{6E^2}\bar{t}\right)\right]\right)$$

damping

General result in (Q)ED

Modification of LPM effect due to medium effects

Concrete implementation:

- μ=1 (we concentrate on transverse modes)
- Radiated quanta follow medium-modified dispersion relations of plasma modes
- View emitted hard (ω>T) quanta as time-like excitations, which obey finite *thermal mass* and which are *damped* within the absorptive medium (Pisarski 1989+1993)
- Lorentzian ansatz for spectral function results in retarded propagator (Peshier 2004-05):

$$-\Delta^{-1}(\omega,\vec{k}) = \omega^2 - k^2 - \underbrace{(m^2 - 2i\Gamma\omega)}_{\Pi}$$

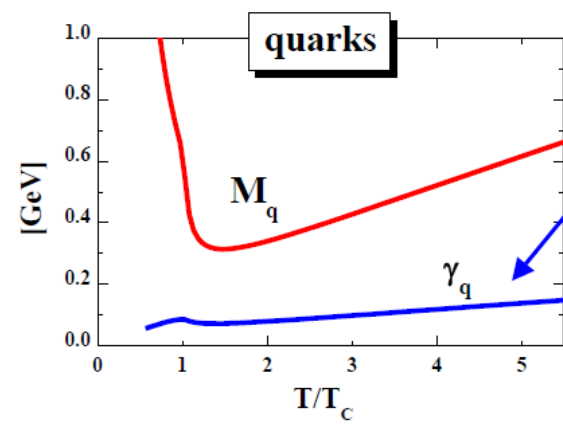
- Corresponding *complex index of refraction* follows via

$$\epsilon = 1 - \Pi/\omega^2$$

Poles for $\Delta^{-1}=0$ $\equiv$ $\varepsilon(\omega)\,\omega^2 - k^2 = 0$

$$n^2(\omega) = 1 - \frac{m^2}{\omega^2} + 2i\frac{\Gamma}{\omega}$$

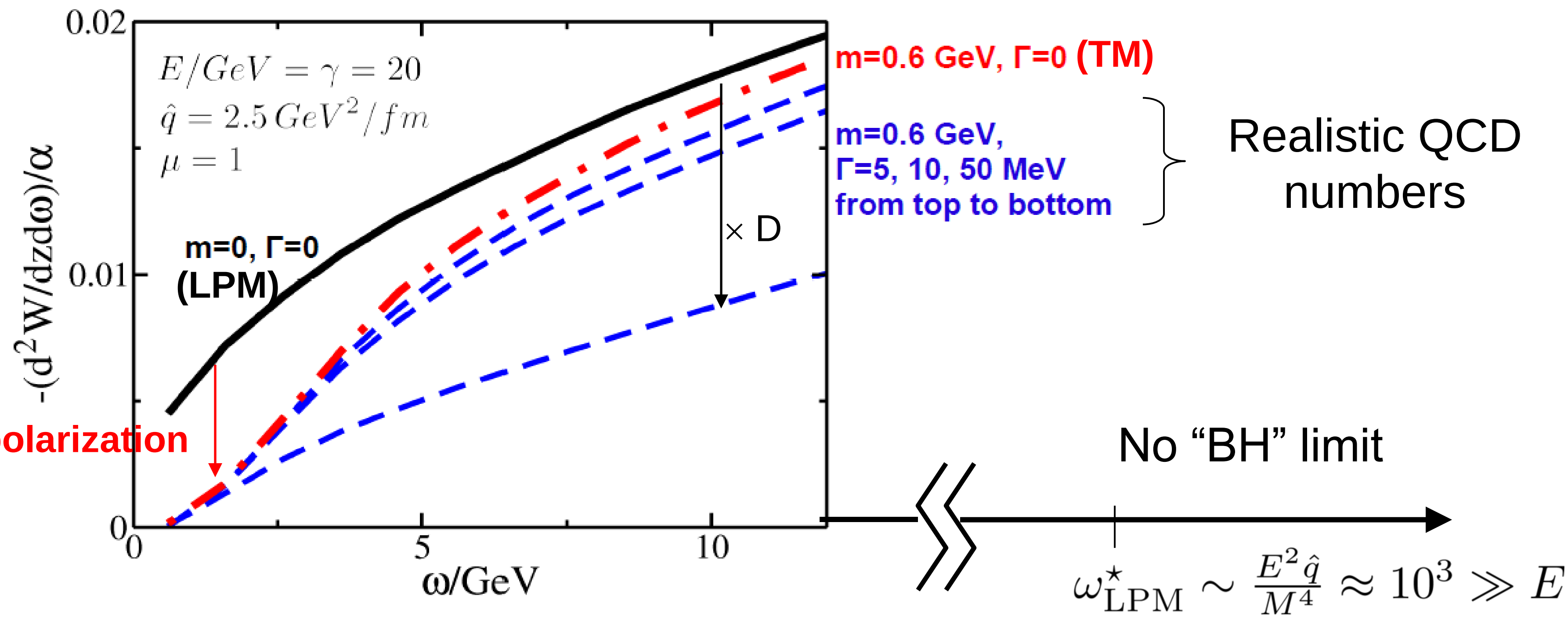
- Plasma modes are time-like, starting from ω=m, with Im(ε)=2Γ/ω in the time-like sector ($\neq$ from HTL, which has a cut in the space-like sector)



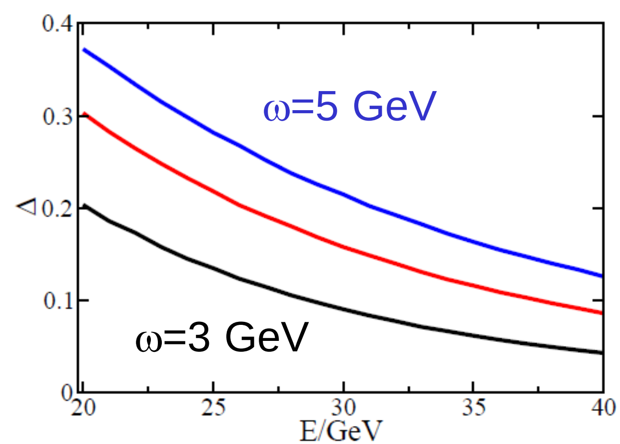
large width for gluons and quarks !

