The lattice Landau gauge gluon propagator at zero and finite temperature

Orlando Oliveira [1], Paulo Silva [1]

[1] Centro de Física Computacional, Universidade de Coimbra, Portugal

8 May 2012

Outline

1. Introduction
2. Gluon propagator at zero temperature
3. Gluon propagator at finite temperature

Gluon confinement in Landau gauge

- Kugo-Ojima confinement criterion $\frac{1}{G(0)} = 1 + u = 0$
- Zwanziger horizon condition $D(0) = 0$

Study of these propagators in the infrared limit requires nonperturbative methods

- Dyson-Schwinger equations (DSE)
- Lattice QCD

	Good features	Bad features
DSE	analitical solution in the IR	truncation of infinite tower of equations
Lattice	include all non-perturbative physics	finite volume and finite lattice spacing

Dyson-Schwinger equations

- Infinite tower of equations relating the QCD Green's functions;
- Finding a solution requires:
  - truncation of the tower of equations;
  - parameterization of the vertices;
- Two DSE solutions have emerged:
  - Scaling solution: D(0)=0, G(0) diverges
    [Alkofer, Fischer, von Smekal,...]

$$Z_{gluon}(p^2) \sim (p^2)^{2\kappa} \, , \; Z_{ghost}(p^2) \sim (p^2)^{-\kappa} \, , \; \kappa = 0.595$$

Review: C.S. Fischer, J. Phys. **G32**, R253 (2006)

  - Decoupling solution: finite non-zero D(0) and G(0)
    [Aguilar, Natale, Papavassiliou,...]

A. C. Aguilar, D. Binosi, J. Papavassiliou, Phys. Rev. **D78** (2008) 025010 and references therein

Refined Gribov-Zwanziger action

- within Gribov-Zwanziger action, takes into account $d = 2$ condensates
- same qualitative IR behaviour as the decoupling solution of DSE

D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel, H. Verschelde, Phys. Rev. D78 (2008) 065047

$$D(q^2) = \frac{q^2 + M^2}{q^4 + (M^2 + m^2)q^2 + 2g^2N\gamma^4 + M^2m^2}$$

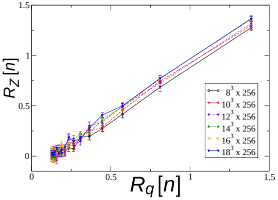
- good description of the infrared lattice gluon propagator

D. Dudal, O. Oliveira, N. Vandersickel, PRD81(2010) 074505
A. Cucchieri, D. Dudal, T. Mendes, arXiv:1111.2327

Zero temperature gluon propagator on the lattice

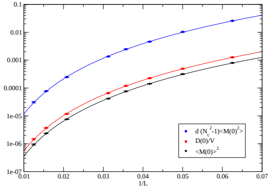
Albeit there were some support for D(0)=0...

- Ratios



O. Oliveira, P. J. Silva, EPJC 62 (2009) 525

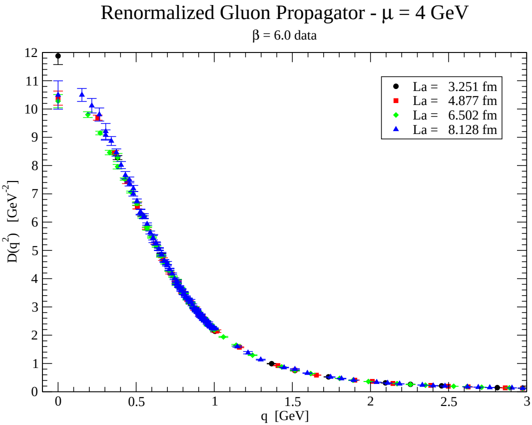
- SU(3) Cucchieri-Mendes bounds



O. Oliveira, P. J. Silva, PRD 79 (2009) 031501(R)

Gluon propagator on the lattice

... scaling solution is not directly observed in large-volume lattice simulations



Gluon propagator on the lattice

- SU(2): $La = 27\text{fm}\, a \sim 0.22\text{fm}$
  A. Cucchieri, T. Mendes, PoS(LAT2007)297
- SU(3): $La = 17\text{fm}\, a \sim 0.18\text{fm}$
  I. Bogolubsky, E. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, Phys. Lett. B676 (2009) 69

- relative large lattice spacing compared with 1 fm
- does this matter?

