Effects of the low lying Dirac modes on excited hadrons in lattice QCD

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Outline

Why are the lowest Dirac eigenmodes interesting?

The Banks-Casher relation

$$\langle \overline{\psi}\psi \rangle = -\pi \rho(0)$$

directly relates the density of the Dirac modes near the origin $\rho(0)$ to the chiral condensate.

Reminder: chiral symmetry and its breaking

When neglecting the two lightest quark masses, the QCD Lagrangian becomes invariant under the symmetry group

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

The axial vector part of the $SU(2)_L \times SU(2)_R$ symmetry is broken spontaneously in the vacuum whereas the vector part is (approximately) preserved. The $U(1)$ axial symmetry is not only broken spontaneously but also explicitly (axial anomaly).

“Unbreaking” chiral symmetry

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  - what is the origin of the $\Delta - N$ splitting?

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  - what is the origin of the $\Delta - N$ splitting?
- Landau gauge quark propagator:
  - what happens to the renormalization function $Z(p^2)$ and the mass function $M(p^2)$ once chiral symmetry is unbroken?

Reducing quark propagators

- Consider the Hermitian Dirac operator $D_5 \equiv \gamma_5 D$ (real eigenvalues)
- Split the quark propagator $S \equiv D^{-1}$ into a low mode (lm) part and a *reduced* (red) part

$$
\begin{aligned}
S &= \sum_{i \leq k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 + \sum_{i > k} \mu_i^{-1} \left| v_i \right\rangle \left\langle v_i \right| \gamma_5 \\
&= S_{\mathrm{lm}(k)} + S_{\mathrm{red}(k)}
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- In this work we investigate the *reduced* (red) part of the propagator

$$S_{\mathrm{red}(k)} = S - S_{\mathrm{lm}(k)}$$

The setup

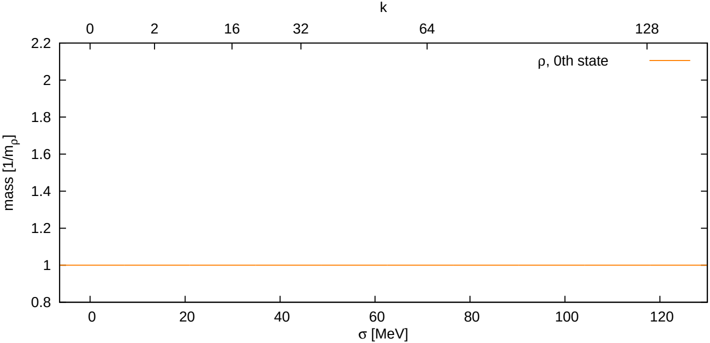
- 161 configurations [Gattringer et al., PRD 79 (2009)]
- size $16^3 \times 32$
- two degenerate flavors of light fermions, $m_\pi = 322(5)\,\mathrm{MeV}$
- lattice spacing $a = 0.1440(12)\,\mathrm{fm}$
- Chirally Improved (CI) Dirac operator [Gattringer, PRD 63 (2001)] (approximate solution of the Ginsparg-Wilson equation)
- three different kinds of quark sources: Jacobi smeared narrow (0.27 fm) and wide (0.55 fm) sources and a P wave like derivative source $\rightarrow$ serves a large operator basis for the variational method.

Mesons

We explore the following isovector mesons which would be related via the chiral symmetry [L.Ya. Glozman, Physics Reports 444 (2007)]

$U(1)_A$	$SU(2)_L \times SU(2)_R$ (axial)
$\rho \longleftrightarrow b_1$	$\rho \longleftrightarrow a_1$

Meson masses vs. Dirac eigenmode reduction level



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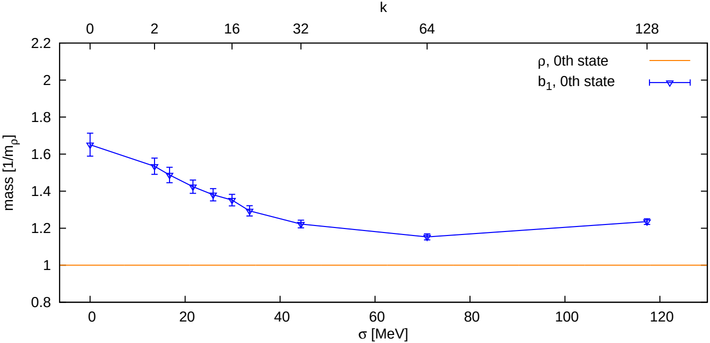

[C.B. Lang, MS: PRD 84 (2011), arXiv:1107.5195]