Modification of LPM effect due to medium effects

Typical Numerical Results:



- Damping significantly reduces the spectrum (as well as the coherence effects)
- with increasing E, relative effect of damping compared to non-damping case increases
- LP: Radiation intensity α t_f



Interpretation based on the concept of formation time

$$\frac{d^2W}{dzd\omega} \sim \frac{2\alpha\omega}{3\pi E^2}\int_0^{+\infty} \sin\Big[\underbrace{(\omega-\beta k(\omega))\,t+\frac{\beta\hat{q}}{6E^2}k(\omega)t^2}_{\text{Phase }\Phi(t)}\Big]\times e^{-\Gamma t}dt$$

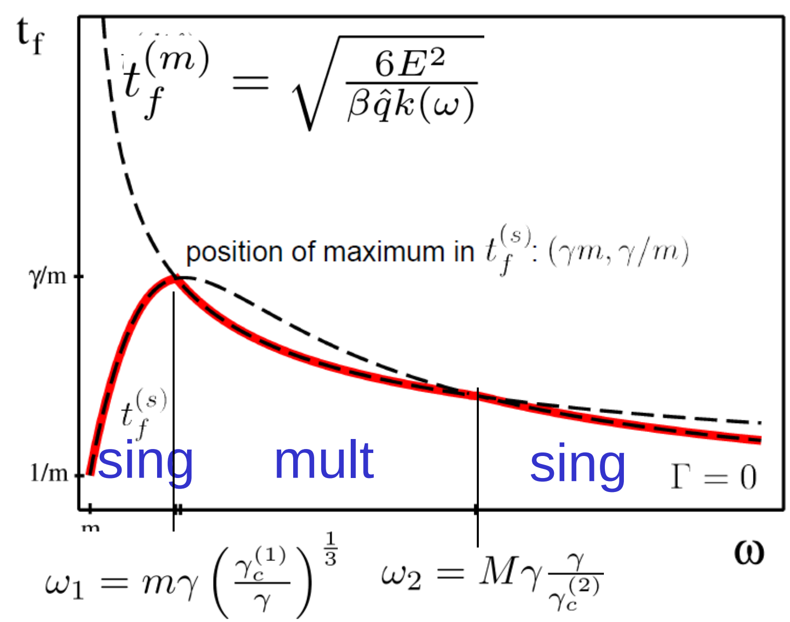
with $k(\omega)=(\omega^2-m^2)^{1/2}$

In the absence of damping, the integral acquires dominant contributions provided $\Phi(t)$ does not become much larger then unity

$$\Phi(t)=\frac{t}{t_f^{(s)}}+\left(\frac{t}{t_f^{(m)}}\right)^2$$

$$\Phi(t)\lesssim 1 \equiv t \lesssim t_f := \min\left(t_f^{(s)},t_f^{(m)}\right)$$

Intermediate γ



Interpretation based on the concept of formation time

No damping:

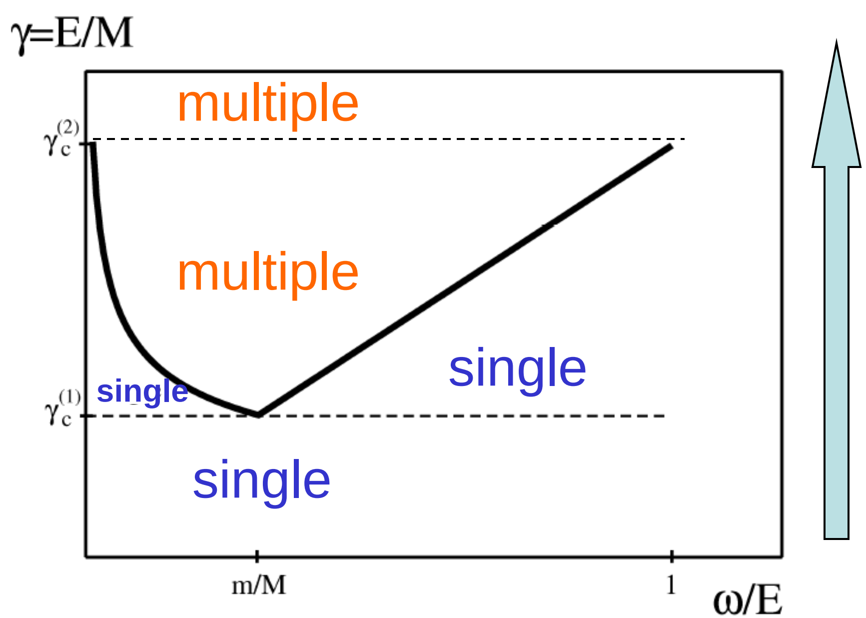
$$\frac{d^2W}{dz\,d\omega} \sim \frac{2\alpha\omega}{3\pi E^2}\int_0^{+\infty} \sin\left[(\omega - \beta k(\omega))\,t + \frac{\beta\hat{q}}{6E^2}k(\omega)t^2\right] dt$$

$$t_{\rm f}^{(s)} \approx \frac{2x}{x^2M^2+m_\gamma^2}\times E \qquad\qquad t_f^{(m)} = \sqrt{\frac{6E^2}{\beta\hat{q}k(\omega)}} \approx \sqrt{\frac{6}{\beta\hat{q}x}}\times E^{\frac{1}{2}}$$

Parameter space

$$\gamma_c^{(2)} = \frac{M}{m}\gamma_c^{(1)} = O\left(\frac{M^3}{g^4T^3}\right)$$

$$\gamma_c^{(1)} = \frac{mM^2}{\hat{q}} = O\left(\frac{M^2}{g^3T^2}\right)$$



Overall increase of t_f

Opening of multiple scattering regime

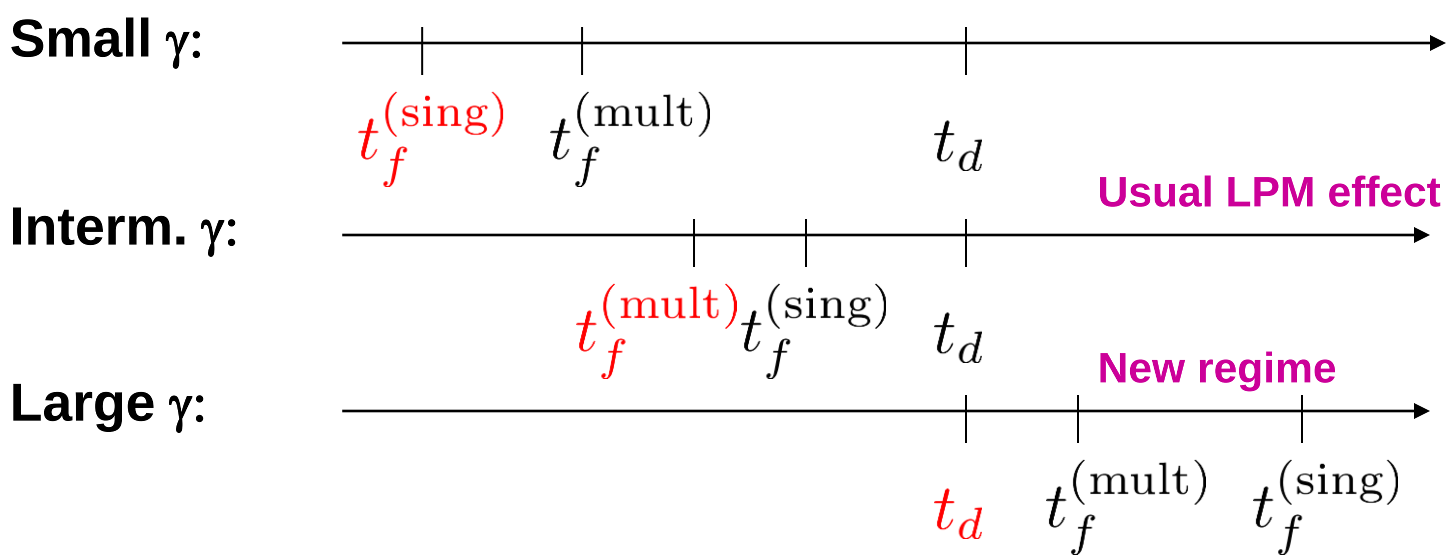
Interpretation based on the concept of formation time

