The QCD Critical End Point in the Context of the Polyakov–Nambu–Jona-Lasinio Model

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Part I

Collaboration with C. A. de Sousa, M. C. Ruivo and H. Hansen

Contents

- Motivation;
- The PNJL model and formalism;
- PNJL *Characteristic* temperatures;
- PNJL vs. lattice results: comparison of thermodynamical quantities with lattice results;
- Nature of the phase transition at $T = 0$ and at finite T and μ;
- Phase diagram;
- The CEP;
- Anomaly strength and the CEP;
- Summary.

Symmetries of QCD

- **QCD**
  - Chiral (χ_S) symmetry $\Leftrightarrow$ $\begin{cases} - \text{hadron masses} \\ - \text{dynamics of hadrons at low energy} \end{cases}$
  - Center ($\mathbb{Z}_3$) symmetry $\Leftrightarrow$ de/confinement

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    - **explicitly** broken (softly) by the presence of dynamical quarks

- **QCD inspired models**
  - NJL model: only chiral symmetry aspects

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    - chiral symmetry is **explicitly** and **spontaneously** broken
    - $U_A(1)$ symmetry breaking is implemented by ´t Hooft interaction
  - PNJL model: **synthesis** between chiral and de/confinement aspects

QCD phase transitions

- **QCD $\rightarrow$ two phase transitions**
  - restoration of chiral symmetry
    order parameters: **quark condensates**

$$\langle \bar{q}_i q_i \rangle \begin{cases} \neq 0 \Leftrightarrow \text{symmetry broken}, T < T_c \\ = 0 \Leftrightarrow \text{symmetry restored}, T > T_c \end{cases}$$

  - deconfinement
    order parameter: **Polyakov loop**

$$\Phi = \frac{1}{N_c} \mathrm{Tr}_c \left\langle \left\langle \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right] \right\rangle \right\rangle$$

$$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase}, T < T_c \\ \neq 0 \Leftrightarrow \text{deconfined phase}, T > T_c \end{cases}$$

- PURPOSE: Consider a model which describes both low and high temperature QCD behavior in a single picture $\rightarrow$ **PNJL model**

Motivation

- **Understand the QCD phase structure is one of the most important topics in the physics of strong interactions**

- **The very first "QCD" phase diagram taken from Cabibbo-Parisi (1975)**

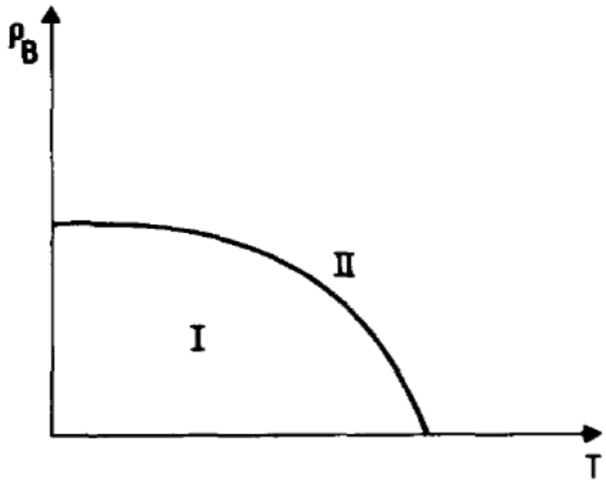
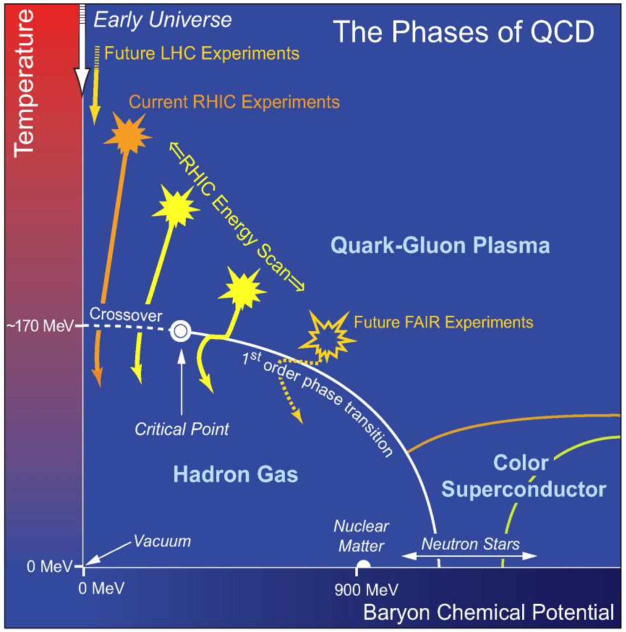


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

- **A schematic outline for the phase diagram of matter at ultrahigh density and temperature**



– N. Cabibbo, G. Parisi, PLB 59 (1975) 67

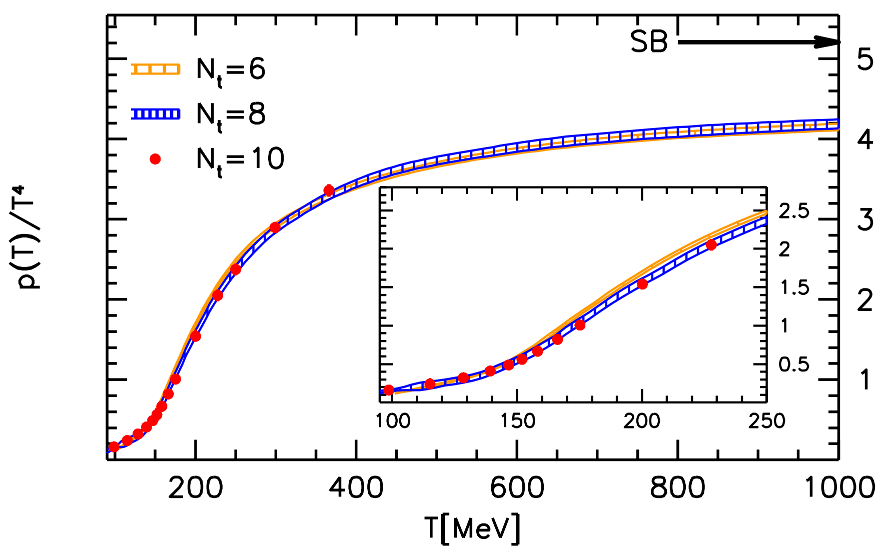
- **Understand the QCD phase structure is one of the most important topics in the physics of strong interactions**

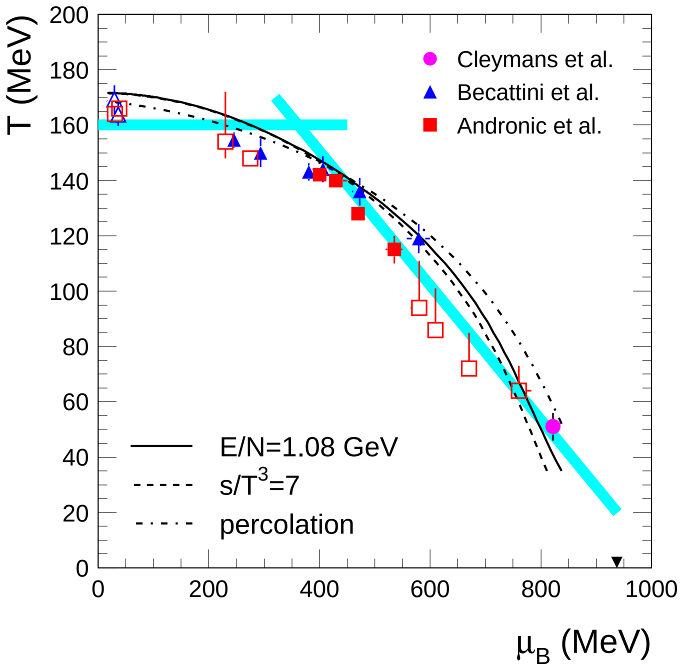
Theoretical point of view:

- Effective model calculations
- Lattice calculations

Experimental point of view:

- Map the QCD phase boundary
- Localization of the CEP





– S. Borsanyi et al., JHEP 1011 (2010) 077 – A. Andronic et al., NPA 837 (2010) 65

Motivation

- **Understand the QCD phase structure is one of the most important topics in the physics of strong interactions**

Experimental point of view:

Beam Energy Scan (BES) program at RHIC[1]:

- map the QCD phase boundary;
- search for the QCD Critical End Point
  - "energy scan" of Au+Au collisions at energies from $\sqrt{s_{NN}} = 7.7 - 200$ GeV
    - higher moments of net-proton multiplicity distributions
    - particle ratio fluctuations (K/π, p/π and K/p)

CEP:

No clear evidence yet!

[1] Xiaofeng Luo, STAR Collaboration, APPB 5 No 2 (2012) 497

Motivation

- **Understand the QCD phase structure is one of the most important topics in the physics of strong interactions**

Much more to come in the future

- CERN (NA61)
- FAIR (CBM)
- NICA (MPD)

NICA: Nuclotron based Ion Collider fAcility Collider with $\sqrt{s_{NN}} = 3.5 - 11$ GeV (Begin 2015)

Program:

- Systems with highest baryon density
- Critical point
- Quarkyonic phase
- Chiral symmetry restoration

If the CEP is found, it would be the first clear indication for the chiral phase transition in the heavy-ion experiments

Model and formalism

Polyakov loop extended NJL model with strange quarks

$$\mathcal{L}_{PNJL} = \bar{q}\,(i\gamma_\mu D^\mu - \hat{m})\,q + \frac{g_S}{2}\sum_{a=0}^{8}\left[(\bar{q}\lambda^a q)^2 + (\bar{q}(i\gamma_5)\lambda^a q)^2\right]$$
$$+ \quad g_D\left[\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right] - \mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right)$$

where $\hat{m} = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix.

Coupling between Polyakov loop and quarks uniquely determined by covariant derivative D^μ

$$D^\mu = \partial^\mu + igA^\mu \text{ and } A^\mu = \delta^\mu_0 A^0 \text{ (Polyakov gauge)}$$

– C. Ratti, et al., PRD 73 (2006) 014019

Model and formalism

Polyakov loop extended NJL model with strange quarks

- **Quarks are coupled** simultaneously to the **chiral condensate** and to the **Polyakov loop**
  - the model includes features of both **chiral** and $\mathbb{Z}_3$ symmetry breaking
  - the coupling is fundamental for reproducing lattice results concerning QCD thermodynamics: it originates a **suppression** of the **unconfined quarks** in the hadronic phase [1] (low temperature)
- A non-zero **Polyakov loop** reflects the **spontaneously broken** $\mathbb{Z}_3$ symmetry characteristic of deconfinement (high temperature)
  - $\mathbb{Z}_3$ is **broken** in the **deconfined phase** ($\Phi \to 1$)
  - $\mathbb{Z}_3$ is **restored** in the **confined one** ($\Phi \to 0$)
- At $T = 0$: $\Phi = \bar{\Phi} = 0 \longmapsto$ both sectors decouple

1– C. Ratti et al., PRD 73 (2006) 014019

Model and formalism

Effective potential $\mathcal{U}\left(\Phi,\bar{\Phi};T\right)$

- Effective potential for the (complex) Φ field: is conveniently chosen to reproduce results obtained in lattice calculations

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{a\left(T\right)}{2}\bar{\Phi}\Phi + b(T)\ln[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2]$$

with

$$a\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

a_0	a_1	a_2	b_3
3.51	-2.47	15.2	-1.75

and $T_0 = 270$ MeV

– S. Roessner, C. Ratti, W. Weise, PRD 75 (2007) 034007

Model and formalism

PNJL model at finite temperature and chemical potential

The thermodynamic potential is:

$$\begin{aligned}\Omega(\Phi,\bar{\Phi},M_i;T,\mu) &= \mathcal{U}(\Phi,\bar{\Phi},T) + 2g_S \sum_{\{i=u,d,s\}} \langle \bar{q}_i q_i \rangle^2 - 2g_D \langle \bar{q}_i q_i \rangle \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle \\ &- 2N_c \sum_{\{i=u,d,s\}} \int_\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} E_i \\ &- 2N_c T \sum_{\{i=u,d,s\}} \int_\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} \left\{ \mathrm{Tr}_c \ln \left[1 + L^\dagger \mathrm{e}^{-(E_i-\mu)/T} \right] \right. \\ &+ \left. \mathrm{Tr}_c \ln \left[1 + L \mathrm{e}^{-(E_i+\mu)/T} \right] \right\}\end{aligned}$$

with $E_i = \sqrt{\mathbf{p}^2 + M_i^2}$

Model and formalism

Methodology:

Minimization of $\Omega(\Phi, \bar{\Phi}, M_i; T, \mu)$ with respect to M_i $(i = u, d, s)$

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“Gap” equations:

$$M_i = m_i - 2 g_S \langle\langle \bar{q}_i q_i \rangle\rangle - 2 g_D \langle\langle \bar{q}_j q_j \rangle\rangle \langle\langle \bar{q}_k q_k \rangle\rangle$$

Effective action for the scalar and pseudoscalar mesons

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Meson propagators, $g_{M\bar{q}q}$, $f_{M\bar{q}q}$,...

Generalization of the PNJL model to finite μ : Matsubara formalism

Model and formalism

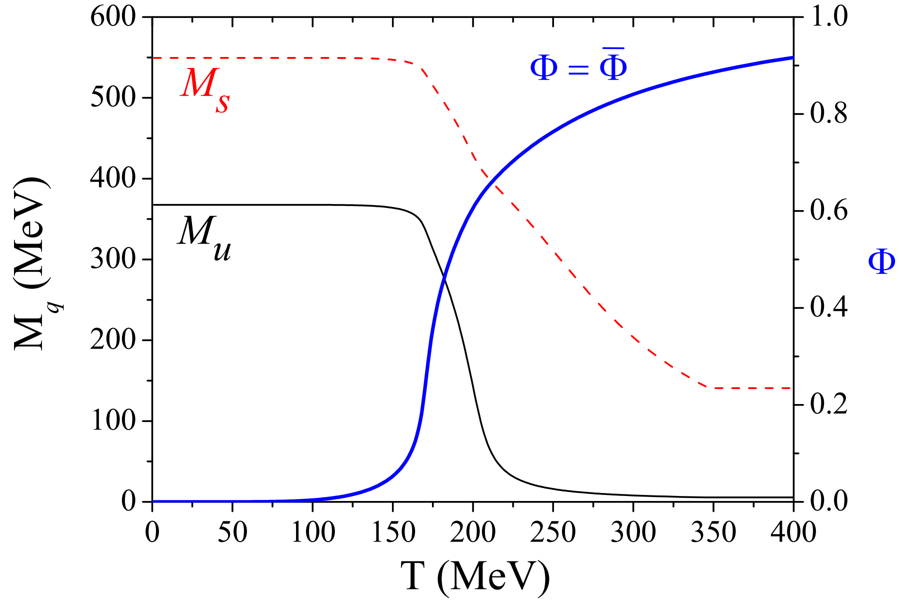
Parameters and results:

