QUARK CONFINEMENT IN THE HEAVY MASS LIMIT

Carina Popovici

JLU Giessen

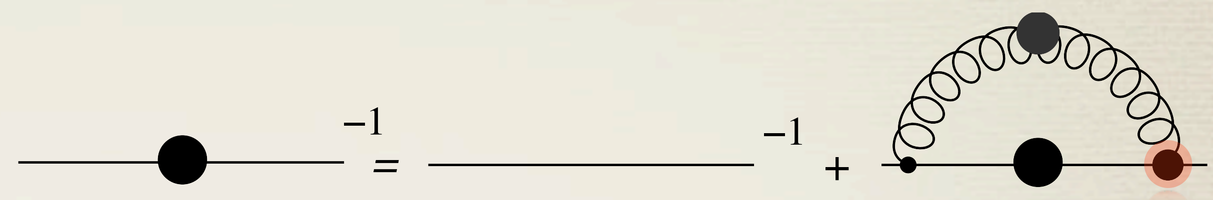
Excited QCD in Peniche, 11 May 2012

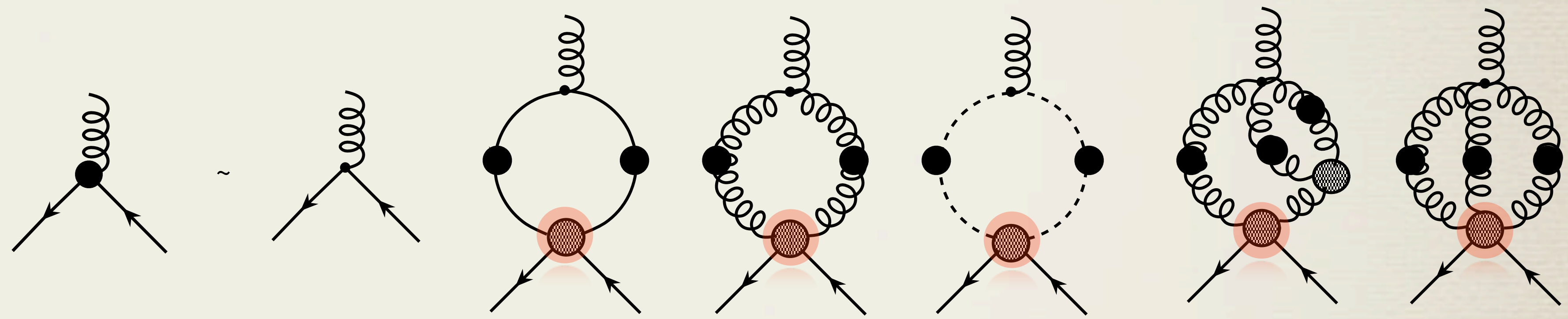
[CP, Watson, Reinhardt, Phys.Rev.D81 (2010)105011; Phys.Rev.D83(2011) 025013;Phys.Rev. D83 (2011) 125018]

- Dyson-Schwinger Equations
- Heavy quarks in coulomb gauge
- mesons/ baryons
- summary

Dyson-Schwinger Equations

- Equations of motion for the n-point Green‘s functions of a theory
- Example: Quark propagator





Dyson-Schwinger Equations

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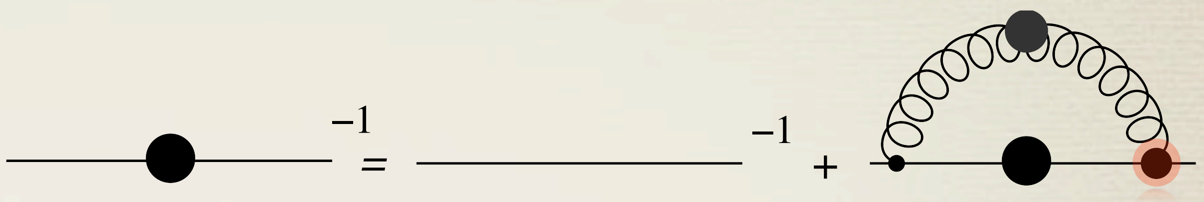

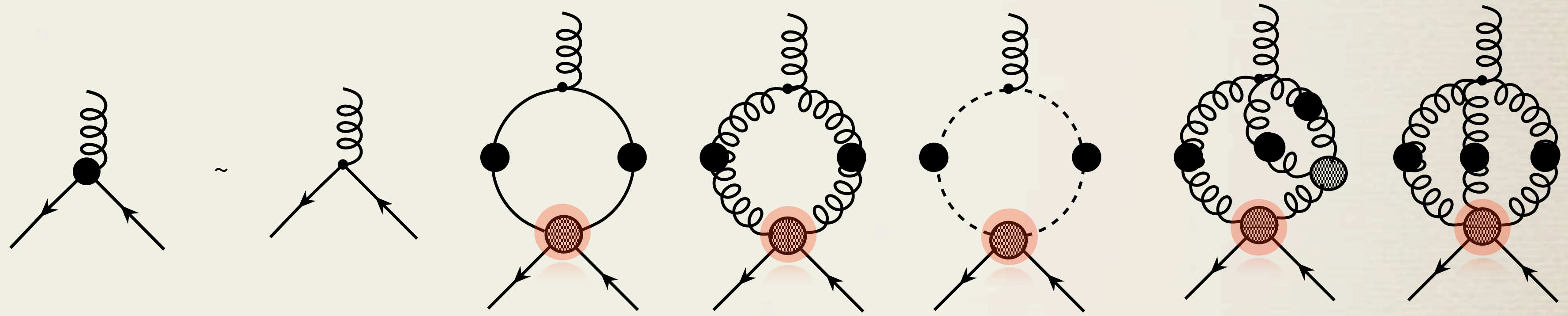

- Infinite tower of coupled integral equations ➡ Truncate the system!
    - Neglect higher-order n-point functions
    - Better: make Ansätze that respect symmetries (WT, ST)

Coulomb Gauge

$$\vec{\nabla} \cdot \vec{A}^a = 0 \qquad \text{(noncovariant!)}$$

➡ Gauge field A_μ: *spatial* ($\vec{A}$) and *temporal* (A^0) (*two* gluon propagators!)

- Understand quark confinement: Gribov-Zwanziger scenario
  - IR: *temporal* gluon propagator enhanced ➡ confining force

    spatial gluon propagator suppressed

  [Gribov, 1978; Zwanziger, 1998]

- Physics is gauge invariant: compare and learn more about covariant, e.g. Landau gauge DSEs studies

  [Alkofer, Binosi, Dudal, Fischer, Llanes-Estrada, Maas, Oliveira,Pawlowski, Papavassiliou, Pennington, Roberts, Silva, von Smekal etc...]

Heavy Quarks

- Heavy quark mass expansion [HQET] of QCD action [Neubert, 1994]
- Truncations
  - restrict to leading order in the mass expansion
  - set the Yang-Mills vertices to zero
- Decomposition

$$q(x) = e^{-\imath m x_0}\left[h(x) + H(x)\right]$$

$$h(x) = e^{\imath m x_0}\frac{\mathbf{1}+\gamma^0}{2}q(x)$$

$$H(x) = e^{\imath m x_0}\frac{\mathbf{1}-\gamma^0}{2}q(x)$$

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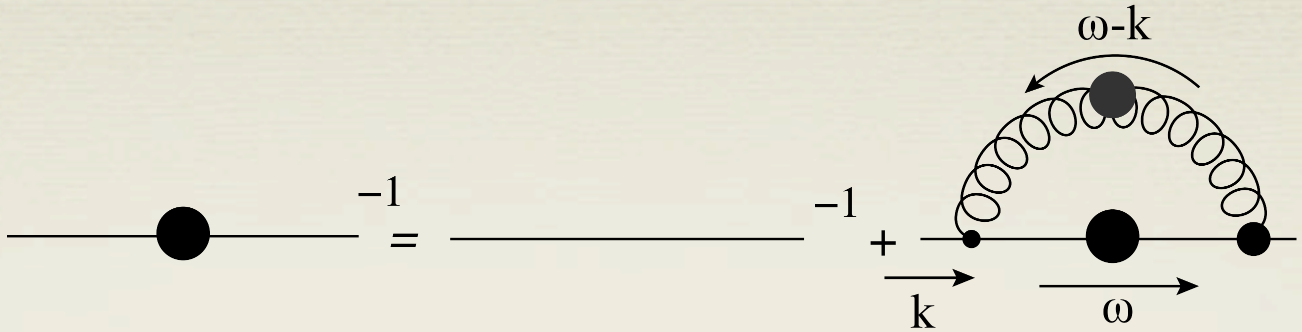
- Quark contribution to QCD action at LO in the mass expansion

$$\mathcal{S}_q = \int d^4x\,\overline{h}\left(\imath\partial_0 + gT^a A_0^a\right)h + \mathcal{O}\left(1/m\right)$$

  - No *gamma* matrices
  - The quark only couples to the temporal component of the gluon
  - Only time derivative, so no $\vec{k}$ dependence

Heavy Quark Propagator



$$[W_{\overline{q}q}(k)]^{-1} = \left[W^{(0)}_{\overline{q}q}(k)\right]^{-1} + gT^a \int \bar{d}\,\omega W_{\overline{q}q}(\omega)\Gamma^b_{\overline{q}qA_0}(\omega, -k, k-\omega) W^{ab}_{A_0}(k-\omega)$$

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$$W^{(0)}_{\bar{q}q}(k_0) = \frac{(-i)}{k_0 - m + \imath\varepsilon} + \mathcal{O}(1/m)$$

$$W^{ab}_{A_0}(k) = \delta^{ab} W_{A_0}(\vec{k})$$

[Cucchieri, Zwanziger, 2001; Quandt et al, 2008]