Gluon propagator on the lattice - technical details

- Landau gauge fixing

$$F_U[g] = C_F \sum_{x,\mu} \mathrm{Re}\{\mathrm{Tr}[g(x)U_\mu(x)g^\dagger(x+\hat{\mu})]\}$$

(Fourier accelerated) Steepest Descent, Overrelaxation

- Gluon propagator

$$D^{ab}_{\mu\nu}(\hat{q}) = \frac{1}{V}\langle A^a_\mu(\hat{q})\, A^b_\mu(-\hat{q})\rangle = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) D\left(q^2\right)$$

Lattice setup

β	a(fm)	1/a (GeV)	L	La (fm)	#Conf
5.7	0.1838	1.0736	44	8.087	55
5.7	0.1838	1.0736	36	6.617	100
5.7	0.1838	1.0736	26	4.780	132
5.7	0.1838	1.0736	18	3.308	149
6.0	0.1016	1.942	80	8.128	55
6.0	0.1016	1.942	64	6.502	121
6.0	0.1016	1.942	48	4.877	104
6.0	0.1016	1.942	32	3.251	126
6.2	0.07260	2.718	80	5.808	70
6.2	0.07260	2.718	64	4.646	99
6.2	0.07260	2.718	48	3.485	87
6.4	0.05445	3.624	80	4.356	52

Lattice setup

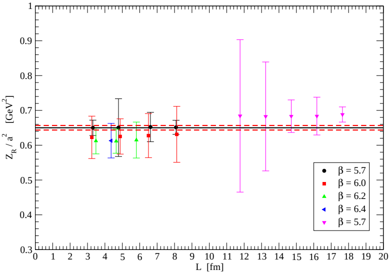
Berlin-Moscow-Adelaide ensembles

β	a(fm)	1/a (GeV)	L	La (fm)
5.7	0.1838	1.0736	64	11.763
5.7	0.1838	1.0736	72	13.234
5.7	0.1838	1.0736	80	14.704
5.7	0.1838	1.0736	88	16.174
5.7	0.1838	1.0736	96	17.645

I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, Phys Lett B676, 69 (2009)

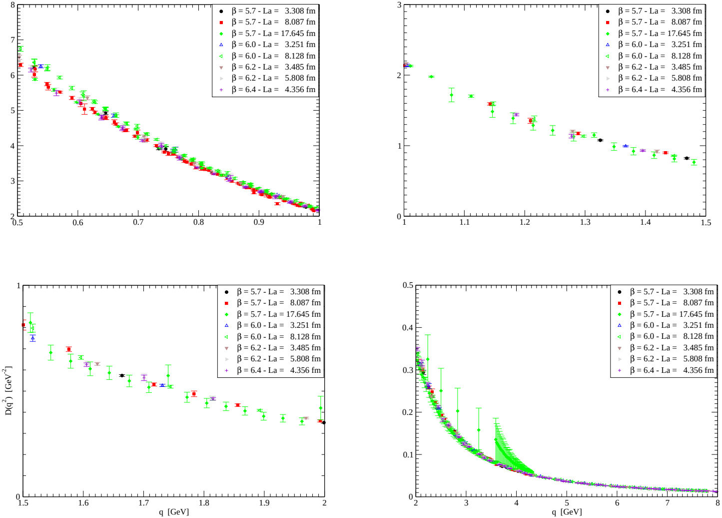
Renormalization

- All lattice data renormalized at $\mu = 4\text{GeV}$
- $D(q^2) = Z_R D_{Lat}(q^2)$
  $D(\mu^2) = 1/\mu^2$
- this procedure removes UV lattice artifacts

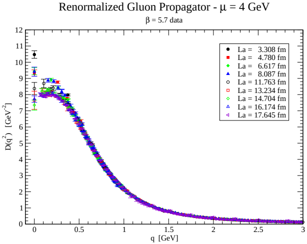


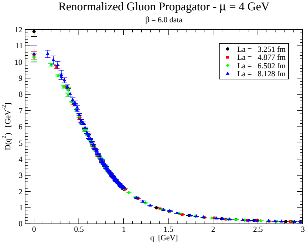
$$\frac{Z_R}{a^2} = 0.6501(65)\ \text{GeV}^2 \text{ is volume independent}$$

