Model Investigation of QCD thermodynamics and phase diagram

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Excited QCD
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arXiv:1108.2941 (Phys. Rev. D 85, 074018 (2012)) and
arXiv:1202.2672

Outline

Outline

Motivation

- Understanding the QCD phase diagram at non zero temperature and baryon density is an essential goal of heavy ion collision experiments
- Theoretically, it is a tough task because the transition between the hadronic and quark gluon phases takes place typically at the scale of Λ_{QCD} where perturbative techniques fail
- Lattice QCD is the ideal candidate in such cases, but there are issues, the most severe being the calculations at nonzero baryon density
- A complementary understanding could be built up by studying QCD-like models that are easier to deal with

Strategy

- Make sure that the symmetry breaking pattern is same
  - $m_q = 0$:$SU_L(N_f) \times SU_R(N_f)$ $(T > T_\chi)(\langle \bar{q}q \rangle = 0)$ $\rightarrow SU_V(N_f)\,(T < T_\chi)(\langle \bar{q}q \rangle \neq 0)$
  - $m_q \rightarrow \infty$: $Z(3)\,(T < T_d)(\langle L \rangle = 0)$ is spontaneously broken for $(T > T_d)(\langle L \rangle \neq 0)$ where L is the Polyakov loop that serves as the order parameter
- The relevant degrees of freedom carry the 'correct' quantum numbers
  - At low T, mesons and baryons dominate
  - At large T, goes to a gas of quarks and gluons

Outline

Model

- (P)QM: Scavenius et al 2002, Schaefer, Pawlowski, Wambach 2007
- Lagrangian
  $\mathcal{L} = \mathrm{Tr}\left(\partial_\mu M^\dagger \partial^\mu M\right) + \bar{q}_f\left(i\gamma^\mu D_\mu - g M_5\right) q_f - \mathcal{U}_M(M) - \mathcal{U}_P\left(\Phi, \bar{\Phi}, T\right)$,
  where $q_f = (u, d, s)^T$, $M = T_a(\sigma_a + i\pi_a)$ are the mesonic fields with T_a as the generators of $U(3)$, the gluon dynamics is described by an effective theory of Polyakov loops where $\Phi = \frac{1}{N_c}\langle \mathrm{Tr}_c L\rangle$
- $\mathcal{U}_M(M) = m^2 \mathrm{Tr}(M^\dagger M) + \lambda_1 \left[\mathrm{Tr}(M^\dagger M)\right]^2 + \lambda_2 \mathrm{Tr}\left(M^\dagger M\right)^2 - c\left[det(M) + det(M^\dagger)\right] - \mathrm{Tr}\left[H(M + M^\dagger)\right]$, where $H = T_a h_a$, to realise the $(2+1)$ scenario all except h_0 and h_8 are 0; with nonzero condensates (σ_0, σ_8) rotated to non strange-strange basis (σ_x, σ_y)

Model

Mean Field Analysis

- Grand Canonical Partition function for a spatially uniform system in thermal equilibrium at finite temperature T and quark chemical potential $\mu_f (f = u, d, s)$: $Z = \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \mathcal{D}\Phi \mathcal{D}\bar{\Phi} \mathcal{D}q \mathcal{D}\bar{q}$ $\exp\left[-\int_0^\beta d\tau \int_V d^3x \left(\mathcal{L}^{\mathcal{E}} + \sum_{f=u,d,s} \mu_f \bar{q}_f \gamma^0 q_f \right) \right]$
- Neglect the thermal and quantum fluctuations of M fields and Φ and $\bar{\Phi}$
- Thus we obtain the grand potential $\Omega\left(T, \mu_x, \mu_y\right) = -\frac{T \ln Z}{V} = \mathcal{U}_{\mathcal{M}}\left(\sigma_x, \sigma_y\right) + \mathcal{U}_P\left(\Phi, \bar{\Phi}, T\right) + \Omega_{\bar{q}q}$

Model

Meson potential

- $\mathcal{U}_{\mathcal{M}}\left(\sigma_x, \sigma_y\right) = \frac{m^2}{2}\left(\sigma_x^2 + \sigma_y^2\right) - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y + \frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}\left(2\lambda_1 + \lambda_2\right)\sigma_x^4 + \frac{1}{8}\left(2\lambda_1 + 2\lambda_2\right)\sigma_y^4 - h_x\sigma_x - h_y\sigma_y$