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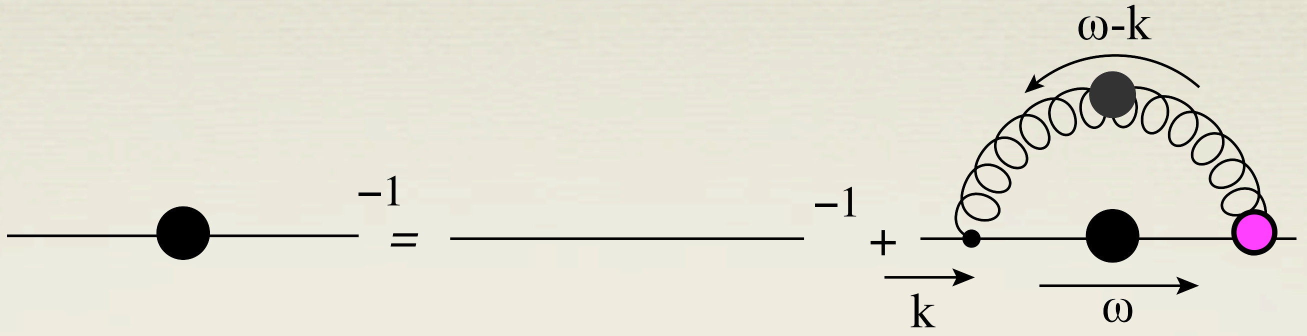

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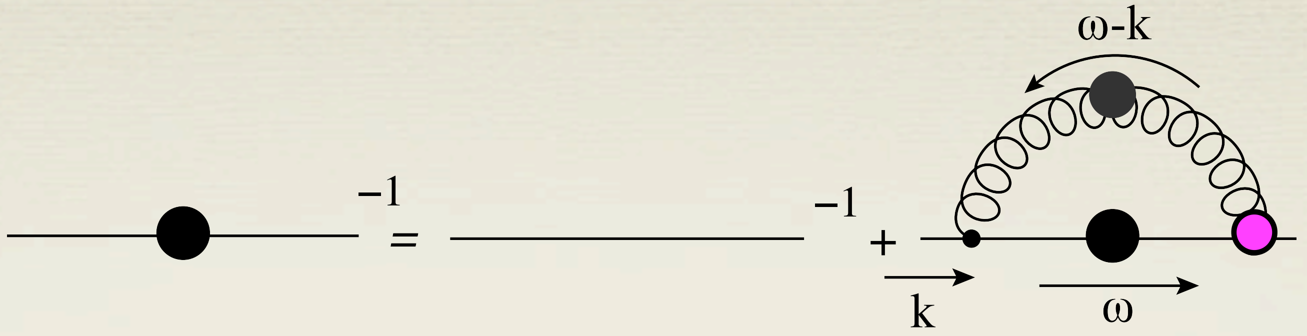
- STid : $\Gamma^d_{\bar{q}qA_0}(k_1, k_2, k_3) = \frac{ig}{k_3^0} T^d \left\{[W_{\bar{q}q}(k_1)]^{-1} - [W_{\bar{q}q}(-k_2)]^{-1}\right\} + \mathcal{O}(1/m)$

Heavy Quark Propagator



$$[W_{\bar q q}(k)]^{-1} = \left[W^{(0)}_{\bar q q}(k)\right]^{-1} + gT^a \int \!\! d\!\!{}^-\omega\, W_{\bar q q}(\omega)\, \Gamma^b_{\bar q q A_0}(\omega, -k, k-\omega)\, W^{ab}_{A_0}(k-\omega)$$

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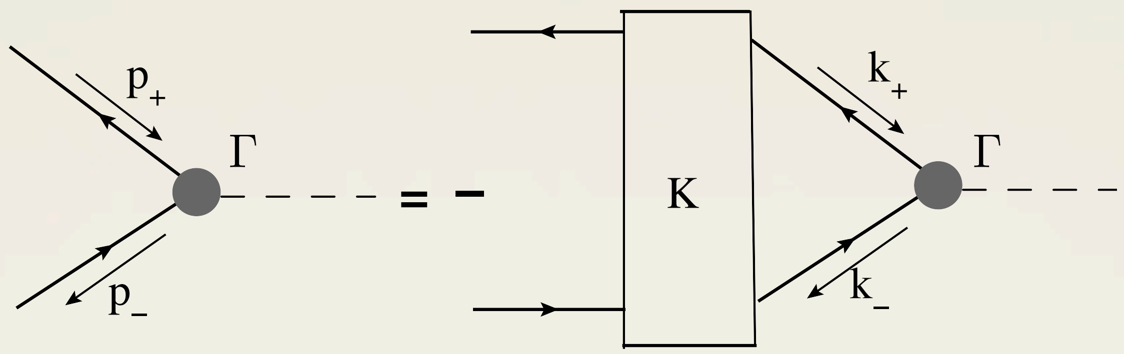
- Exact solution (under truncation): $[W_{\bar q q}(k)]^{-1} = i\left[k_0 - m - \frac{1}{2} g^2 C_F \int d\!\!{}^-\vec\omega\, W_{A_0}(\vec\omega)\right]$
  - Temporal quark-gluon vertex remains ***bare***: $\Gamma^d_{\bar q q A_0}(k_1, k_2, k_3) = gT^d + \mathcal{O}(1/m)$
  - Gap equation reduces to rainbow truncation!

✤ Use these results to study bound state equations

Mesons (Bethe-Salpeter Equation)

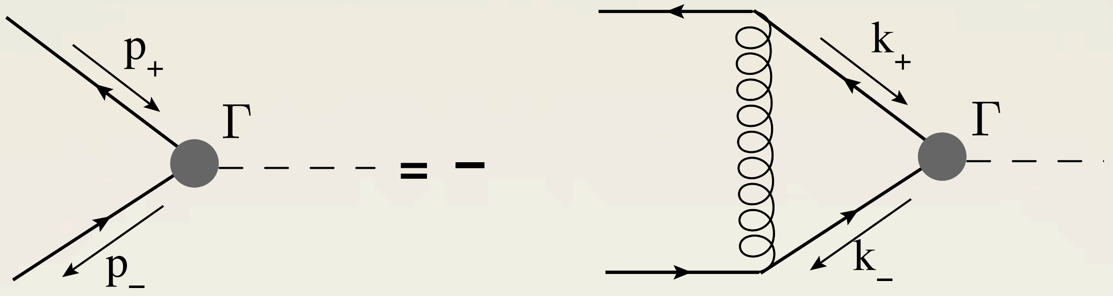
$$\Gamma(p) = -\int \bar{d}\, k\, K(p,k) W_{\overline{q}q}(k_+)\Gamma(k) W_{\overline{q}q}(k_-)$$



Mesons (Bethe-Salpeter Equation)

$$\Gamma(p) = g^2 \int đk\, W_{A_0}(\vec{p} - \vec{k})[T^a \Gamma(k) T^a] W_{\bar{q}q}(k_+) W_{\bar{q}q}(k_-)$$

- Ladder kernel



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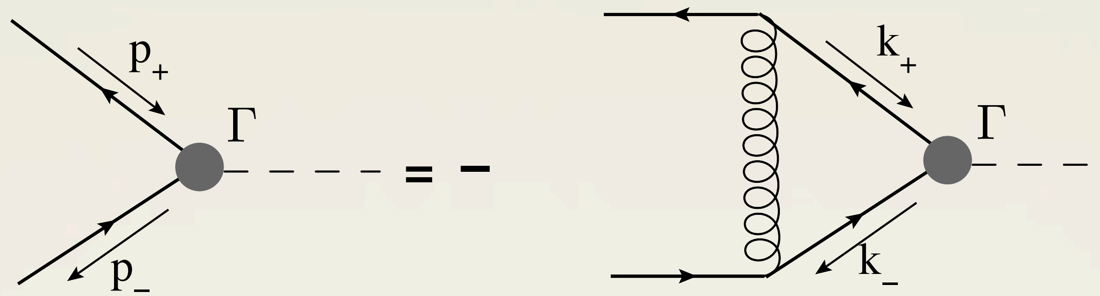

- No assumption on the meson vertex : $[T^a \Gamma(\vec{r}) T^a]_{\alpha\beta} = C_M \Gamma_{\alpha\beta}(\vec{r})$

- Total energy of the $\bar{q}q$ pair:

$$P_0 = g^2 \int đ\vec{\omega}\, W_{A_0}(\vec{\omega}) \left[C_F - e^{\imath \vec{\omega}\cdot\vec{r}} C_M \right]$$

[C_F Casimir operator; C_M, $W_{A_0}(\vec{\omega})$ to be identified]

➡ infrared confining: $P_0 = \sigma|\vec{r}|$

- temporal gluon propagator: $W_{A_0}(\vec{\omega}) \sim 1/\vec{\omega}^4$
- only ***color singlet*** meson states physically allowed: $C_F \equiv C_M$

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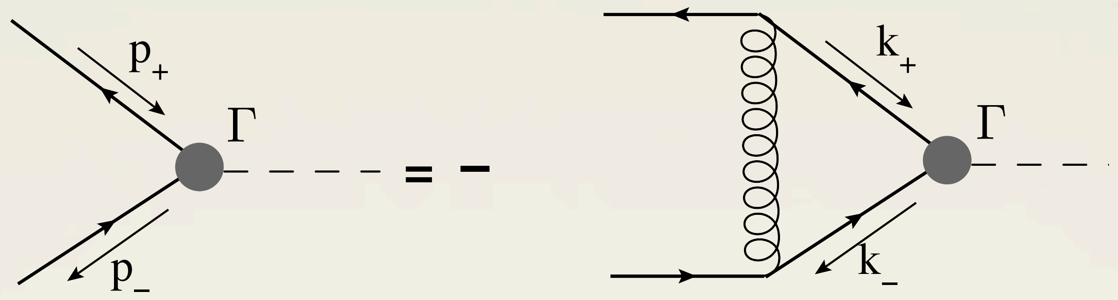

