The CGC and some applications

Dionysis Triantafyllopoulos
ECT*/FBK, Trento, Italy

Excited QCD, Peniche, May 2012

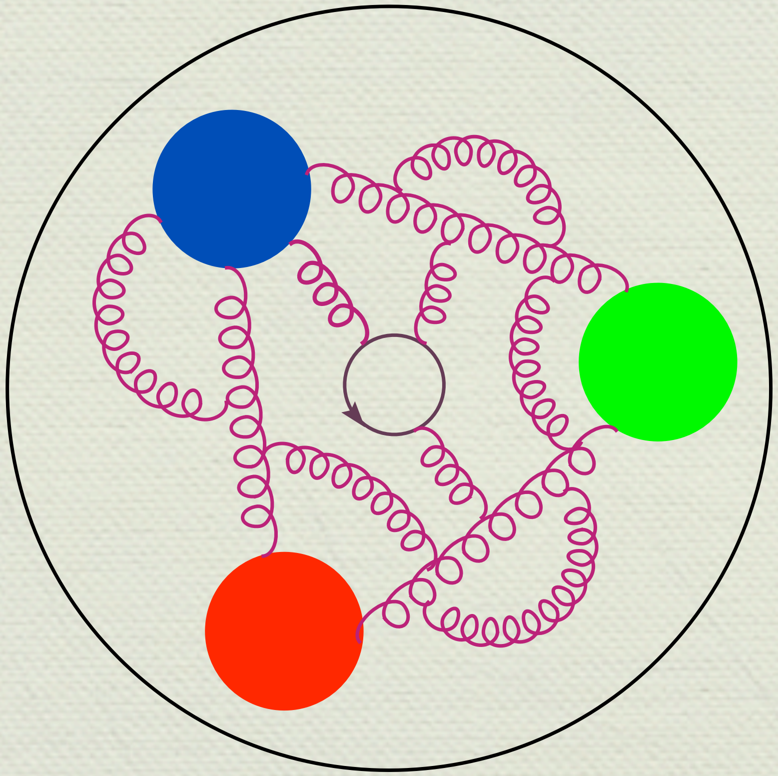
E. Iancu, DNT: JHEP 11 (2011) 105 [arXiv:1109:0302]
E. Iancu, DNT: JHEP 04 (2012) 025 [arXiv:1112.1104]
M. Alvioli, G. Soyez, DNT: in progress
E. Iancu: arXiv:1205.0579 “QCD in HIC”

Outline

- ☐ Parton evolution in QCD and the partonic “Phase diagram”
- ☐ The Color Glass Condensate and Wilson lines
- ☐ Particle production in pA (dilute-dense) collisions
- ☐ Dihadron production in forward region in pA collisions
- ☐ JIMWLK evolution of dipoles, quadrupoles, ...
- ☐ More applications and dense-dense collisions
- ☐ Conclusion and outlook

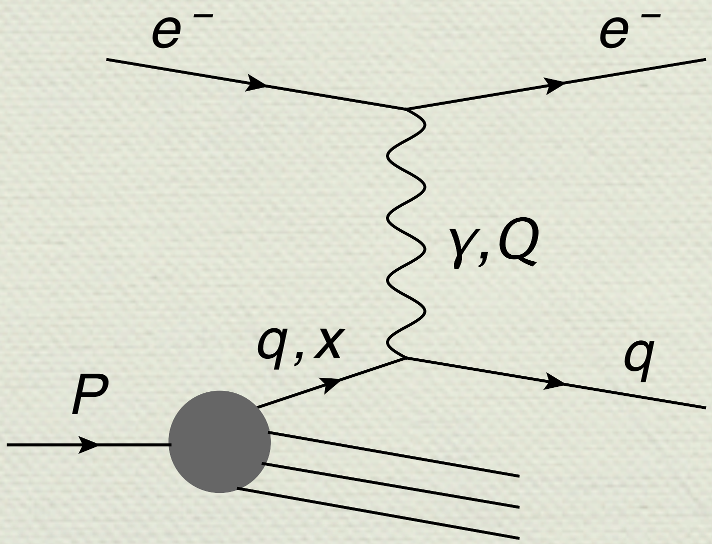
Constituents of a hadron

- Proton, or generic hadron, is complicated in rest frame
- Hadronic and vacuum fluctuations
- Non-perturbative with same lifetime $\Delta t_{\mathrm{RF}} \sim 1/\Lambda_{\mathrm{QCD}}$



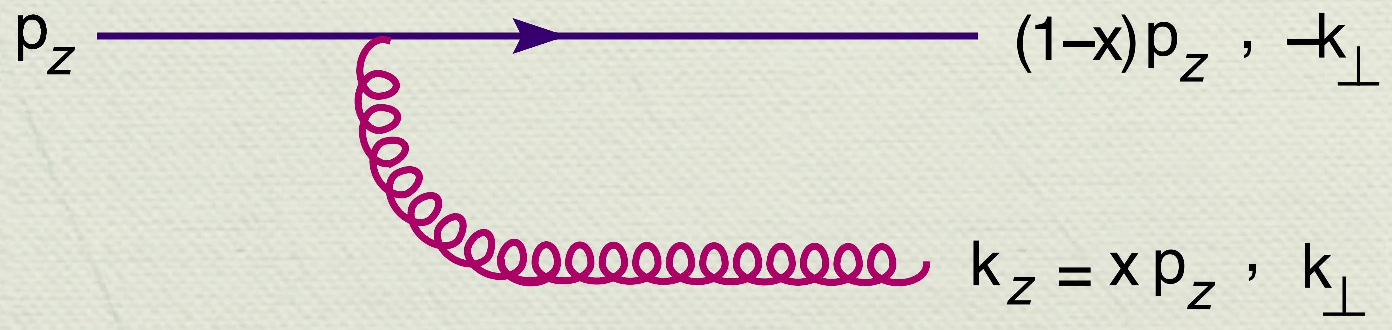
Infinite momentum frame and DIS

- IMF: Hadronic fluctuations live longer $\Delta t_{\mathrm{IMF}} \sim \gamma/\Lambda_{\mathrm{QCD}}$
- Longer than vacuum fluctuations
- Longer than collision time, e.g. in DIS $\Delta t_{\mathrm{coll}} \sim 2xP/Q^2$
- Quark with $\Delta t_{\mathrm{fluct}} \sim 2xP/k_\perp^2 \gtrsim \Delta t_{\mathrm{coll}}$ seen by photon



Soft and collinear gluons

- $\mathrm{d}P = C_R \frac{\alpha_s(k_\perp^2)}{\pi^2} \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}x}{x}$
- Emission of soft and collinear gluons is favored
- Large logs can overcome smallness of coupling
- Source lives longer than emitted parton: frozen
- Gluons dominate at small-x



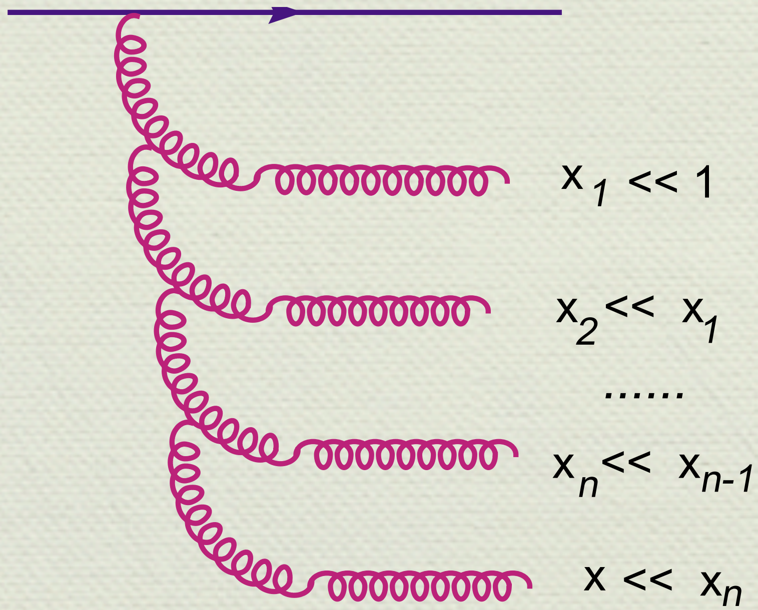
Cascades and evolution

- Successive emissions: DGLAP or BFKL cascade

$$\left(\frac{\alpha_s N_c}{\pi}\right)^n \int_x^1 \frac{\mathrm{d}x_n}{x_n} \int_{x_n}^1 \frac{\mathrm{d}x_{n-1}}{x_{n-1}} \cdots \int_{x_2}^1 \frac{\mathrm{d}x_1}{x_1} = \frac{1}{n!}\left(\bar{\alpha}_s \ln\frac{1}{x}\right)^n$$

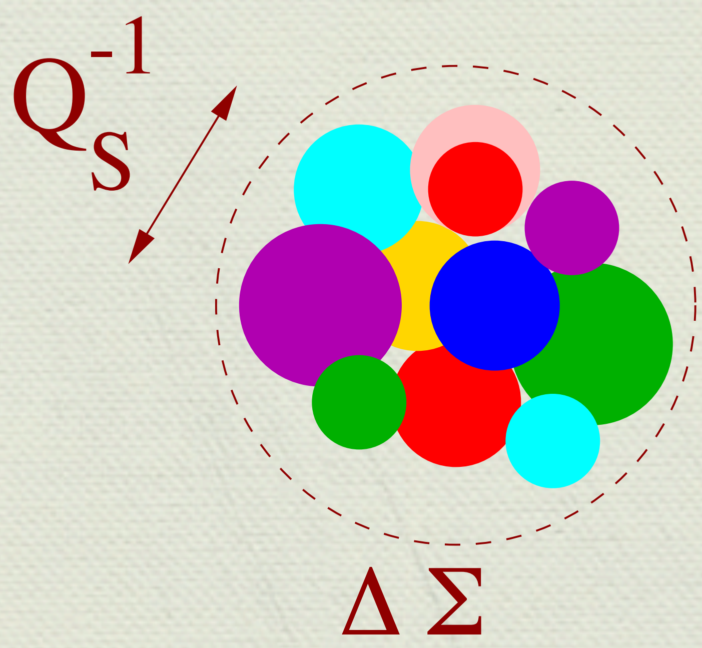
- Resum all diagrams $f_g = \frac{\mathrm{d}N_g}{\mathrm{d}Y\,\mathrm{d}k_\perp^2} \sim \frac{\alpha_s C_F}{\pi}\frac{1}{k_\perp^2}\exp(\omega\bar{\alpha}_s Y)$

