Electroweak Interaction, Hadrons and Semileptonic τ-Decay

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H-QM Helmholtz Research School Quark Matter Studies

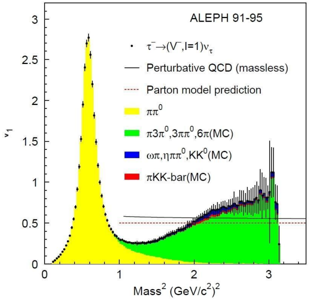
Motivation

- QCD not analytically solvable, reduce complexity with hadronic models that apply the symmetries known from the QCD Lagrangian:
- τ-decay involves strong and weak interactions
- τ spectral functions from ALEPH collaboration

  ALEPH Collaboration, S. Schael et al., Branching Ratios and Spectral Functions of Tau Decays: Final ALEPH measurements and physics implications, Phys.Rept. 421 (2005) 191284, [hep-ex/0506072]
- Determine effective electroweak interactions for hadronic degrees of freedom in the vacuum.
- Results can be used to perform calculations at nonzero temperature and density (e.g. dilepton decay rate).
- Understand the nature of resonances such as a_1, e.g. $\bar{q}q$ or $\rho\pi$-state?

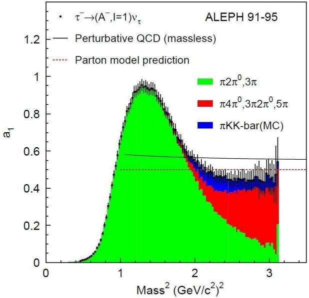
ALEPH Inclusive Spectral Functions

Vector Channel $\tau \to \nu_\tau 2\pi\nu_\tau$



$$B(\tau \to \pi^-\pi^0\nu_\tau) = 25.47\%$$
$$\vec{\rho} \simeq \bar{q}\gamma_\mu\vec{\tau}q$$
$$J^{PC} = 1^{--}$$

Axial-Vector Channel $\tau \to \nu_\tau 3\pi\nu_\tau$

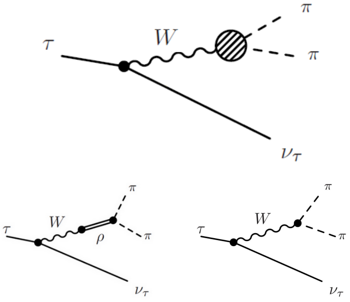


$$B(\tau \to \pi^-2\pi^0\nu_\tau + 2\pi^-\pi^+\nu_\tau) = 18.28\%$$
$$\vec{a}_1 \simeq \bar{q}\gamma_\mu\vec{\tau}\gamma_5 q$$
$$J^{PC} = 1^{++}$$

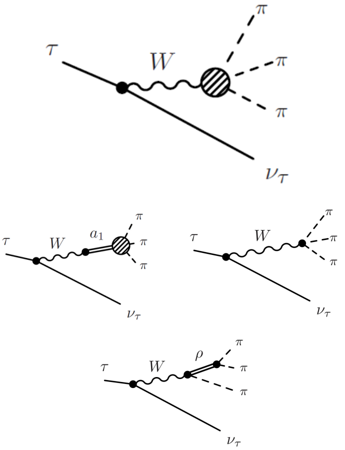
- Resonances are associated with $q\bar{q}$ states in the $N_f = 2$ and $I = 0, 1$ meson multiplets of $U(2)_L \times U(2)_R$ according to the quantum numbers of their decay products.

τ-Decay

Vector Channel



Axial-Vector Channel

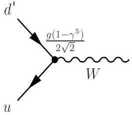


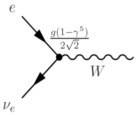
What do we know about weak interaction?

- $SU(2)_L \times U(1)_Y$ gauge symmetry with gauge fields W^μ and B^μ
- Weinberg mixing between $SU(2)_L \times U(1)_Y$ gauge fields B^μ, W_3^μ to physical interaction fields A^μ, Z_0^μ .
- Cabibbo mixing, flavour eigenstates are not weak eigenstates.
- Charged interaction violates symmetry under Charge and Parity transformations but preserves combined CP; $W_\pm^\mu$ act on left-handed particles, right-handed antiparticles only

$$P_L = \frac{1-\gamma_5}{2} \qquad P_R = \frac{1+\gamma_5}{2} .$$

- Charged bosons induce flavour-changing processes.