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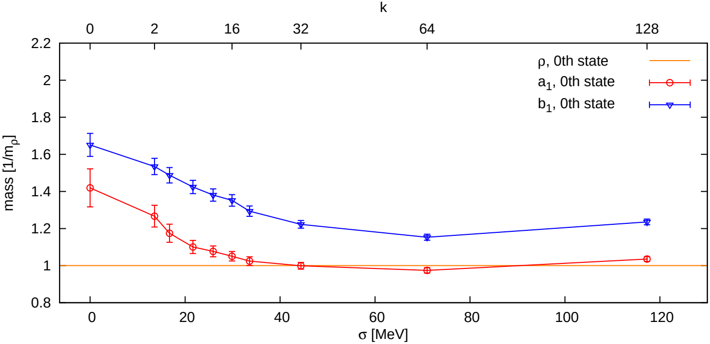

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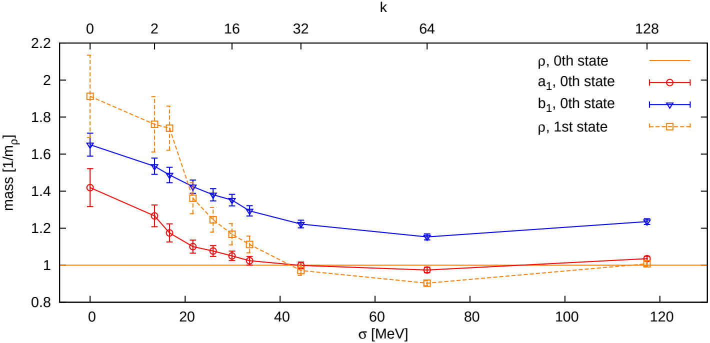

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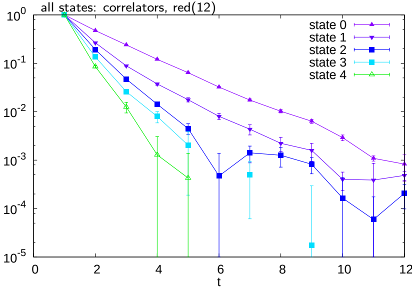
$\rho\,(J^{PC} = 1^{--})$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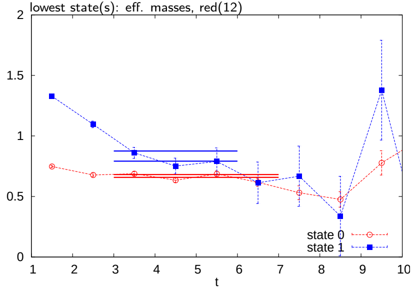
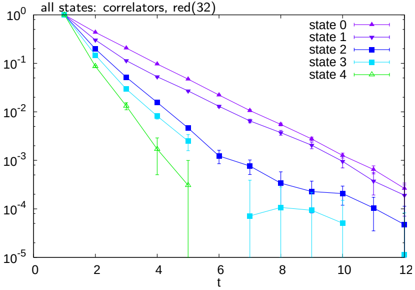
state 0
state 1
state 2
state 3
state 4
t