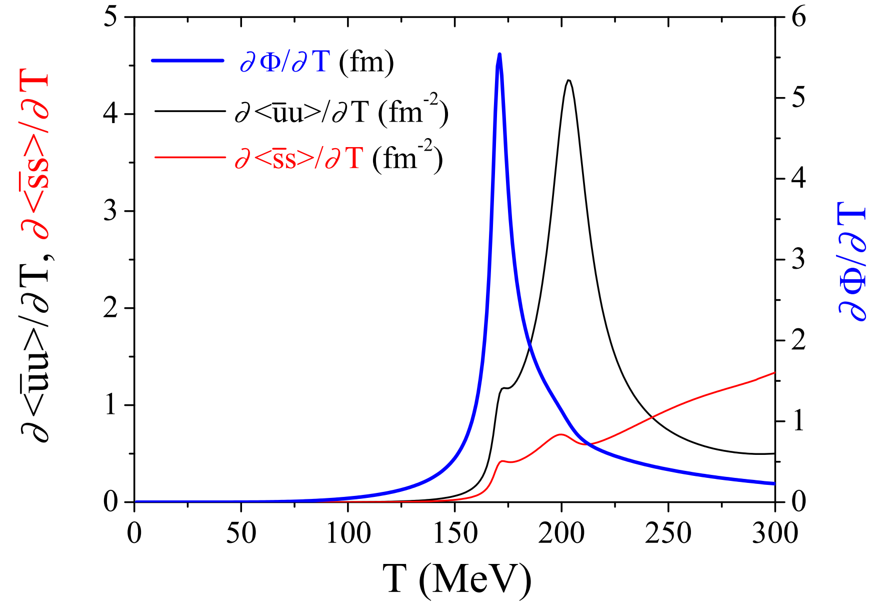
Physical quantities	Parameter set and constituent quark masses
$f_\pi = 92.4$ MeV	$m_u = m_d = 5.5$ MeV
$M_\pi = 135.0$ MeV	$m_s = 140.7$ MeV
$M_K = 497.7$ MeV	$\Lambda = 602.3$ MeV
$M_{\eta'} = 960.8$ MeV	$g_S \Lambda^2 = 3.67$
$M_\eta = 514.8$ MeV*	$g_D \Lambda^5 = -12.36$
$f_K = 97.7$ MeV*	$M_u = M_d = 367.7$ MeV*
$M_\sigma = 728.8$ MeV*	$M_s = 549.5$ MeV*
$M_{a_0} = 873.3$ MeV*	
$M_\kappa = 1045.4$ MeV*	
$M_{f_0} = 1194.3$ MeV*	
$\theta_P = -5.8^{o*}$; $\theta_S = 16^{o*}$	

– S.P. Klevansky et al., PRC 53 (1996) 410
– P. Costa, et al., PRD 71 (2005) 116002

PNJL *characteristic* temperatures

We rescale T_0 from **270 MeV** to **210 MeV**:





- Smooth **crossover** from the chirally broken to the chirally symmetric phase around T_c^χ: ***partial* restoration of chiral symmetry**
- **Polyakov loop**: still **good** (approximate) **order parameter**
- $\Phi \to 1$ as T **increases**: **deconfinement**

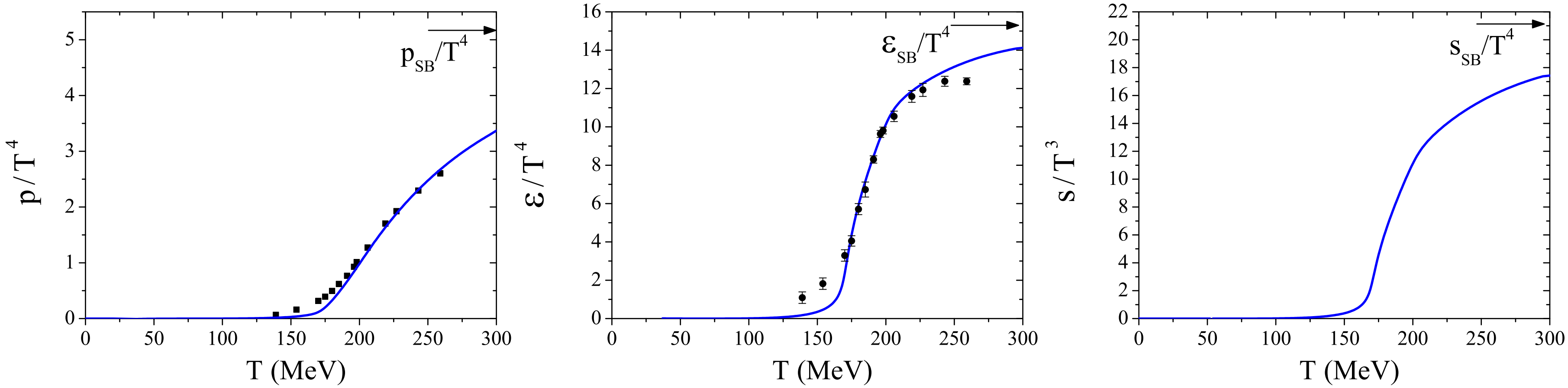
– P. Costa, et al., Symmetry 2 (2010) 1338

PNJL *characteristic* temperatures

T_0 [MeV]	T_c^χ [MeV]	T_c^Φ [MeV]	**T$_c$** [MeV]	$T_c^\chi - T_c^\Phi$ [MeV]
210	203	171	187	32
270	222	210	216	12

- **T_0 = 210 MeV** to compare with lattice calculations (next slide)
  - $T_c^\chi - T_c^\Phi = 32$ MeV
- **T_0 = 270 MeV** (rest of the presentation)
  - chiral/deconfinement transitions almost **coincidence** ($T_c^\chi - T_c^\Phi = 12$ MeV)
  - *characteristic* temperatures are **larger** than lattice results for chiral/deconfinement transitions
- Results are qualitatively **similar** for both choices

PNJL vs. lattice results



- ***Pressure***, ***energy*** and ***entropy*** densities are continuous functions of the temperature: a crossover takes place
- In the three curves there is a sharp increase in the vicinity of the **transition temperature and then a tendency to saturate. The corresponding ideal gas limit is**