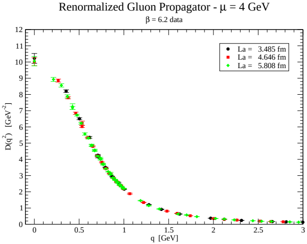
Well defined for momenta above 900 MeV

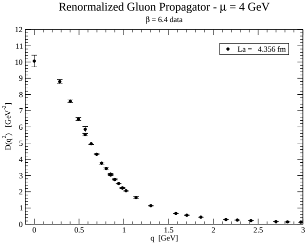


IR propagator decreases with lattice volume

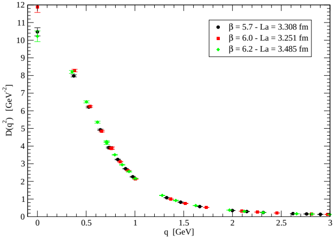


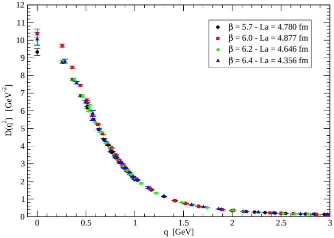


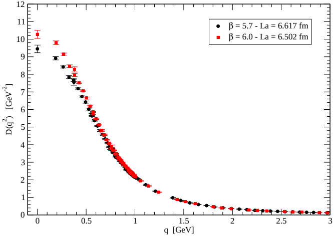


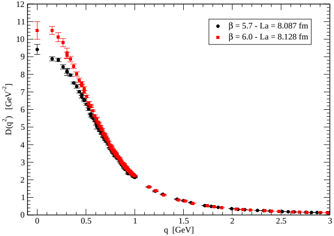


Larger *a* underestimates IR propagator



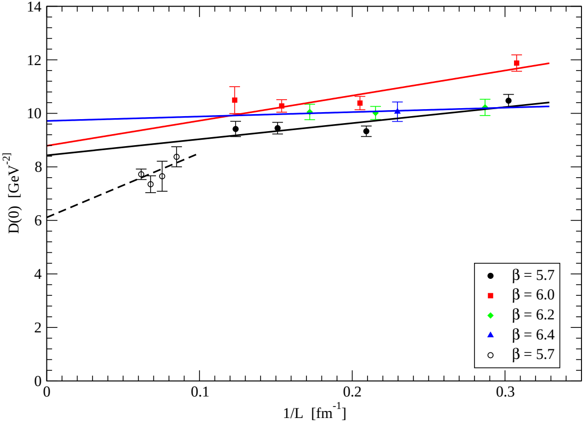






Extrapolation of D(0)

$$D(0) = \frac{c}{L} + D_\infty(0)$$



Summary - Zero temperature

- Good description of the gluon propagator in the UV
- be careful with large lattice spacing simulations
    - $\beta = 5.7$ results seem to be a lower bound
- Linear extrapolations $D_\infty(0) \sim 8 - 10 GeV^{-2}$
    - Always possible to use an anzatz such that $D_\infty(0) = 0$
    - Will this remain a unsolved question?

Gluon propagator at finite temperature

Propagator splitted into two components

- transverse D_T
- longitudinal D_L

$$D^{ab}_{\mu\nu}(\hat{q}) = \delta^{ab} \left(P^T_{\mu\nu} D_T(q_4^2, \vec{q}) + P^L_{\mu\nu} D_L(q_4^2, \vec{q}) \right)$$

Transverse and longitudinal projectors in the Landau gauge

$$P^T_{\mu\nu} = (1 - \delta_{\mu 4})(1 - \delta_{\nu 4}) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^2} \right)$$

$$P^L_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) - P^T_{\mu\nu} \tag{1}$$

Gluon propagator at finite temperature

$$D_T(q^2) = \frac{1}{2V(N_c^2-1)} \left(\langle A_i^a(q) A_i^a(-q) \rangle - \frac{q_4^2}{\vec{q}^2} \langle A_4^a(q) A_4^a(-q) \rangle \right)$$

$$D_L(q^2) = \frac{1}{V(N_c^2-1)} \left(1 + \frac{q_4^2}{\vec{q}^2} \langle A_4^a(q) A_4^a(-q) \rangle \right)$$

