Effects of the low lying Dirac modes on excited hadrons in lattice QCD

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Outline

Motivation and introduction

Mesons

Baryons

Quark propagator

Conclusions

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Effects of the low lying Dirac modes on excited hadrons in lattice QCD
Why are the lowest Dirac eigenmodes interesting?

The Banks-Casher relation

\[ \langle \psi \psi \rangle = -\pi \rho(0) \]

directly relates the density of the Dirac modes near the origin \( \rho(0) \) to the chiral condensate.
Reminder: chiral symmetry and its breaking

When neglecting the two lightest quark masses, the QCD Lagrangian becomes invariant under the symmetry group

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

The axial vector part of the $SU(2)_L \times SU(2)_R$ symmetry is broken spontaneously in the vacuum whereas the vector part is (approximately) preserved. The $U(1)$ axial symmetry is not only broken spontaneously but also explicitly (axial anomaly).
"Unbreaking" chiral symmetry

- Our goal is to construct hadron correlators out of reduced quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., [DeGrand, PRD 69 (2004)]).
“Unbreaking” chiral symmetry

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- Mesons:
  - can we restore the chiral symmetry and if, what happens to confinement?
  - what happens to the broken $U(1)$ axial symmetry?
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  - what happens to the broken $U(1)$ axial symmetry?
- Baryons:
  - is the $N(1535)$ the chiral partner of the nucleon?
  - what is the origin of the $\Delta - N$ splitting?
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- Baryons:
  - is the $N(1535)$ the chiral partner of the nucleon?
  - what is the origin of the $\Delta - N$ splitting?

- Landau gauge quark propagator:
  - what happens to the renormalization function $Z(p^2)$ and the mass function $M(p^2)$ once chiral symmetry is unbroken?
Reducing quark propagators

- Consider the Hermitian Dirac operator $D_5 \equiv \gamma_5 D$ (real eigenvalues)
- Split the quark propagator $S \equiv D^{-1}$ into a low mode (lm) part and a reduced (red) part

$$S = \sum_{i \leq k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5 + \sum_{i > k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5$$

$$= S_{lm}(k) + S_{red}(k)$$
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\[
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\]

\[
= S_{\text{lm}(k)} + S_{\text{red}(k)}
\]

- In this work we investigate the reduced (red) part of the propagator

\[
S_{\text{red}(k)} = S - S_{\text{lm}(k)}
\]
The setup

- 161 configurations [Gattringer et al., PRD 79 (2009)]
- size $16^3 \times 32$
- two degenerate flavors of light fermions, $m_\pi = 322(5) \text{MeV}$
- lattice spacing $a = 0.1440(12) \text{fm}$
- Chirally Improved (CI) Dirac operator [Gattringer, PRD 63 (2001)] (approximate solution of the Ginsparg-Wilson equation)
- three different kinds of quark sources: Jacobi smeared narrow (0.27 fm) and wide (0.55 fm) sources and a $P$ wave like derivative source → serves a large operator basis for the variational method.
Mesons

We explore the following isovector mesons which would be related via the chiral symmetry [L.Ya. Glozman, Physics Reports 444 (2007)]

\[
\begin{array}{c|c}
U(1)_A & SU(2)_L \times SU(2)_R \text{ (axial)} \\
\rho \longleftrightarrow b_1 & \rho \longleftrightarrow a_1 \\
\end{array}
\]
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Meson masses vs. Dirac eigenmode reduction level