Model

Polyakov loop potential

- A Landau-Ginzburg type polynomial potential[1]
  $\mathcal{U}_{Poly}\left(\Phi, \bar{\Phi}, T\right) = T^4 - \frac{b_2(T)}{2}\Phi\bar{\Phi} - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\Phi\bar{\Phi}\right)^2$ where
  $b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$
- the scenario can be even bettered by taking care of the Jacobian of transformation from L to Φ which results in the following modified ansatz[2]: $\mathcal{U}_{Poly-VM}/T^4 = \mathcal{U}_{Poly}/T^4 - \kappa log[J(\Phi, \bar{\Phi})]$ where
  $J(\Phi, \bar{\Phi}) = (1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2)$

[1] Ratti et al. 2006

[2] Ratti et al. 2007, Ghosh et al. 2008

Model

Including vacuum fluctuations

- $\Omega_{\bar{q}q} = \Omega^{\mathrm{v}}_{\bar{q}q} + \Omega^{\mathrm{th}}_{\bar{q}q}$,
  - $\Omega^{\mathrm{v}}_{\bar{q}q} = -2N_c \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_f$
  - $\Omega^{\mathrm{th}}_{\bar{q}q} = -2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left[\ln g_f^+ + \ln g_f^-\right]$ where $g_f^+ = \left[1 + 3\Phi e^{-E_f^+/T} + 3\bar{\Phi} e^{-2E_f^+/T} + e^{-3E_f^+/T}\right]$. Here $\mathrm{E}_f^\pm = E_f \mp \mu_f$ where E_f is the single particle energy of a quark/antiquark, $E_f = \sqrt{p^2 + {m_f}^2}$ and $m_x = g\frac{\sigma_x}{2}$, $m_y = g\frac{\sigma_y}{\sqrt{2}}$
- In the 2 flavor massless case, the order of the phase transition changes from first to second order[3]
- Modifies the shape of the isentropic trajectories near the critical end point[4]
- In the presence of magnetic field, the phase diagram is shown to be considerably affected by the vacuum term[5]

[3] Phys. Rev. D 82, 034029 (2010)
[4] Phys. Lett. B 682 401407 (2010)
[5] Phys. Rev. D 82, 105016 (2010)

Model

Including vacuum fluctuations

- Study the effect of including the vacuum fluctuations on thermodynamics of the $(2+1)$ PQM model and compare with lattice
- Use dim. reg. to regularise the diverging vacuum integral: $\Omega^{\mathrm{v}}_{\bar{q}q} = \Omega^{\mathrm{reg}}_{\bar{q}q}(\Lambda) = -\frac{N_c}{8\pi^2}\sum_{f=u,d,s} m_f^4 \log\left[\frac{m_f}{\Lambda}\right] \sim -\sigma^4 \log(\sigma)$
- Ω can be shown to be independent of $\Lambda \Rightarrow \Lambda$ is an arbitrary parameter and physical observables do not depend on it.
- Solve the following gap eqns. to determine the mean field configurations $(\sigma_x, \sigma_y, \Phi, \bar{\Phi})$ to be put into Ω and hence obtain results on thermodynamics and susceptibilities: $\frac{\partial\Omega}{\partial\sigma_x} = \frac{\partial\Omega}{\partial\sigma_y} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\bar{\Phi}} = 0$

Model

Model Parameters

- Fixed by inputs from lattice and experiments
- The parameters of the mesonic potential namely m^2, λ_1, $\lambda_2\,(\Lambda)$, c, g, h_x and h_y are fixed by comparing the vacuum properties to the following experimentally known quantities: m_π and f_π, m_K and f_K, $\left(m_\eta^2 + m_{\eta'}^2\right)$ and m_σ. g is fixed by using a light quark constituent mass $m_x = 300$ MeV. This gives a strange quark constituent mass $m_s \approx 433$ MeV.
- Parameters of the Polyakov potential are fixed from lattice inputs of both pure gauge theory as well as $(2+1)$ QCD.

