Holography and the Generalized Langevin Process

Francesco Nitti

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Work with E. Kiritsis, U. Gursoy, L. Mazzanti 1006.3261, 1111.1008

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This talk: I will review aspects of the Langevin dynamics, and show how it can be computed holographically in a generic setup.

Introduction: Generalized Langevin Process

The diffusion of a Heavy quark through a deconfined plasma is a dissipative process that can be described by a generalized Langevin process:

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- $G_R(\tau)$ and $G_{sym}(\tau)$ can be computed holographically. This involves a *dressing* procedure to cure UV problems.

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The motion of a heavy quark through the QCD plasma can be described by an effective action obtained by integrating out the plasma degrees of freedom, including a coupling to the instantaneous force $\mathcal{F}(t)$ acting on the quark:

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These correlators can be computed holographycally, once we identify the bulk field dual to the operator $\mathcal{F}(t)$

Dispersion Relations

The equation of motion is in Fourier space:

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- If this does not happen, the process is dominated by wild random kicks at short time separation.
- The "bare" holographic spectral density $ImG_R(\omega) \sim \omega^3$ in the UV. $\Rightarrow$ this cannot be the actual physical quantity.

Recovering Local Langevin process

$$M_q \ddot{X} + \int_0^{+\infty} dt' \, \gamma(t') \dot{X}(t-t') = \xi(t') \qquad \begin{cases} \langle \xi(t)\xi(t') \rangle = G_{sym}(t,t'), \\ \dot{\gamma}(t') \equiv G_{asym}(t') \end{cases}$$

$$G_{sym}(t) = -i\langle \{\mathcal{F}(t), \mathcal{F}(0)\} \rangle, \qquad G_{asym}(t) = -i\langle [\mathcal{F}(t), \mathcal{F}(0)] \rangle.$$

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The viscous friction η and diffusion coefficient κ are the low-frequency limit of the Langevin Green's functions.

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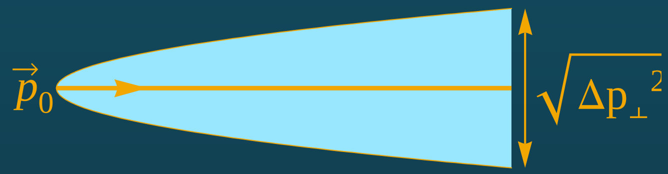

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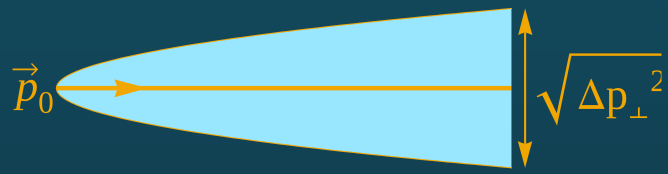
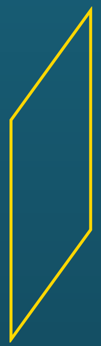


Define the *jet quenching parameter*

$$\hat{q} \equiv \frac{\langle (p^{\perp})^2 \rangle}{mean \, free \, path} = \frac{(p^{\perp})^2}{v \, t} = 2 \frac{\kappa^{\perp}}{v}$$

AdS/CFT Correlators

The $4D$ theory "lives" on the **boundary** of a $(4+n)D$ curved **bulk** space-time with some fields $\Phi(x, r)$



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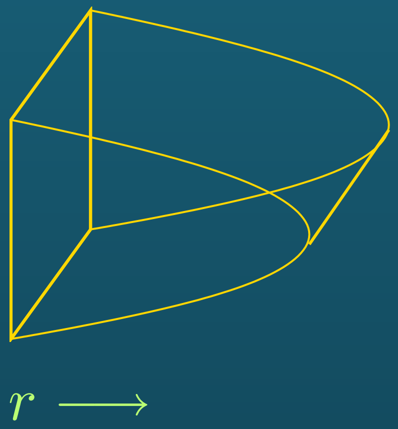

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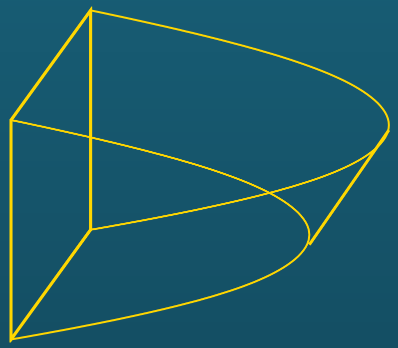