lowest state(s): eff. masses, red(12)
state 0
state 1
t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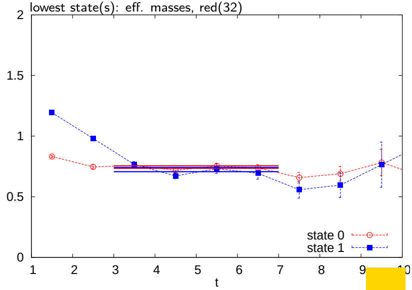
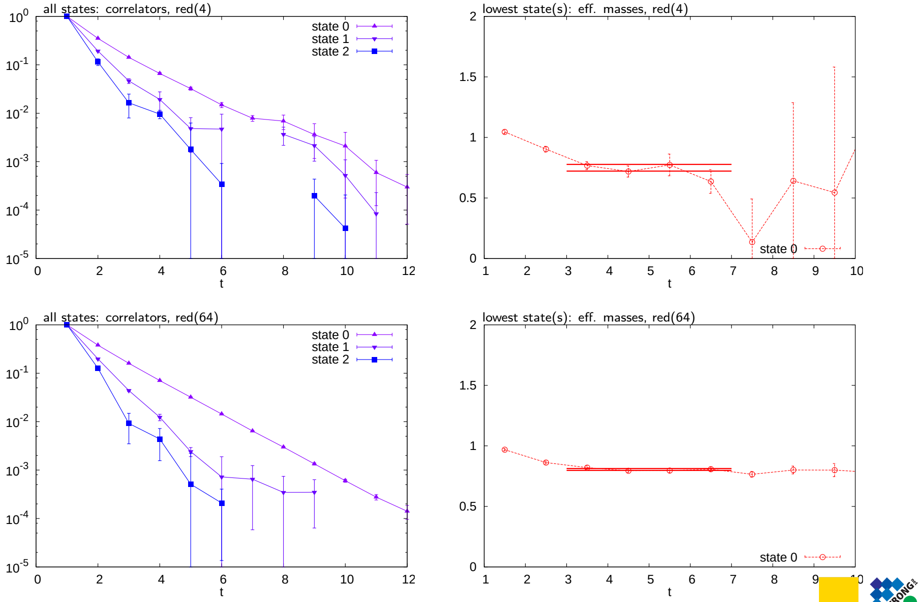
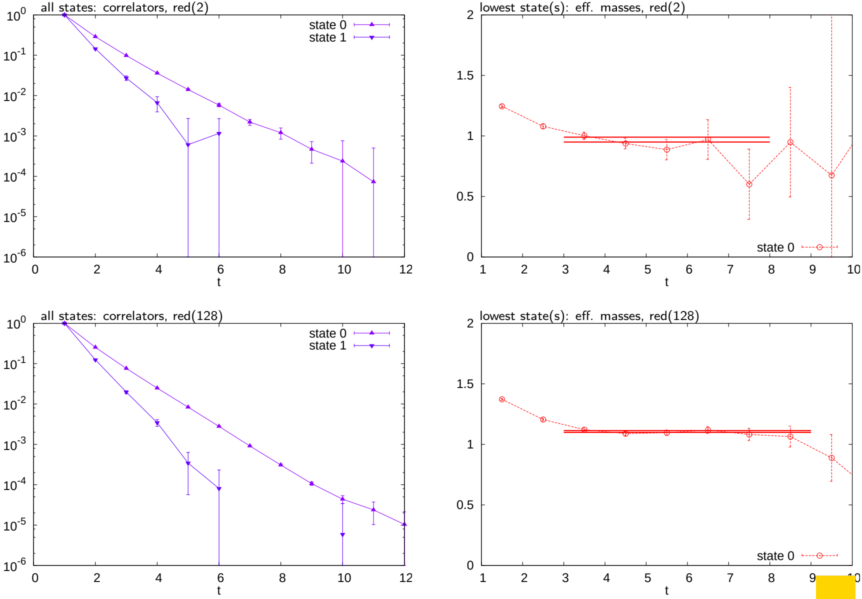
state 1
state 2
state 3
state 4
t


lowest state(s): eff. masses, red(32)
state 0
state 1
t

a_1 $(J^{PC} = 1^{++})$


all states: correlators, red(4)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(4)
state 0
t
all states: correlators, red(64)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(64)
state 0
t

b_1 ($J^{PC} = 1^{+-}$)


all states: correlators, red(2)
state 0
state 1
t
lowest state(s): eff. masses, red(2)
state 0
t
all states: correlators, red(128)
state 0
state 1
t
lowest state(s): eff. masses, red(128)
state 0
t

Baryons

The $\Delta - N$ splitting is usually attributed to the hyperfine spin-spin interaction between valence quarks. The realistic candidates for this interaction are

- the spin-spin color-magnetic interaction
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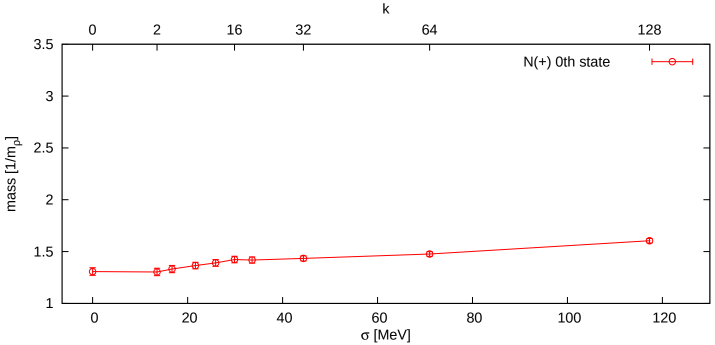
What happens to the $\Delta - N$ splitting after restoration of the chiral symmetry?

Do the masses of the nucleon and the $N(1535)$ meet?

Baryon masses vs. Dirac eigenmode reduction level



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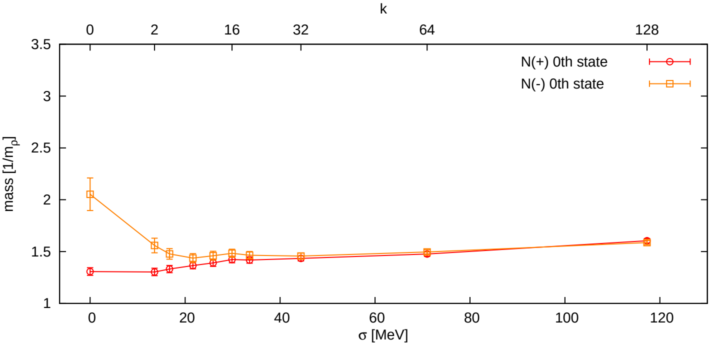