Linear Sigma Model

- Hadronic degrees of freedom are the light $N_f = 2$ meson multiplets, represented by the matrix-valued fields Φ , L^μ , R^μ:

$$\begin{aligned} \Phi : &\quad \text{scalar } S\ (\sigma, \vec{a}_0), &\quad \text{pseudoscalar } P\ (\eta, \vec{\pi}). \\ L^\mu(R^\mu): &\quad \text{vector } V^\mu\ (\omega^\mu, \vec{\rho^\mu}), &\quad \text{axial-vector } A^\mu\ (f_1^\mu, \vec{a_1^\mu}). \end{aligned}$$

- Strong isospin algebra with generators

$$t_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \quad t_i = \frac{\sigma_i}{2} , \quad i = 1, 2, 3$$

$$\Phi = (\sigma + i\eta)t_0 + (\vec{a}_0 + i\vec{\pi})\vec{t}, \quad L^\mu(R^\mu) = (\omega^\mu \pm f_1^\mu)t_0 + (\vec{\rho}^\mu \pm \vec{a}_1)\vec{t}$$

- τ-decay can be calculated within Linear σ Model after including electroweak interactions

Lagrangian

$$\mathscr{L}_{\mathrm{L}\sigma\mathrm{M}} = \mathrm{Tr}[(D^{\mu}\Phi)^{\dagger}(D^{\mu}\Phi)] - m^2\mathrm{Tr}(\Phi^{\dagger}\Phi) - \lambda_1[\mathrm{Tr}(\Phi^{\dagger}\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^{\dagger}\Phi)^2$$
$$-\frac{1}{4}\mathrm{Tr}[(L_0^{\mu\nu})^2 + (R_0^{\mu\nu})^2] + \frac{m_1^2}{2}\mathrm{Tr}[(L^{\mu})^2 + (R^{\mu})^2] + \mathrm{Tr}[H(\Phi + \Phi^{\dagger})]$$
$$+ c(\det\Phi + \det\Phi^{\dagger}) + \text{higher order couplings of } \Phi\,,\ L^{\mu}\,,\ R^{\mu}$$

$$D_{\mu}\Phi = \partial_{\mu}\Phi - ig_1(L_{\mu}\Phi - \Phi R_{\mu})\,, \quad (L,R)_0^{\mu\nu} = \partial^{\mu}(L,R)^{\nu} - \partial^{\nu}(L,R)^{\mu}$$

1 Invariant under global $U(2)_L \times U(2)_R$ transformations

$$\begin{aligned} \Phi &\to \Phi' = U_L \Phi U_R^{\dagger} \\ L^{\mu} &\to L^{\mu\,\prime} = U_L L^{\mu} U_L^{\dagger} \\ R^{\mu} &\to R^{\mu\,\prime} = U_R R^{\mu} U_R^{\dagger}\,. \end{aligned}$$

2 Symmetry is explicitly broken to $U(1)_V$, baryon number conservation.

Spontaneous Symmetry Breaking and Mass Splitting

- ρ and a_1 are chiral partners, $m_\rho = 0.7755$ GeV , $m_{a_1} \simeq 1.23$ GeV
- for $m^2 < 0$ we obtain infinite number of degenerate groundstates

  in the (pseudo-)scalar sector shifting $\sigma \to \sigma + \phi_0$ generates

  i) mass difference between the chiral partners ρ and a_1.
  $m_\rho^2 = m_1^2 + \frac{\phi_0^2}{2}(h_1 + h_2 + h_3)$, $\quad m_{a_1}^2 = m_1^2 + (g_1\phi_0)^2 + \frac{\phi_0^2}{2}(h_1 + h_2 - h_3)$

  ii) 3 point interaction vertices and mixing terms in $(D^\mu\Phi)^\dagger D_\mu\Phi$ that are proportional to the VEV ϕ_0

$SU(2)_L \times U(1)_y$ Transformation

- mesons are $\bar{q}q$ composite fields
  - a subject the composite quark fields to a $SU(2)_L \times U(1)_Y$ transformation
  - b apply Vector Meson Dominance and consider left-handed fields L_μ as left-handed hadronic vacuum fluctuations of W boson
- transformation laws

$$
\begin{array}{ccccl}
\Phi & \longrightarrow & \Phi' & = & U_L \Phi U_Y^\dagger \\
L^\mu & \longrightarrow & L^{\mu\,\prime} & = & U_L L^\mu U_L^\dagger \;, \\
R^\mu & \longrightarrow & R^{\mu\,\prime} & = & U_Y R^\mu U_Y^\dagger \;, \\
B^\mu & \longrightarrow & B^{\mu\,\prime} & = & U_Y B^\mu U_Y^\dagger + \frac{i}{g'} U_Y \partial^\mu U_Y^\dagger \;, \\
W^\mu & \longrightarrow & W^{\mu\,\prime} & = & U_L W^\mu U_L^\dagger + \frac{i}{g} U_L \partial^\mu U_L^\dagger \;.
\end{array}
$$