With damping:

$$\frac{d^2W}{dz\,d\omega} \sim \frac{2\alpha\omega}{3\pi E^2}\int_0^{+\infty} \sin\left[\left(\omega-\beta k(\omega)\right)t+\frac{\beta\hat{q}}{6E^2}k(\omega)t^2\right]\times e^{-\Gamma t}\,dt$$

New time scale: $t_d=1/\Gamma \Rightarrow$ 3 possible regimes

γ - hierarchy:

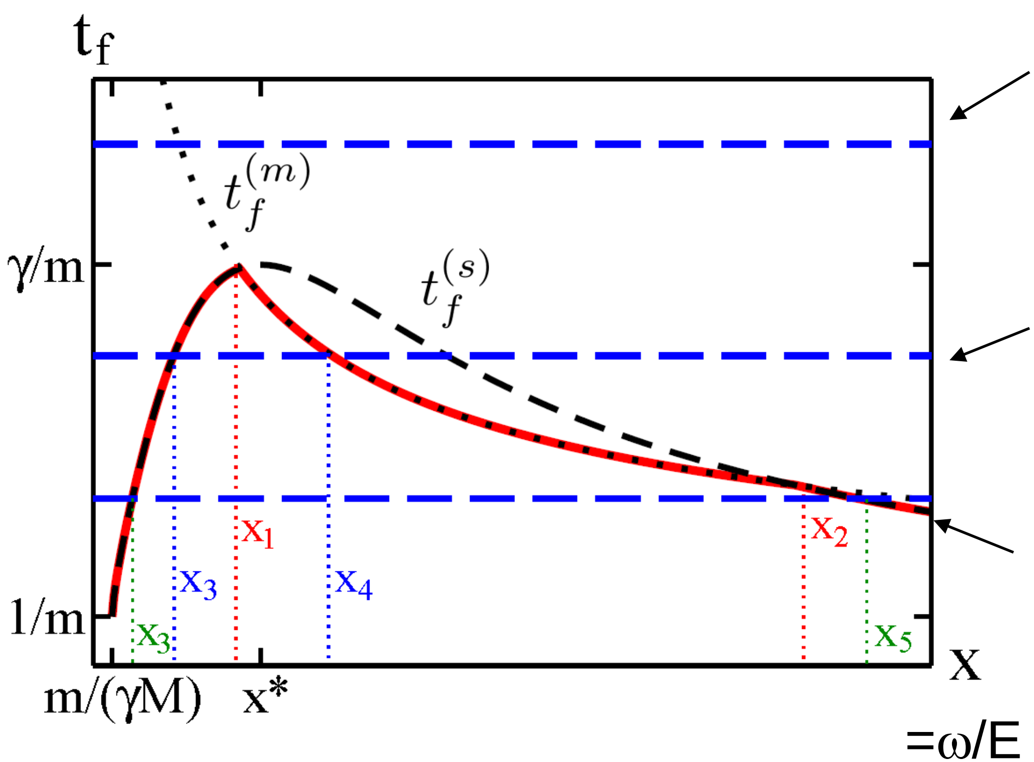


Interpretation based on the concept of formation time

With damping:

$$\frac{d^2W}{dzd\omega} \sim \frac{2\alpha\omega}{3\pi E^2}\int_0^{+\infty} \sin\left[(\omega - \beta k(\omega))\, t + \frac{\beta\hat{q}}{6E^2}k(\omega)t^2\right] \times e^{-\Gamma t} dt$$

New time scale: $t_d = 1/\Gamma \Rightarrow$ 3 possible regimes



0) Low Γ or low E: 1/Γ exceeds max(t_f), i.e $t_f(\omega_1)$: no damping effect

$$\gamma \lesssim \gamma_c^{(3)} \approx \frac{m}{\Gamma} = O(1/g)$$

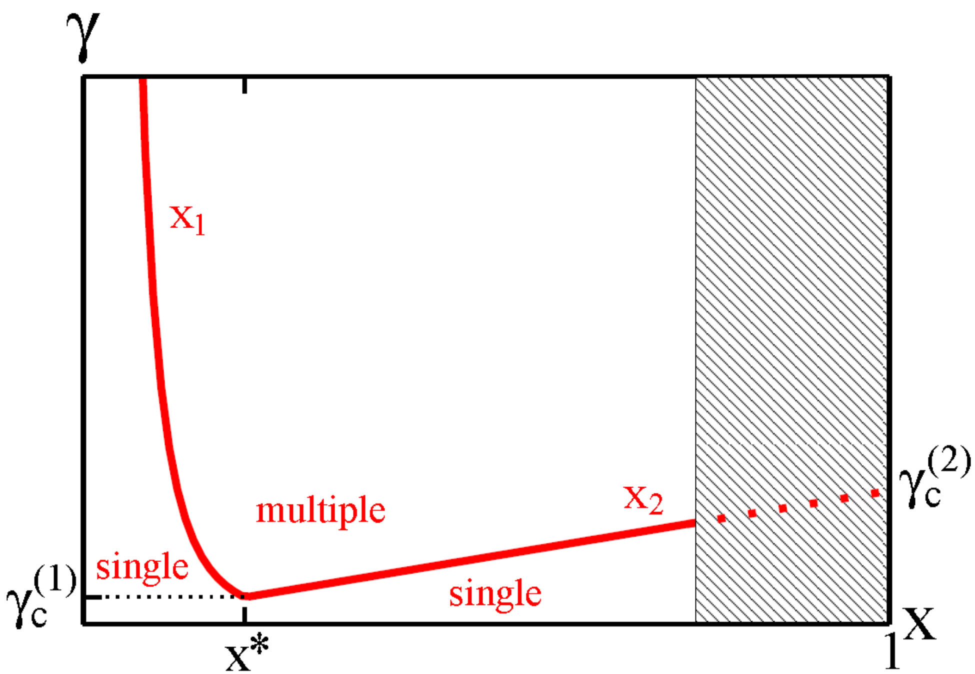
1) Intermediate Γ or intermediate E: 1/ Γ smaller then $t_f(\omega_1)$ but larger then $t_f(\omega_2)$: damping sets in for the mult. scattering case