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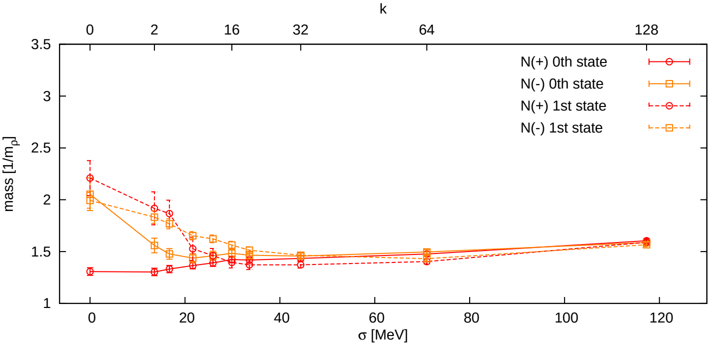

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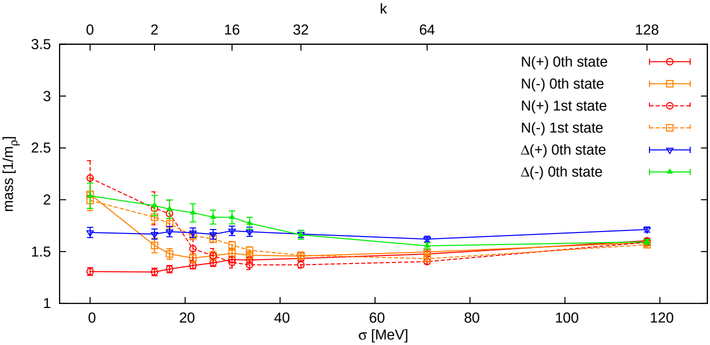

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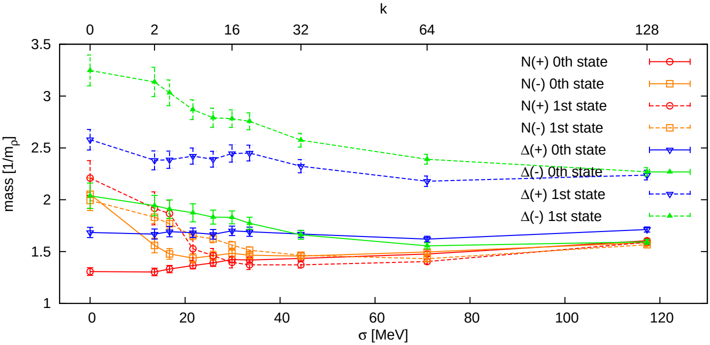
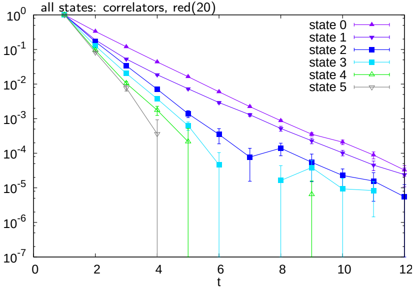


$N\,(J^P = 1/2^+)$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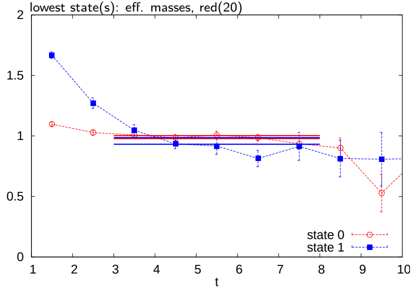
state 0
state 1
state 2
state 3
state 4
state 5
t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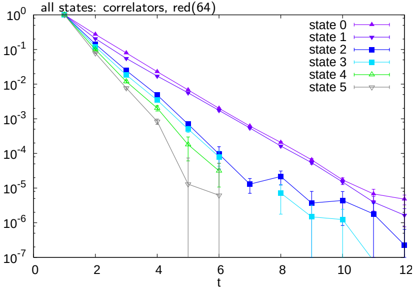
state 0
state 1
t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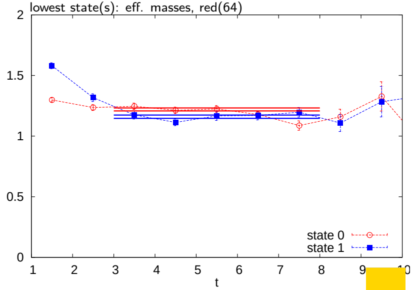
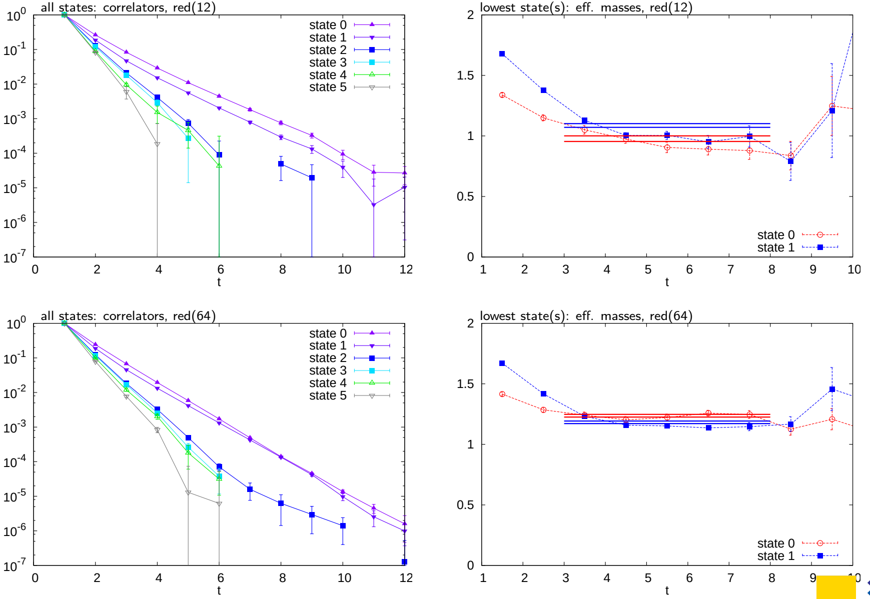
state 0
state 1
state 2
state 3
state 4
state 5
t