Model

Model Parameters

- Inconsistencies between HotQCD and WB:[6]
  - While in HotQCD $T_\chi \sim T_d$, in WB $T_\chi < T_d$
  - The peak value of the conformal symmtery breaking measure, Δ in the case of HotQCD is almost 150 % greater than that of WB
- We tune T_0 to fix the relative order of the chiral and deconfinement crossovers and adjust κ to tune the peak value of Δ
- For WB: $T_0 = 270$ MeV and $\kappa = 0.2$, for HotQCD: $T_0 = 210$ MeV and $\kappa = 0.1$

[6] latest results from HotQCD (arXiv:1111.1710 [hep-lat]) tend to agree better with WB

Fixing κ

$(\epsilon - 3P)/T^4$

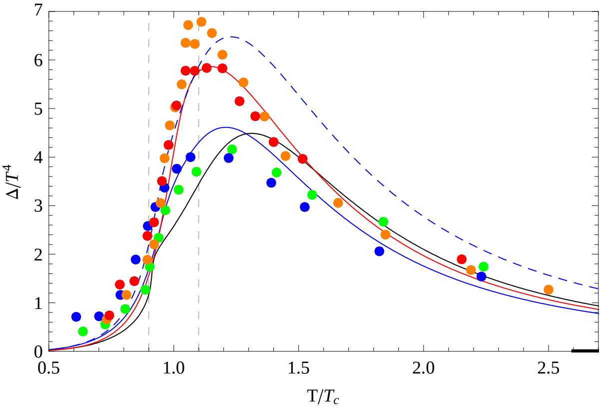
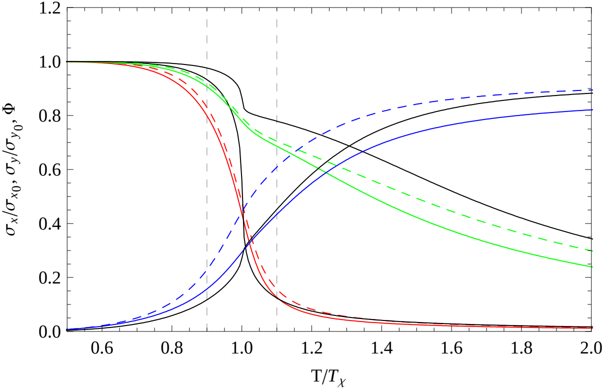


Figure: Plots of Δ in PQMVT.

Outline

Results

Condensates



Results

Condensates

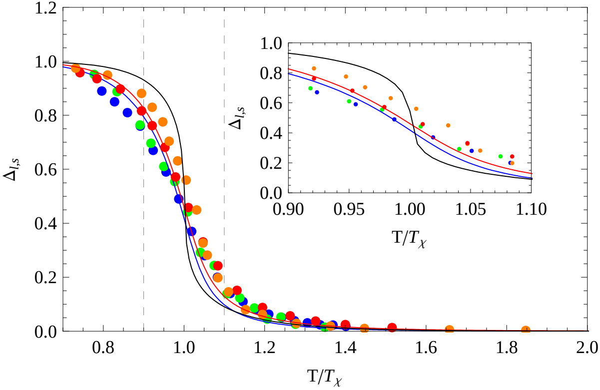


Figure: Plots of $\Delta_{l,s}$ in ModelWB (blue) and ModelHotQCD (red) as obtained in PQMVT are shown.