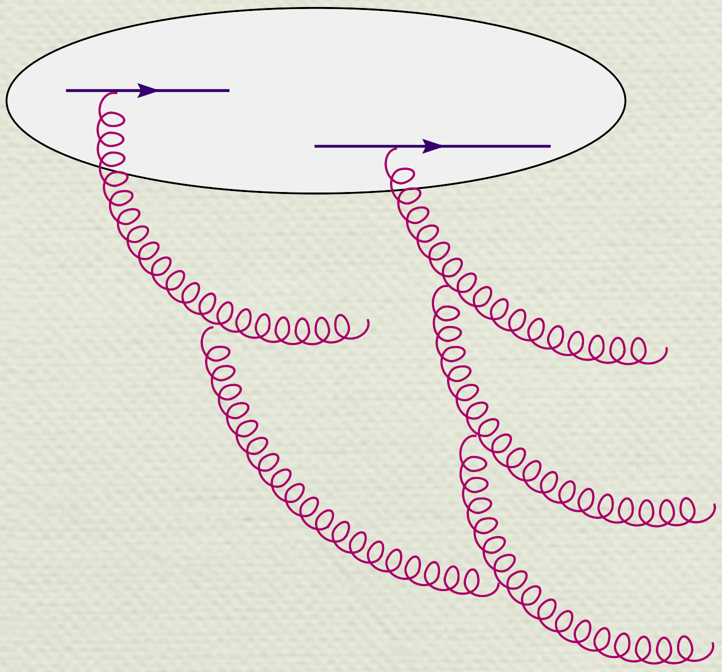
- Evolution equation $\frac{\mathrm{d}}{\mathrm{d}Y} f_g = \omega\bar{\alpha}_s f_g$



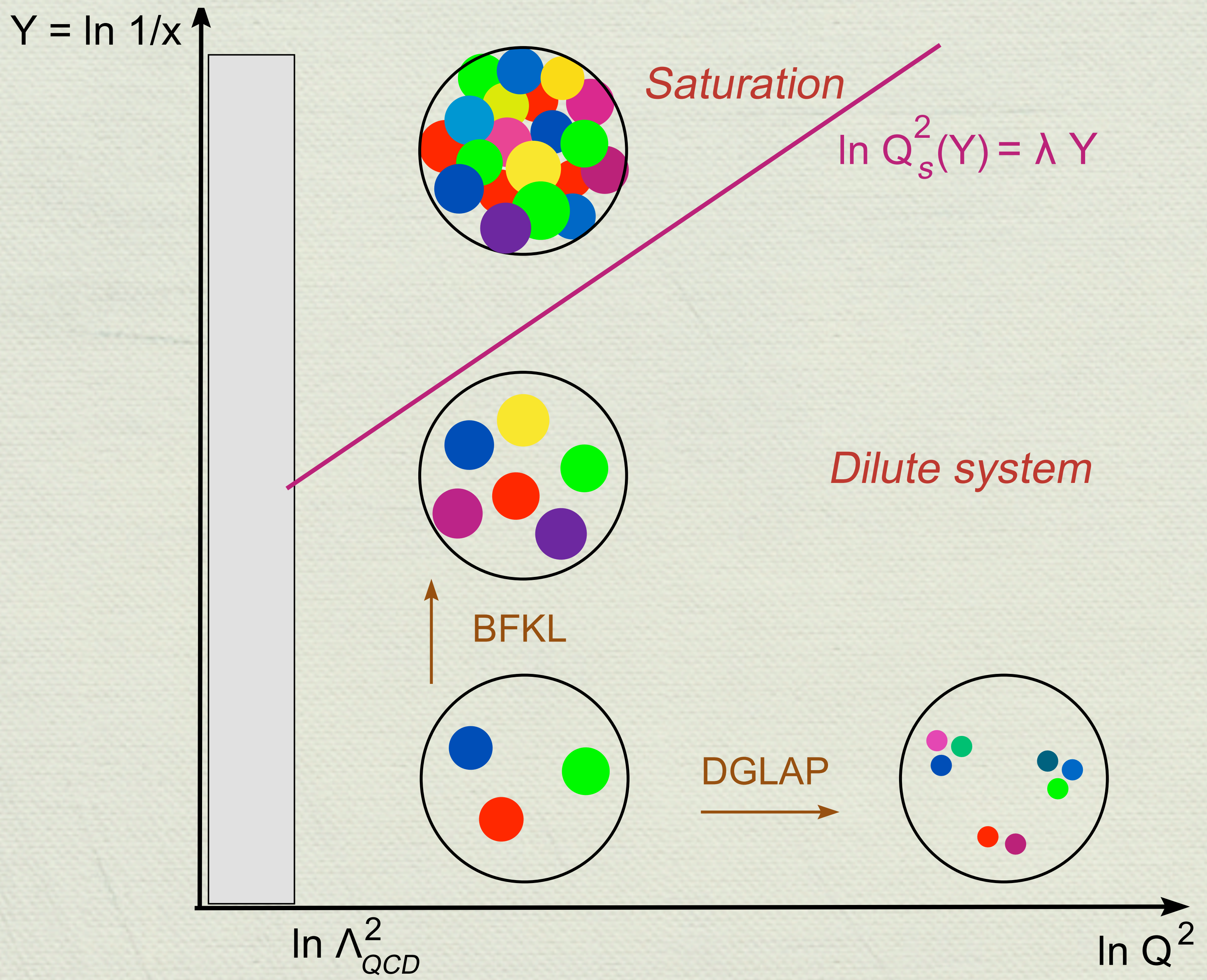
Kinematic regimes

- BFKL and DGLAP: linear, incoherent emissions
- DGLAP: smaller and smaller partons of size $1/Q^2$
- BFKL: typically same size partons
- Partons will “overlap”, coherent, non-linear evolution



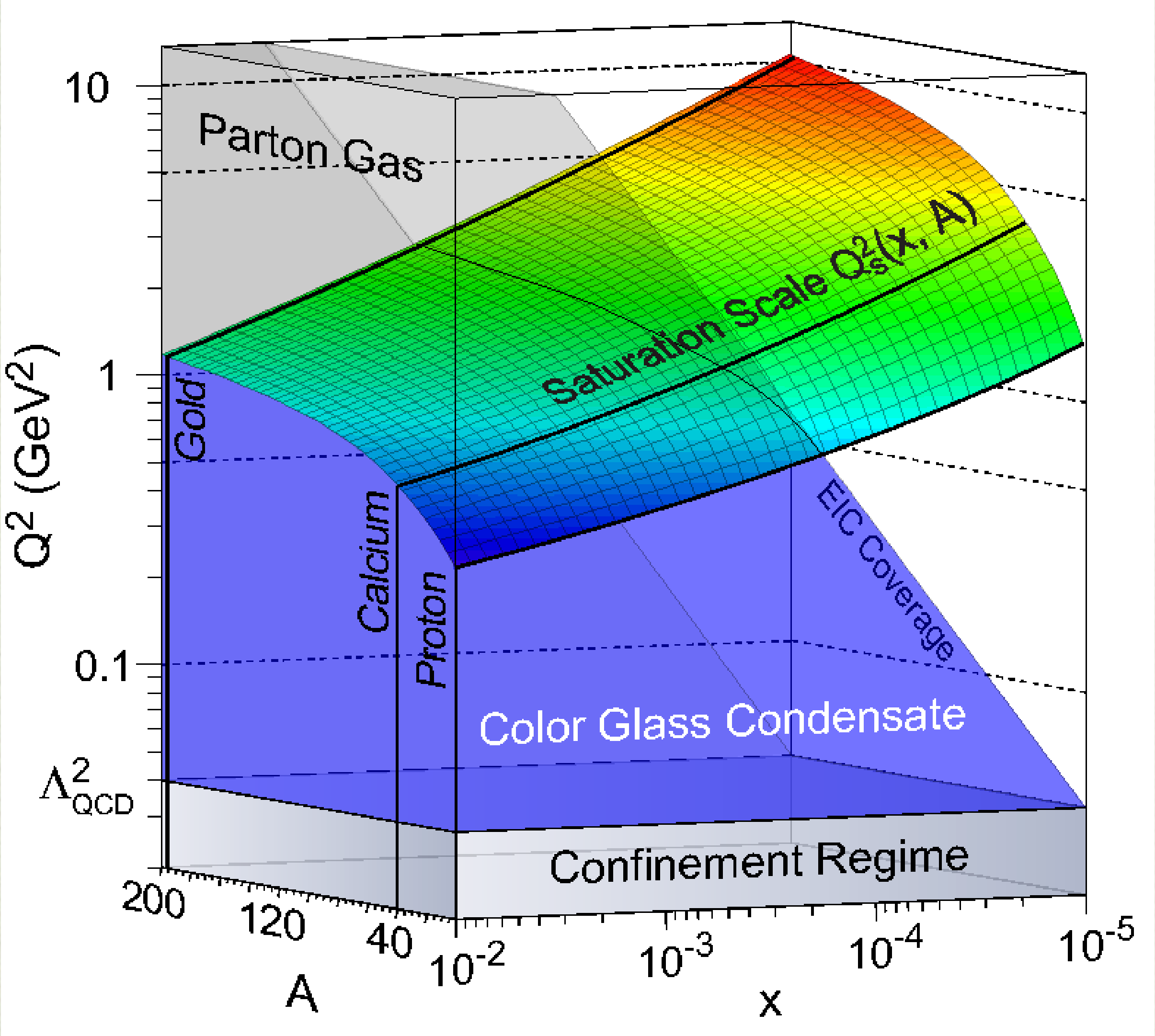


Partonic "phase diagram"