lowest state(s): eff. masses, red(64)
state 0
state 1
t

$N\,(J^P = 1/2^-)$


all states: correlators, red(12)
state 0
state 1
state 2
state 3
state 4
state 5
t
lowest state(s): eff. masses, red(12)
state 0
state 1
t
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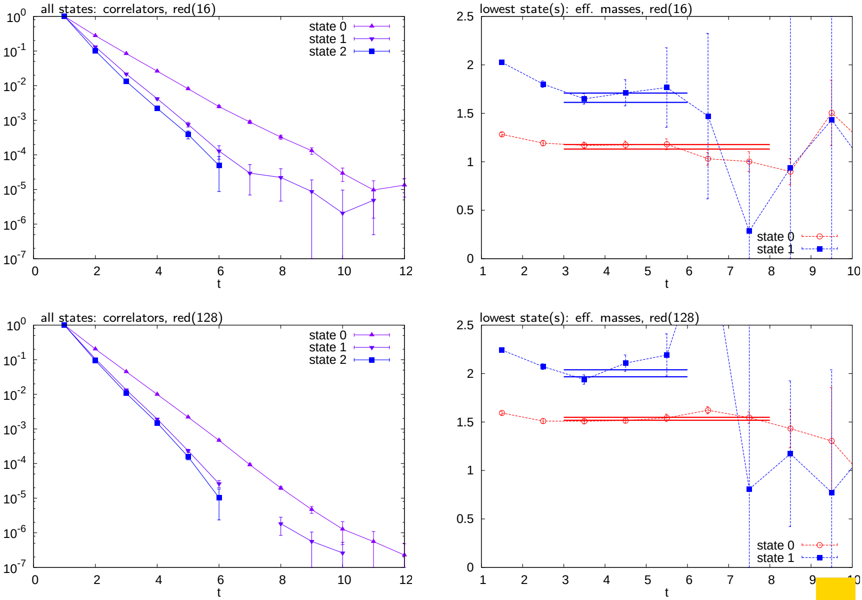
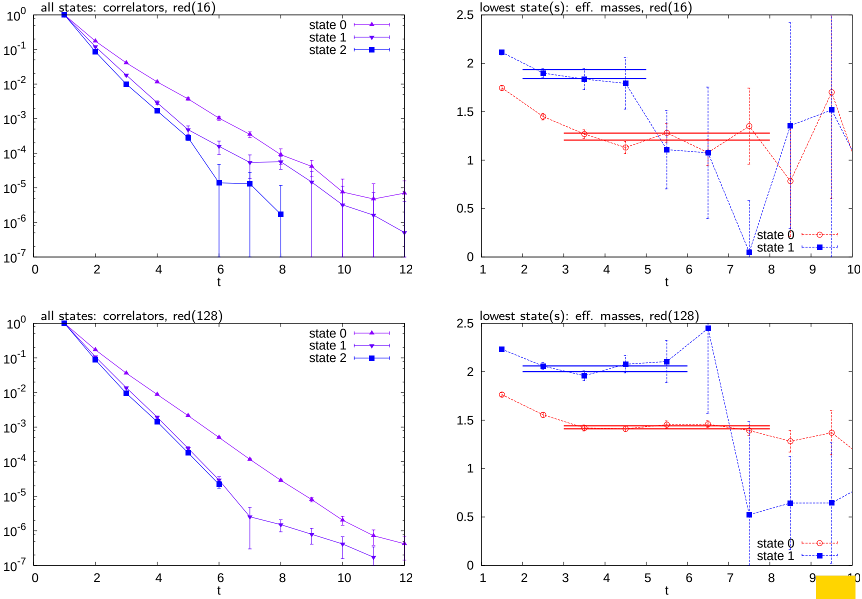
state 0
state 1
state 2
state 3
state 4
state 5
t
lowest state(s): eff. masses, red(64)
state 0
state 1
t