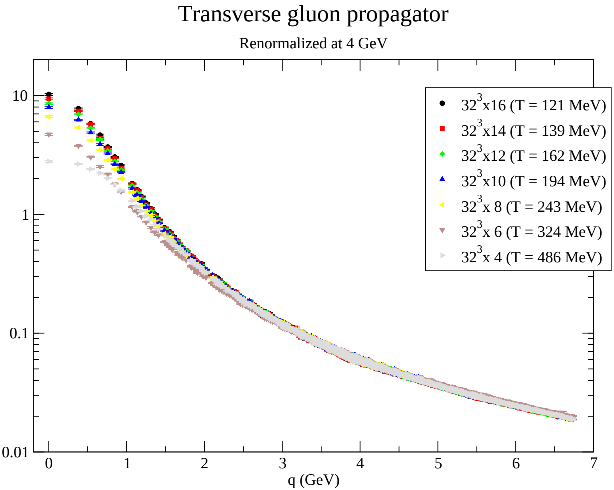
On the lattice, finite temperature is introduced by reducing the extent of temporal direction: $L_t << L_s$

$$T = \frac{1}{aL_t}$$

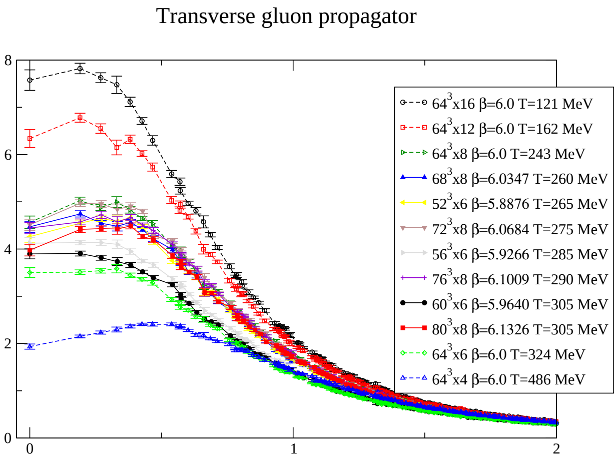
Lattice setup

Temp. (MeV)	β	L_s	L_t	a [fm]	1/a (GeV)
121	6.0000	32,64	16	0.1016	1.9426
162	6.0000	32,64	12	0.1016	1.9426
243	6.0000	32,64	8	0.1016	1.9426
260	6.0347	68	8	0.09502	2.0767
265	5.8876	52	6	0.1243	1.5881
275	6.0684	72	8	0.08974	2.1989
285	5.9266	56	6	0.1154	1.7103
290	6.1009	76	8	0.08502	2.3211
305	5.9640	60	6	0.1077	1.8324
305	6.1326	80	8	0.08077	2.4432
324	6.0000	32,64	6	0.1016	1.9426
486	6.0000	32,64	4	0.1016	1.9426

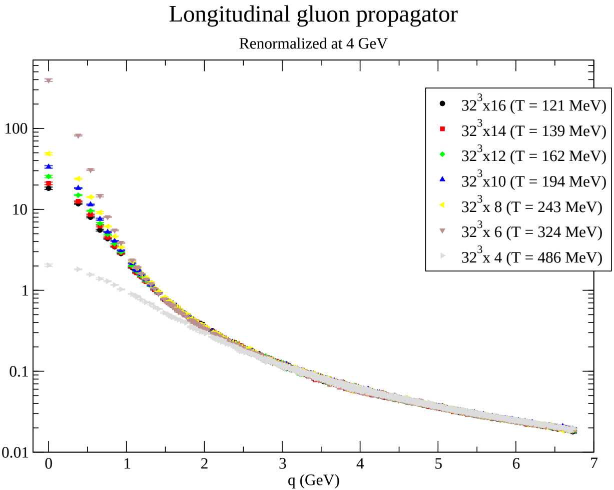
Tranverse gluon propagator - $L \sim 3.3fm$



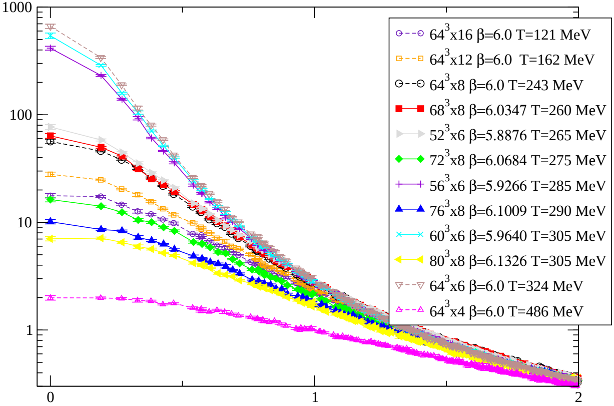
Transverse gluon propagator - $L \sim 6.5fm$