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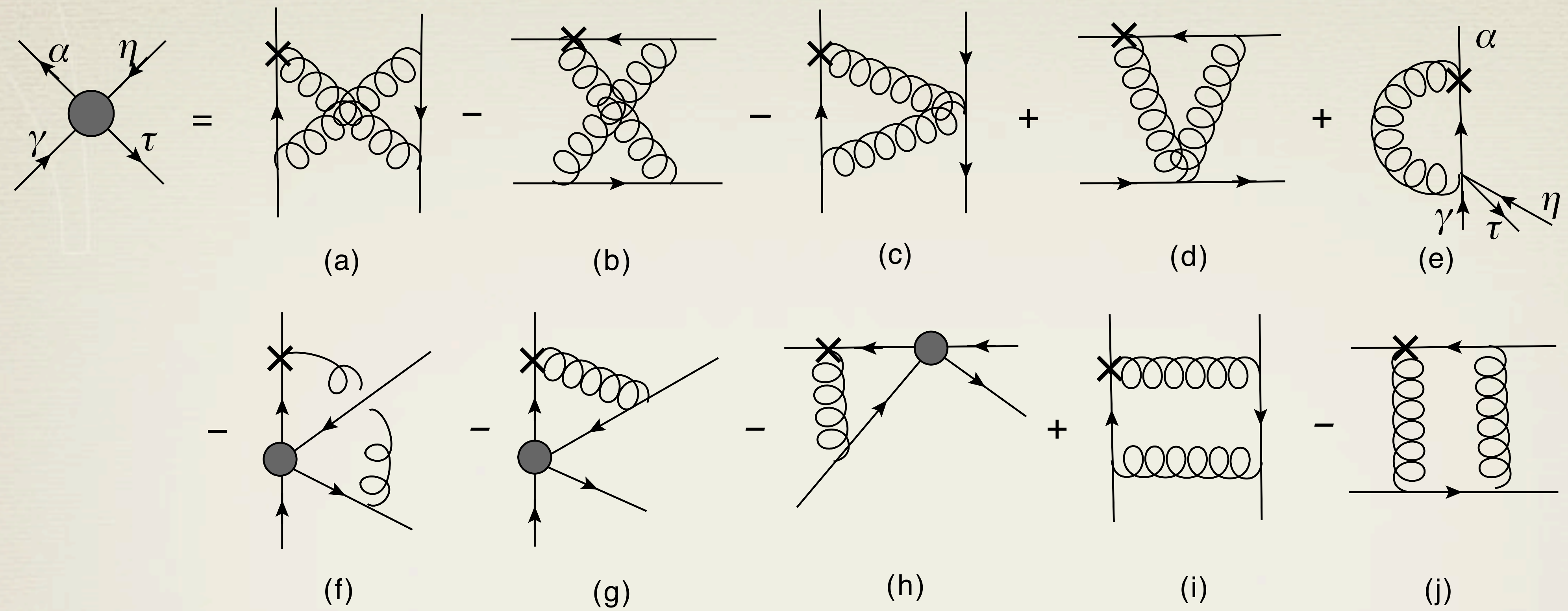
- More general: 4-pct Greens' functions

Four-point Green's Function



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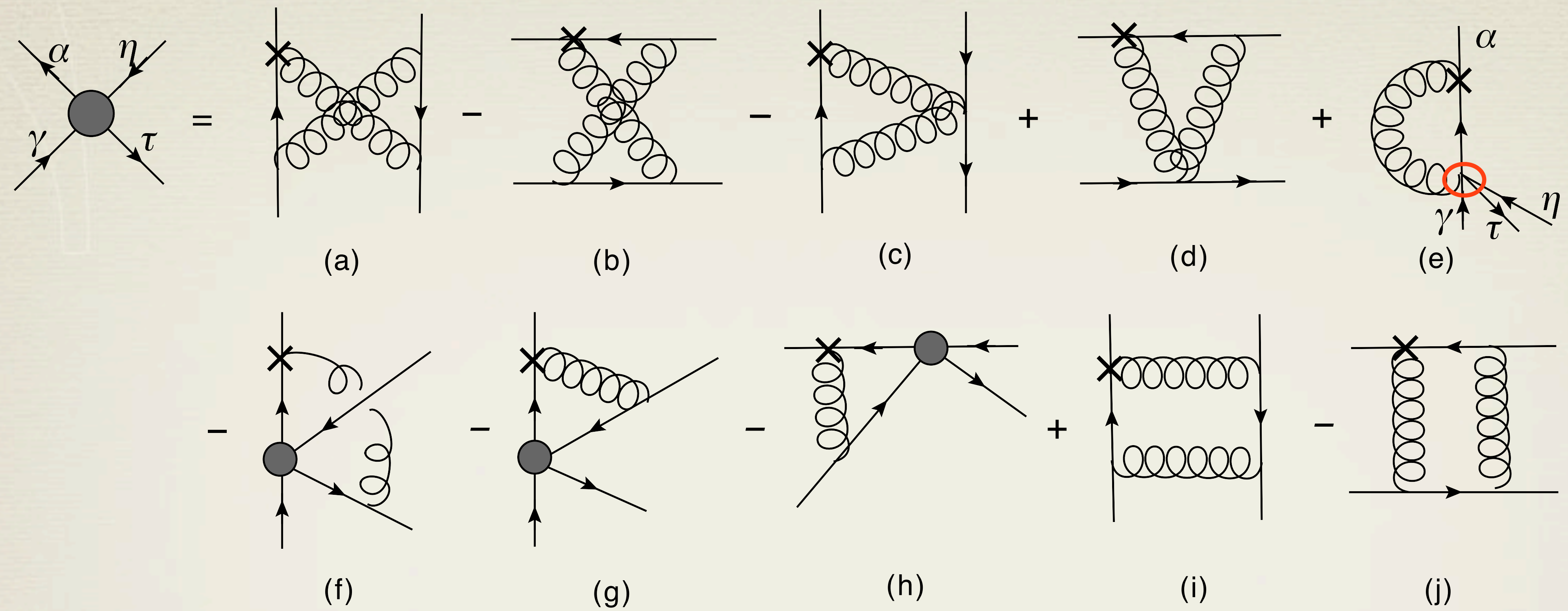