$\Delta\ (J^P = 3/2^+)$


all states: correlators, red(16)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(16)
state 0
state 1
t
all states: correlators, red(128)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(128)
state 0
state 1
t

$\Delta\,(J^P = 3/2^-)$


all states: correlators, red(16)
state 0
state 1
state 2
t
lowest state(s): eff. masses, red(16)
state 0
state 1
t
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state 1
state 2
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The lattice quark propagator

The tree-level quark propagator is

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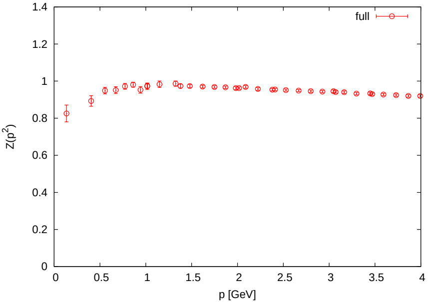
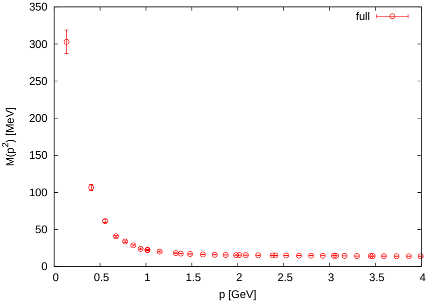
We calculate $S_{\rm bare}(a;p)$ in Landau gauge on the lattice and therefrom extract

- the renormalization function $Z(\mu;p^2)$
- the renormalization point independent mass function $M(p^2)$

The quark propagator under eigenmode reduction





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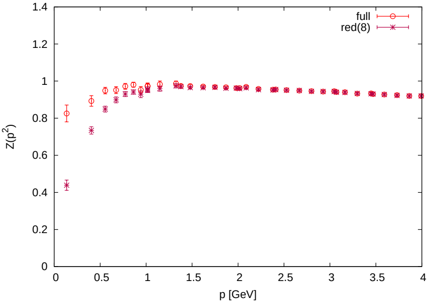

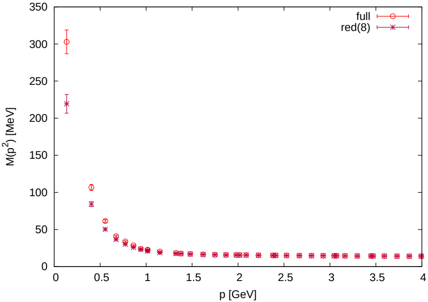

[MS: PLB 711 (2012), arXiv:1112.5107]

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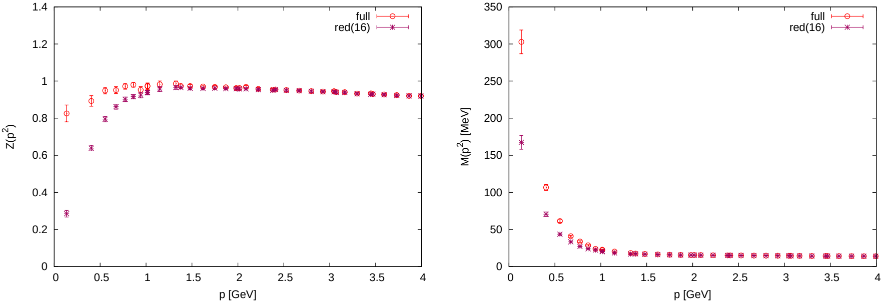

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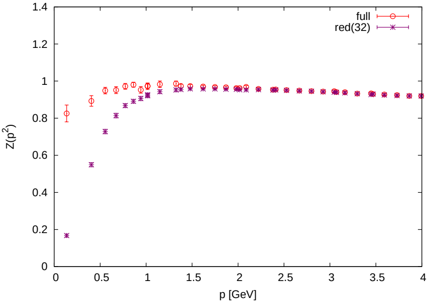

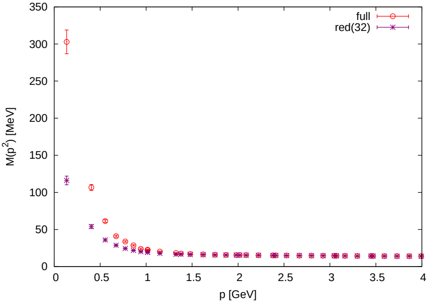

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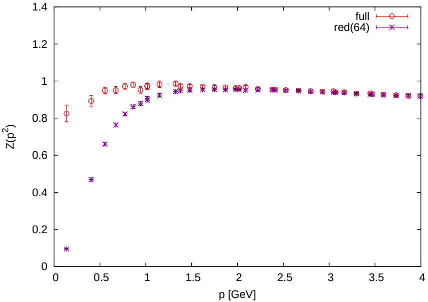