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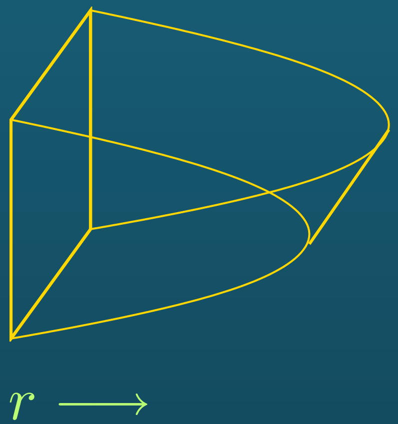

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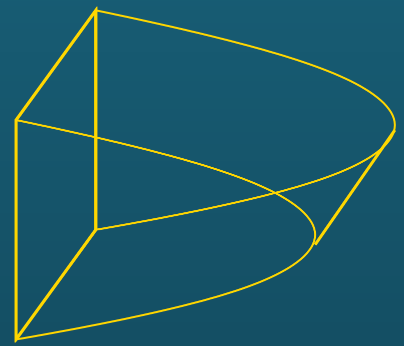

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- The 2-point function for O is:

$$\langle O(p) O(-p) \rangle \sim B(p)/A(p).$$

5D Holographic setup

We consider a generic 5D black hole background

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx_i\right]$$

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- Dual to a non-conformal gauge theory in thermal equilbrium at a temperature T_h, in a deconfined phase.

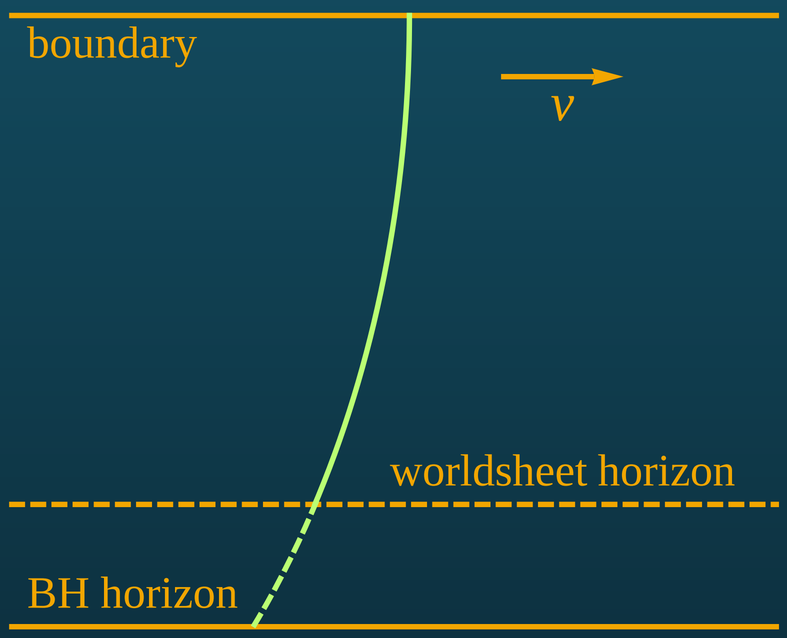
Trailing string

Holographic description of a heavy quark (Gubser 2006) :

$\Updownarrow$

String attached at the AdS boundary and trailing in the interior, described by the embedding:

$$\vec{X}(t,r) = \left(vt + \xi(r)\right)\frac{\vec{v}}{v}, \qquad \xi(0) = 0$$



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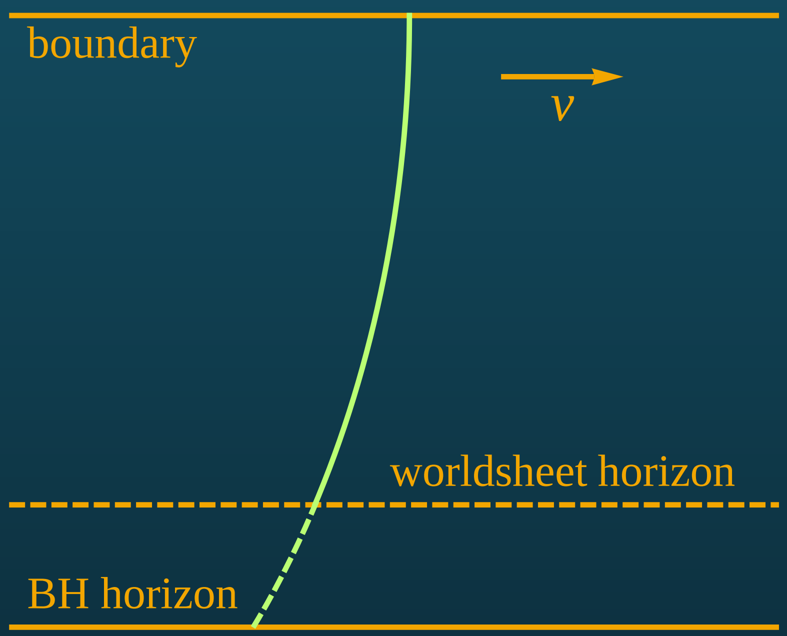

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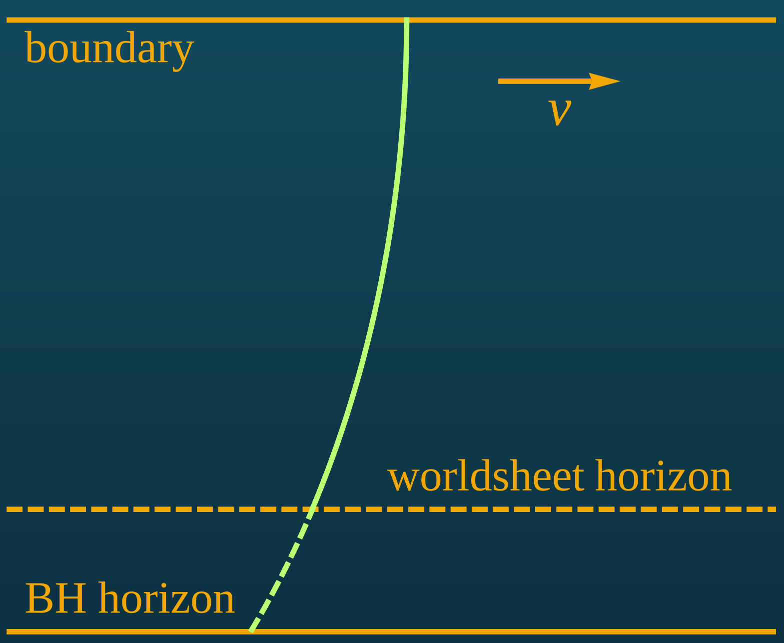

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Pure AdS: $T_s = T_h/\sqrt{\gamma}$, $\gamma \equiv 1/\sqrt{1-v^2}$

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$\delta\vec{X}(r,t)$ is the bulk field dual to the boundary operator $\vec{\mathcal{F}}$

$\Rightarrow$ According to the AdS/CFT prescription correlators of $\vec{\mathcal{F}}$ are obtained from the solutions of the wave equation for $\delta\vec{X}(r,t)$

Retarded Correlator

Expanding the trailing string action to quadratic order in $\Psi(r,t) \equiv \{\delta X^{\perp}, \delta X^{\parallel}\}$ one finds the wave equation:

$$\partial_\alpha \left(\mathcal{H}^{\alpha\beta}(r) \partial_\beta \Psi \right) = 0 \qquad \alpha, \beta = r, t$$

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Prescription for the real-time retarded propagator (Son + Starinets, '02):

$$G_R(\omega) = \mathcal{H}^{r\alpha}\, \Psi_R^*(r,\omega)\, \partial_\alpha\, \Psi_R(r,\omega) \Big|_{Boundary}$$

$\Psi_R(r,\omega)$ is the solution with boundary conditions:

$$\Psi_R(0,\omega) = 1, \qquad \Psi_R(r,\omega) \sim (r_s - r)^{-\frac{i\omega}{4\pi T_s}}, \quad r \sim r_s$$

Diffusion coefficient

In the zero-frequency limit we get the diffusion coefficients:

$$\eta^{\perp} = \frac{b^2(r_s)}{2\pi\ell_s^2}, \qquad \kappa^{\perp} = 2T_s\eta^{\perp}$$

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- The local approximation of the full generalized Langevin requires we are in a *long time* regime, where only low frequencies are relevant:
- Consistent only if the relaxation time of the process $(1/\eta_D)$ is much slower than the correlation time $(\tau_c \sim 1/T_s)$ of $G_R(t)$ (effectively $G_R(t) \approx \delta(t)$).