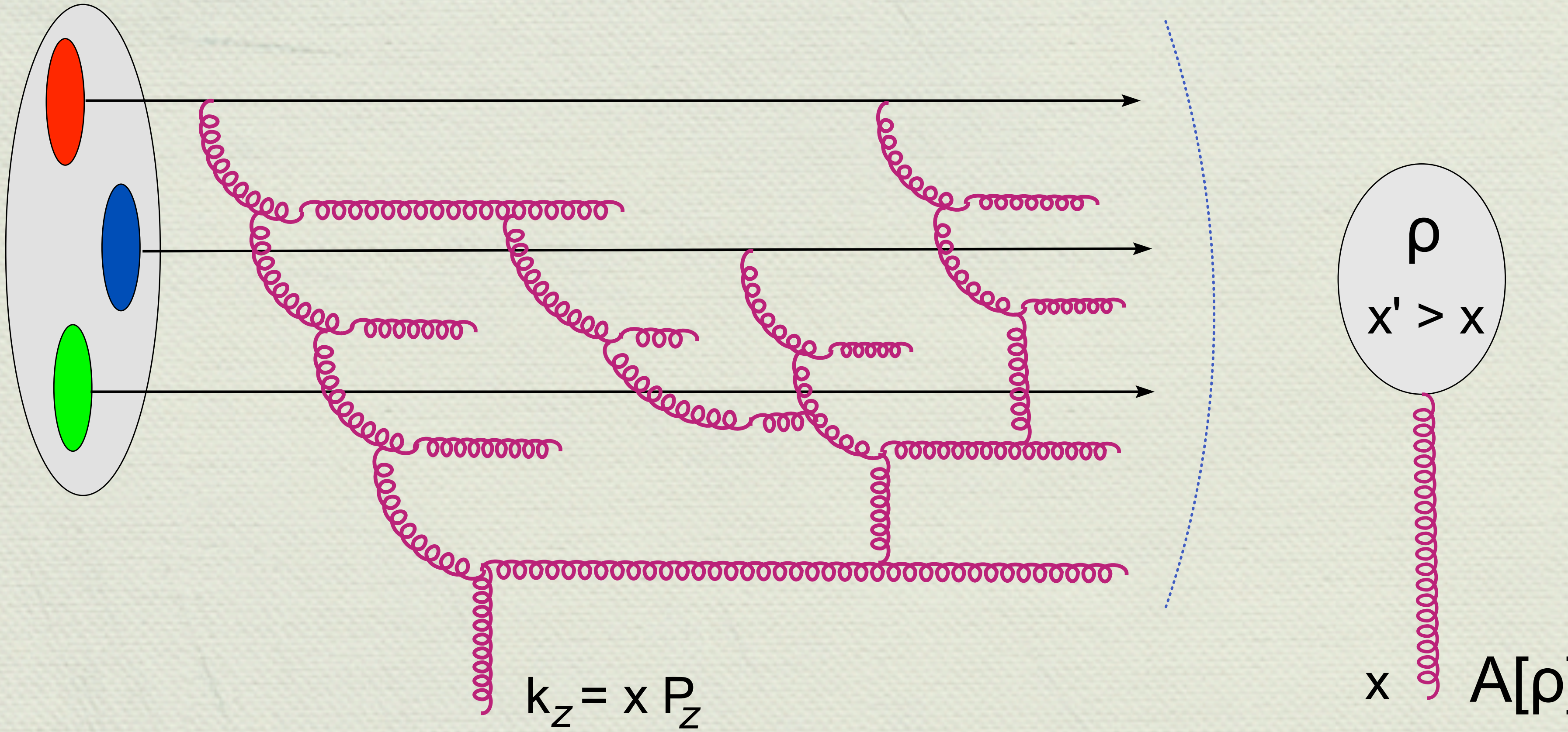
Saturation momentum

- Saturation when $\frac{xg(x, Q_s^2)}{Q_s^2 R^2} \sim \frac{1}{\alpha_s}$
- $Q_s^2(x, A) \sim Q_0^2 A^{1/3} \left(\frac{x_0}{x}\right)^\lambda$ with $\lambda = 0.2 \div 0.3$



Color Glass Condensate

- QCD, frozen sources, occupation numbers of order $1/\alpha_s$
- All orders in $\alpha_s \ln 1/x$ and classical field $A_a^\mu \sim \mathcal{O}(1/g)$



JIMWLK evolution

□ Evolution for functional of color sources

$$\frac{\partial W_Y[\rho]}{\partial Y} = H_{\mathrm{JIMWLK}} W_Y[\rho]$$

□ Average over color sources

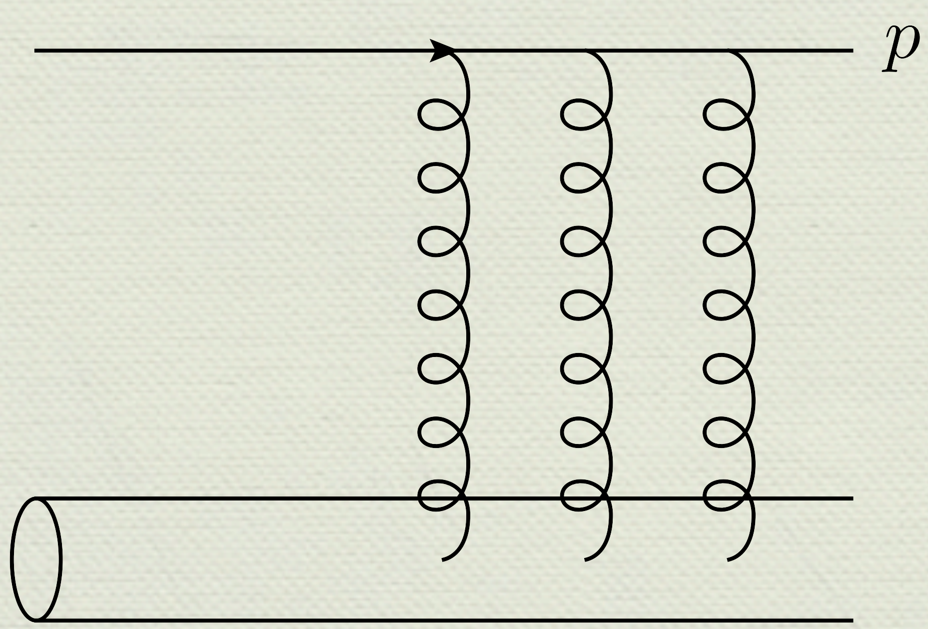
$$\langle \mathcal{O}[A] \rangle_Y = \int \mathcal{D}\rho W_Y[\rho] \mathcal{O}[A(\rho)]$$

□ Solve classical Yang-Mills equations to get the field

$$D_\nu^{ab} F_b^{\nu\mu} = \delta^{\mu +} \rho^a(x^-, \boldsymbol{x}_\perp)$$

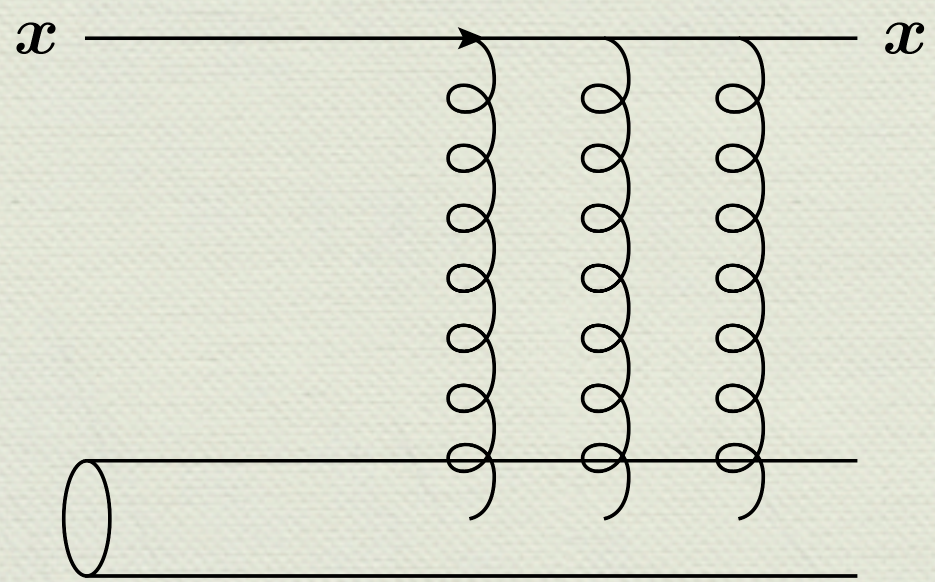
Single particle production

- Large-x quark from proton: eikonal trajectory
- Interacts with soft components of nucleus
- Quark "measured" in forward region



Wilson lines

- Mixed representation: transverse momenta → coordinates
- Nucleus viewed as large classical color field
- Eikonal interaction → Wilson lines: $V_{\boldsymbol{x}}^{\dagger} = P \exp\left[\mathrm{i}\, \mathrm{g} \int \mathrm{d}x^{-} t^{a} \mathcal{A}_{\boldsymbol{x}}^{+a}(x^{-})\right]$



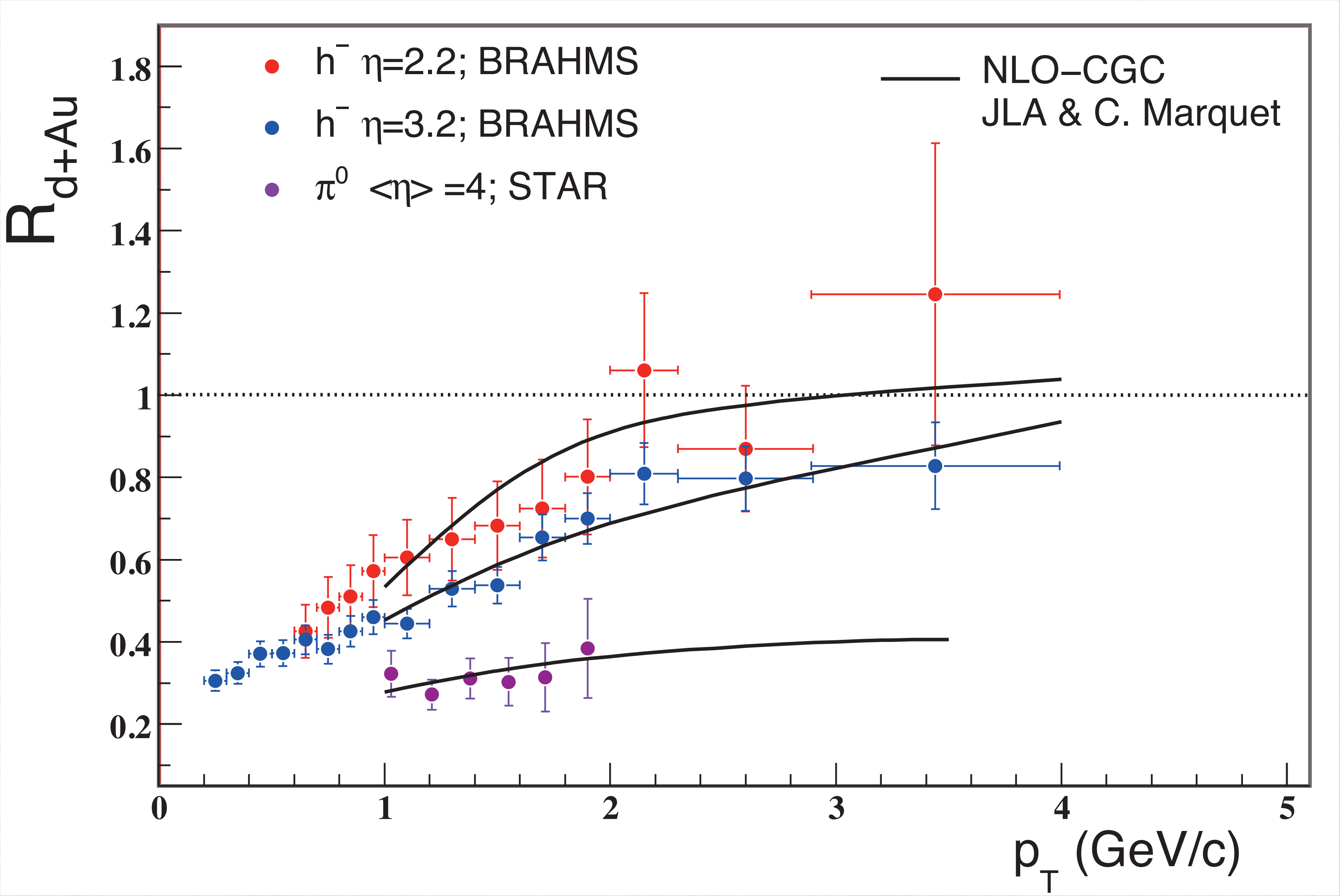
The cross section

- Multiply by c.c. and F.T. to calculate cross section

$$\frac{\mathrm{d}\sigma^{qA\to qX}}{\mathrm{d}y\mathrm{d}^2\boldsymbol{p}} = x_1 f_q(x_1,\boldsymbol{p}^2)\int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{r}\cdot\boldsymbol{p}}\frac{1}{N_c}\Big\langle \mathrm{tr}[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})] + \dots\Big\rangle_Y$$