- covariant derivative

$$D^\mu \Phi = \partial^\mu \Phi - i g_1 (L^\mu \Phi - \Phi R^\mu) + i g' \Phi B^\mu - i g W^\mu \Phi \;.$$

Weinberg Mixing and Cabibbo Mixing

- neutral bare $SU(2)_L \times U(1)_Y$ gauge fields B^μ, W_3^μ are related to the physical fields A^μ, Z^μ by

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

and

$$e = g' \cos\theta_W = g \sin\theta_W$$

- strong isospin eigenstates d, s, b are related to the weak eigenstates by the CKM matrix
- Cabibbo mixing

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Covariant Derivative and Field Strength Tensors

- covariant derivative in terms of physical weak interaction fields and weak eigenstates

$$\begin{aligned} D^\mu \Phi &= \partial^\mu \Phi - i g_1 (L^\mu \Phi - \Phi R^\mu) - i g \cos\theta_C (W_1^\mu t_1 + W_2^\mu t_2)\Phi \\ &\quad - ieA^\mu [t_3, \Phi] - i g \cos\theta_W \left(Z^\mu \Phi + \tan^2\theta_W \Phi Z^\mu\right) \end{aligned}$$

- also redefinition of the field strength tensors for left- and right-handed fields

$$L^{\mu\nu} = \frac{i}{g}\left(\partial^\mu L^\nu - ig[W^\mu, L^\nu] - \partial^\nu L^\mu + ig[W^\nu, L^\mu]\right)$$

- chirally invariant term that generates W-ρ mixing

$$\mathscr{L}_{W\rho} = \frac{\delta}{2} g \cos\theta_C \mathrm{Tr}[W_{\mu\nu} L^{\mu\nu}]$$

with

$$W^{\mu\nu} = \partial_\mu W^\nu - \partial_\nu W^\mu - ig[W^\mu, W^\nu]$$

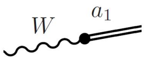
Lagrangian with weak interaction

$$\mathscr{L}_{L\sigma Mew} = \mathrm{Tr}[(D^{\mu}\Phi)^{\dagger}(D^{\mu}\Phi)] + \frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2] + \frac{\delta}{2} g \cos\theta_C \mathrm{Tr}[W_{\mu\nu} L^{\mu\nu}] + \frac{g}{2\sqrt{2}}\left(W_{\mu}^{-}\bar{u}_{\nu_\tau}\gamma_{\mu}(1-\gamma_5)u_{\tau} + \mathrm{h.c.}\right)$$

from local $SU(2)_L \times U(1)_Y$ symmetry

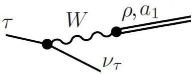
generates W-ρ-mixing and also contribution to W-a_1-mixing

from Glashow-Salam-Weinberg Model of electroweak interaction



$\sim g_1\phi_0^2 + \delta s$

$\sim \delta s$



exchanged momenta small compared to W mass

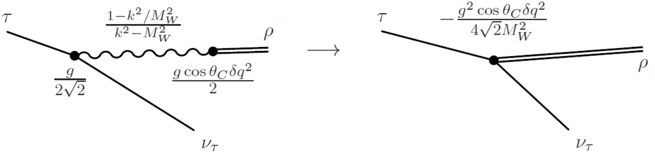
Fermi coupling constant

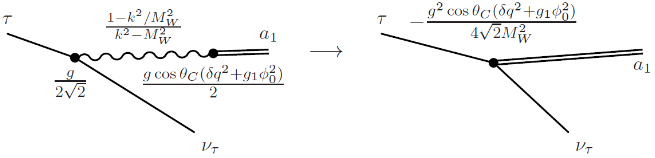
$$G_F = \frac{g}{2\sqrt{2}M_W^2}$$

Effective Couplings

$$W\rho: \quad \frac{g\cos\theta_C}{2}\ \delta s\ W^-_\mu \rho^{\mu+} + \text{ h.c.}$$

$$Wa_1: \quad \frac{g\cos\theta_C}{2}\ (\delta s - g_1\phi_0^2) W^-_\mu a_1^{\mu\,+} + h.c.$$