$\sigma$ [MeV]

mass $[1/m_\rho]$}

$k$

$\rho$, 0th state

C.B. Lang, MS: PRD 84 (2011), arXiv:1107.5195

L.Ya. Glozman, C.B. Lang, MS: in preparation

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\[ \rho(J^{PC} = 1^{--}) \]

\begin{align*}
&\text{all states: correlators, red(12)} \\
&\text{state 0} \quad \text{state 1} \quad \text{state 2} \\
&\text{state 3} \quad \text{state 4} \\
&\text{state 0} \quad \text{state 1} \\
&\text{all states: correlators, red(32)} \\
&\text{state 0} \quad \text{state 1} \quad \text{state 2} \\
&\text{state 3} \quad \text{state 4} \\
&\text{lowest state(s): eff. masses, red(12)} \\
&\text{state 0} \quad \text{state 1} \\
&\text{lowest state(s): eff. masses, red(32)} \\
&\text{state 0} \quad \text{state 1} \\
\end{align*}

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\[ a_1 \left( J^{PC} = 1^{++} \right) \]

all states: correlators, red(4)

lowest state(s): eff. masses, red(4)

all states: correlators, red(64)

lowest state(s): eff. masses, red(64)

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\[ b_1 \left( J^{PC} = 1^{+-} \right) \]

all states: correlators, red(2)

lowest state(s): eff. masses, red(2)

all states: correlators, red(128)

lowest state(s): eff. masses, red(128)

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Effects of the low lying Dirac modes on excited hadrons in lattice QCD
The $\Delta - N$ splitting is usually attributed to the hyperfine spin-spin interaction between valence quarks. The realistic candidates for this interaction are:

- the spin-spin color-magnetic interaction
- the flavor-spin interaction related to the spontaneous chiral symmetry breaking
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What happens to the $\Delta - N$ splitting after restoration of the chiral symmetry?
Baryons

The $\Delta - N$ splitting is usually attributed to the hyperfine spin-spin interaction between valence quarks. The realistic candidates for this interaction are

- the spin-spin color-magnetic interaction
- the flavor-spin interaction related to the spontaneous chiral symmetry breaking

What happens to the $\Delta - N$ splitting after restoration of the chiral symmetry?
Do the masses of the nucleon and the $N(1535)$ meet?
Baryon masses vs. Dirac eigenmode reduction level

The graph shows the relationship between the mass of the N(+) 0th state and the Dirac eigenmode reduction level, represented by the parameter $\sigma$ in MeV. The mass is plotted against $\sigma$ for different values of $k$. The data points indicate an upward trend as $\sigma$ increases.
Baryon masses vs. Dirac eigenmode reduction level

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Effects of the low lying Dirac modes on excited hadrons in lattice QCD
Baryon masses vs. Dirac eigenmode reduction level

\[ \text{mass \: [1/m}_\rho \text{]} \]

\[ \sigma \text{ [MeV]} \]

\[ k \]

- \( N(+) \) 0th state
- \( N(-) \) 0th state
- \( N(+) \) 1st state
- \( N(-) \) 1st state

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Effects of the low lying Dirac modes on excited hadrons in lattice QCD
Baryon masses vs. Dirac eigenmode reduction level

![Graph showing the relationship between baryon masses and Dirac eigenmode reduction level](image.png)

- N(+) 0th state
- N(+) 1st state
- N(-) 0th state
- N(-) 1st state
- ∆(+) 0th state
- ∆(-) 0th state

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Baryon masses vs. Dirac eigenmode reduction level
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\[ N(J^P = 1/2^+) \]

\[ \text{all states: correlators, red(20)} \]

\[ \text{lowest state(s): eff. masses, red(20)} \]

\[ \text{all states: correlators, red(64)} \]

\[ \text{lowest state(s): eff. masses, red(64)} \]

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\[ N(J^P = 1/2^-) \]

all states: correlators, red(12)

\[ \text{state 0, state 1, state 2, state 3, state 4, state 5} \]

\[ \text{state 0, state 1} \]

all states: correlators, red(64)

\[ \text{state 0, state 1, state 2, state 3, state 4, state 5} \]

\[ \text{state 0, state 1} \]

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\[ \Delta (J^P = 3/2^+) \]

all states: correlators, red(16)

lowest state(s): eff. masses, red(16)

all states: correlators, red(128)

lowest state(s): eff. masses, red(128)