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The local Langevin description is valid if

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Not so fast...

UV behavoir of the full correlator

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- Look at the large ω limit. It can be computed analytically in a WKB approximation, in the regime $\omega r_s \gamma \gg 1$:

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- Remark: the leading behavior is temperature-independent.

Dressed Correlators

Proposal to have UV-safe spectral densities: subtract the correlator obtain from the vacuum background. No black hole, $f(r) \equiv 1$.

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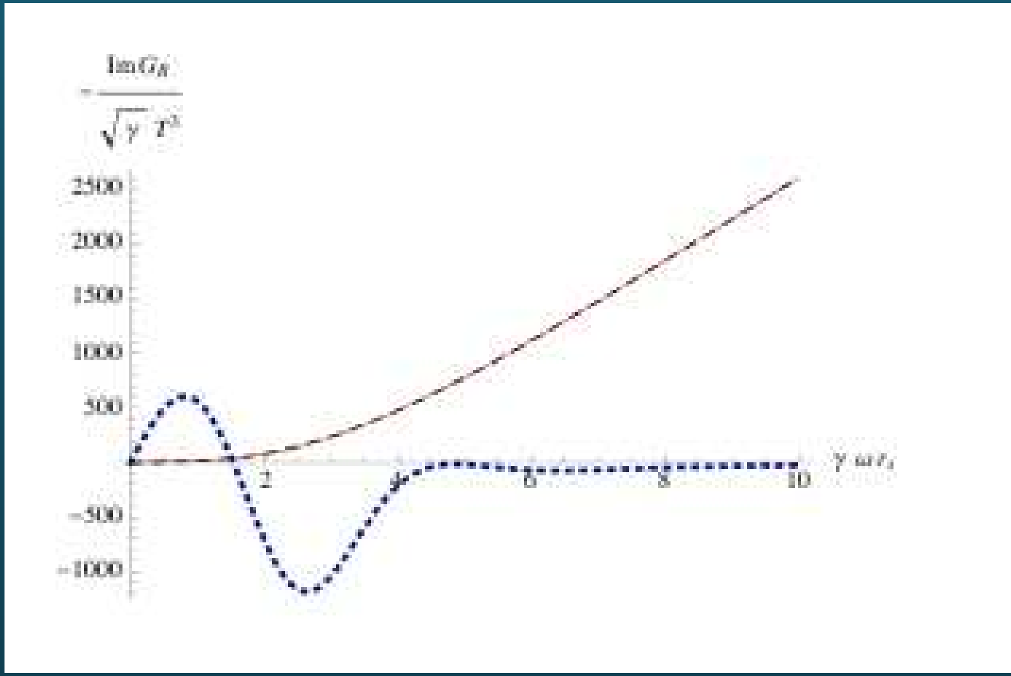
- Trailing string in the vacuum $f(r) = 1$: straight line starting at the boundary $r = 0$ and stretching to $r \to \infty$.
- The leading $O(\omega^3)$ and subleading $O(\omega)$ terms cancel exactly.

$$G_R^{ph}(\omega) \sim 1/\omega \qquad \omega \to \infty$$

Dispersion relations safe, FT exists, short-time limit regular.

Subtracted correlator



$$G_R^{ph}(\omega) \sim 1/\omega \qquad \omega \to \infty$$

Peak at $\omega_c \sim (\gamma r_s)^{-1} = T_s \quad \Rightarrow \quad \tau_c \approx 1/T_s$

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  In this case the vacuum Langevin coefficient diverges!
- Resolution: the straight string is not the minimal embedding in the confining vacuum. (Work in progress with E. Kiritsis and L.Mazzanti)

Outlook

- We obtained a prescription to compute physical correlators for the Generalized Langevin process of a heavy quark in the plasma in the AdS/CFT approach.
- For non-confining background this does not affect the zero-frequency limit.
- More work needed to obtain fully consistent correlators in the physically interesting case (confining backgrounds).
- Ultimate goal: use correlators in numerical simulation of the LHC plasma and compare results to experiment.