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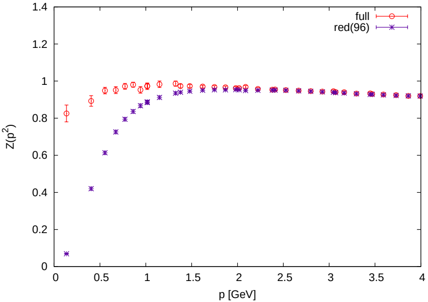

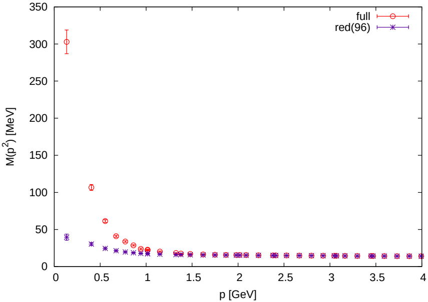

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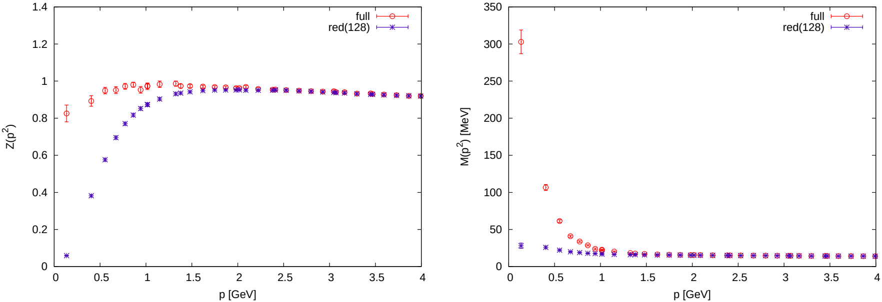

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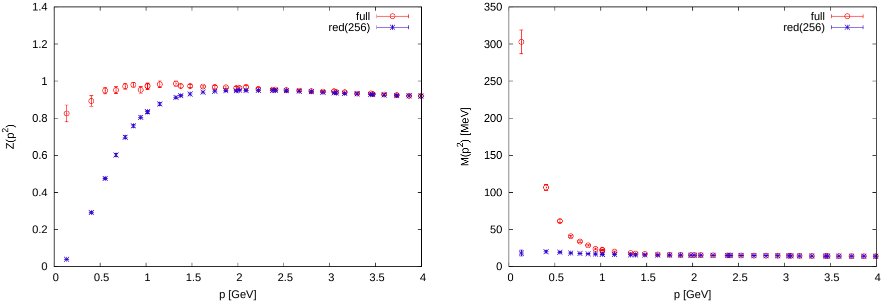

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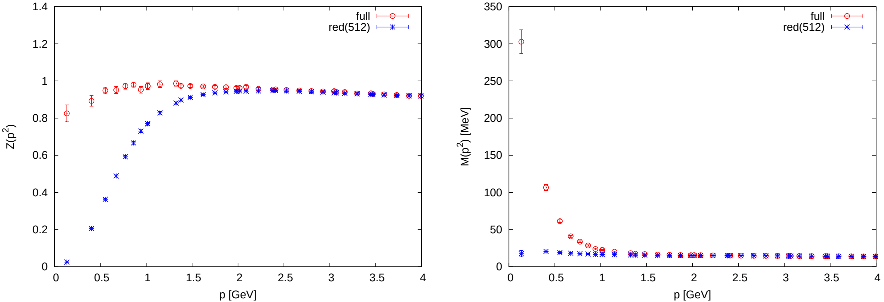

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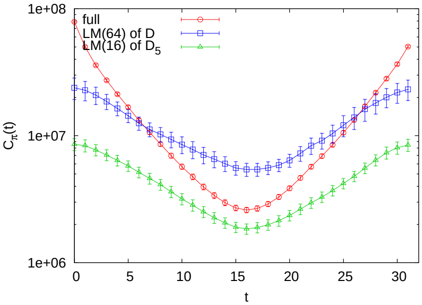
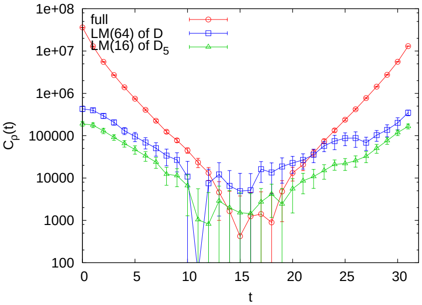
Conclusions