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\[ \Delta \left( J^P = \frac{3}{2}^- \right) \]

all states: correlators, red(16)

lowest state(s): eff. masses, red(16)

all states: correlators, red(128)

lowest state(s): eff. masses, red(128)
The lattice quark propagator

The tree-level quark propagator is

$$S_0(p) = \frac{1}{i\not{p} + m_0}$$
The lattice quark propagator

The tree-level quark propagator is

\[ S_0(p) = \frac{1}{i\not{p} + m_0} \]

\[ S_0(p) \rightarrow S_{\text{bare}}(a; p) = Z_2(\mu; a)S(\mu; p) \]
The lattice quark propagator

The tree-level quark propagator is

\[
S_0(p) = \frac{1}{i\not{p} + m_0}
\]

\[
S_0(p) \rightarrow S_{\text{bare}}(a; p) = Z_2(\mu; a)S(\mu; p)
\]

the renormalized quark propagator

\[
S(\mu; p) = \frac{1}{i\not{p}A(\mu; p^2) + B(\mu; p^2)} = \frac{Z(\mu; p^2)}{i\not{p} + M(p^2)}.
\]
The lattice quark propagator

The tree-level quark propagator is

$$S_0(p) = \frac{1}{i\not{p} + m_0}$$

$$S_0(p) \rightarrow S_{\text{bare}}(a; p) = Z_2(\mu; a)S(\mu; p)$$

the renormalized quark propagator

$$S(\mu; p) = \frac{1}{i\not{p}A(\mu; p^2) + B(\mu; p^2)} = \frac{Z(\mu; p^2)}{i\not{p} + M(p^2)}.$$ 

We calculate $S_{\text{bare}}(a; p)$ in Landau gauge on the lattice and therefrom extract

- the renormalization function $Z(\mu; p^2)$
- the renormalization point independent mass function $M(p^2)$
The quark propagator under eigenmode reduction

\[ Z(p^2) \]

\[ M(p^2) \] [MeV]

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The quark propagator under eigenmode reduction

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Effects of the low lying Dirac modes on excited hadrons in lattice QCD
The quark propagator under eigenmode reduction

The quark propagator under eigenmode reduction

Conclusions

- low lying eigenvalues of the Dirac operator are associated with chiral symmetry breaking
- we have computed hadron propagators while removing increasingly more of the low lying eigenmodes of the Dirac operator
- the confinement properties remain intact, i.e., we still observe clear bound states for most of the studied hadrons
- the mass values of the vector meson chiral partners $a_1$ and $\rho$ approach each other: restoration of $SU(2)_L \times SU(2)_R$
- no degeneracy between $a_1$ and $b_1$: $U(1)_A$ axial anomaly untouched
- the nucleon and the $N(1535)$ become degenerate
- the spin-spin color-magnetic interaction and the flavor-spin interaction are of equal importance for the $\Delta - N$ splitting
- the dynamical mass generation of quarks as seem from the IR behavior of $M(p^2)$ unimportant for chiral symmetric hadrons
Low-mode contribution of $D$ and $D_5$ to the $\pi$ and $\rho$ correlators

![Graphs showing the correlators $C_{\pi}(t)$ and $C_{\rho}(t)$ for different modes and lattice QCD calculations.](image)
Low-mode contribution of $D$ and $D_5$ to the $\pi$ and $\rho$ correlators
### ρ interpolators

<table>
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<th>#_ρ</th>
<th>interpolator(s)</th>
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<tr>
<td>1</td>
<td>$\bar{a}_n \gamma_k b_n$</td>
</tr>
<tr>
<td>8</td>
<td>$\bar{a}_w \gamma_k \gamma_t b_w$</td>
</tr>
<tr>
<td>12</td>
<td>$\bar{a}_{\partial_k} b_w - \bar{a}<em>w b</em>{\partial_k}$</td>
</tr>
<tr>
<td>17</td>
<td>$\bar{a}<em>{\partial_i} \gamma_k b</em>{\partial_i}$</td>
</tr>
<tr>
<td>22</td>
<td>$\bar{a}<em>{\partial_k} \epsilon</em>{ijk} \gamma_j \gamma_5 b_w - \bar{a}<em>w \epsilon</em>{ijk} \gamma_j \gamma_5 b_{\partial_k}$</td>
</tr>
</tbody>
</table>