- Summing over final color, average initial: 1/Nc tr ...
- QCD dynamics in $\langle\dots\rangle_Y = \int D\mathcal{A}^+ W_Y[\mathcal{A}^+]\dots$
- $\mathrm{e}^{-Y} = x_2 = \frac{|\boldsymbol{p}|\mathrm{e}^{-y}}{\sqrt{s}} \ll 1$ in forward region, $x_1 = \frac{|\boldsymbol{p}|\mathrm{e}^{y}}{\sqrt{s}}$

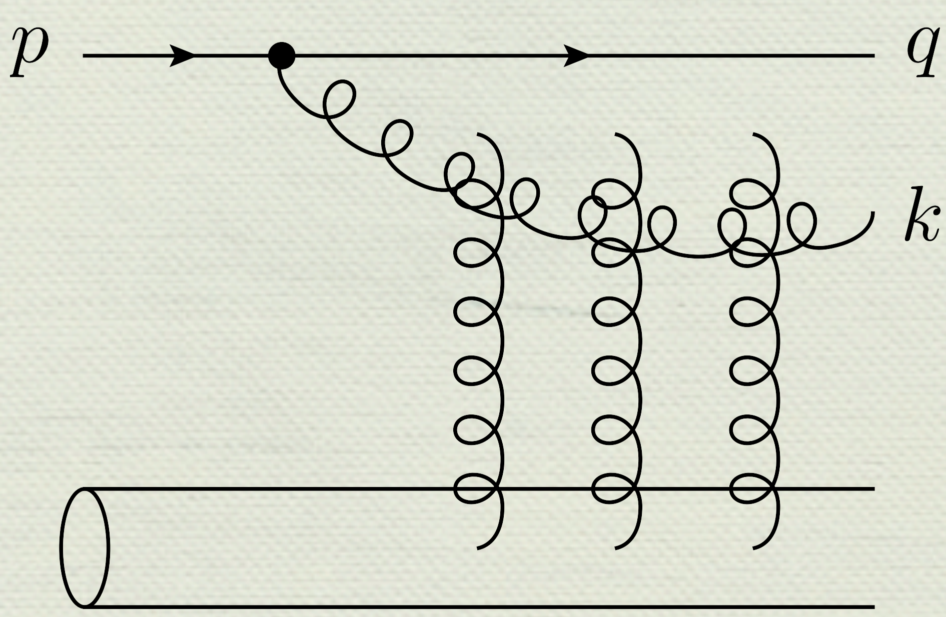
Comparing with data



$$R_{pA}(\eta, \boldsymbol{p}) \equiv \frac{1}{A^{1/3}} \frac{\mathrm{d}N_h/\mathrm{d}\eta \mathrm{d}^2\boldsymbol{p}\big|_{pA}}{\mathrm{d}N_h/\mathrm{d}\eta \mathrm{d}^2\boldsymbol{p}\big|_{pp}}$$

Dihadron Production

- Large-x quark from proton splits into quark-gluon pair



$$|\Psi_{\text{out}}\rangle = \int DA^+ \, \Phi_Y[\mathcal{A}^+] \int_{\boldsymbol{x},\boldsymbol{b}} \mathrm{d}z \, p^- g \, \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{b}} \sum_{j\beta c\lambda} \phi^{\lambda}_{\alpha\beta}(p, zp^-, \boldsymbol{x}-\boldsymbol{b})$$

$$[T^d V(\boldsymbol{b})\tilde{V}^{dc}(\boldsymbol{x}) - V(\boldsymbol{b}+z(\boldsymbol{x}-\boldsymbol{b}))T^c]_{ij}$$

$$|(1-z)p^-, \boldsymbol{b}, j, \beta; zp^-, \boldsymbol{x}, c, \lambda\rangle \otimes |\mathcal{A}^+\rangle$$

The cross section

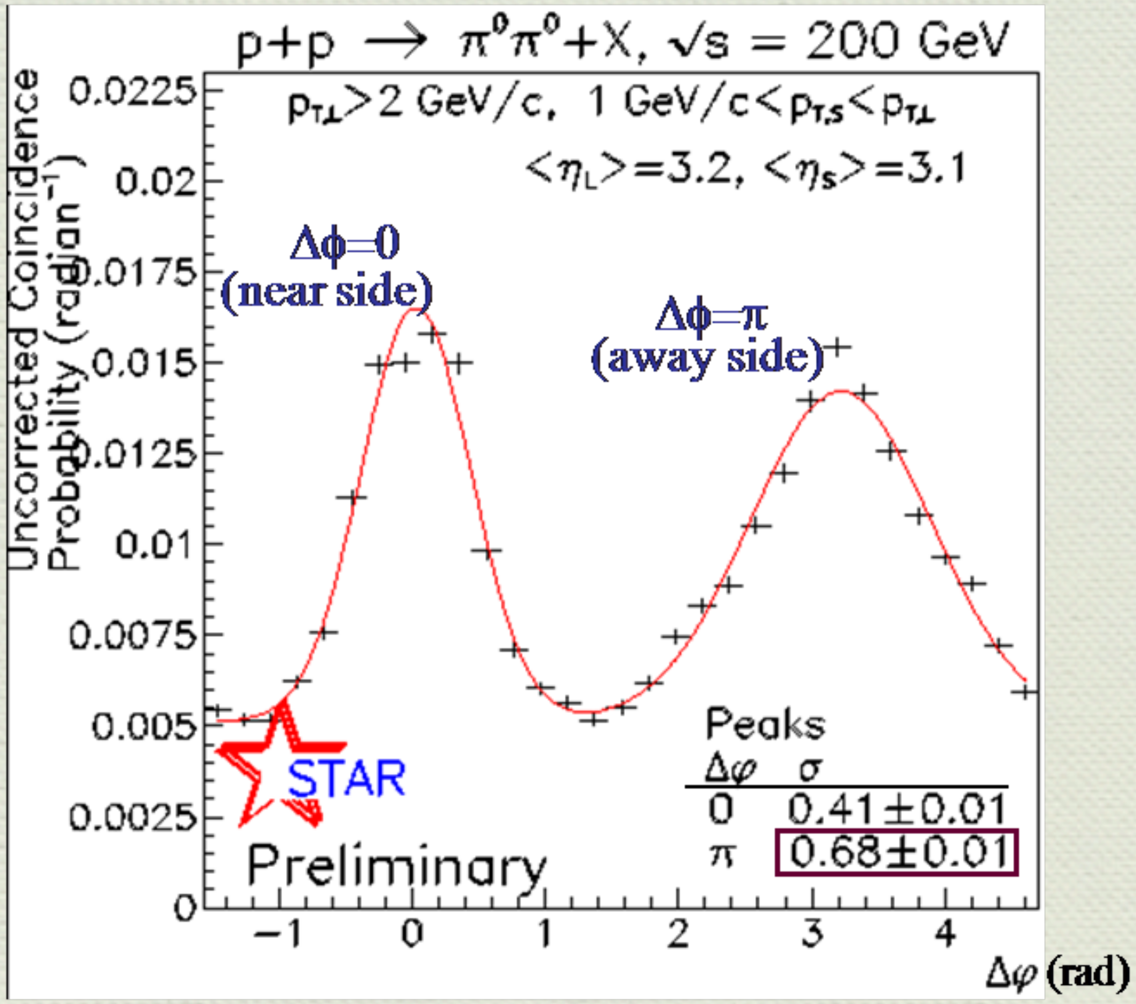
- From $\langle \Psi_{\text{out}} | N_q(q) N_g(k) | \Psi_{\text{out}} \rangle$ calculate cross section