- Heavy quarks, in Coulomb gauge:
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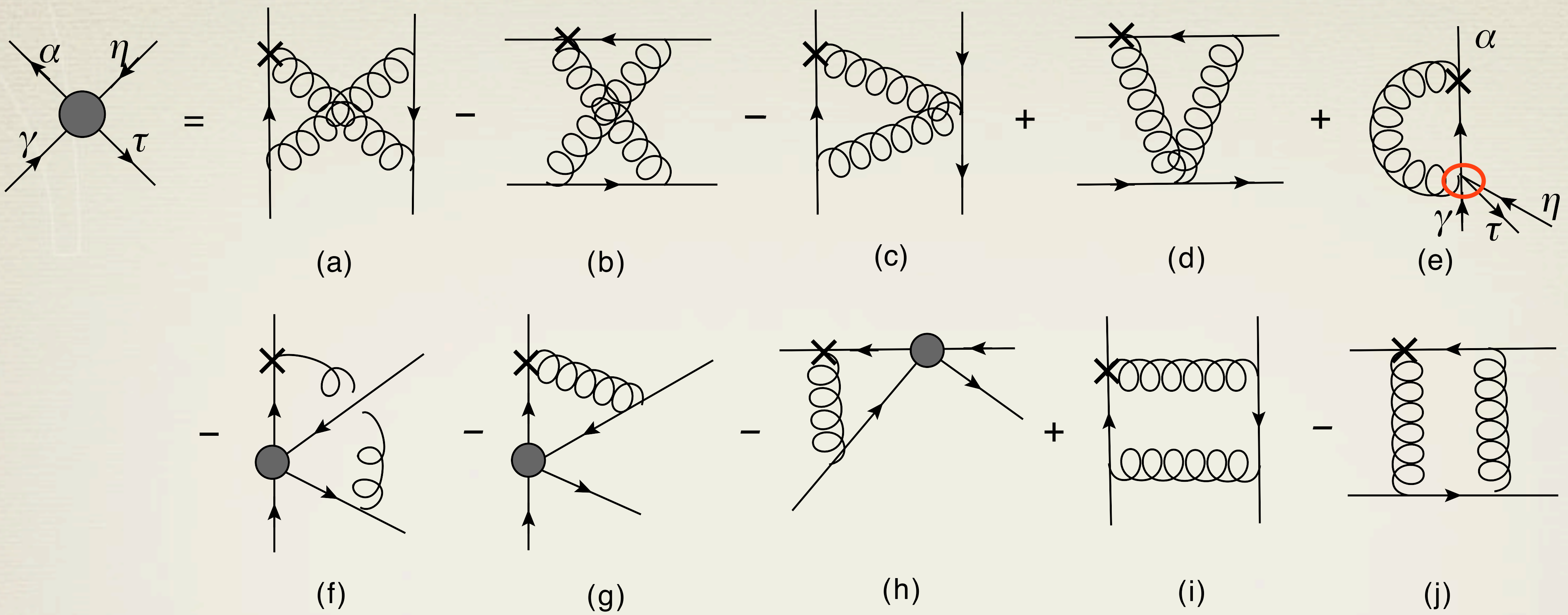
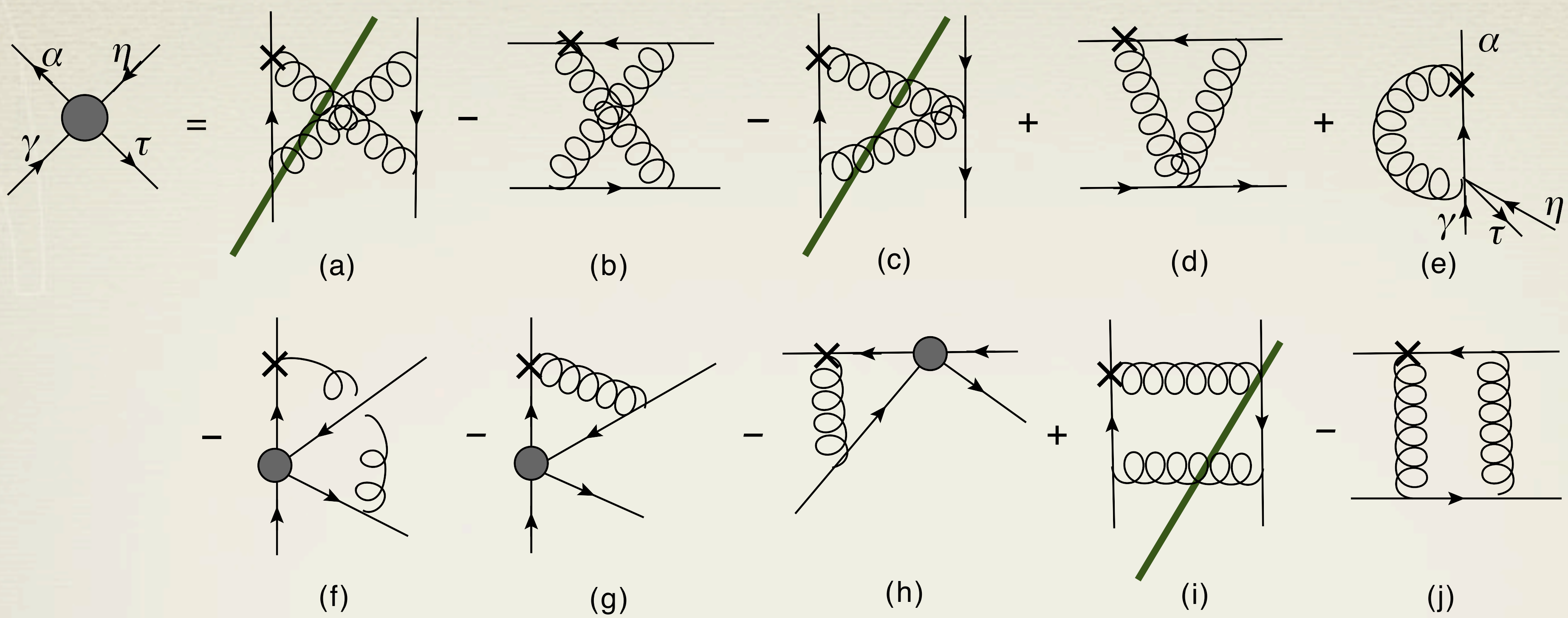


- Heavy quarks, in Coulomb gauge:
  - 5-point function vanishes
  - Input: quark propagator
  - Output: resonance position (bound state energy as function of separation)

    \+ offshell information

Solution in the Heavy Mass Limit



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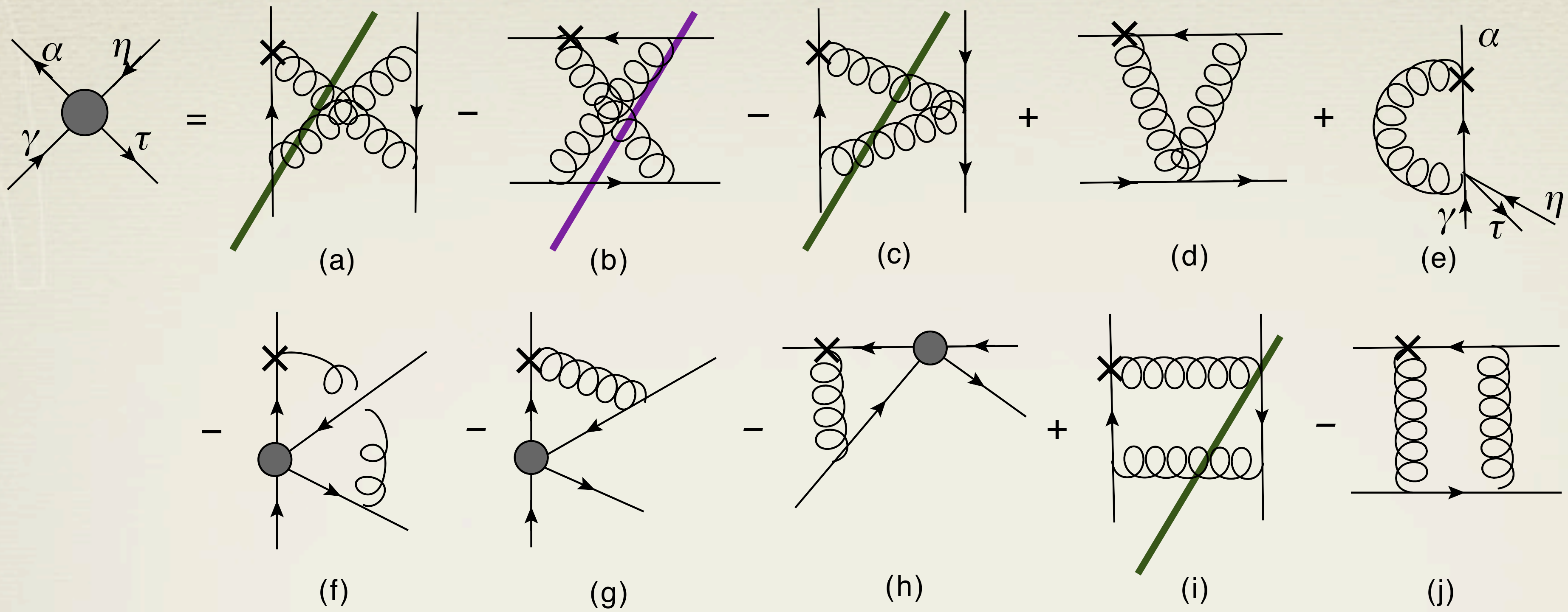

- flavor nonsinglet
- energy integral

$$\int \frac{dk_0}{\left[k_0 - m - \frac{g^2 C_F}{2}\int_r đ\,\vec{\omega}\, W_{A_0}(\vec{\omega}) + \imath\varepsilon\right]\left[k_0 + p_0 - m - \frac{g^2 C_F}{2}\int_r đ\,\vec{\omega}\, W_{A_0}(\vec{\omega}) + \imath\varepsilon\right]} = 0$$

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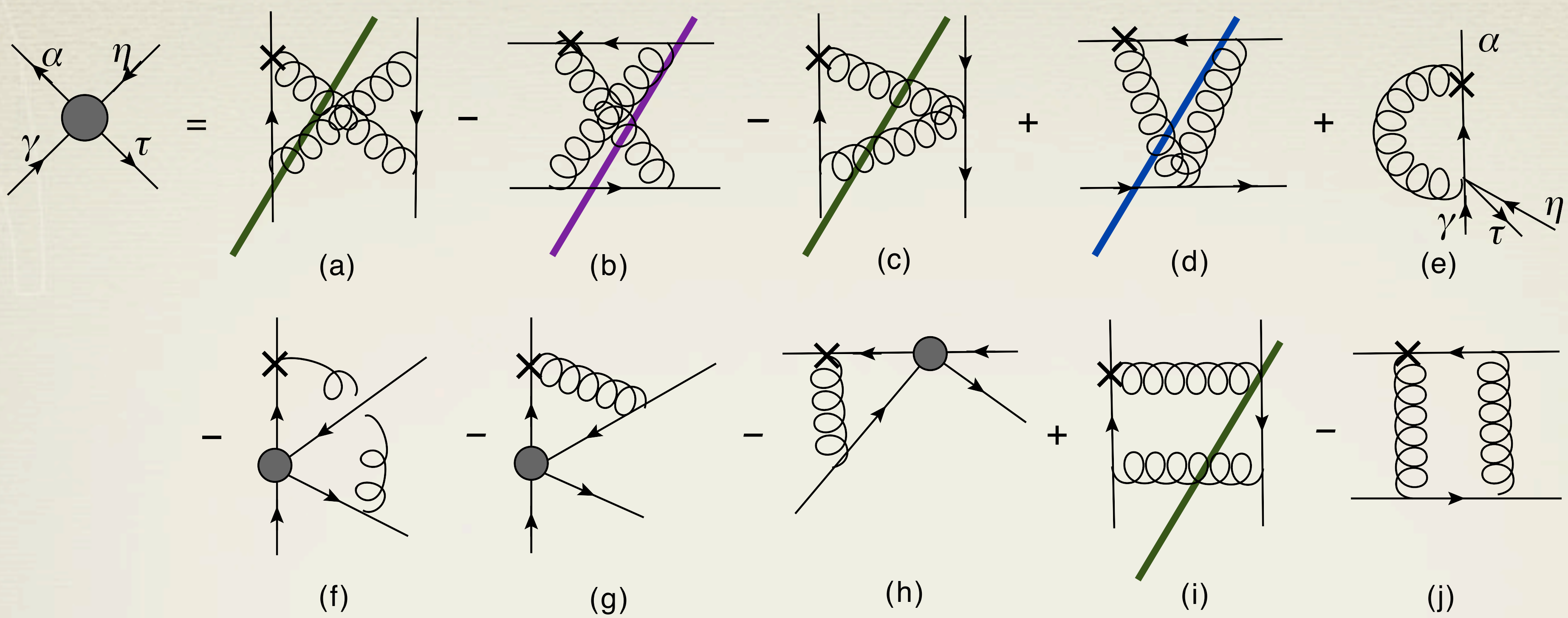