Interpolators for the $\rho$-meson, $J^{PC} = 1^{--}$. The first column shows the number, the second shows the explicit form of the interpolator. cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]
$a_1$ interpolators

<table>
<thead>
<tr>
<th>$#_{a_1}$</th>
<th>interpolator(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\bar{a}_n \gamma_k \gamma_5 b_n$</td>
</tr>
<tr>
<td>2</td>
<td>$\bar{a}_n \gamma_k \gamma_5 b_w + \bar{a}_w \gamma_k \gamma_5 b_n$</td>
</tr>
<tr>
<td>4</td>
<td>$\bar{a}_w \gamma_k \gamma_5 b_w$</td>
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</table>

$a_1$-meson, $J^{PC} = 1^{++}$, cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]
$b_1$ interpolators

<table>
<thead>
<tr>
<th>$#_{b_1}$</th>
<th>interpolator(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$\bar{a}_\partial b_n - \bar{a}<em>n \gamma_5 b</em>\partial$</td>
</tr>
<tr>
<td>8</td>
<td>$\bar{a}_\partial \gamma_5 b_w - \bar{a}<em>w \gamma_5 b</em>\partial$</td>
</tr>
</tbody>
</table>

$b_1$-meson, $J^{PC} = 1^{-+}$, cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]
\( N \) interpolators

- \( N(i) = \epsilon_{abc} \Gamma_1^{(i)} u_a \left( u_b^T \Gamma_2^{(i)} d_c - d_b^T \Gamma_2^{(i)} u_c \right) \)
- \( N(+) \): 1, 2, 4, 14, 15, 18
- \( N(-) \): 1, 7, 8, 9

\[
\begin{array}{c|c|c|c|c}
\chi^{(i)} & \Gamma_1^{(i)} & \Gamma_2^{(i)} & \text{smearing} & \#N \\
\hline
\chi^{(1)} & 1 & C \gamma_5 & (nn)n & 1 \\
 & & & (nn)w & 2 \\
 & & & (nw)n & 3 \\
 & & & (nw)w & 4 \\
 & & & (ww)n & 5 \\
 & & & (ww)w & 6 \\
\hline
\chi^{(2)} & \gamma_5 & C & (nn)n & 7 \\
 & & & (nn)w & 8 \\
 & & & (nw)n & 9 \\
 & & & (nw)w & 10 \\
 & & & (ww)n & 11 \\
 & & & (ww)w & 12 \\
\hline
\chi^{(3)} & i 1 & C \gamma_t \gamma_5 & (nn)n & 13 \\
 & & & (nn)w & 14 \\
 & & & (nw)n & 15 \\
 & & & (nw)w & 16 \\
 & & & (ww)n & 17 \\
 & & & (ww)w & 18 \\
\end{array}
\]

cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]
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Δ interpolators

- $\epsilon_{abc} u_a (u_b^T C \gamma_k u_c)$
- $\Delta(+)$: 1, 2, 3
- $\Delta(−)$: 1, 2, 3

<table>
<thead>
<tr>
<th>smearing</th>
<th>#(\Delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(nn)n</td>
<td>1</td>
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<tr>
<td>(nn)w</td>
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<tr>
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</tr>
<tr>
<td>(nw)w</td>
<td>4</td>
</tr>
<tr>
<td>(ww)n</td>
<td>5</td>
</tr>
<tr>
<td>(ww)w</td>
<td>6</td>
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</tbody>
</table>

cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]