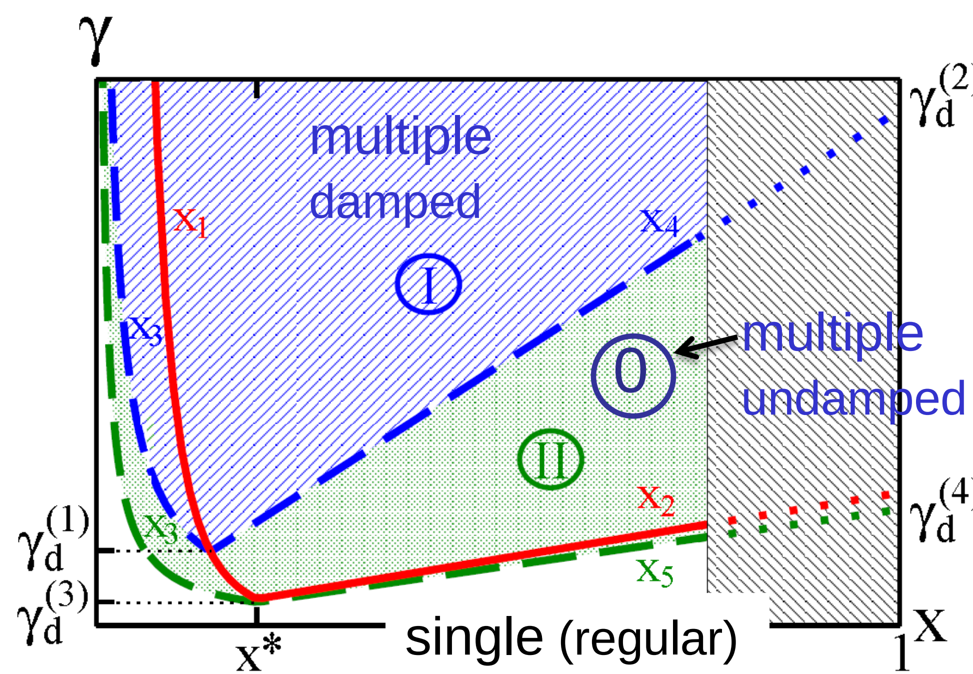
2) "Large" Γ : 1/ Γ smaller then $t_f(\omega_2)$: damping totally affects the coherent regime

$$\frac{1}{\Gamma} \lesssim \frac{M^2}{\hat{q}} = O\left(\frac{M^2}{g^4T^3}\right)$$

Interpretation based on the concept of formation time

Parameter space (including damping):





Main conclusion: In QED-like, damping of time-like excitations in the plasma might prevent their emission through multiple scattering processes.

Scaling law of radiation spectra:

$$\frac{\frac{dN}{d\omega}}{\frac{dN_{\text{sing}}}{d\omega}} \approx \frac{\min(t_d, t_f^{(\text{sing})}, t_f^{(\text{mult})})}{t_f^{\text{sing}}}$$

Allows for first phenomenological study in the QCD case

Genuine QCD case

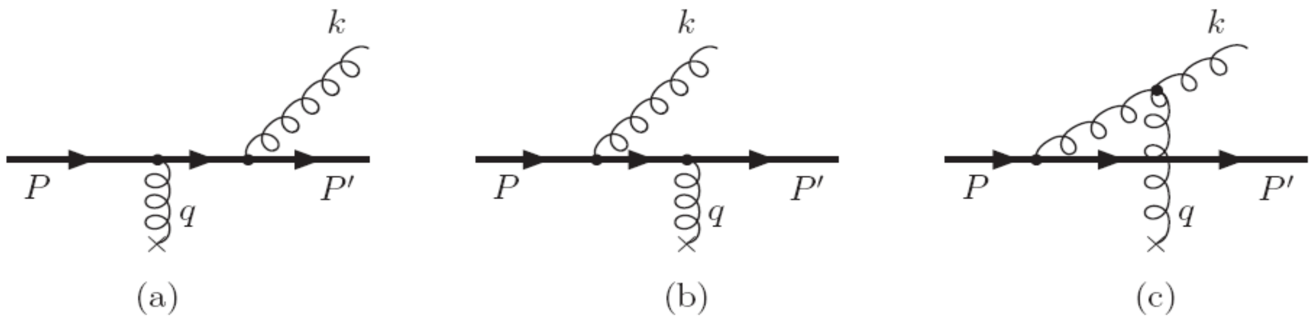
From M. Bluhm, PBG, T. Gousset & J. Aichelin,

arXiv:1204.2469v1

Important facts about radiative *induced* energy loss

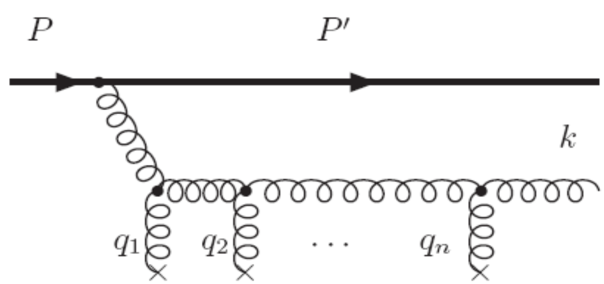
QCD:

4. QCD analog of Bethe Heitler result established by Gunion & Bertsch (M=0) at high energy; third diagram involved…



… important as it contributes to populate the mid rapidity gap (large angle radiation)

5. QCD analog of LPM effects: BDMPS; main difference: dominant process are the ones for which the emitted gluon is rescattered:



$$\Delta E \propto \hat{q} L^2$$

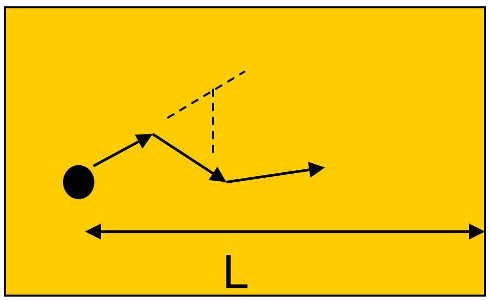
Yes, but…

… leads to a complete modification of the formation times and radiation spectra, but these concepts still apply

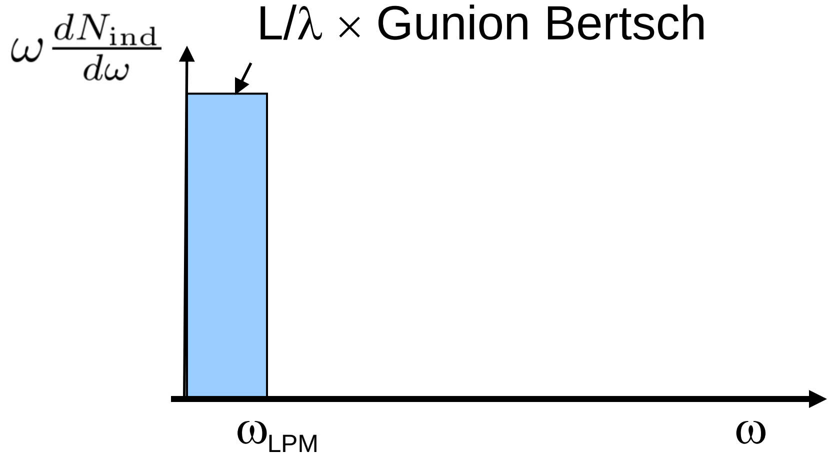
Important facts about radiative *induced* energy loss

LHC: the realm for coherence !