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- vanishing quark-2 gluon vertex (STid)

$$\Gamma^{de}_{\bar{q}q\sigma\sigma} \sim \imath[T^e, T^d] + f^{eda} T^a = 0$$

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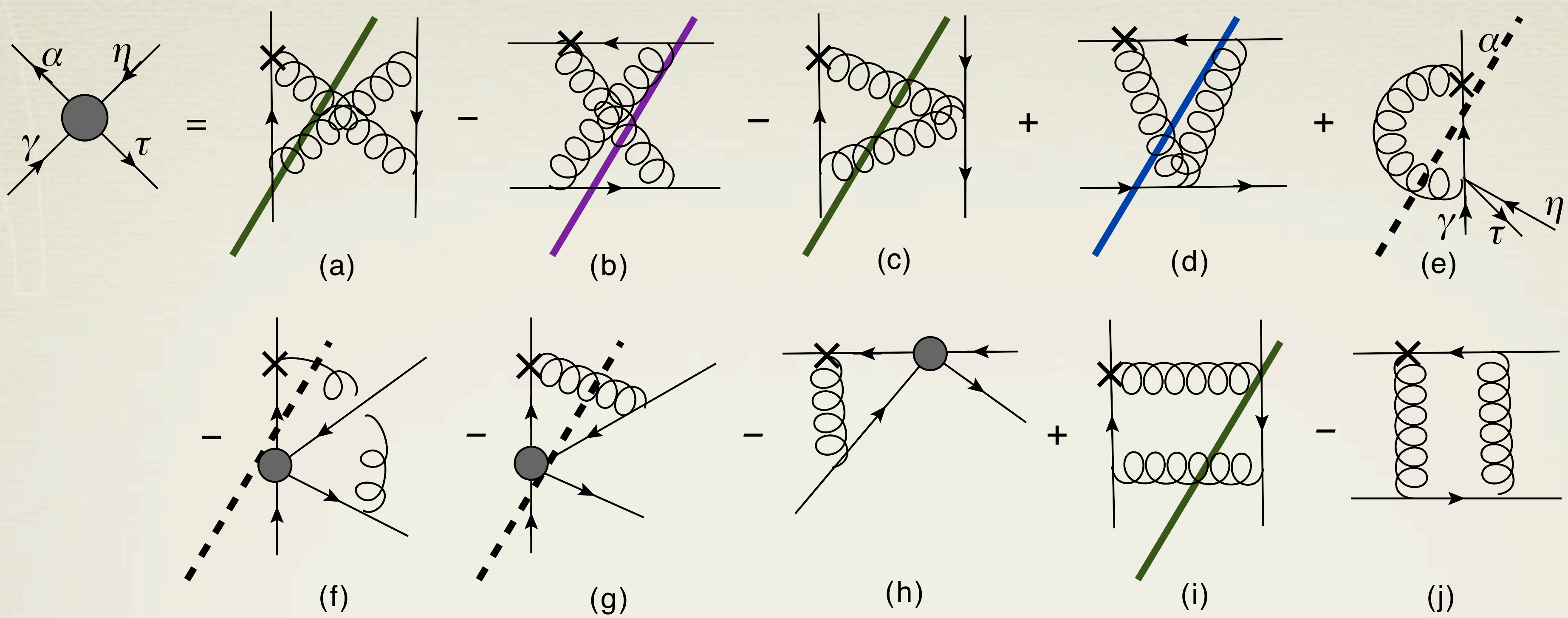

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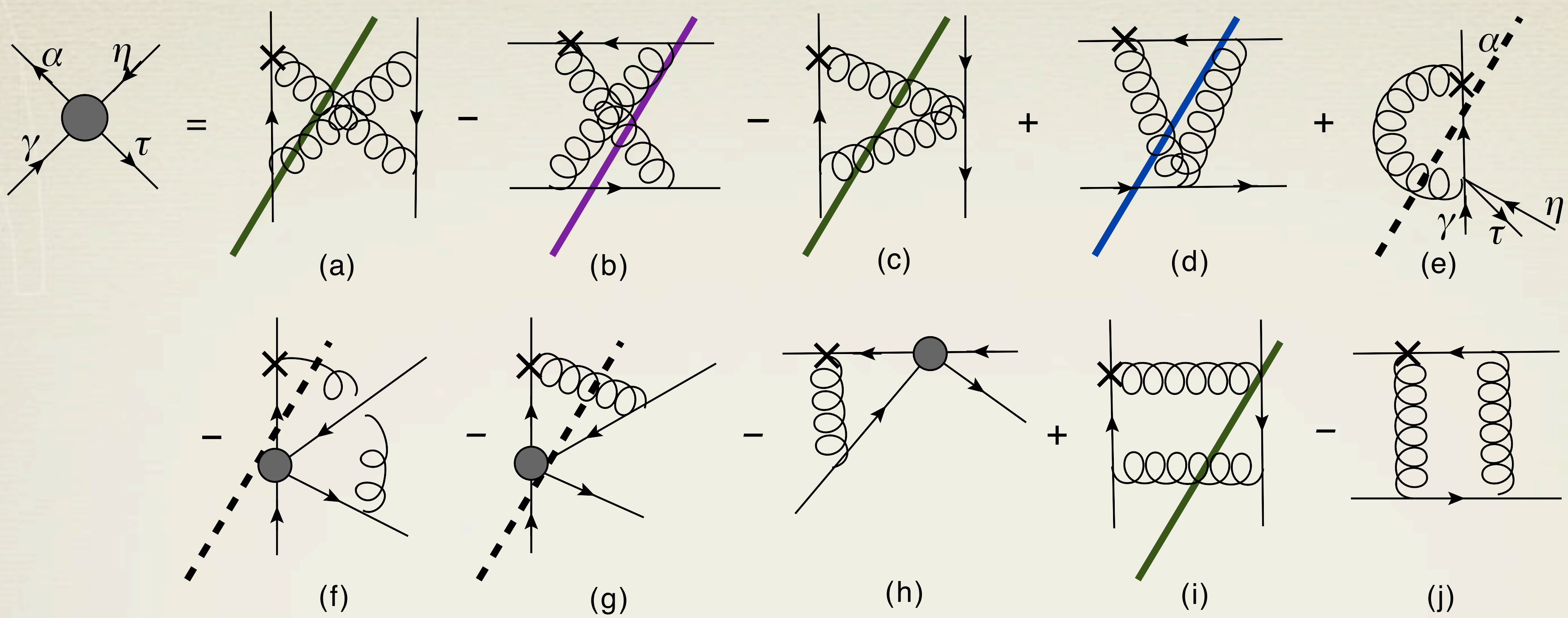

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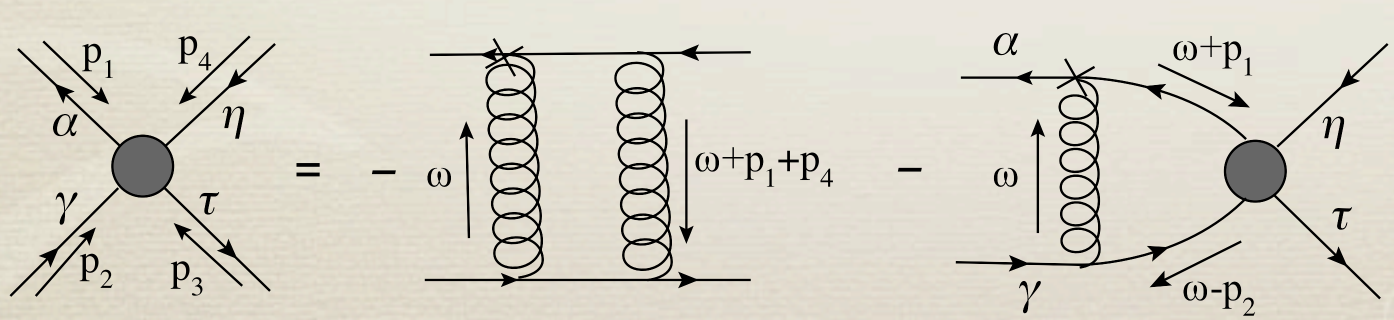