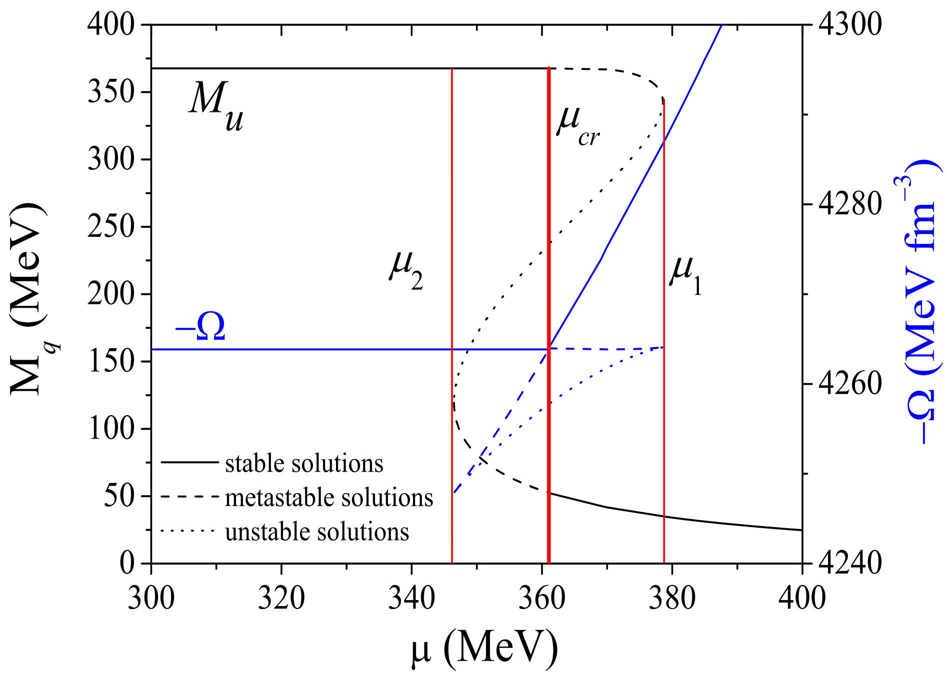
$$\frac{p_{SB}}{T^4} = (N_c^2 - 1)\,\frac{\pi^2}{45} + N_c\,N_f\,\frac{7\,\pi^2}{180}$$

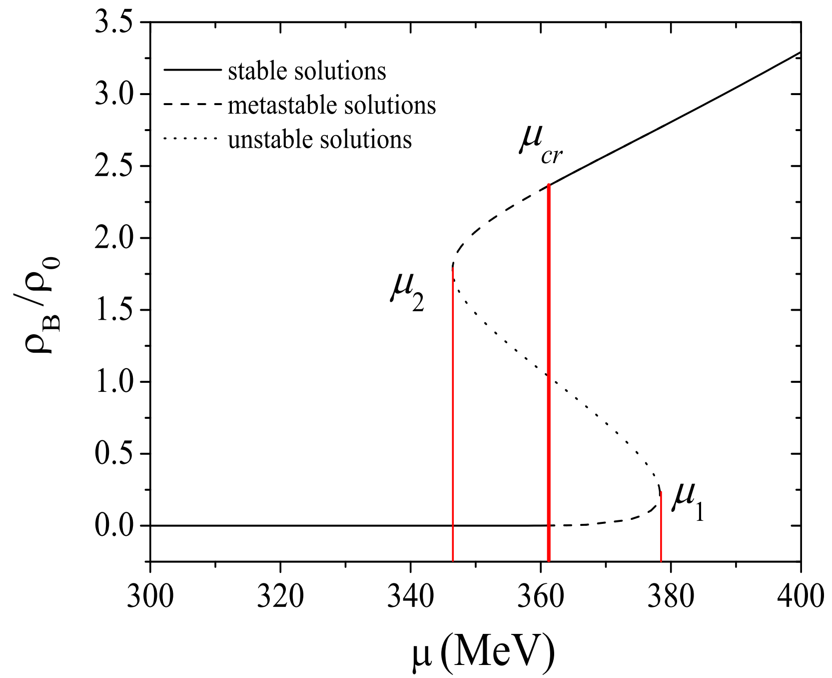
- The inclusion of the Polyakov loop effective potential $\mathcal{U}(\Phi, \bar{\Phi})$ (it can be seen as an effective pressure term mimicking the gluonic degrees of freedom of QCD) is required to approach the Stefan–Boltzmann limit

– M. Cheng, et al., PRD 81 (2010) 054504

Nature of the phase transition

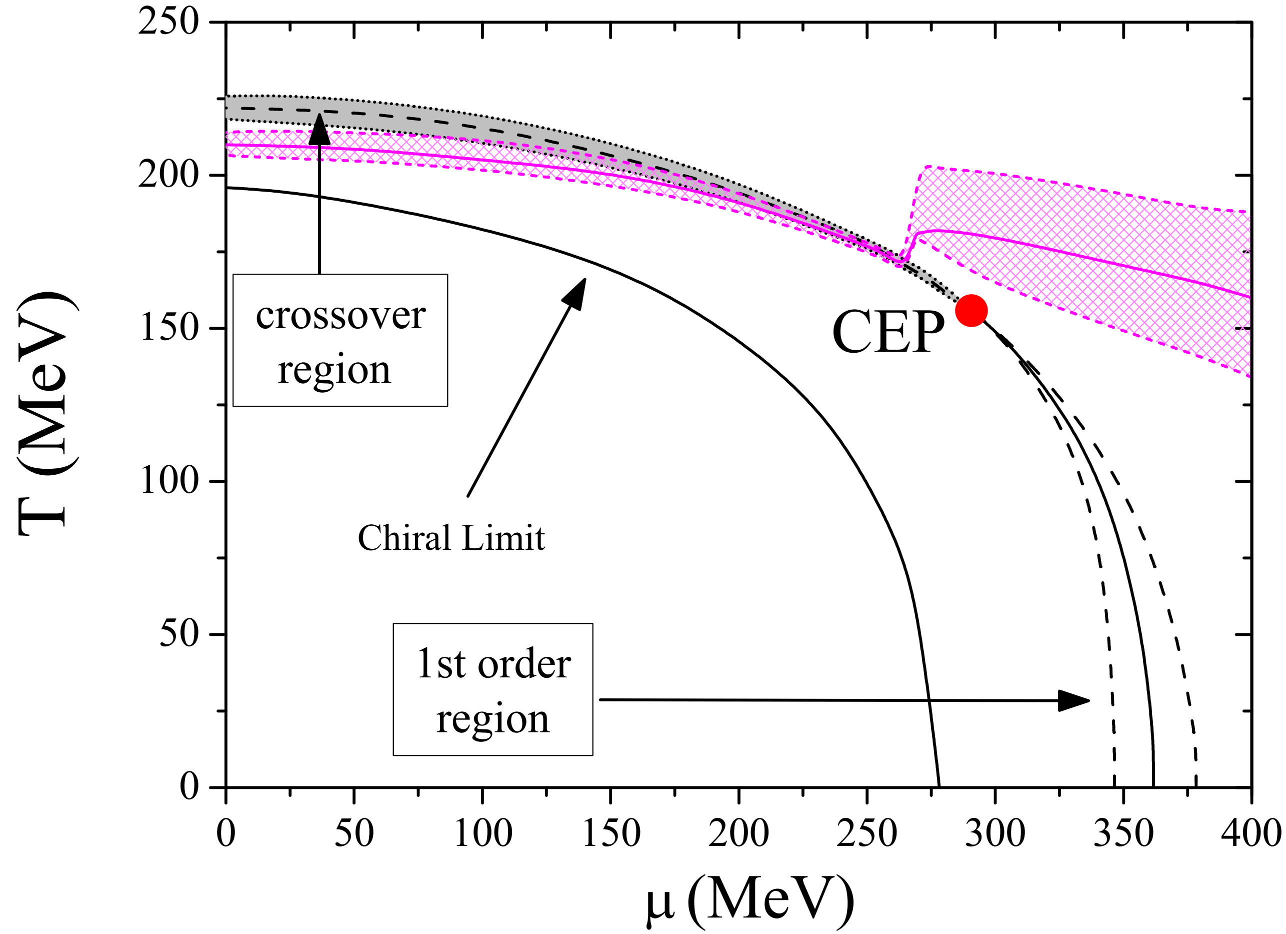
1st order phase transition at $T = 0$ and $\mu \neq 0$