3 regimes and various path length (L) dependences : (light q)



QGP brick



→ a) Low energy gluons: Typical formation time ω/k_t^2 is smaller than mean free path λ:

$\omega < \omega_{\text{LPM}} := \frac{\hat{q}\lambda^2}{2}$ Incoherent Gunion-Bertsch radiation

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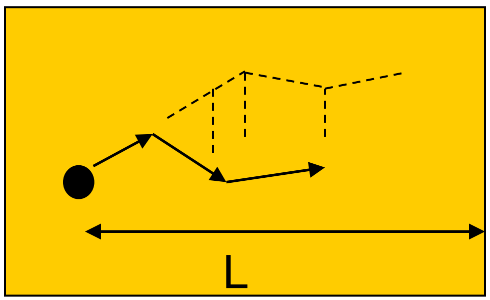

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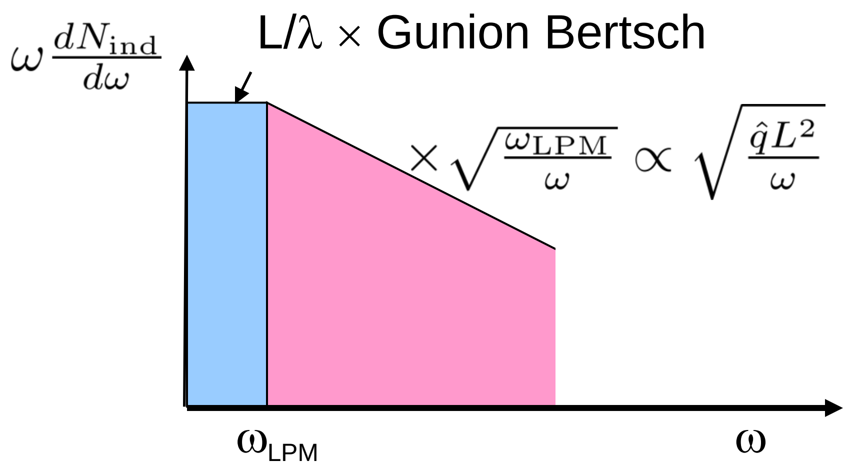


a) Low energy gluons: Typical formation time ω/k_t^2 is smaller than mean free path λ:

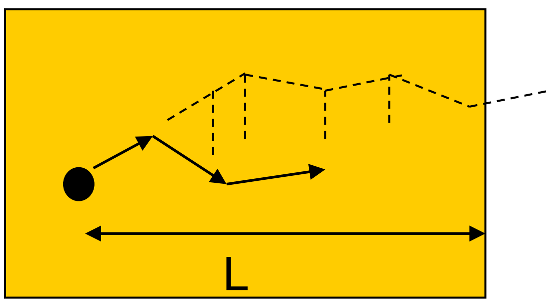
$\omega < \omega_{\rm LPM} := \frac{\hat{q}\lambda^2}{2}$ **Incoherent** Gunion-Bertsch radiation

→ b) Inter. energy gluons: Produced **coherenty** on $N_{\rm coh}$ centers after typical formation time $t_f = \sqrt{\frac{\omega}{\hat{q}}} \Rightarrow N_{\rm coh} = \frac{t_f}{\lambda} = \sqrt{\frac{\omega}{\omega_{\rm LPM}}}$ leading to an effective reduction of the GB radiation spectrum by a factor $1/N_{\rm coh}$

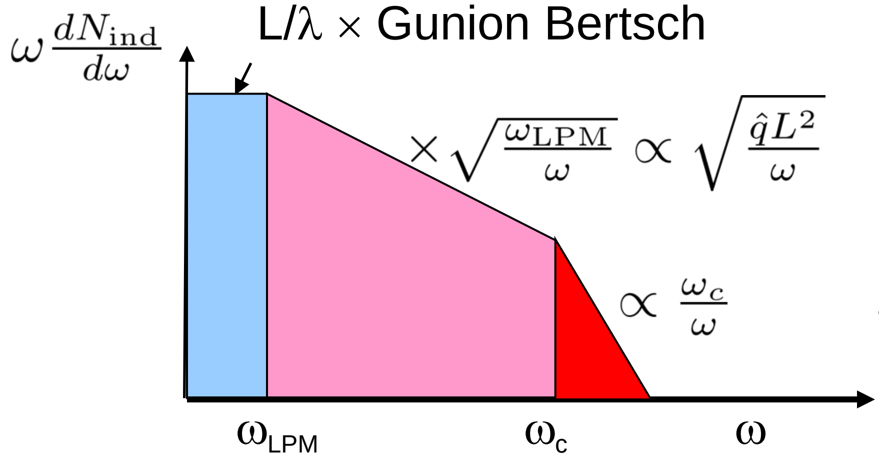
Important facts about radiative *induced* energy loss

LHC: the realm for coherence !

3 regimes and various path length (L) dependences : (light q)



QGP brick



GLV (2001), Zakharov (2001)

a) Low energy gluons: **Incoherent** Gunion-Bertsch radiation

b) Inter. energy gluons: Produced **coherenty** on $N_{\rm coh}$ centers after typical formation time $t_f = \sqrt{\frac{\omega}{\hat{q}}}$

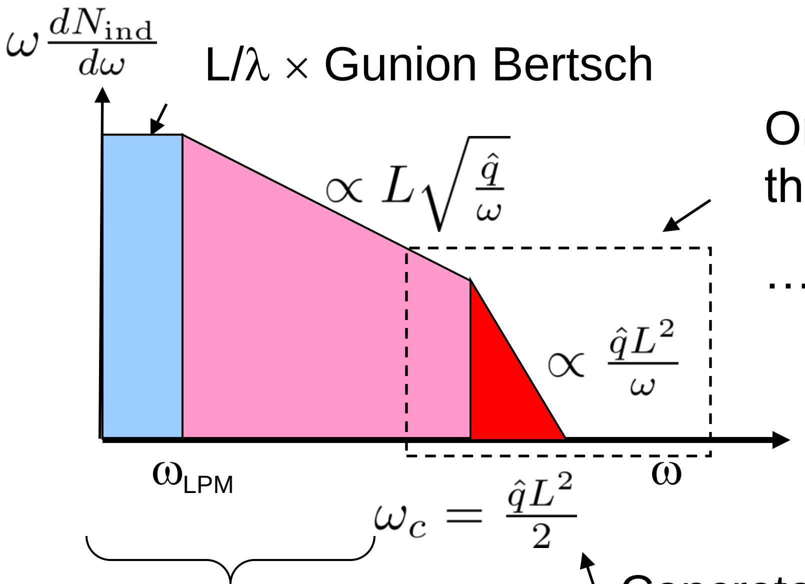
→ c) High energy gluons: Produced mostly outside the QGP… nearly as in vacuum **do not contribute significantly to the induced energy loss**

$\sqrt{\frac{\omega}{\hat{q}}} > L \Rightarrow \omega > \omega_c := \frac{\hat{q}L^2}{2}$

Important facts about radiative *induced* energy loss

LHC: the realm for coherence !