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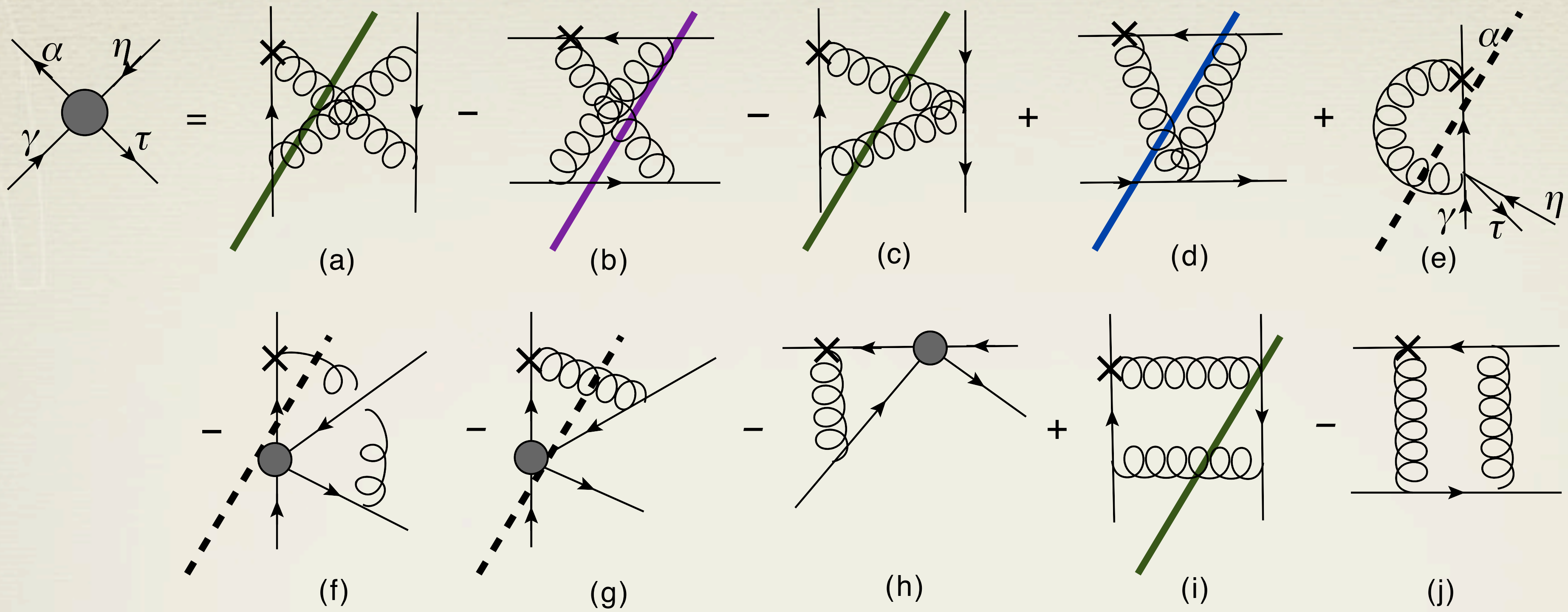

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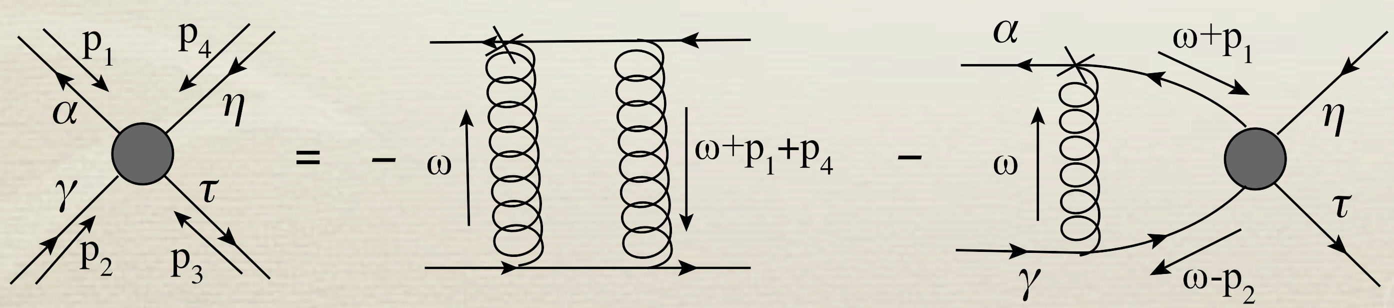
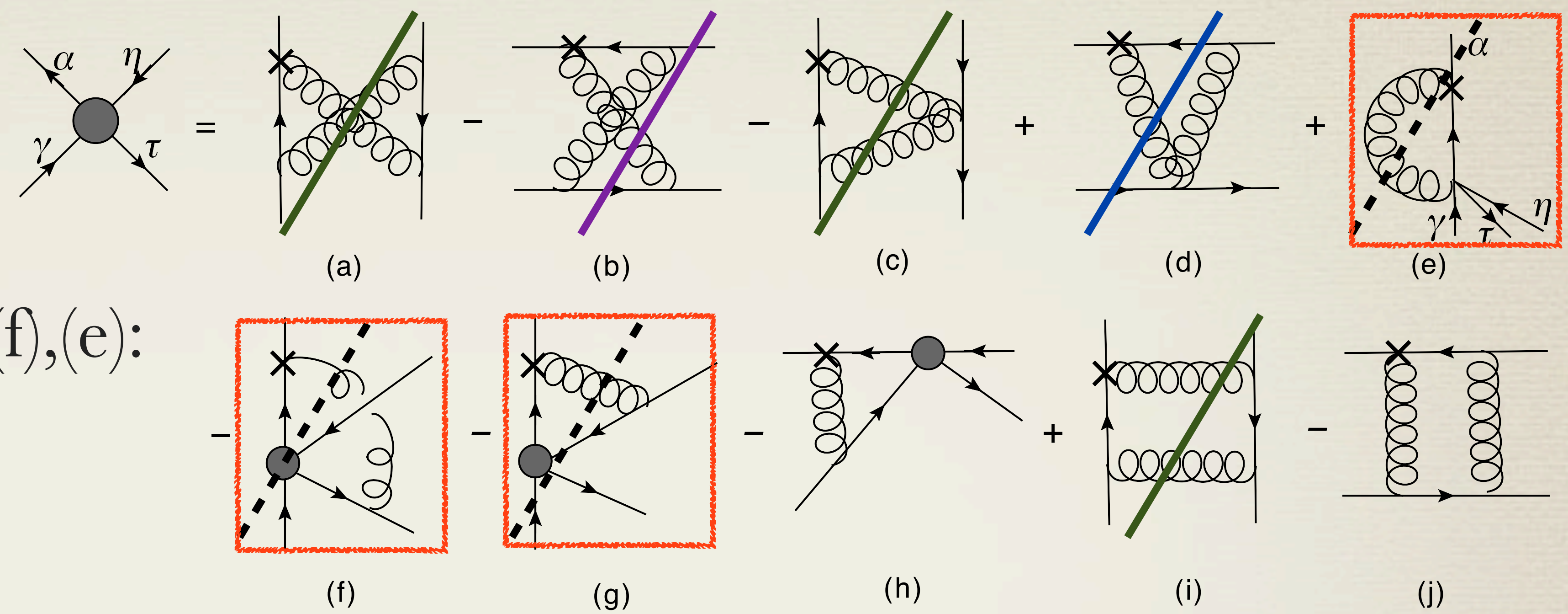


➡ Energy dependence:

$$\Gamma^{(4)}(P_0 + \omega_0) \sim \frac{\omega_0^m}{[\omega_0 + X + \imath\varepsilon]^n}$$

Solution in the Heavy Mass Limit

- Return to diagrams (g), (f),(e):



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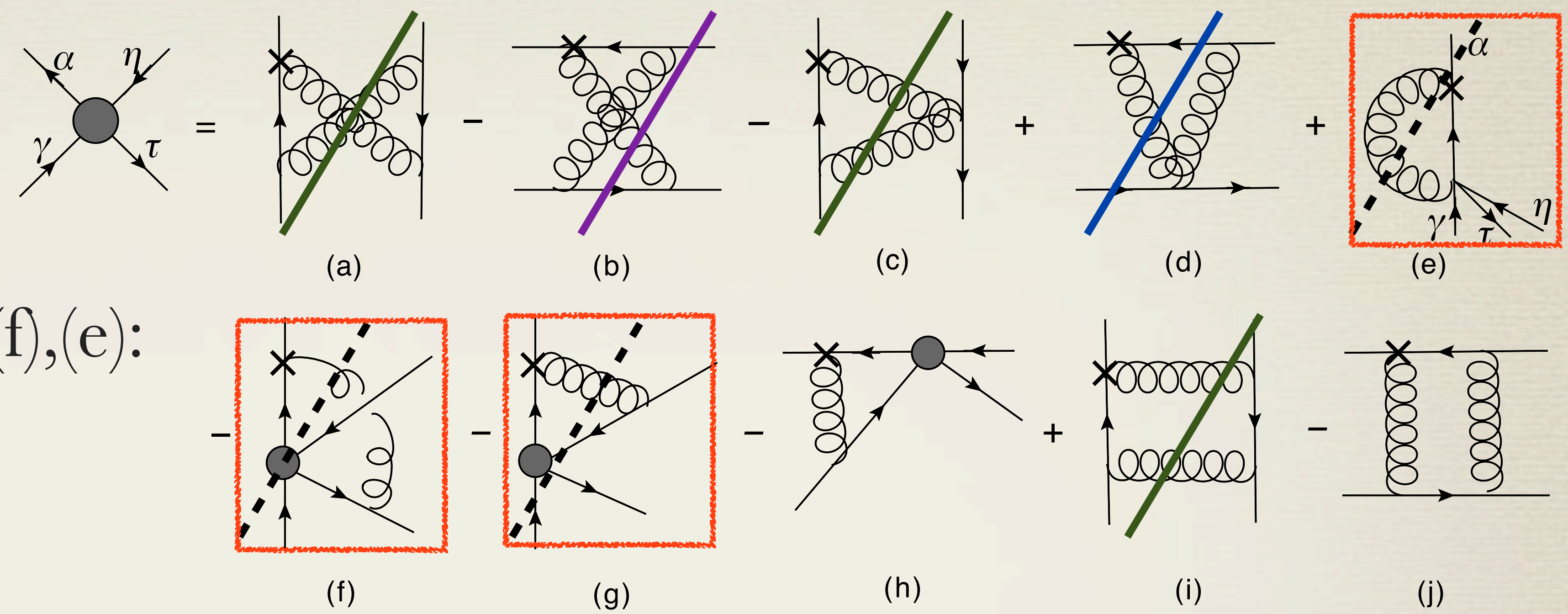