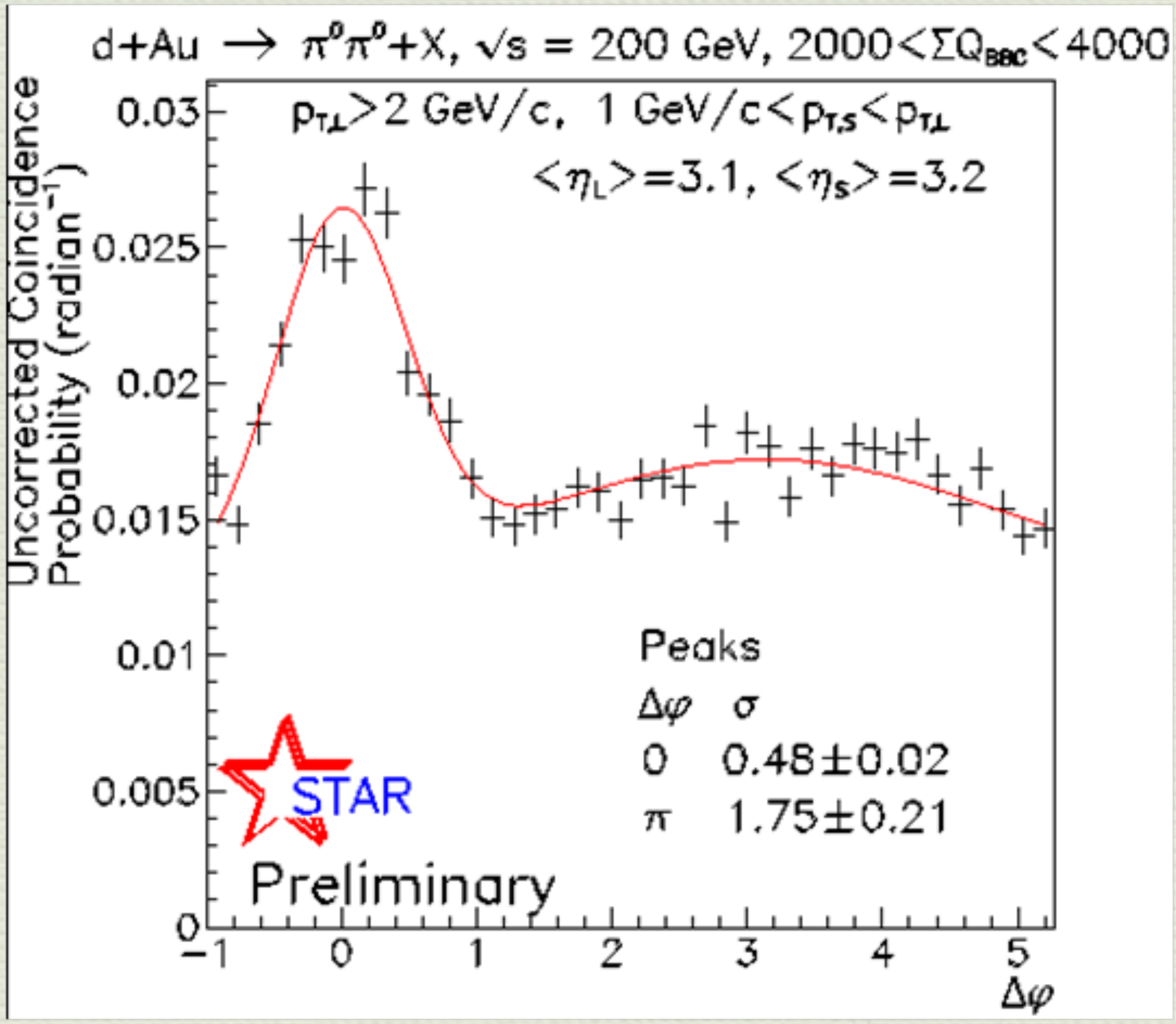
$$\frac{\mathrm{d}\sigma^{qA\to qgX}}{\mathrm{d}^3k\,\mathrm{d}^3q} = \frac{\alpha_s N_c}{2} \int_{\boldsymbol{x}\acute{\boldsymbol{x}}\boldsymbol{b}\acute{\boldsymbol{b}}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\acute{\boldsymbol{x}})+\mathrm{i}(\boldsymbol{q}-\boldsymbol{p})\cdot(\acute{\boldsymbol{b}}-\boldsymbol{b})}$$
$$\sum_{\lambda\alpha\beta} \phi^{\lambda *}_{\alpha\beta}(p, zp^-, \acute{\boldsymbol{x}} - \acute{\boldsymbol{b}}) \phi^{\lambda}_{\alpha\beta}(p, zp^-, \boldsymbol{x} - \boldsymbol{b})$$
$$\left\langle \frac{1}{N_c} \mathrm{tr}[V^\dagger(\boldsymbol{x}) V(\boldsymbol{b}) V^\dagger(\acute{\boldsymbol{b}}) V(\acute{\boldsymbol{x}})] \frac{1}{N_c} \mathrm{tr}[V^\dagger(\acute{\boldsymbol{x}}) V(\boldsymbol{x})] + \dots \right\rangle_Y$$

- QCD dynamics in $\langle \dots \rangle_Y = \int D\mathcal{A}^+ W_Y[\mathcal{A}^+] \dots$

- $\mathrm{e}^{-Y} = x = \dfrac{|\boldsymbol{k}|\mathrm{e}^{-y_k} + |\boldsymbol{q}|\mathrm{e}^{-y_q}}{\sqrt{s}} \ll 1$ in forward region

Di-hadron azimuthal correlations





Wilson line correlators

- Dipole operator: $\hat{S}_{12} = \frac{1}{N_c} \mathrm{tr}(V_1^\dagger V_2)$
- Quadrupole operator: $\hat{Q}_{1234} = \frac{1}{N_c} \mathrm{tr}(V_1^\dagger V_2 V_3^\dagger V_4)$
- 2n-point operator: $\hat{S}^{(2n)}_{12\ldots(2n-1)2n} = \frac{1}{N_c} \mathrm{tr}(V_1^\dagger V_2 \ldots V_{2n-1}^\dagger V_{2n})$
- Finite Nc, given wavefunction, calculate each correlator
- Large Nc, factorization: $\langle \hat{S}^{2n_1} \ldots \hat{S}^{2n_k} \rangle_Y \to \langle \hat{S}^{2n_1} \rangle_Y \ldots \langle \hat{S}^{2n_k} \rangle_Y$
- Still infinite number of correlators, e.g. $\langle \hat{Q}_{1234} \rangle_Y = ?$

Evolution of correlators

- QCD dynamics encoded in JIMWLK Hamiltonian

$$H = -\frac{1}{16\pi^3} \int_{\boldsymbol{uvz}} \mathcal{M}_{\boldsymbol{uvz}} [1 + \widetilde{V}^\dagger_{\boldsymbol{u}} \widetilde{V}_{\boldsymbol{v}} - \widetilde{V}^\dagger_{\boldsymbol{u}} \widetilde{V}_{\boldsymbol{z}} - \widetilde{V}^\dagger_{\boldsymbol{z}} \widetilde{V}_{\boldsymbol{v}}]^{ab} \frac{\delta}{\delta \alpha^a_{\boldsymbol{u}}} \frac{\delta}{\delta \alpha^b_{\boldsymbol{v}}}$$

- Known kernel M: one dipole -> two dipoles

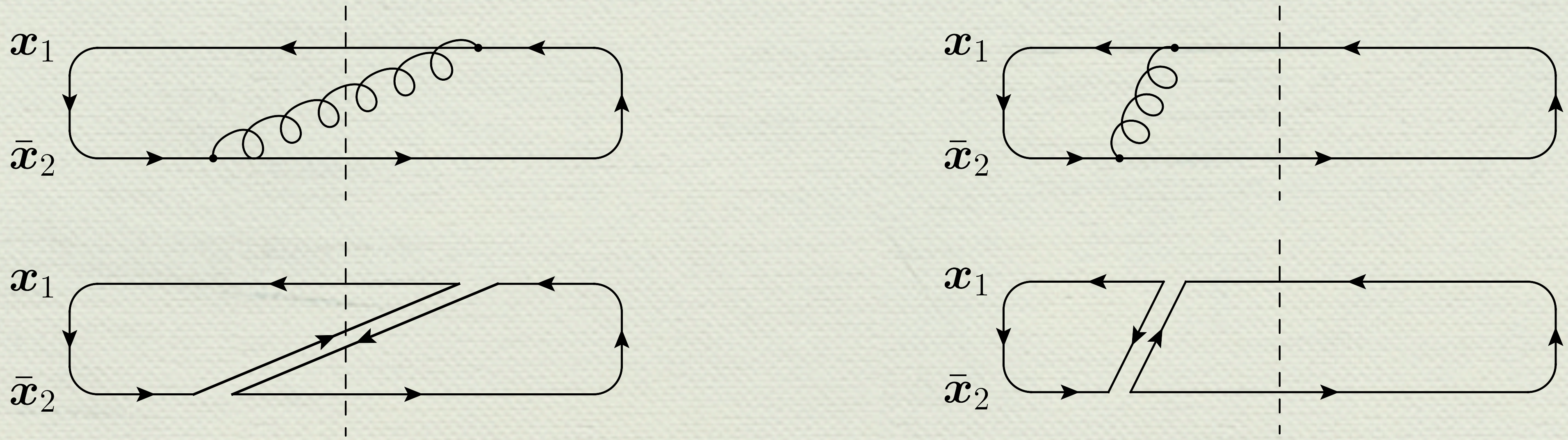
- Evolution of expectation value of arbitrary correlator

$$\frac{\partial \langle \hat{\mathcal{O}} \rangle_Y}{\partial Y} = \langle H \hat{\mathcal{O}} \rangle_Y$$

- Easy to work out: act on end-point, use Fierz identities.

The Dipole

- Well-known eqn: $\dfrac{\partial \langle \hat{S}_{12} \rangle_Y}{\partial Y} = \dfrac{\bar{\alpha}_s}{2\pi} \displaystyle\int_{\boldsymbol{z}} \mathcal{M}_{12\boldsymbol{z}} \langle \hat{S}_{1\boldsymbol{z}} \hat{S}_{\boldsymbol{z}2} - \hat{S}_{12} \rangle_Y$



- Weak scattering T = 1 - S small: linear, BFKL, easy to solve

- Strong scattering, assume large Nc: linear in S

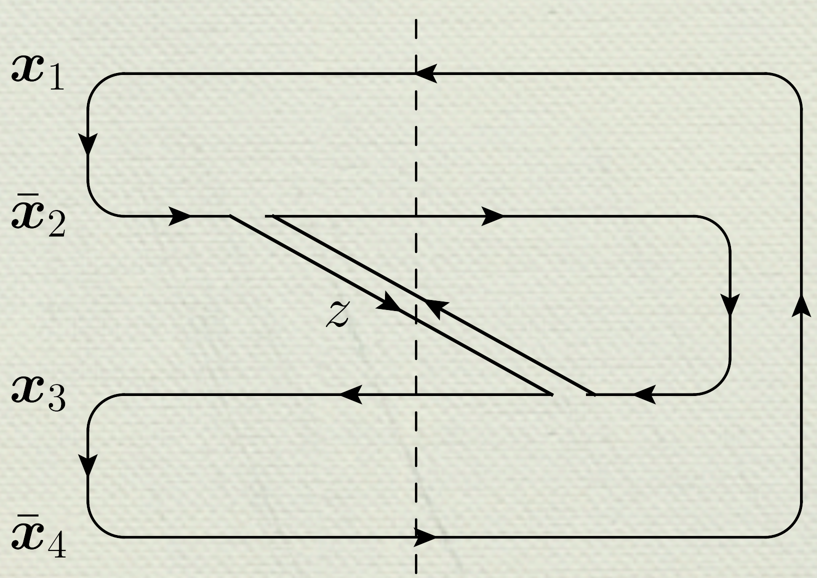
$$\frac{\partial \langle \hat{S}_{12} \rangle_Y}{\partial Y} = -\bar{\alpha}_s \int_{1/Q_s^2}^{r_{12}^2} \frac{\mathrm{d}z^2}{z^2} \langle \hat{S}_{12} \rangle_Y = -\bar{\alpha}_s \ln(r_{12}^2 Q_s^2) \langle \hat{S}_{12} \rangle_Y$$