3 regimes and various path length (L) dependences : (light q)



Only this tail makes the L^2 dependence in the average Eloss integral …

…provided the higher boundary ω=E > ω_c.

Otherwise, everything α L

Concrete values @ LHC $\begin{cases} \hat{q} \sim 25\text{GeV}^2/\text{fm} \\ L \sim 2\text{fm} \end{cases}$

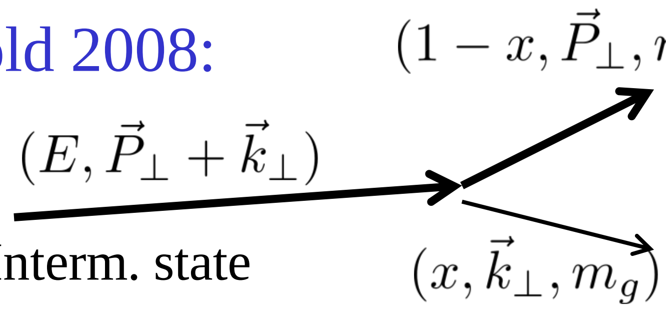
Bulk part of the spectrum still scales like path length L

$\omega_c \sim 500\text{GeV}$ Huge value !

A large part of radiative energy loss @ LHC still scales like the path length
=> **Still makes sense to speak about energy loss per unit length**

Formation time of radiated gluon (from HQ)

Arnold 2008:



$$t_f \left[\frac{\langle p_B^2 \rangle + x^2 m_s^2 + (1-x) m_g^2}{2x(1-x)E} \right] \simeq 1$$

$$p_B^2 := \left((1-x)\vec{k}_\perp + x\vec{P}_\perp \right)^2 \quad \Rightarrow \quad \langle p_B^2 \rangle \approx (1-x)^2 \hat{q}_g t_f$$

In QCD: mostly **gluon** rescattering

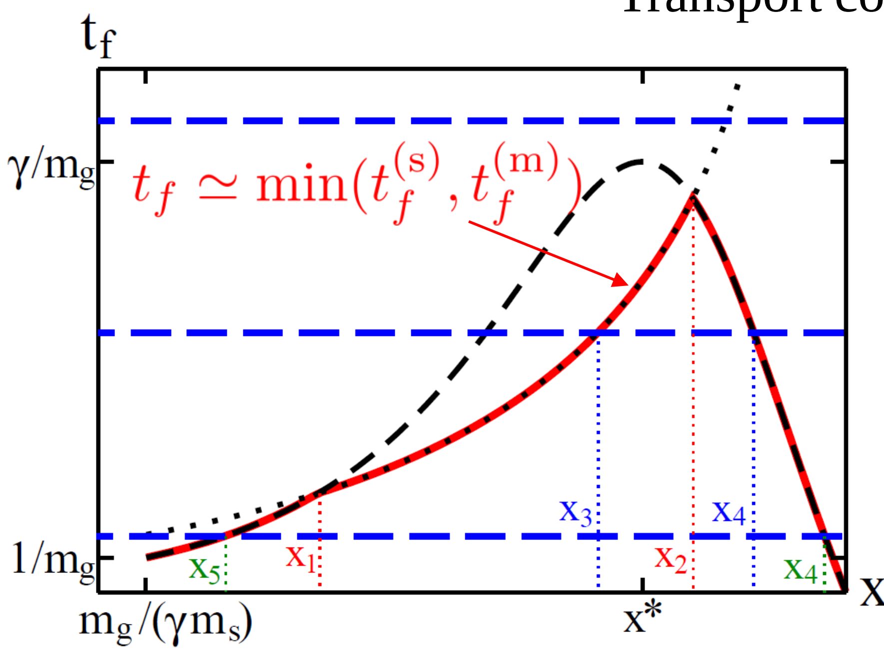
Transport coefficient: [GeV^2/fm]