- Decay rates of τ to ρ and a_1

$$\Gamma_{\tau\to\rho\nu_\tau}(s)\,, \quad \Gamma_{\tau\to a_1\nu_\tau}(s)\,.$$

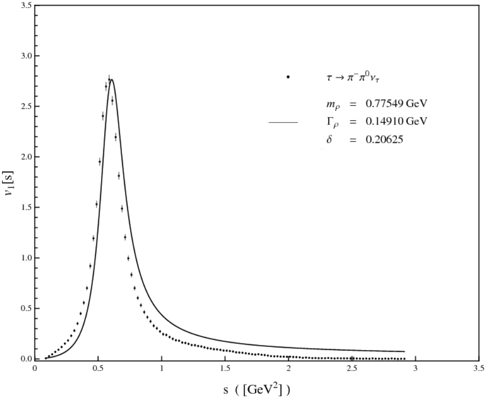
- Spectral densities (in a form suitable to compare to ALEPH data)

$$v_1(s) = C_A\ (\delta s)^2\ \frac{\rho_V(s)}{s}$$
$$a_1(s) = C_V\ (\delta s - g_1\phi_0^2)^2\ \frac{\rho_A(s)}{s}$$

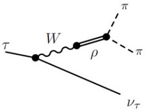
- Chiral condensate is related to the pion renormalisation constant Z and the pion vacuum decay constant f_π as

$$\phi_0 = Zf_\pi\,.$$

Vectorchannel Spectral Function

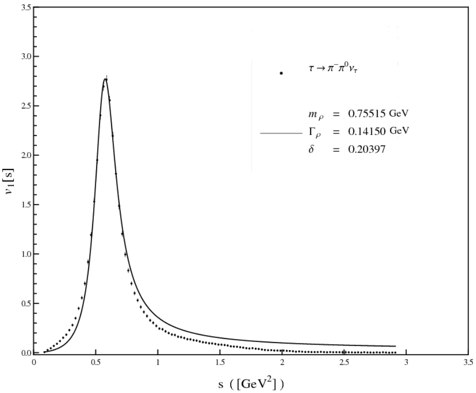


Spectral density $v_1(s)$ based on $\Gamma_{\rho^- \to \pi^- \pi^0}(s)$ calculated from global $U(2)_L \times U(2)_R$ Linear Sigma Model by Parganlija, Giacosa, Rischke.

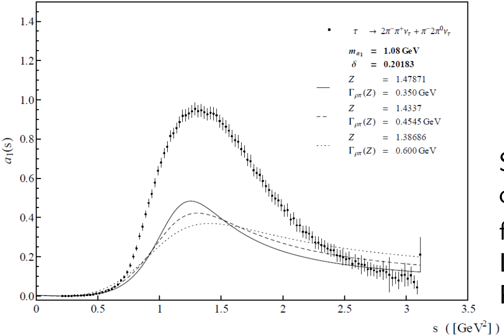
- $m_\rho = 0.77549$ GeV, $\Gamma_{\rho^- \to \pi^- \pi^0}(m_\rho^2) = 0.149$ GeV
- Only one free parameter δ, fixed to obtain peak value.
- Only 

Fitted Parameters

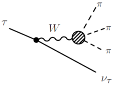
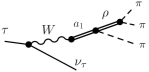
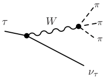
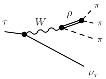
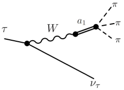
Improve agreement by refitting ρ mass and width:
$\delta = 0.20397\,,\ m_\rho = 0.75515$ GeV $,\ \Gamma_{\rho^- \to \pi^- \pi^0}(m_\rho) = 0.1415$ GeV



Axial-Vector Spectral Density



Spectral density $a_1(s)$ based on $\Gamma_{a_1^- \to 3\pi}(s)$ calculated from global $U(2)_L \times U(2)_R$ Linear Sigma Model by Parganlija, Giacosa, Rischke.

- $m_{a_1} = 1.08$ GeV
- No free parameter, δ fixed in vector-channel
- From  only 
- There are also   

Conclusion and Outlook

- Vector Spectral Density is well described with weak hadron-coupling $\delta \approx 0.2$ within the global $U(2)_L \times U(2)_R$ Linear Sigma Model.
- Consider other contributions to Axial-Vector Spectral Density:
  - δ may in fact be larger due to destructive interference effects from additional contributions in the vectorchannel, that will be considered.
  - other contributions in the axial-vector channel that are not included yet.
- After fixing the parameters:
  - calculate dilepton decay rate within LσM.
  - go to $N_f = 3$ and calculate $K \to 2\pi$ and $K \to 3\pi$ to study parity-violating effects.