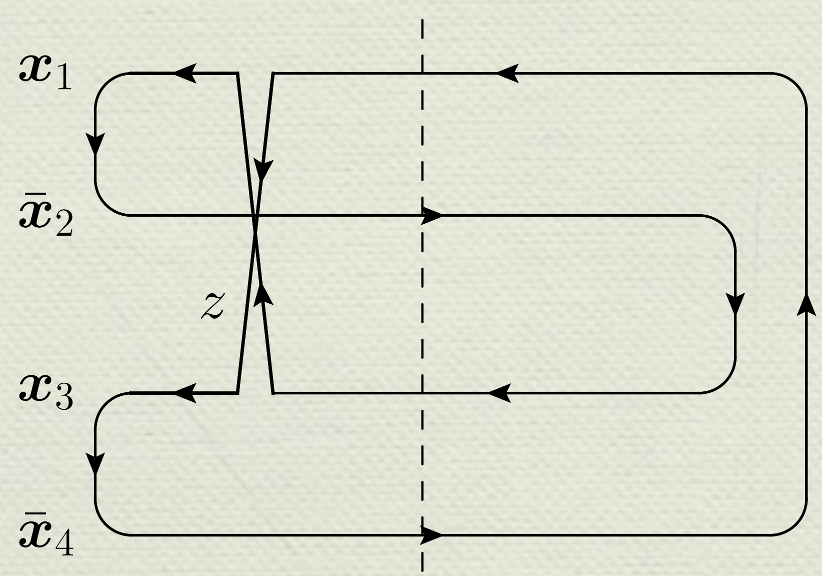
- Local in S, trivially solved if we know Qs(Y)

The Quadrupole

- The evolution equation...

$$\frac{\partial \langle \hat{Q}_{1234} \rangle_Y}{\partial Y} = \frac{\bar{\alpha}_s}{4\pi} \int_{\boldsymbol{z}} (\mathcal{M}_{12\boldsymbol{z}} + \mathcal{M}_{14\boldsymbol{z}} - \mathcal{M}_{24\boldsymbol{z}}) \langle \hat{S}_{1\boldsymbol{z}} \hat{Q}_{\boldsymbol{z}234} \rangle_Y$$
$$+ (\mathcal{M}_{12\boldsymbol{z}} + \mathcal{M}_{23\boldsymbol{z}} - \mathcal{M}_{13\boldsymbol{z}}) \langle \hat{S}_{\boldsymbol{z}2} \hat{Q}_{1\boldsymbol{z}34} \rangle_Y$$
$$+ (\mathcal{M}_{23\boldsymbol{z}} + \mathcal{M}_{34\boldsymbol{z}} - \mathcal{M}_{24\boldsymbol{z}}) \langle \hat{S}_{3\boldsymbol{z}} \hat{Q}_{12\boldsymbol{z}4} \rangle_Y$$
$$+ (\mathcal{M}_{14\boldsymbol{z}} + \mathcal{M}_{34\boldsymbol{z}} - \mathcal{M}_{13\boldsymbol{z}}) \langle \hat{S}_{\boldsymbol{z}4} \hat{Q}_{123\boldsymbol{z}} \rangle_Y$$
$$- (\mathcal{M}_{12\boldsymbol{z}} + \mathcal{M}_{14\boldsymbol{z}} + \mathcal{M}_{23\boldsymbol{z}} + \mathcal{M}_{14\boldsymbol{z}}) \langle \hat{Q}_{1234} \rangle_Y$$
$$- (\mathcal{M}_{12\boldsymbol{z}} + \mathcal{M}_{34\boldsymbol{z}} - \mathcal{M}_{13\boldsymbol{z}} - \mathcal{M}_{24\boldsymbol{z}}) \langle \hat{S}_{12} \hat{S}_{34} \rangle_Y$$
$$- (\mathcal{M}_{14\boldsymbol{z}} + \mathcal{M}_{23\boldsymbol{z}} - \mathcal{M}_{13\boldsymbol{z}} - \mathcal{M}_{24\boldsymbol{z}}) \langle \hat{S}_{14} \hat{S}_{23} \rangle_Y$$



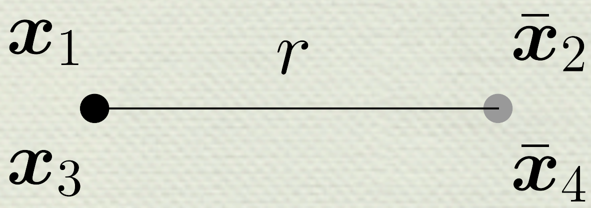


Quadrupole in terms of dipole

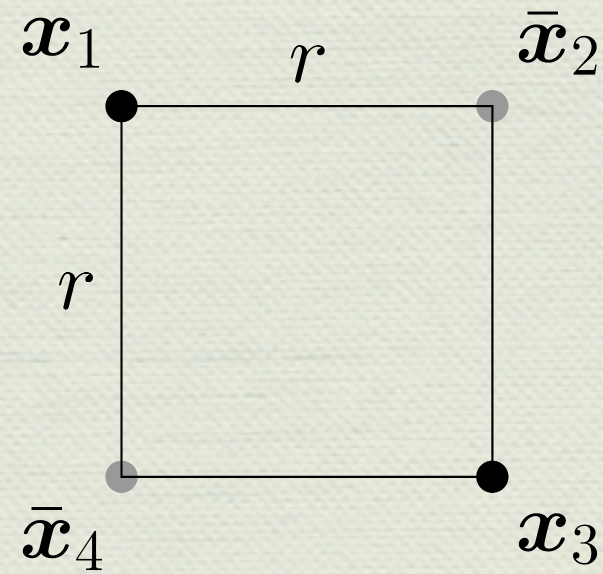
$$\langle \hat{Q}_{1234} \rangle_Y = \frac{\ln[\langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y / \langle \hat{S}_{13} \rangle_Y \langle \hat{S}_{24} \rangle_Y]}{\ln[\langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y / \langle \hat{S}_{14} \rangle_Y \langle \hat{S}_{23} \rangle_Y]} \langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y + \frac{\ln[\langle \hat{S}_{14} \rangle_Y \langle \hat{S}_{23} \rangle_Y / \langle \hat{S}_{13} \rangle_Y \langle \hat{S}_{24} \rangle_Y]}{\ln[\langle \hat{S}_{14} \rangle_Y \langle \hat{S}_{23} \rangle_Y / \langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y]} \langle \hat{S}_{14} \rangle_Y \langle \hat{S}_{23} \rangle_Y$$

- Easily extended at finite number of colors
- T. Ulrich requested in eQCD 2011 a solution to JIMWLK
- Here is analytical one in terms of dipole

vs numerical solution



(a)