=> Self consistent expression for t_f

$$t_f^{(s)} \simeq \frac{2x(1-x)E}{x^2 m_s^2 + m_g^2 (1-x)}$$

$$t_f^{(m)} \simeq \sqrt{\frac{2xE}{(1-x)\hat{q}_g}}$$

$$t_d = \frac{1}{\Gamma}$$



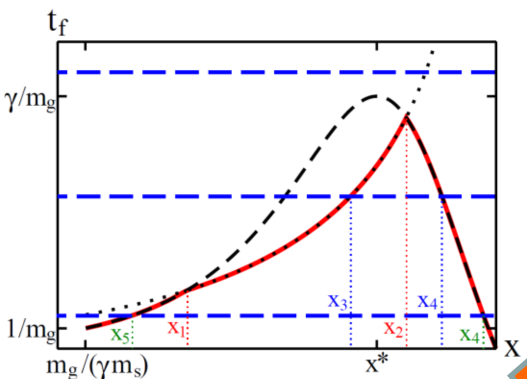
Small Γ

Interm. Γ

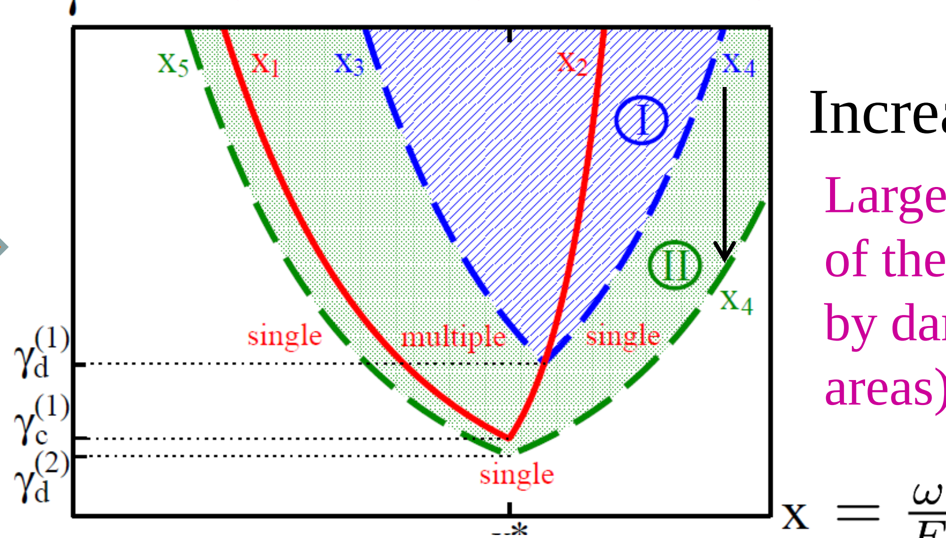
Large Γ

New regimes when including gluon damping

x-γ space for $\hat{q} < m_g^3$



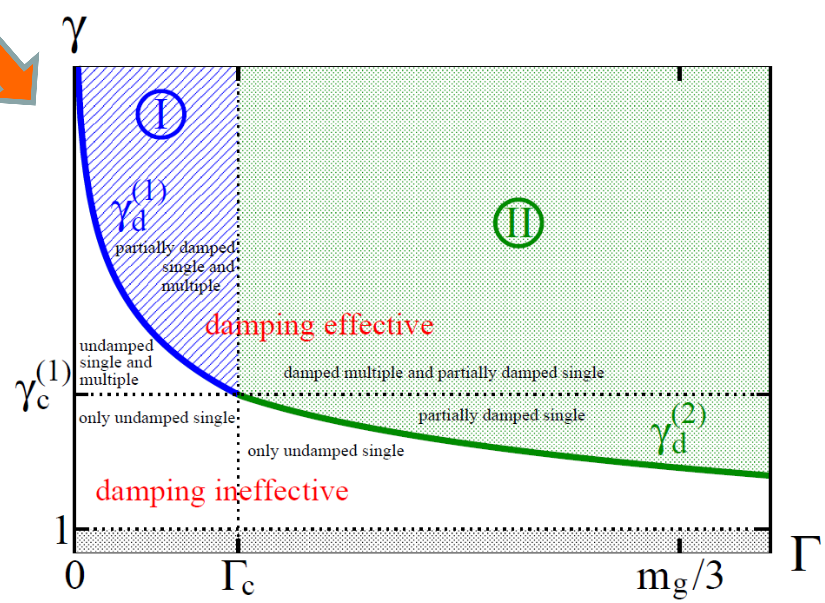
Larger damping effect at large γ



Increasing Γ

Larger and larger part of the spectrum affected by damping (shaded areas)

Γ–γ space

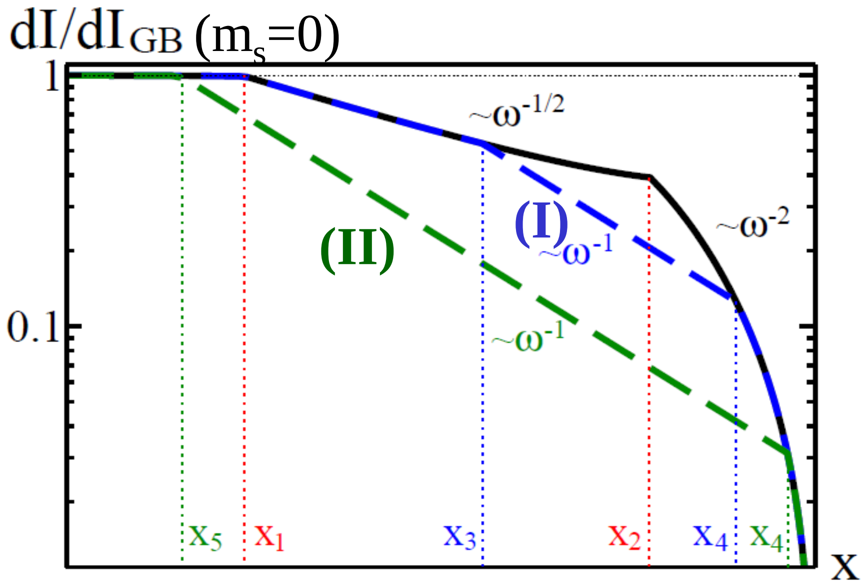


γ-scales
$\gamma_c^{(1)} \sim m_g^3/\hat{q}_g$
$\gamma_d^{(1)} \sim \sqrt{\hat{q}_g/\Gamma^3}$
$\gamma_d^{(2)} \sim m_g/\Gamma$

For $\Gamma > \Gamma_c \approx \frac{\hat{q}_g}{m_g^2}$ coherent radiation is totally superseeded by damping

Consequences on the spectra

$\hat{q} < m_g^3$



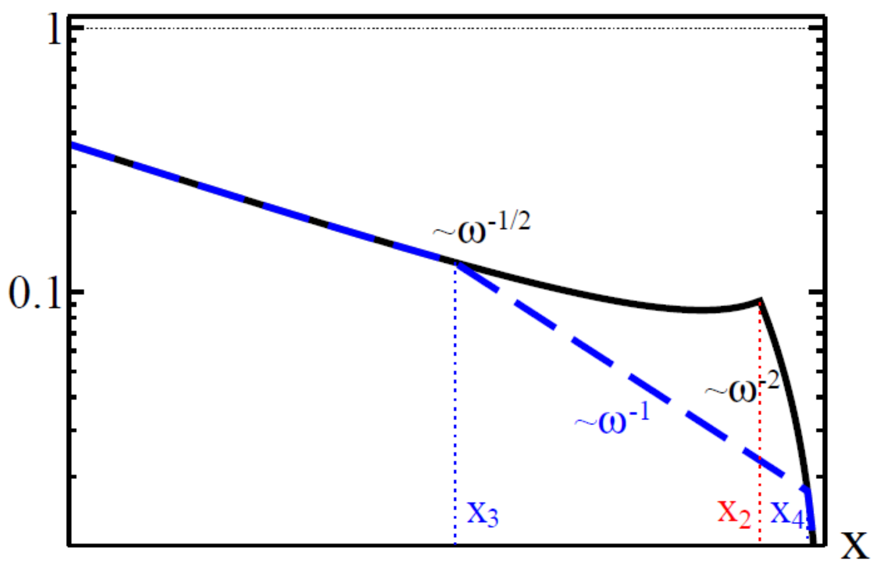
(I) and (II): moderate and large damping (see previous slide)

E= 45 GeV, m_s=1.5 GeV

m_g=0.6 GeV, $\hat{q} = 0.1\mathrm{GeV}^2/\mathrm{fm}$

Γ=0.05 GeV (I) & 0.15 GeV (II)

$\hat{q} > m_g^3$



Same but

$\hat{q} = 2\mathrm{GeV}^2/\mathrm{fm}$

Γ=0.25 GeV

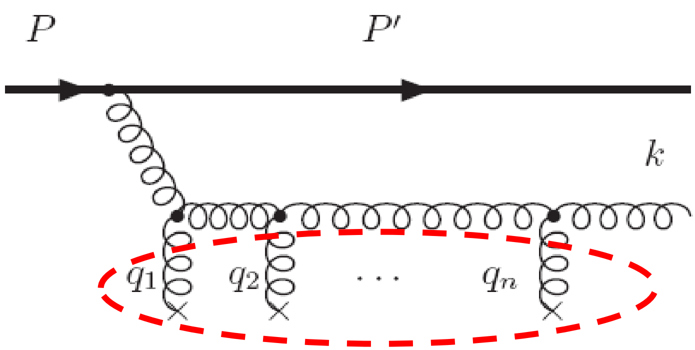
(High energy) gluon damping in pQCD and estimates for Γ