- **First-order phase transition** found at $\mu^{cr} = 361.7$ MeV
- **Three solutions of the gap equation** in the domain $\mu_2 < \mu < \mu_1$: allow for regions of *stability*, *metastability* and *instability*
- The stable solutions are realized by the minimum of the thermodynamic potential
- $\mu < \mu^{cr}$: phase of broken χ_S symmetry; $\mu > \mu^{cr}$: “symmetric” phase
- At $\mu^{cr} = 361.7$ MeV ρ_B **jumps** from 0 to $\rho_B^{cr} = 2.36\rho_0$

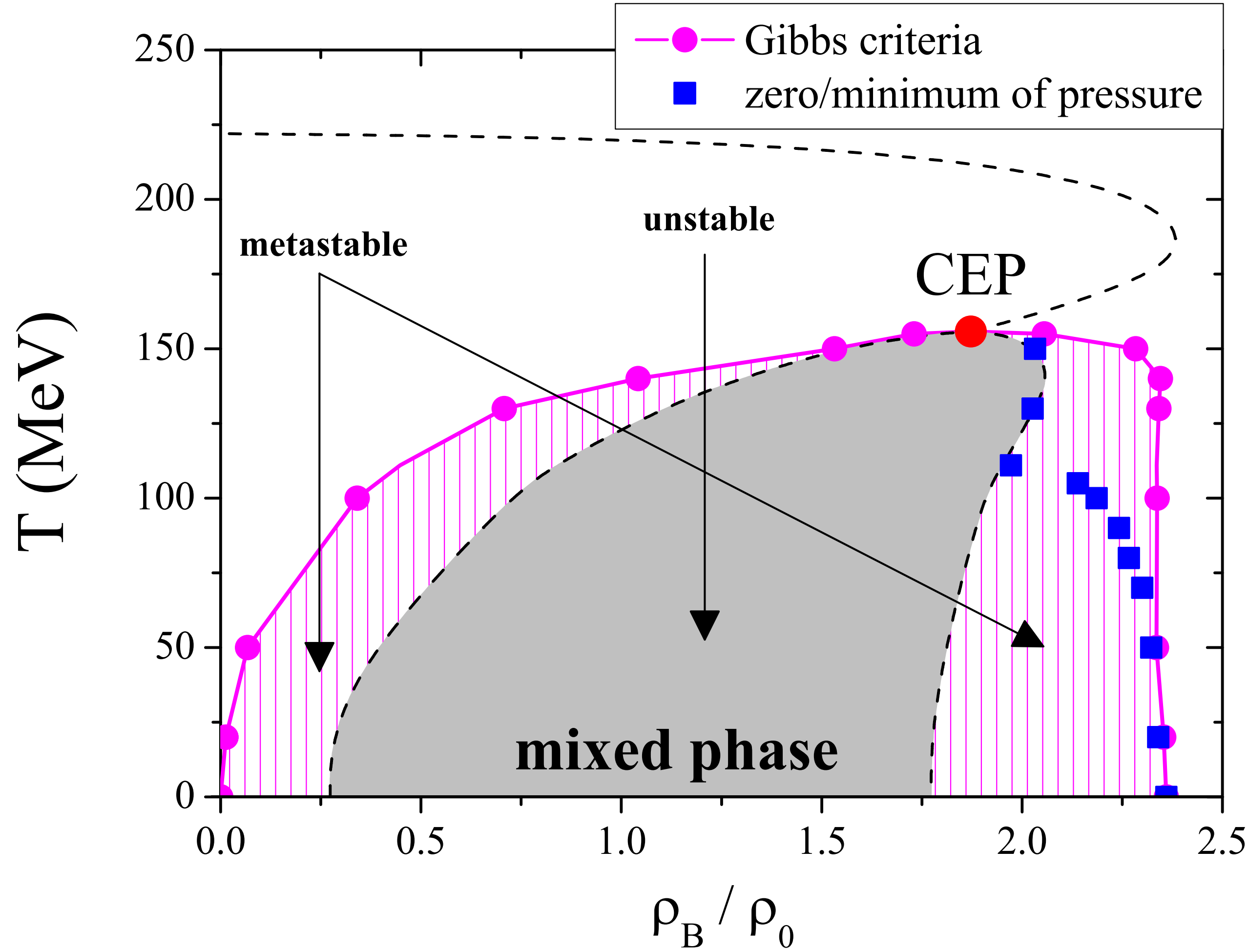
Phase diagram



CEP:

$$T^{CEP} = 155.80 \text{ MeV}$$
$$\mu^{CEP} = 290.67 \text{ MeV}$$

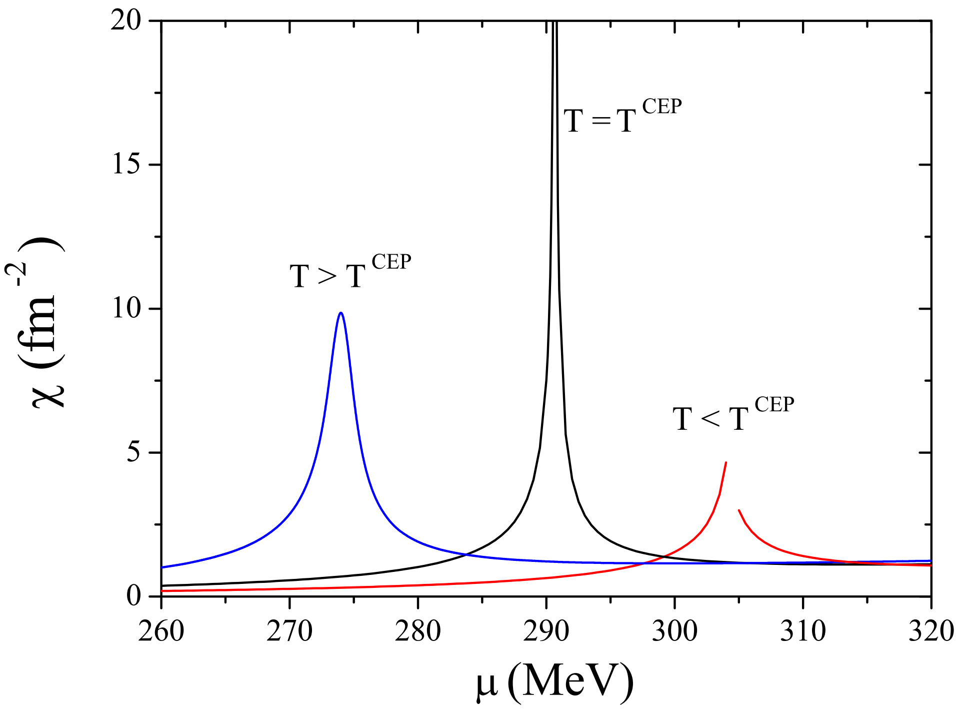
Phase diagram



- The phase diagram: it is represented the zero/minimum of the pressure. The different types of solutions for the mixed phase are also represented