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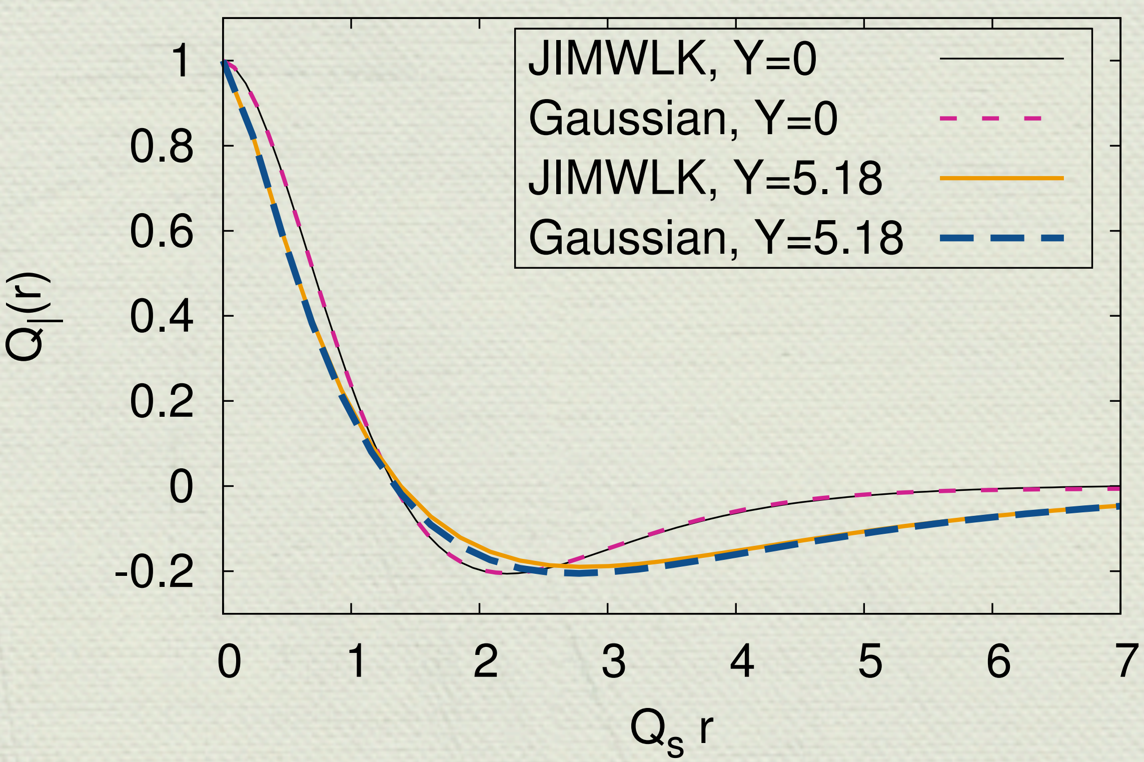
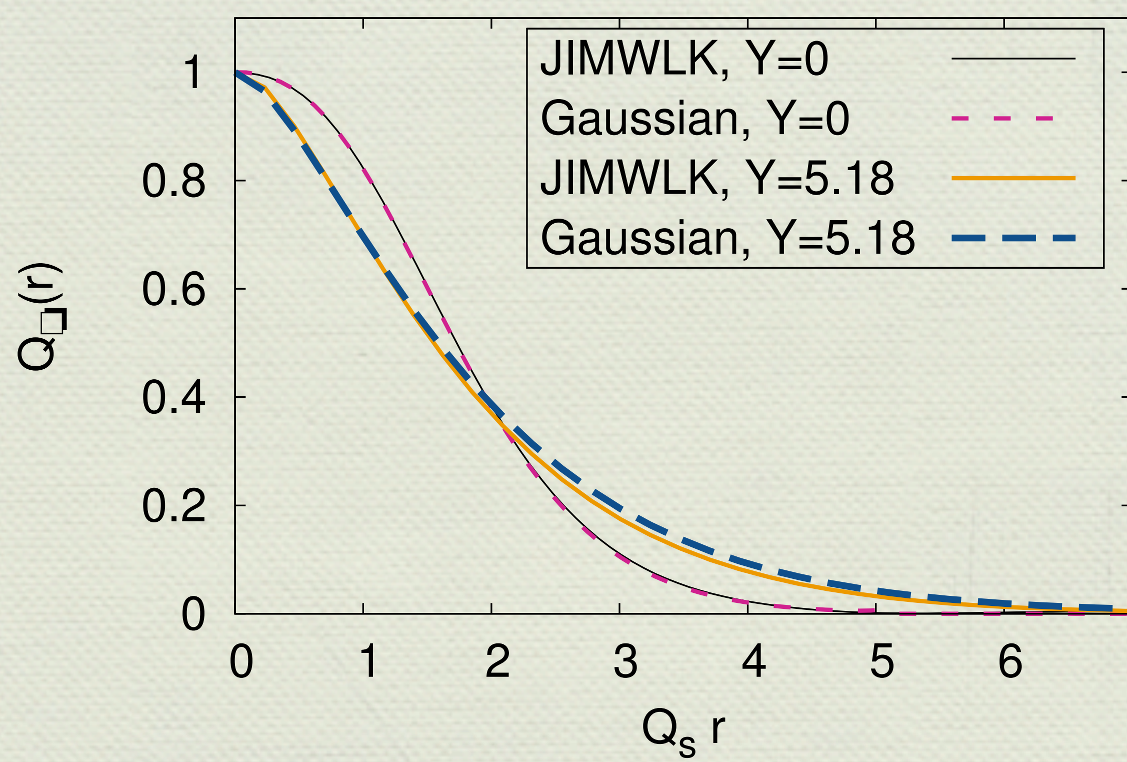
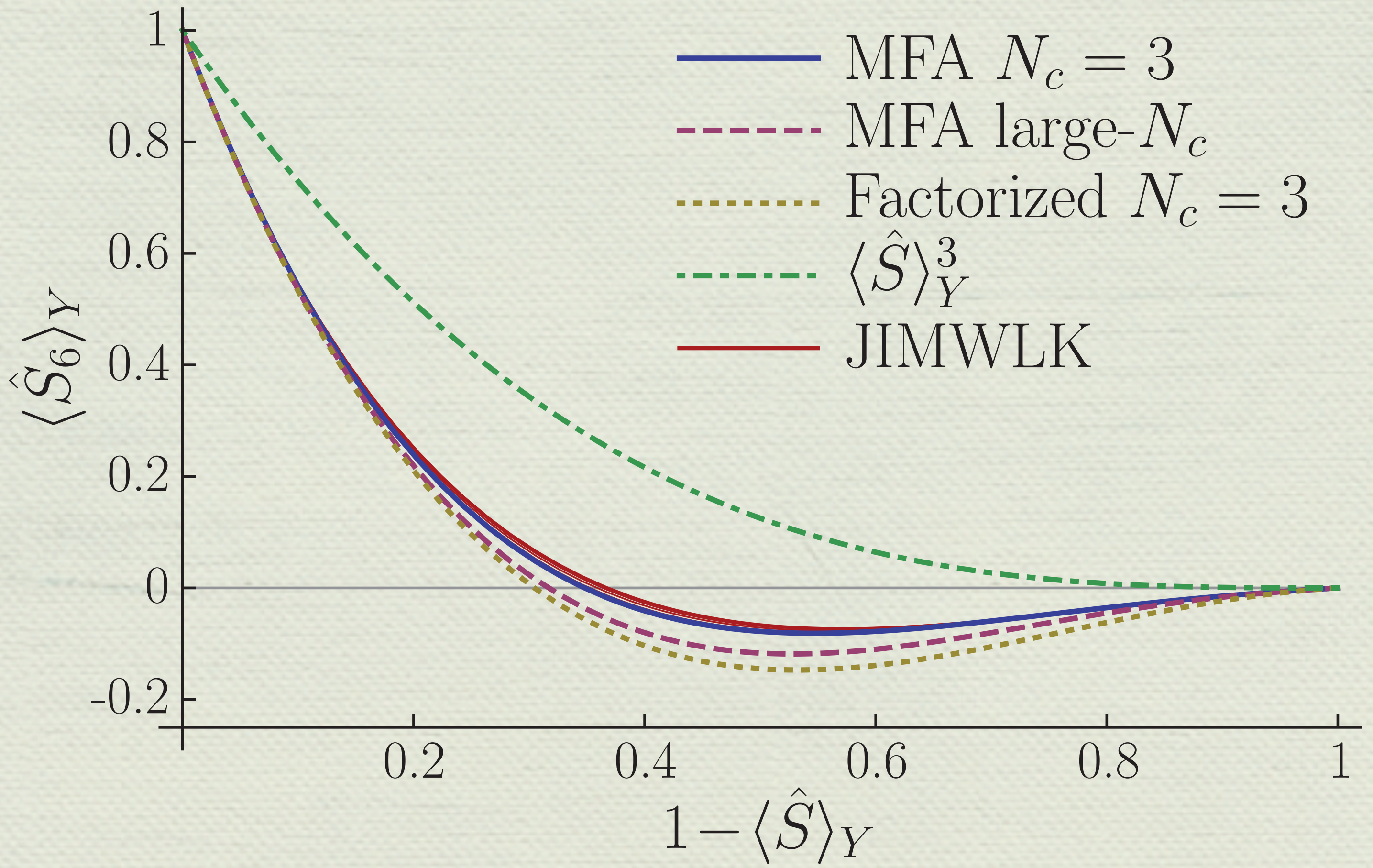




vs numerical solution



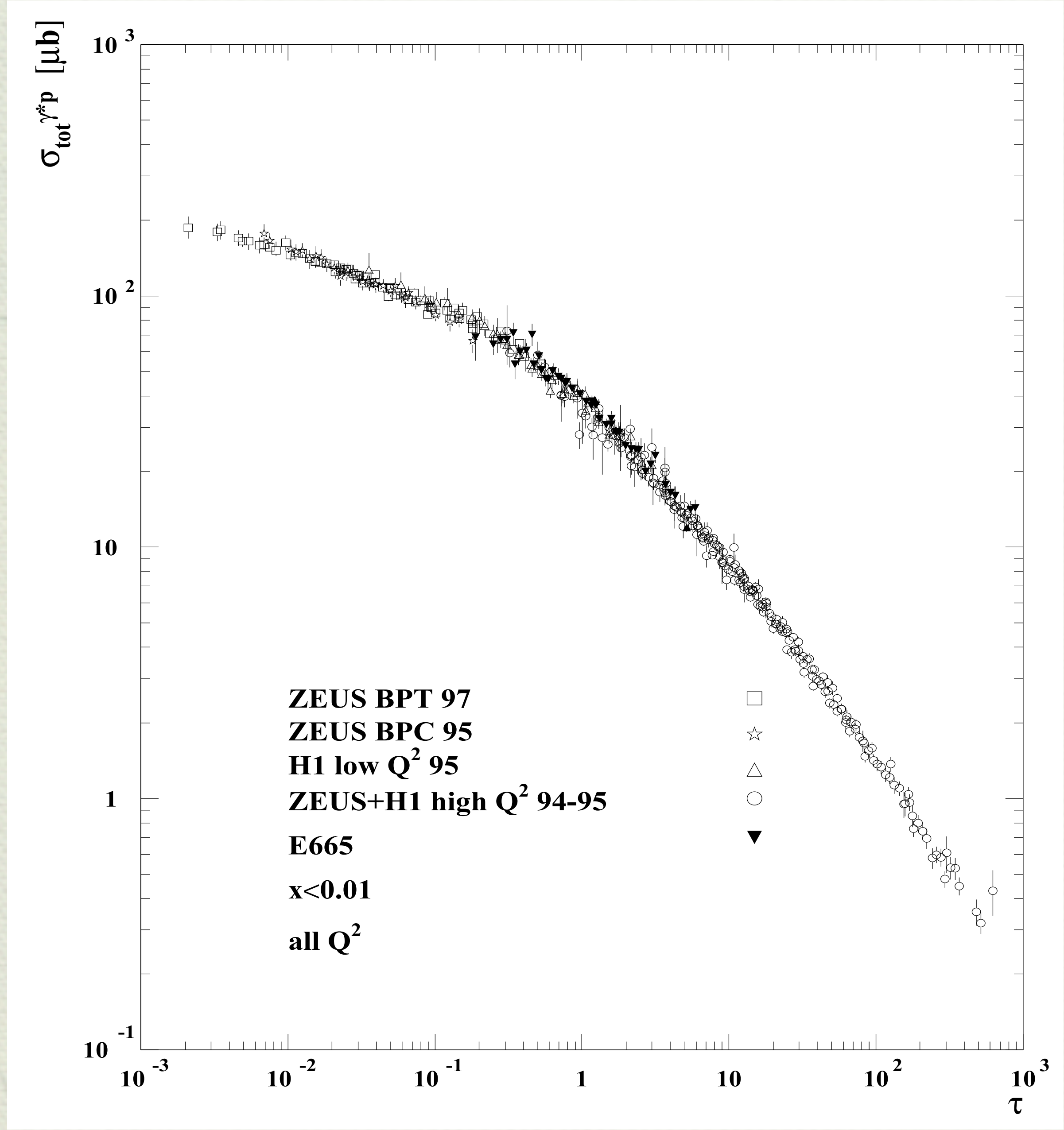
□ Can calculate 6-point functions. Line: easiest configuration

$$\hat{S}_{6\,\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4} = \frac{N_c^2}{N_c^2-1}\,\hat{Q}_{1234}\hat{S}_{43} - \frac{1}{N_c^2-1}\,\hat{S}_{12}$$

$$\langle\hat{Q}\hat{S}\rangle_Y = \frac{(N_c+2)(N_c-1)}{2N_c}\,\langle\hat{S}\rangle_Y^{\frac{3N_c-1}{N_c-1}} - \frac{(N_c+1)(N_c-2)}{2N_c}\,\langle\hat{S}\rangle_Y^{\frac{3N_c+1}{N_c+1}}$$