Results

p, E, s, c_V

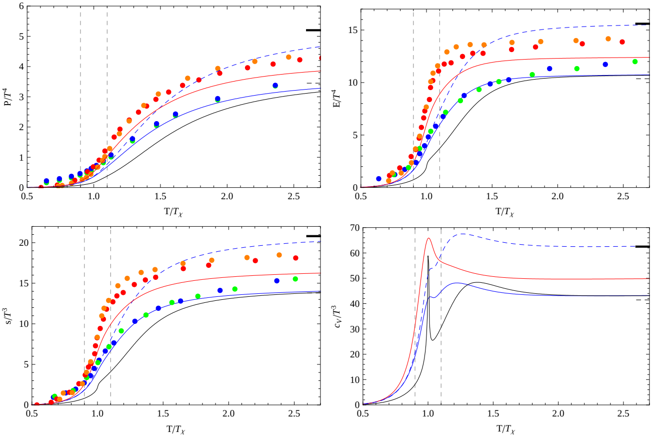
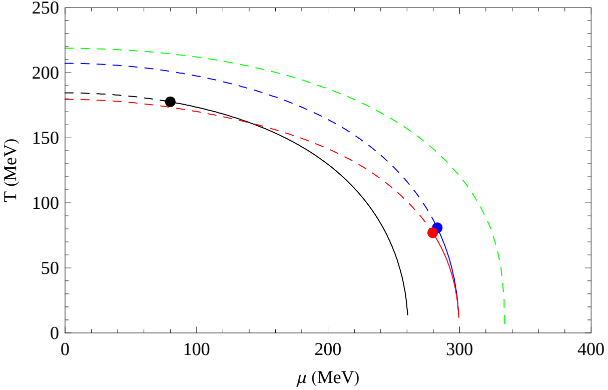


Figure: Plots of p, E, s and c_V in PQMVT.

Results

CEP pushed deeper into nonzero μ

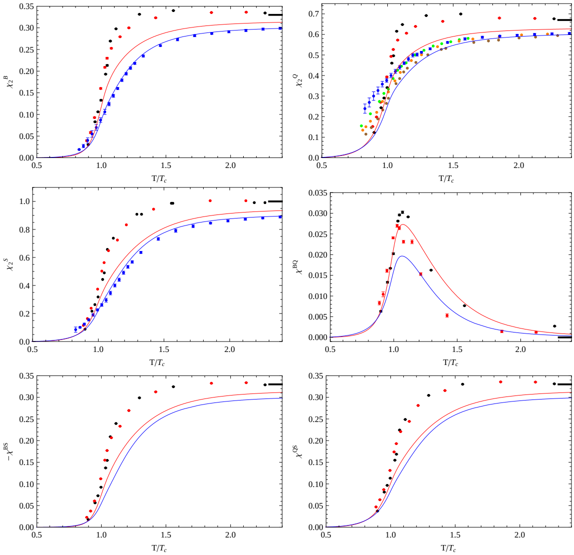


Results

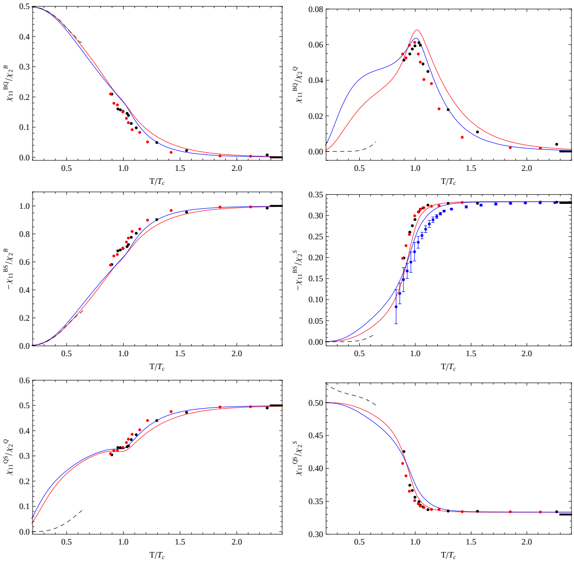
Susceptibilities

- QCD has three conserved charges: baryon number B, strangeness S and electric charge Q
- Fluctuations and correlations of the conserved charges are expected to provide telltale signatures of the CEP (Stephanov 1998)
- Low and high T behaviour:
  - High T: $\frac{P^{SB}}{T^4} = \sum_{f=u,d,s} \left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right]$
  - Low T: $p^{\mathrm{HRG}}/T^4 = \frac{1}{VT^3}\sum_{i\in\mathrm{hadrons}} \ln Z_i\left(T, V, \mu_B, \mu_Q, \mu_S\right)$
  - $\chi_{h}{}_{ijk}^{BQS} \approx \frac{d_l}{\pi^2}\left(\frac{m_l}{T}\right)^2 K_2\left(\frac{m_l}{T}\right) B_l^i Q_l^j S_l^k$
  - Similarly in PQM at low T: $p^{\mathrm{PQM}}/T^4 \sim K_2\left(3m_q/T\right) cosh\left(3\mu_q/T\right)$
  - Then ratios like $\chi_{h}{}_X^i/\chi_{h}{}_X^j$ become simply X_l^{i-j} where $X \in B, Q, S$ and X_l are the B, Q, S quantum numbers of h_l. Since the quantum numbers carried by the relevant degrees of freedom are the same in both HRG and PQM at low T, they agree well.