High energy: ω>>T

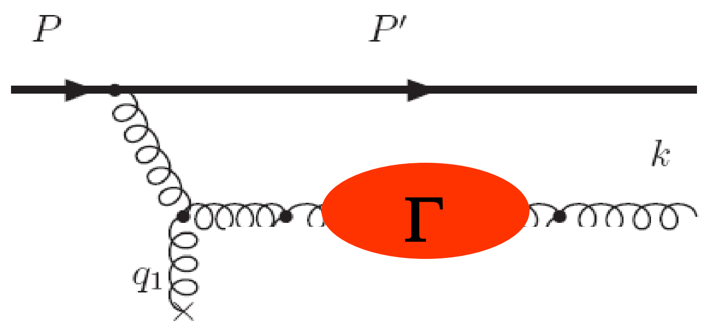
- Elastic process (collisionnal broadening): $\Gamma \approx g^2\, T\, (\ln 1/g)$ for $\omega = O(T)$;

  R. D. Pisarski, Phys. Rev. D 47 (93); no known result for ω>>T

- But double counting with original BDMPS description:

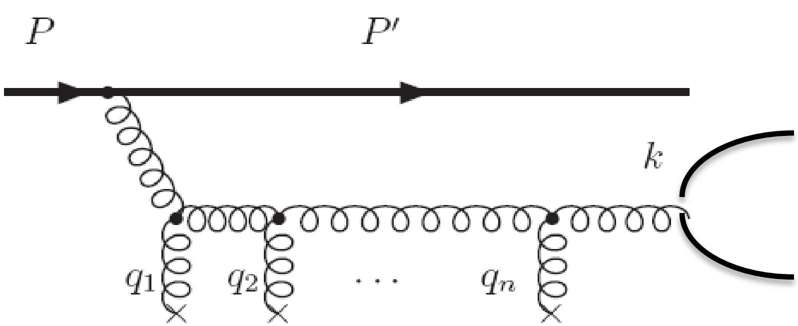


or



But not both !

- Genuine gluon absorption



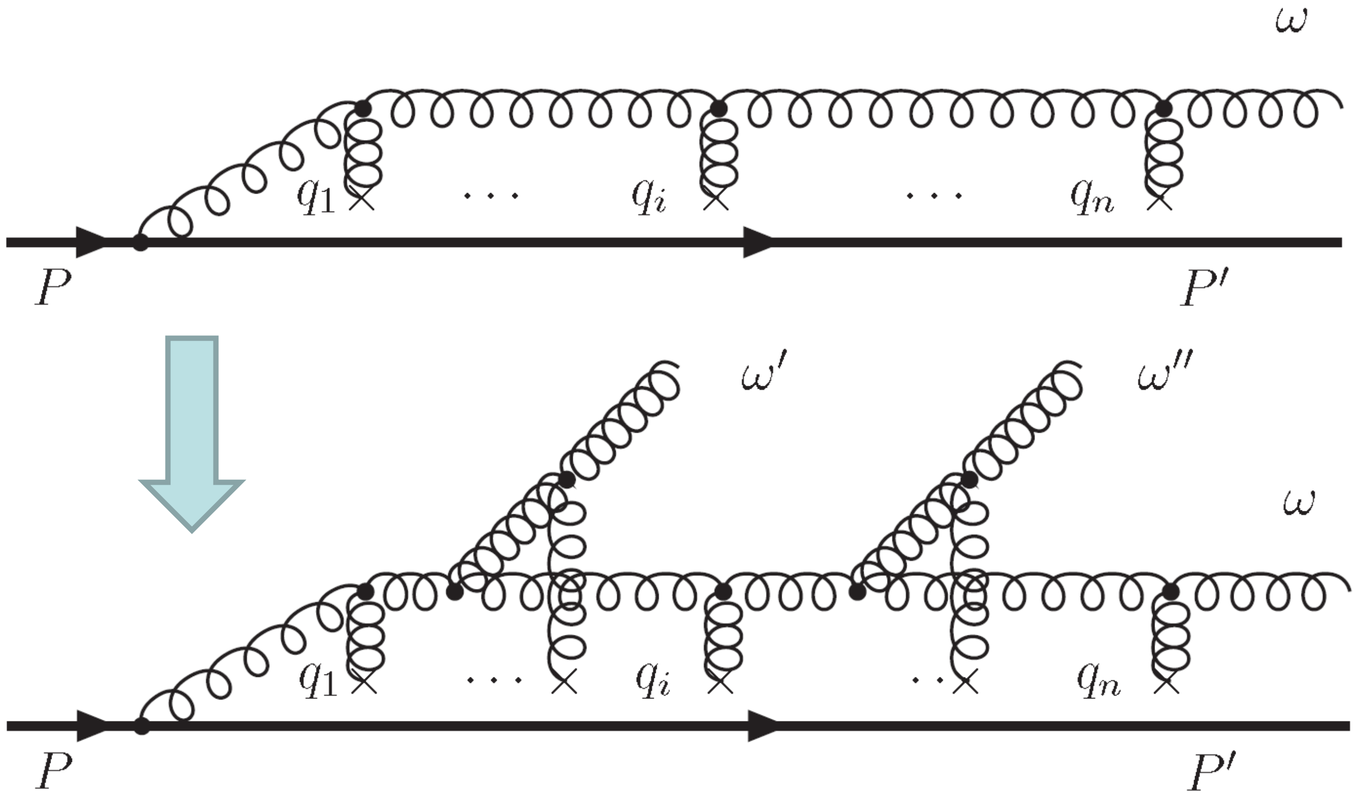
Hints that $\Gamma(\omega)\ \alpha\ g^4\, T^2/\omega$

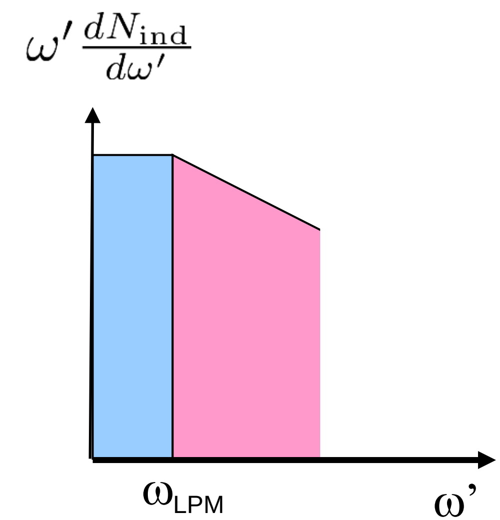
« damping rate of hard photon… »

Thoma, PRD51 (95)

(High energy) gluon damping in pQCD and estimates for Γ

- Considering the "pre-gluon" as a radiator itself and iterate (consistent if $\omega' < \omega$)





Emission of low energy quasi-isotropic gluons (ω', ω'')

$\Gamma \approx g^4\, T$

Possible candidate mechanism for di-jets imbalance and jet isotropisation observed by CMS !

Summary and perspectives

$\Gamma \approx g^4 T$ => In QED or pQCD, damping is indeed a NLO process (neglected in BDMPS-Z):

$$r_{\text{Debye}} = O\left(\frac{1}{gT}\right) < \lambda = O\left(\frac{1}{g^2 T}\right) < t_d = O\left(\frac{1}{g^4 T}\right)$$

However: formation time of radiation t_f increases with boost factor γ of the charge, so that t_d can play a significant role provided $t_d < L$

Evaluation of the power spectrum including effect of damping

Consequences on the observables under study !