Susceptibility near the CEP

Behavior of the quark number susceptibility around the CEP:
$T^{CEP} = 155.80$ MeV and $T = T^{CEP} \pm 10$ MeV

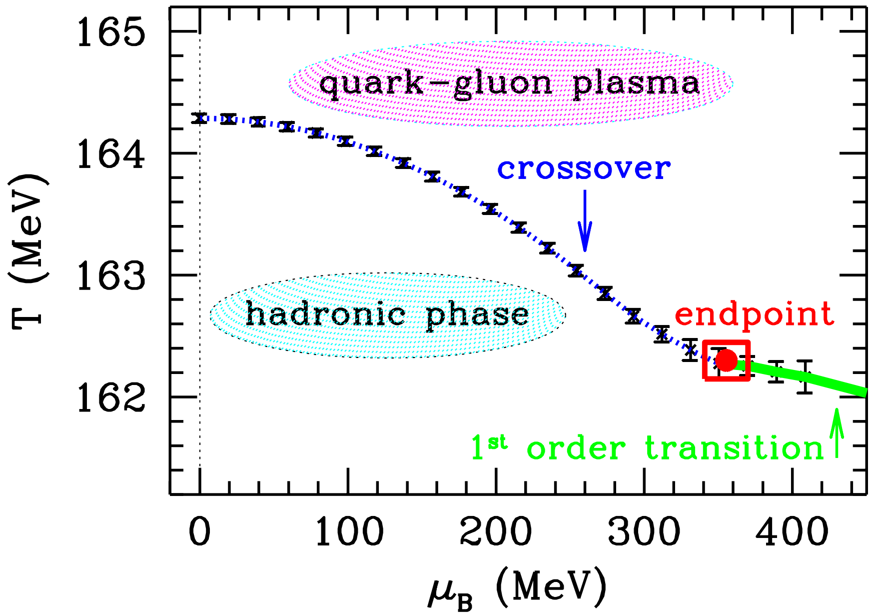


- $T < T^{CEP}$: the transition is first-order and χ has a discontinuity
- $T = T^{CEP}$: χ diverges
- $T > T^{CEP}$: the discontinuity disappears at the transition line (crossover)

Lattice calculations:

- **CEP: Is it there or not?**

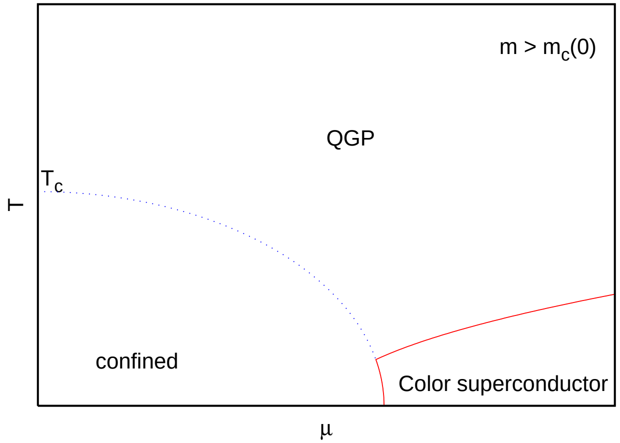
Yes[1]

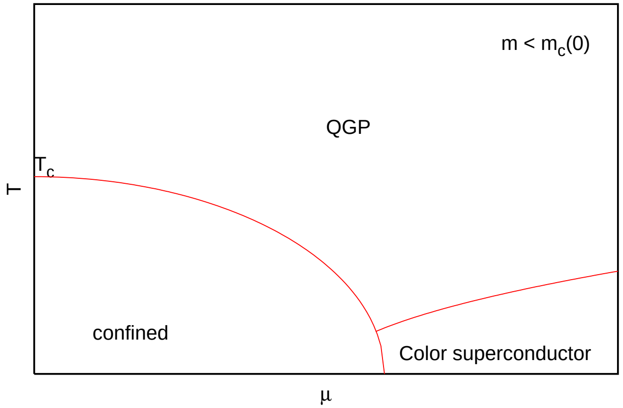


CEP:

$$T^{CEP} = 162 \pm 2 \quad \text{MeV}$$
$$\mu_B^{CEP} = 360 \pm 40 \text{ MeV}$$

No[2]