The s-dependent Decay Widths

- $\Gamma_{\rho^- \to \pi^- \pi^0}(s)$:

$$\Gamma_{\rho^- \to \pi^- \pi^0}(s) = \frac{m_\rho^5}{48\pi m_{a_1}^4} \left[1 - \left(\frac{2m_\pi}{\sqrt{s}}\right)^2\right]^{\frac{3}{2}} \left[g_1 Z^2 + \frac{g_2}{2}(1 - Z^2)\right]^2$$

$$v_1(s) = \frac{(2\pi)^2}{NS_{EW}} (\delta s)^2 \, \frac{1}{s} \, \frac{\sqrt{s}\,\Gamma_{\rho^- \to \pi^- \pi^0}(s)}{(s - m_\rho^2)^2 + (\sqrt{s}\,\Gamma_{\rho^- \to \pi^- \pi^0}(s))^2}$$

- $\Gamma_{a_1 \to \rho\pi}(s)$:

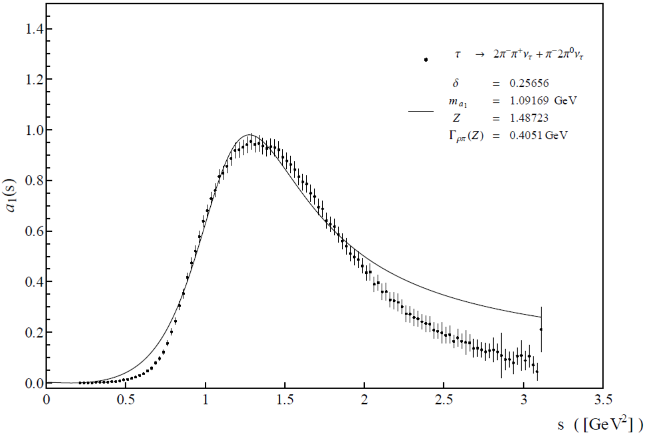
$$\Gamma_{a_1 \to \rho\pi}(s) = \frac{k(\sqrt{s}, m_\rho, m_\pi)}{12\pi m_{a_1}^2} \left[h_{\mu\nu}^2 - \frac{(h_{\mu\nu} K_1^\nu)^2}{m_\rho^2} - \frac{(h_{\mu\nu} P^\mu)^2}{m_{a_1}^2} + \frac{(h_{\mu\nu} P^\mu K_1^\nu)^2}{m_\rho^2 m_{a_1}^2}\right]$$

$$a_1(s) = \frac{(2\pi)^2}{NS_{EW}} (\delta s + g_1 \phi_0^2)^2 \, \frac{1}{s} \, \frac{\sqrt{s}\,\Gamma_{a_1 \to \rho\pi}(s)}{(s - m_{a_1}^2)^2 + (\sqrt{s}\,\Gamma_{a_1 \to \rho\pi}(s))^2}$$

$\Gamma_{\rho^- \to \pi^- \pi^0}(s)$ and $\Gamma_{a_1 \to \rho\pi}(s)$calculated from global $U(2)_L \times U(2)_R$ Linear Sigma Model in D. Parganlija, F. Giacosa, and D. H. Rischke, Vacuum Properties of Mesons in a Linear Sigma Model with Vector Mesons and Global Chiral Invariance, Phys. Rev. D82 (2010) 054024, [arXiv:1003.4934]

Axial-Vector Spectral Density

Fitted axial-vector spectral density



Representation of the Fields

S scalar, **P** pseudoscalar:

$$\Phi = \sum_{a=0}^{3}(S_a + iP_a)t_a \left(\sim \sqrt{2}\sum_{a=0}^{3}(\bar{q}t_a q + \bar{q}t_a\gamma_5 q)t_a\right)$$

$$= \frac{1}{2}\begin{pmatrix} \sigma + a_0^0 + i(\eta + \pi^0) & a_0^+ + i\pi^+ \\ a_0^- + i\pi^- & \sigma - a_0^0 + i(\eta - \pi^0) \end{pmatrix}$$

$\mathbf{V}_\mu$ vector, $\mathbf{A}_\mu$ axialvector:

$$(L,R)_\mu = \sum_{a=0}^{3}(V_\mu^a \pm A_\mu^a)t_a \left(\sim \sqrt{2}\sum_{a=0}^{3}(\bar{q}\gamma_\mu t_a q \pm \bar{q}t_a\gamma_\mu\gamma_5 q)t_a\right)$$

$$= \frac{1}{2}\begin{pmatrix} (\omega_\mu + \rho_\mu^0) \pm (f_{1\mu} + a_{1\mu}^0) & \rho_\mu^+ \pm a_{1\mu}^+ \\ \rho_\mu^- \pm a_{1\mu}^- & (\omega_\mu - \rho_\mu^0) \pm (f_{1\mu} - a_{1\mu}^0) \end{pmatrix}$$