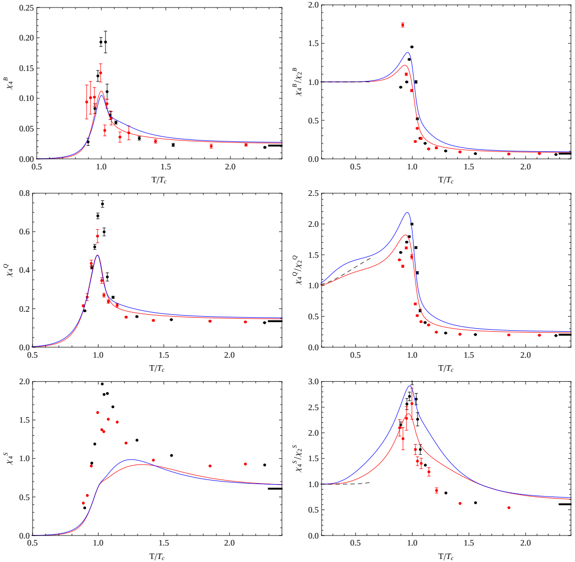
Susceptibilities ($i + j + k = 2$)



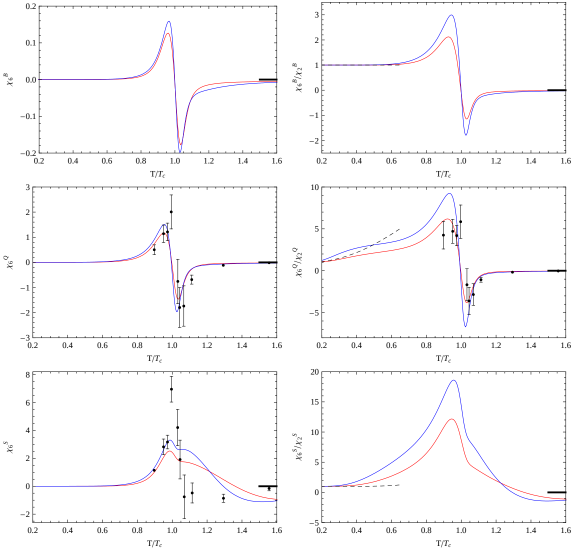
Ratios


χ_{11}^{BQ}/χ_2^B
χ_{11}^{BQ}/χ_2^Q
$-\chi_{11}^{BS}/\chi_2^B$
$-\chi_{11}^{BS}/\chi_2^S$
χ_{11}^{QS}/χ_2^Q
χ_{11}^{QS}/χ_2^S
T/T_c

Susceptibilities $(i + j + k = 4)$



Susceptibilities $(i + j + k = 6)$



Outline

Conclusions and Outlook

- The conventional PQM model suffers from a rapid phase transition contrary to what is found through lattice.
- This could be due to the use of a Polyakov potential which carries with it a remnant first order phase transition of the pure glue theory. Another possible reason could be the sudden release of quark degrees of freedom in PQM with increase in temperature.
- Addition of the vacuum term in PQMVT addresses the latter. This tames the rapid transitions that we see in the PQM model and significantly improves the model's agreement to lattice data.
- The resulting model describes qualitatively the thermodynamics obtained in LQCD.

Conclusions and Outlook

- The susceptibilities as obtained in the model compare well with those of LQCD at 0 μ, particularly the ratios where undetermined normalisation effects get cancelled as well as the details of the spectrum, they interpolate well between the low T HRG limit and high T ideal gas limit.
- There are two possible ways ahead
  - Improve the model to include gluon dynamics beyond the potential description, include fluctuations, effect of baryons
  - Apply the model to non-zero μ, include finite size effects, beyond equilibrium physics