- low lying eigenvalues of the Dirac operator are associated with chiral symmetry breaking
- we have computed hadron propagators while removing increasingly more of the low lying eigenmodes of the Dirac operator
- the confinement properties remain intact, i.e., we still observe clear bound states for most of the studied hadrons
- the mass values of the vector meson chiral partners a_1 and ρ approach each other: restoration of $SU(2)_L \times SU(2)_R$
- no degeneracy between a_1 and b_1: $U(1)_A$ axial anomaly untouched
- the nucleon and the $N(1535)$ become degenerate
- the spin-spin color-magnetic interaction and the flavor-spin interaction are of equal importance for the $\Delta - N$ splitting
- the dynamical mass generation of quarks as seem from the IR behavior of $M(p^2)$ unimportant for chiral symmetric hadrons

Low-mode contribution of D and D_5 to the π and ρ correlators





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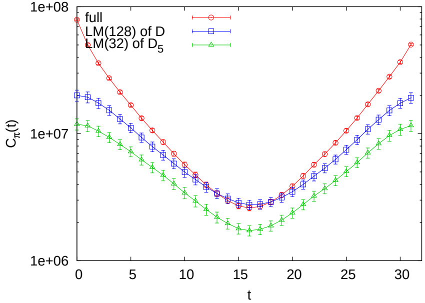

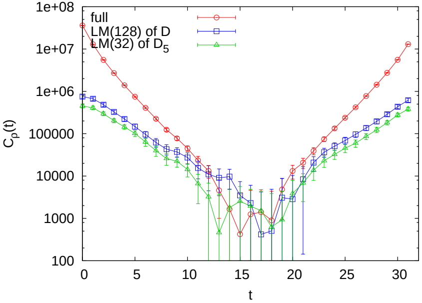

ρ interpolators

$\#_\rho$	interpolator(s)
1	$\overline{a}_n \gamma_k b_n$
8	$\overline{a}_w \gamma_k \gamma_t b_w$
12	$\overline{a}_{\partial_k} b_w - \overline{a}_w b_{\partial_k}$
17	$\overline{a}_{\partial_i} \gamma_k b_{\partial_i}$
22	$\overline{a}_{\partial_k} \epsilon_{ijk} \gamma_j \gamma_5 b_w - \overline{a}_w \epsilon_{ijk} \gamma_j \gamma_5 b_{\partial_k}$

Interpolators for the ρ-meson, $J^{PC} = 1^{--}$. The first column shows the number, the second shows the explicit form of the interpolator. cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]

a_1 interpolators

$\#_{a_1}$	interpolator(s)
1	$\overline{a}_n \gamma_k \gamma_5 b_n$
2	$\overline{a}_n \gamma_k \gamma_5 b_w + \overline{a}_w \gamma_k \gamma_5 b_n$
4	$\overline{a}_w \gamma_k \gamma_5 b_w$

a_1-meson, $J^{PC} = 1^{++}$, cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]

b_1 interpolators

$\#_{b_1}$	interpolator(s)
6	$\overline{a}_{\partial_k}\gamma_5 b_n - \overline{a}_n\gamma_5 b_{\partial_k}$
8	$\overline{a}_{\partial_k}\gamma_5 b_w - \overline{a}_w\gamma_5 b_{\partial_k}$

b_1-meson, $J^{PC} = 1^{+-}$, cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]

N interpolators

- $N^{(i)} = \epsilon_{abc} \, \Gamma_1^{(i)} \, u_a \left(u_b^T \, \Gamma_2^{(i)} \, d_c - d_b^T \, \Gamma_2^{(i)} \, u_c \right)$
- $N(+)$: 1, 2, 4, 14, 15, 18
- $N(-)$: 1, 7, 8, 9

$\chi^{(i)}$	$\Gamma_1^{(i)}$	$\Gamma_2^{(i)}$	smearing	$\#_N$
$\chi^{(1)}$	$\mathbb{1}$	$C\,\gamma_5$	$(nn)n$	1
			$(nn)w$	2
			$(nw)n$	3
			$(nw)w$	4
			$(ww)n$	5
			$(ww)w$	6
$\chi^{(2)}$	γ_5	C	$(nn)n$	7
			$(nn)w$	8
			$(nw)n$	9
			$(nw)w$	10
			$(ww)n$	11
			$(ww)w$	12
$\chi^{(3)}$	$i\,\mathbb{1}$	$C\,\gamma_t\,\gamma_5$	$(nn)n$	13
			$(nn)w$	14
			$(nw)n$	15
			$(nw)w$	16
			$(ww)n$	17
			$(ww)w$	18

cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]

Δ interpolators

- $\epsilon_{abc}\, u_a \left(u_b^T\, C\, \gamma_k\, u_c\right)$
- $\Delta(+)$: 1, 2, 3
- $\Delta(-)$: 1, 2, 3

smearing	$\#_\Delta$
$(nn)n$	1
$(nn)w$	2
$(nw)n$	3
$(nw)w$	4
$(ww)n$	5
$(ww)w$	6

cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]