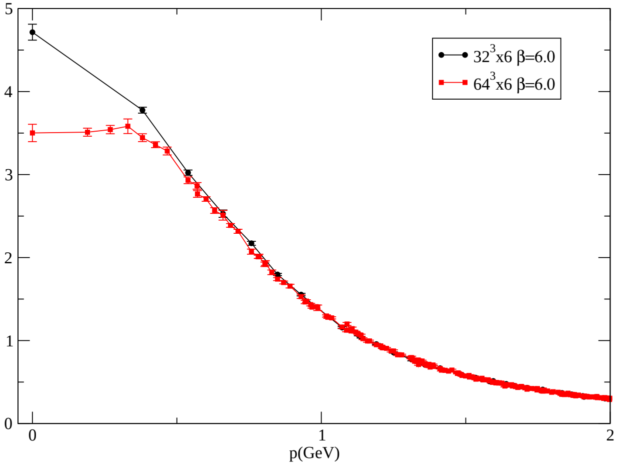
Longitudinal gluon propagator - $L \sim 3.3fm$



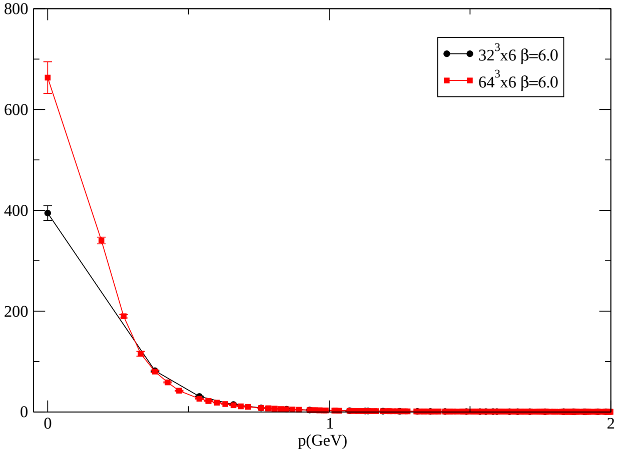
Longitudinal gluon propagator - $L \sim 6.5fm$



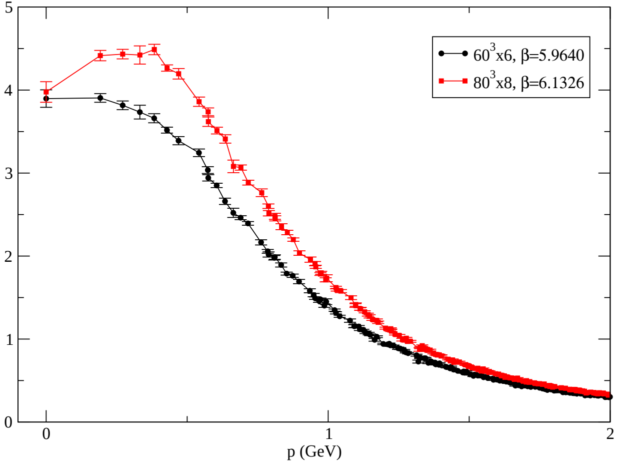
Finite volume effects – tranverse propagator @ 324 MeV



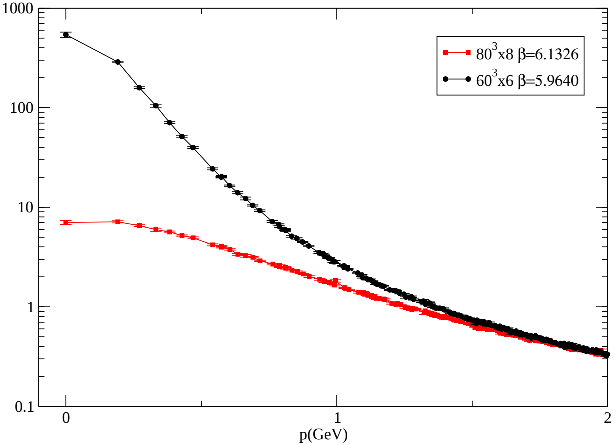
Finite volume effects – longitudinal propagator @ 324 MeV



Finite lattice spacing effects – tranv. propagator @ 305 MeV



Finite lattice spacing effects – longit. propagator @ 305 MeV



Summary - Finite temperature

- Preliminary results for the Landau gauge gluon propagator at finite temperature
- Need to understand lattice effects

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This work is funded by FEDER, through the Programa Operacional Factores de Competitividade- COMPETE and by national funds through FCT – Fundação para a Ciência e Tecnologia in the frame of project CERN/FP/123620/2011. Paulo Silva is also supported by FCT grant SFRH/BPD/40998/2007.

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