- Return to diagrams (g), (f),(e):

- Energy integral of diagrams (g), (f):

$$\int đ\,\omega_0\,\omega_0^m \prod_{i=1}^{2+n} \frac{1}{[\omega_0 + X_i + \imath\varepsilon]} = 0$$

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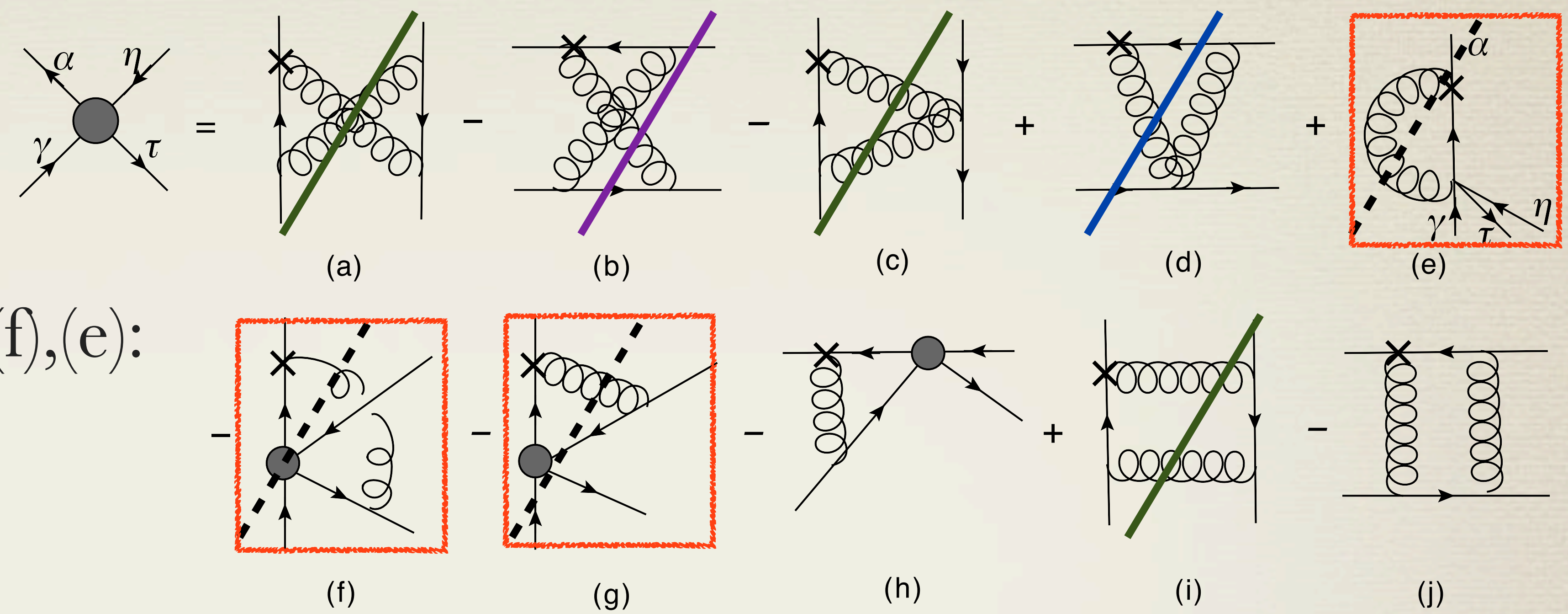

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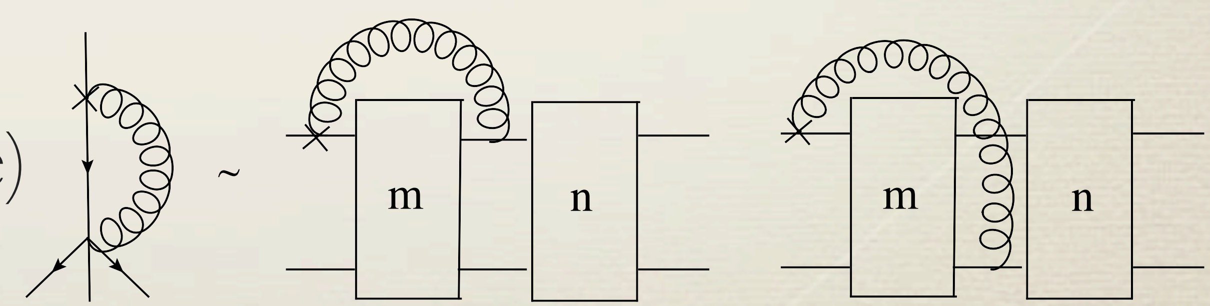

$= 0$

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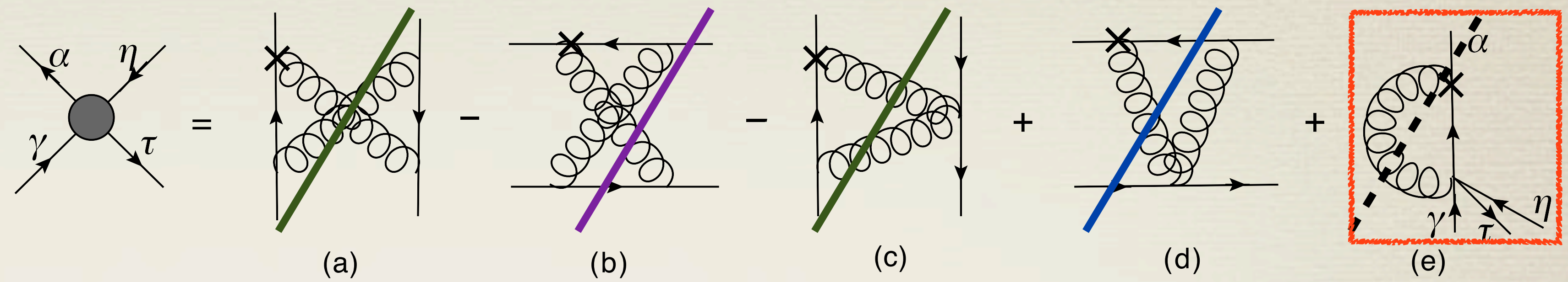

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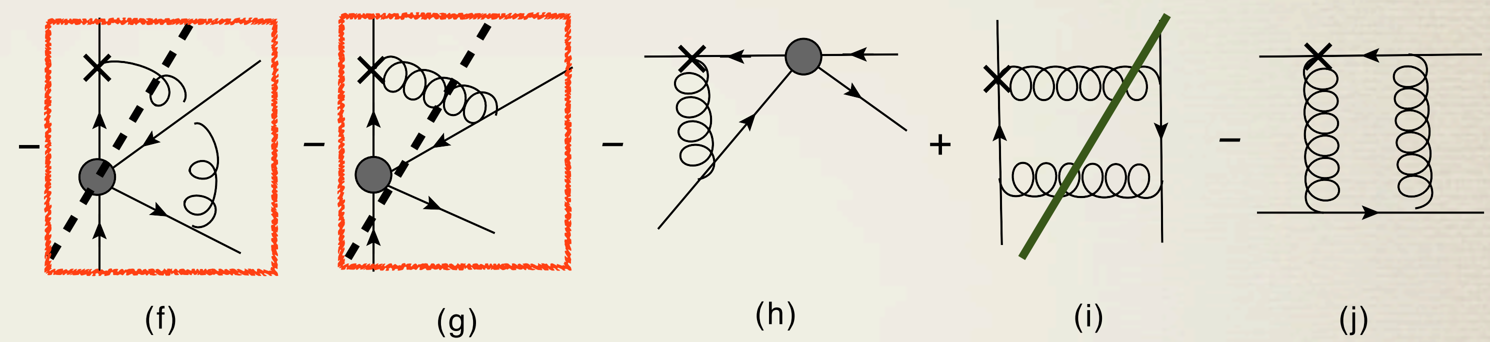

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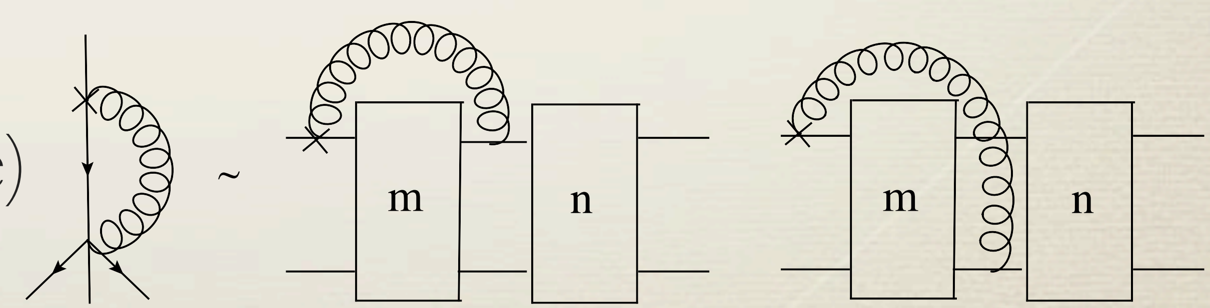
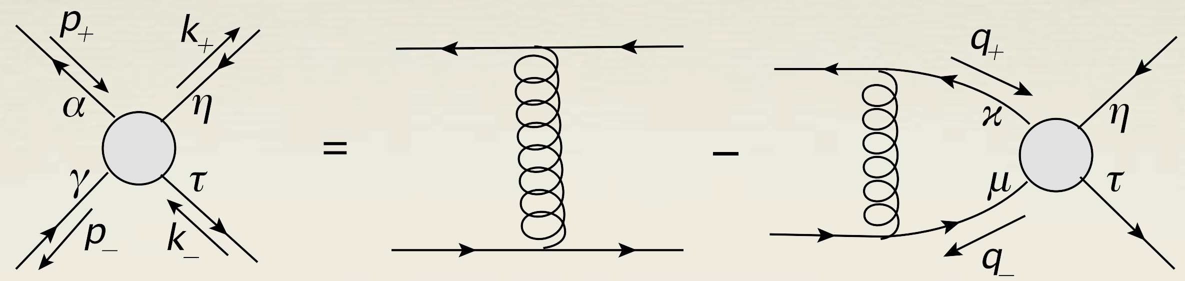