Scaling

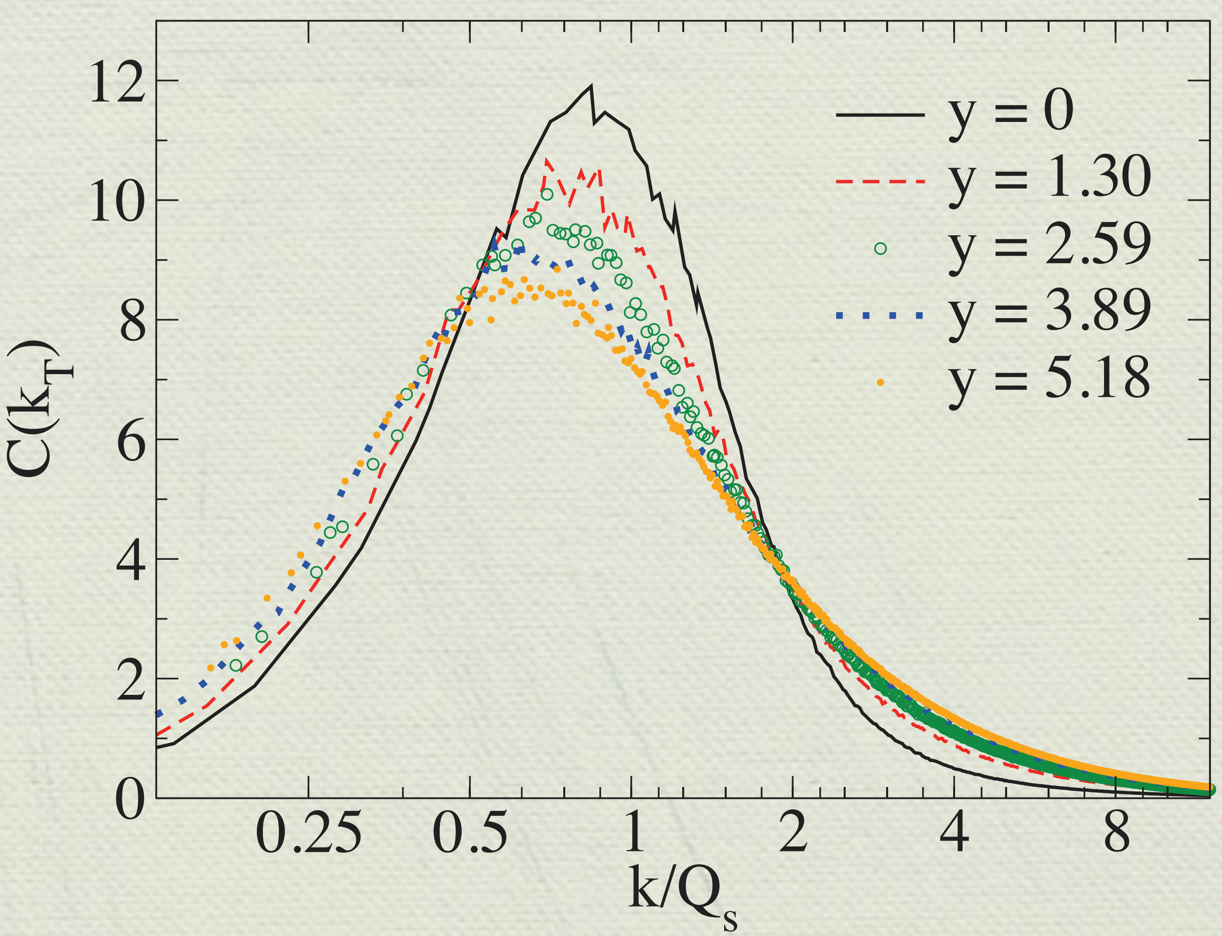
- Qs(Y) sets the scale: dependence only on rQs(Y) around Qs

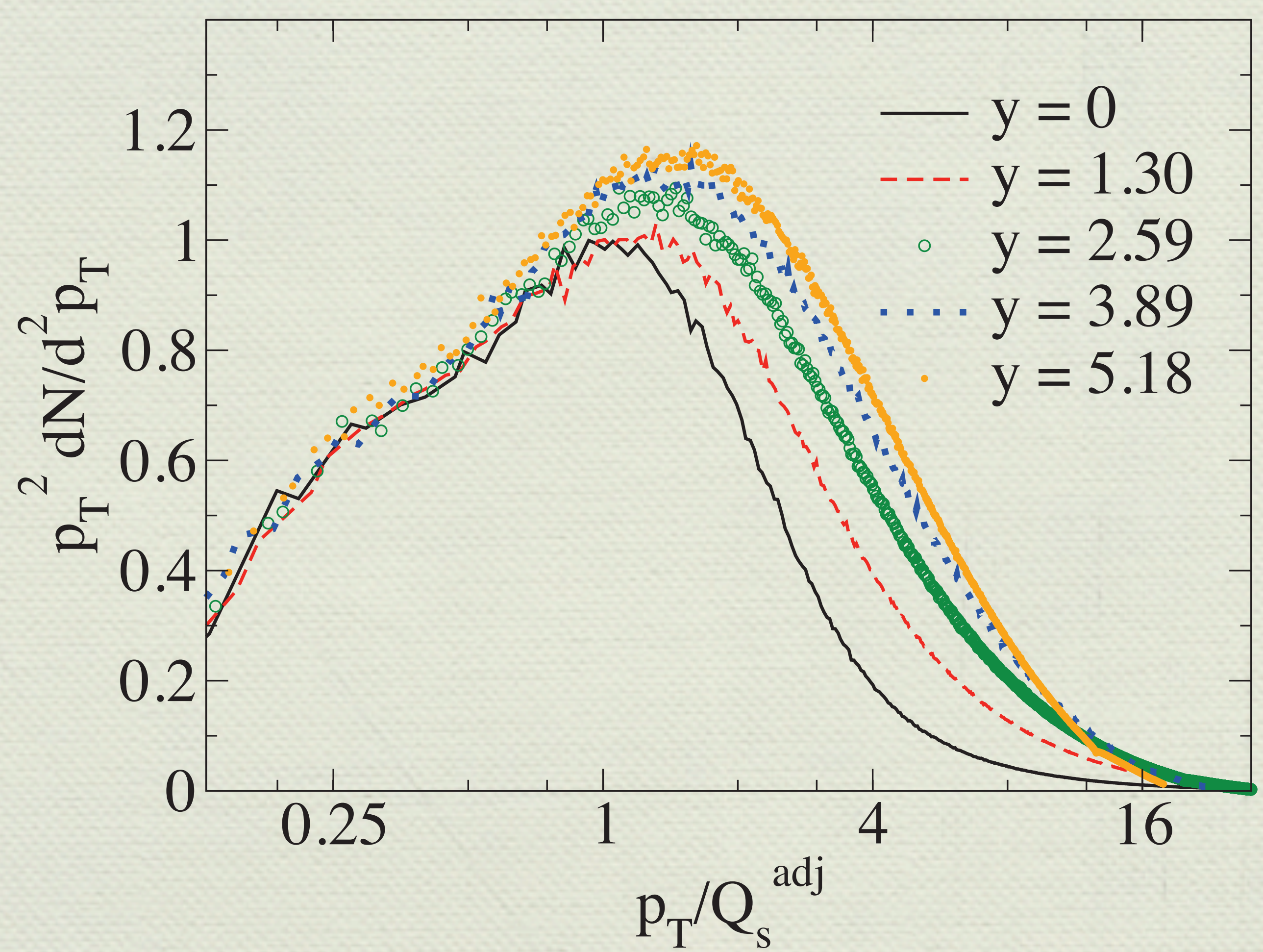


Gluon distribution and gluon production in AA

□ Distribution $$C(Y,\boldsymbol{p}) = \boldsymbol{p}^2 \int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{r}\cdot\boldsymbol{p}} \frac{1}{N_c} \left\langle \mathrm{tr}[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})] + \dots \right\rangle_Y$$

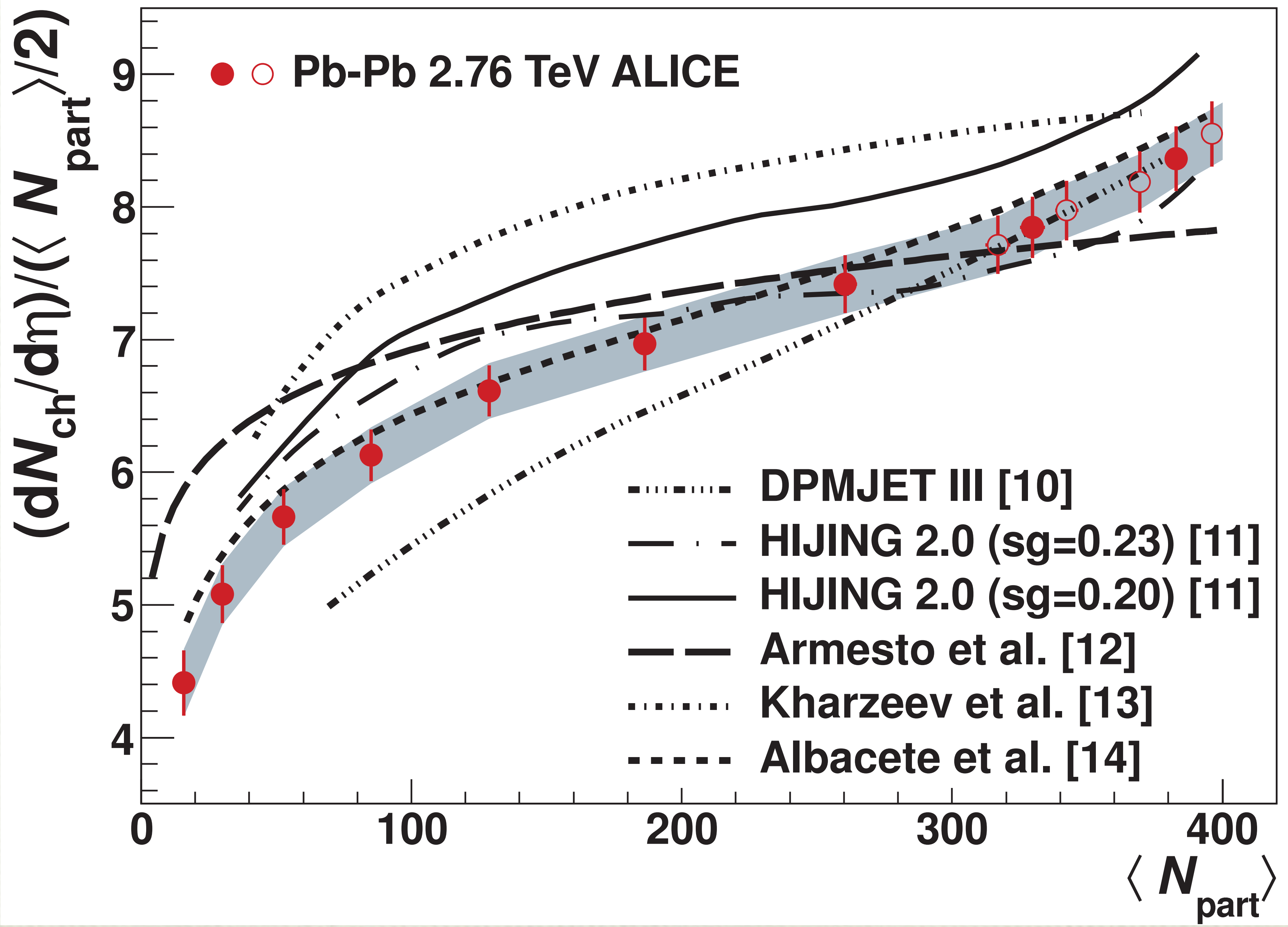
□ Production: collide two "color glasses"





Multiplicities in AA

- Multiplicities should not be affected by final state



Conclusion

- □ Color Class Condensate <-> Parton saturation
- □ JIMWLK evolution under control
- □ Applications in Heavy Ion Collisions
- □ Consistent where it is supposed to work