$= 0$

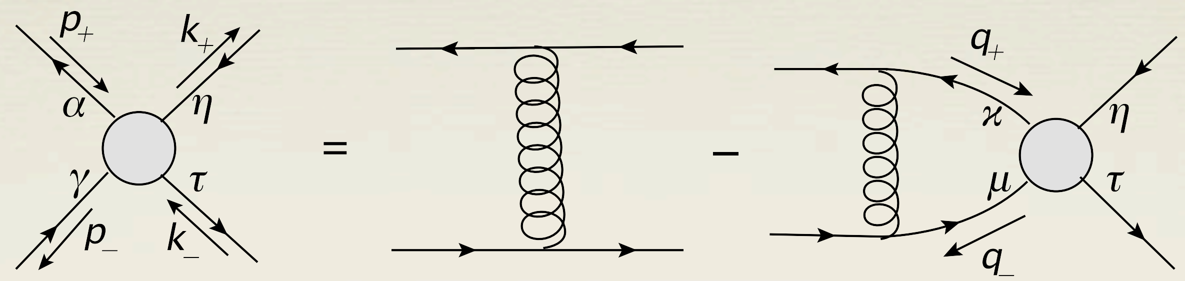
➡ Solution for the Green's function valid at every order perturbatively!

Amputated Green's function



$$G^{(4)}_{\alpha\gamma;\tau\eta}(x) = \frac{g^2}{2}\frac{g_1(x)}{P_0 - g^2\int d\!\!{}^-\vec{\omega}\, W_{A_0,\vec{\omega}}\left[C_F + \frac{e^{i\vec{\omega}\cdot\vec{x}}}{2N}\right] + i\varepsilon}\left\{\delta_{\alpha\gamma}\delta_{\tau\eta}\frac{g_2(x)}{P_0 - g^2 C_F\int d\!\!{}^-\vec{\omega}\, W_{A_0,\vec{\omega}}\,[1 - e^{i\vec{\omega}\cdot\vec{x}}] + i\varepsilon} - \delta_{\alpha\eta}\delta_{\tau\gamma}\frac{1}{N}\right\}$$

Amputated Green's function



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- **Poles:**
  - bound state (infrared confining) energy
    - temporal gluon propagator: $W_{A_0}(\vec{\omega}) \sim 1/\vec{\omega}^4$

$$P_{0\,res}(x) = \sigma|\vec{x}|$$

  - total energy of the $\bar{q}q$ pair

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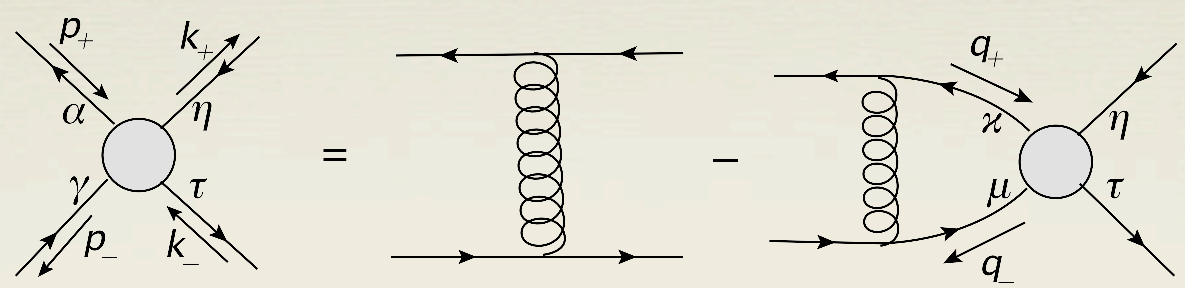

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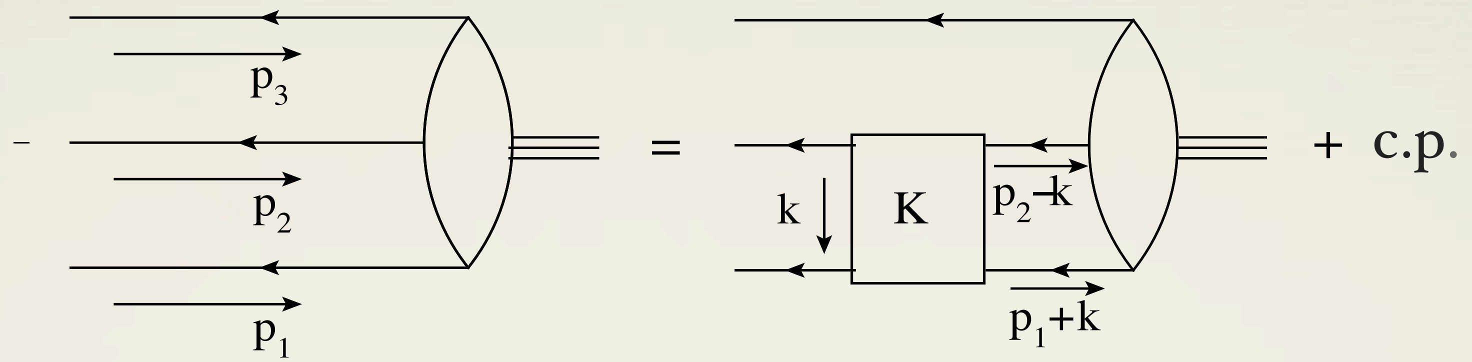
- **Poles:**
  - bound state (infrared confining) energy
    - temporal gluon propagator: $W_{A_0}(\vec{\omega}) \sim 1/\vec{\omega}^4$

$$P_{0\;res}(x) = \sigma|\vec{x}|$$

  - total energy of the $\bar{q}q$ pair
    - part of the normalization
    - does not appear in the homogeneous BSE ➔ ***tetraquarks?***

Baryons (Faddeev Equation)

$$\Gamma(p_1,p_2,p_3) = -\int đk\, K(k) W_{\bar{q}q}(p_1+k) W_{\bar{q}q}(p_2-k)\Gamma(p_1+k,p_2-k,p_3) + cp$$



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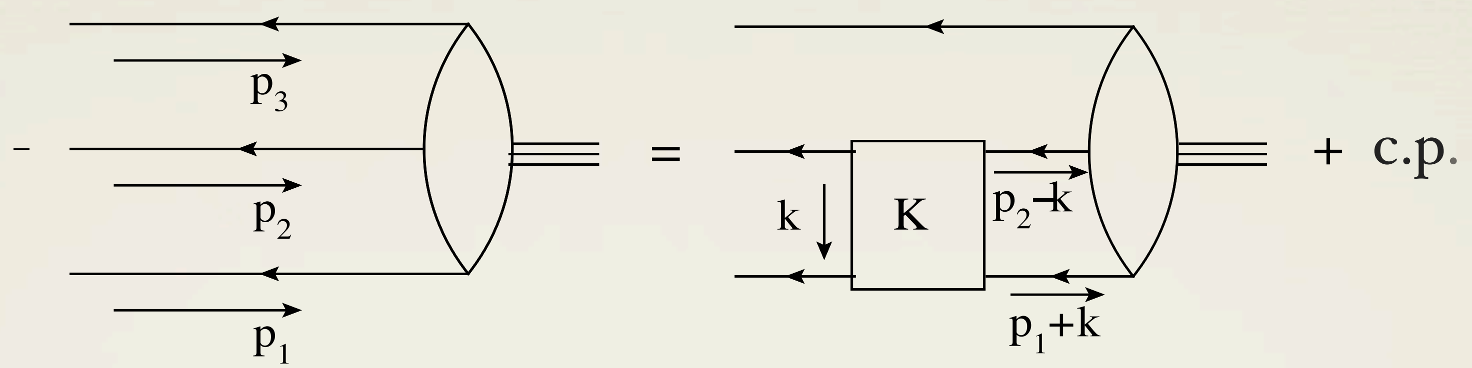

- **Ladder kernel**, no assumption on baryon vertex $T^a_{\alpha\delta}T^a_{\beta\eta}\Psi_{\delta\eta\gamma} = C_B\Psi_{\alpha\beta\gamma}$

- Total energy of qqq system:

  (equal quark separations)

$$P_0 = 3m + \frac{3}{2}g^2 \int đ\vec{\omega}\, W_{A_0}(\vec{\omega}) \left[C_F - 2C_B e^{\imath\vec{\omega}\cdot\vec{r}} \right]$$

[C_F Casimir operator; C_B, $W_{A_0}(\vec{\omega})$ to be identified]

➡ infrared confining: $P_0 = \sigma|\vec{r}|$

- temporal gluon propagator: $W_{A_0}(\vec{\omega}) \sim 1/\vec{\omega}^4$
- only ***color singlet*** baryon states physically allowed: $C_F \equiv 2C_B$

Summary

- Heavy quarks in Coulomb gauge QCD via DSEs
- Rainbow-ladder approximation to DS/BS/F equations exact
- Confinement for color singlet meson and baryon states
- 4-pct Green's functions
  - exact analytic solution
  - disentangle physical (*confining*) and unphysical poles
- Future work:
  - include $O(1/m^2)$ terms
  - use off-shell information to calculate tetraquark states

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In progress: DSEs in condensed matter systems **[graphene]**