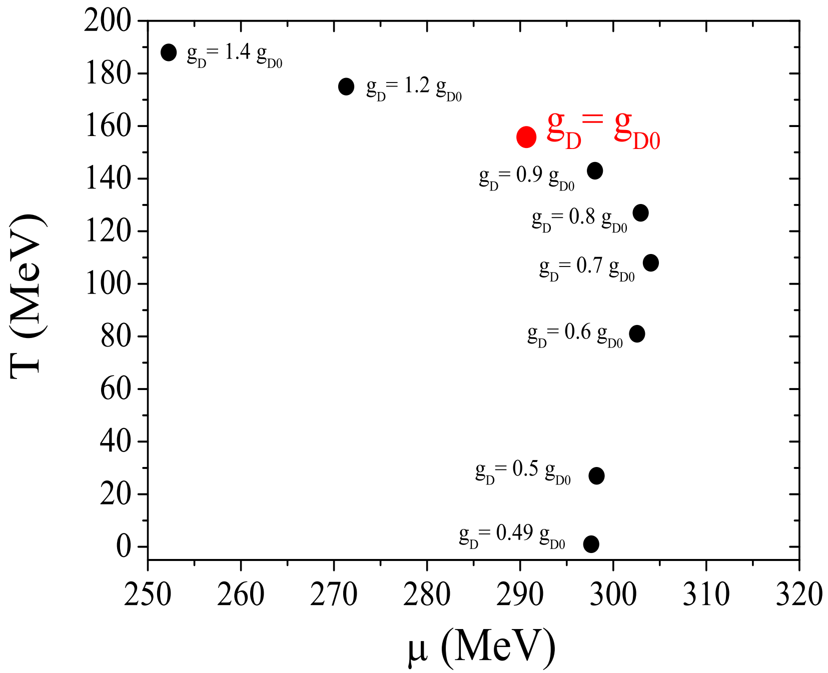


[1] Z. Fodor, S.D. Katz, JHEP 0404 (2004) 050

[2] P. de Forcrand, O. Philipsen, JHEP 0811 (2008) 012

Anomaly strength and the CEP

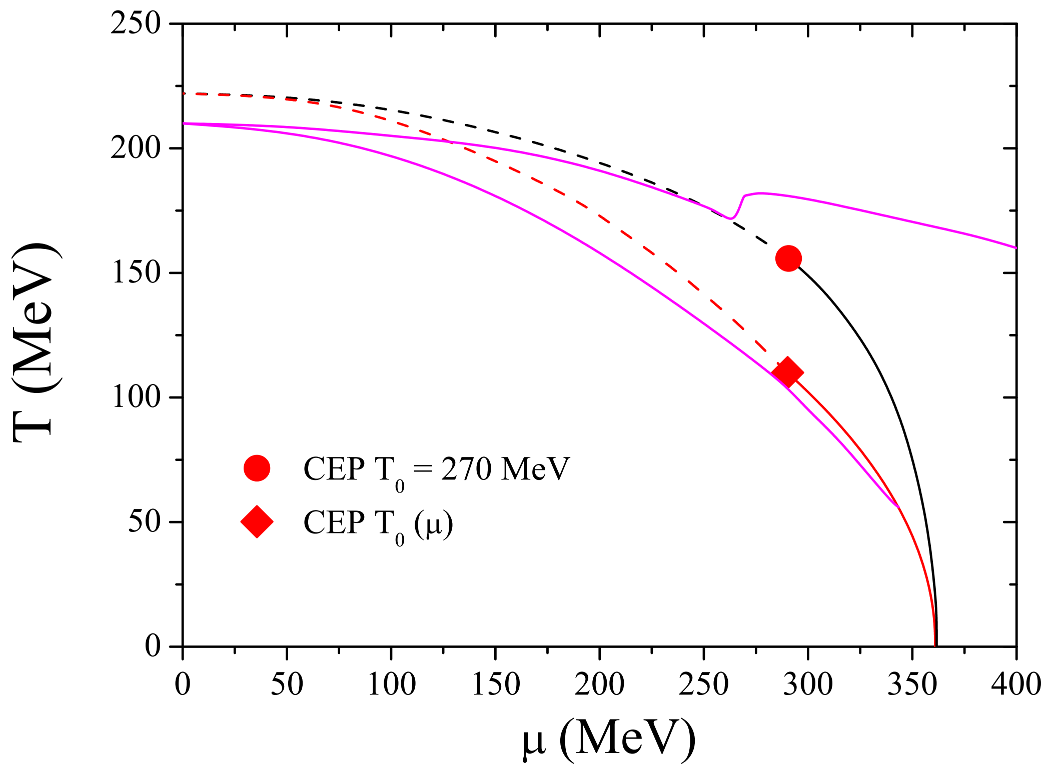
In a hot and dense medium g_D might take a different (presumably smaller) value then g_{D0} (fixed in the vacuum)



- the location of the QCD critical end point depends on the value of g_D
  - as $g_D \to 0$, the CEP ***disappears*** from the phase diagram
  - the first-order region becomes wider with larger g_D and narrower with smaller g_D

Phase diagram

[1] $\boxed{T_0(\mu) = T_\tau e^{-1/(\alpha_0 b(\mu))}}$; $b(\mu) = \frac{11N_c}{6\pi} - \frac{16N_f}{\pi}\frac{\mu^2}{T_\tau^2}$; $\alpha_0 = 0.304$; $T_\tau^2 = 1.770$ GeV

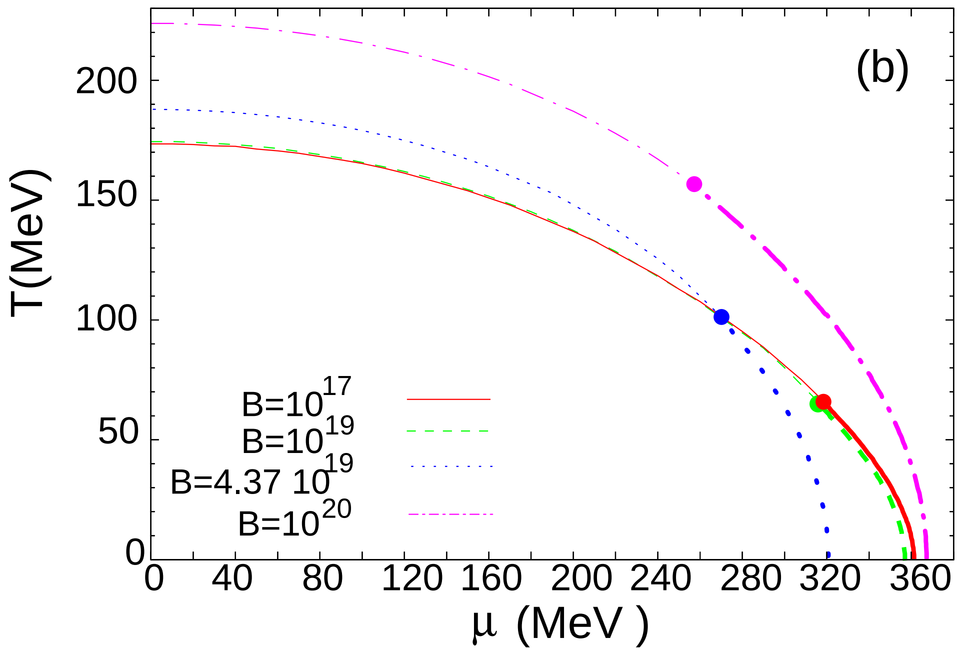


CEP (T_0 = 270 MeV):
$T^{CEP} = 155.80$ MeV
$\mu^{CEP} = 290.67$ MeV

CEP ($T_0(\mu)$):
$T^{CEP} = 110.0$ MeV
$\mu^{CEP} = 290.2$ MeV

[1] – B. J. Schaefer et al., PRD 76 (2007) 074023
– T. Kähärä et al., PRD 82 (2010) 114026

The CEP Under Strong Magnetic Fields in NJL model[1]:

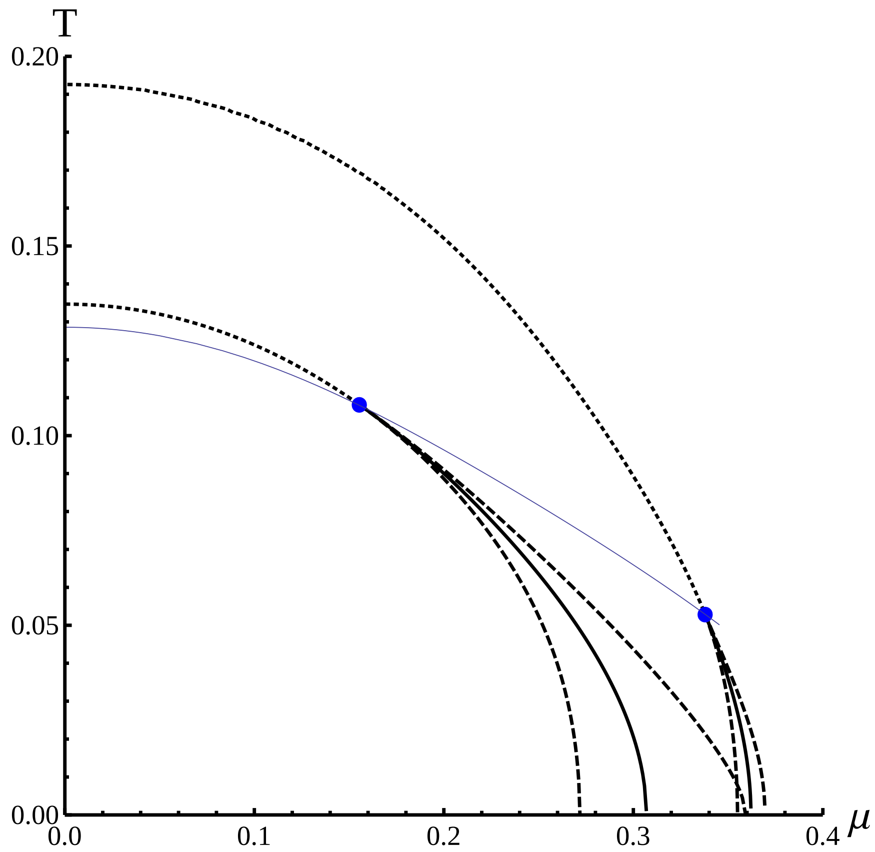


- The size of the first order transition line increases as the magnetic field strength increases
- The location of the CEP is also affected by the presence of magnetic fields

[1] – S. S. Avancini et al., accepted for publication in PRD(R), arXiv:1202.5641v1 [hep-ph]

Phase diagram

Polyakov-NJL model with eight quark interactions[1]:



[1] See talk by B. Hiller

Summary Part I

- The effects of the Polyakov loop on the restoration of symmetries have been investigated, in the SU(3) PNJL model;
- The model incorporates symmetry breaking of χ_S, $\mathbb{Z}_3$, and $U_A(1)$;
- The most reliable parametrization of PNJL model predicts the CEP in the phase diagram, together with the formation of stable quark droplets in the vacuum state at $T = 0$;
- The comparison with lattice results shows that the model provides a convenient tool to obtain information for systems at finite T;
- The location of the CEP depends on the value of g_D: as $g_D \to 0$, the CEP disappears from the phase diagram;
- The physical observables are strongly influenced by the nature of the phase transitions: baryon number susceptibility diverges at the CEP.

Part II

Collaboration with O. Oliveira and P. J. Silva

Low Energy Physics and The Gluon Propagator

To connect the IR gluon propagator with low energy phenomenology an effective low energy chiral quark model of the NJL type can be built.
The interaction between quarks and gluons is:

$$\mathcal{L}_{\overline{\psi}\psi A} = g\,\overline{\psi}\,\gamma^{\mu}\,A^{a}_{\mu}\,\frac{\lambda^{a}}{2}\,\psi$$

Expanding the term containing $\mathcal{L}_{\overline{\psi}\psi A}$ up to g^2 and integrating the gluon fields, the theory becomes an effective nonlocal fermionic theory

$$\begin{aligned} S[\overline{\psi},\psi] &= \int d^4x d^4y \Big\{ \overline{\psi}(y)\,\delta(y-x)\,\left(i\gamma^{\mu}\partial_{\mu}-m\right)\psi(x) \\ &+ \frac{g^2}{8} J(x,y) D(x-y) J(y,x) - \frac{g^2}{8} J_5(x,y) D(x-y) J_5(y,x) \Big\} \end{aligned}$$

$J(x,y) = \overline{\psi}(x)\psi(y)$, $J_5(x,y) = \overline{\psi}(x)\gamma_5\psi(y)$

Low Energy Physics and The Gluon Propagator

First principles calculations of the G P from lattice QCD:

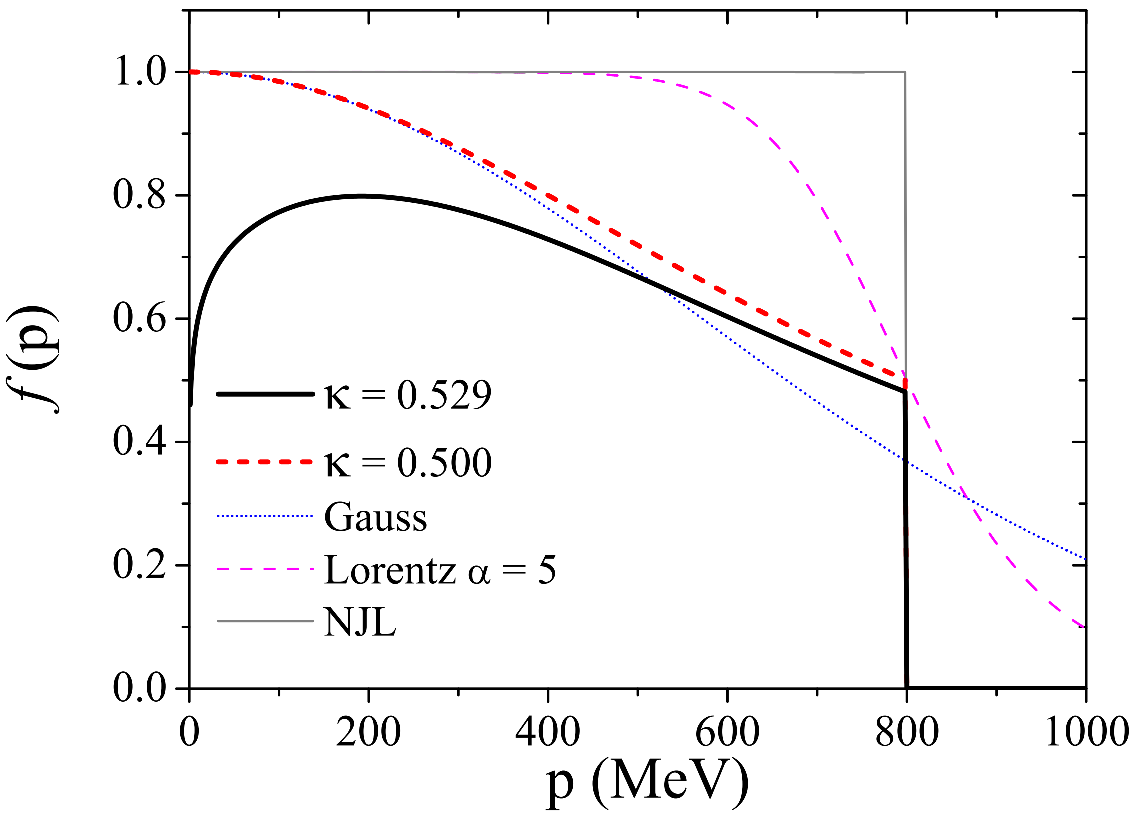
$$D(p^2) = Z \frac{\left(p^2\right)^{2\kappa-1}}{\left(p^2 + \Lambda_{QCD}^2\right)^{2\kappa}}$$

- describes both the scaling ($\kappa > 0.5$) and decoupling ($\kappa = 0.5$) infrared DSE solutions
- the lattice data up to $p \sim 800$ MeV [1]
- Λ_{QCD} stands for an infrared mass scale

Let us define the dimensionless form factor in momentum space as

$$f(\mathrm{p}^2) = \Lambda^2 D(\mathrm{p}^2) = \frac{\Lambda^2}{\mathrm{p}^2}\left(\frac{\mathrm{p}^2}{\mathrm{p}^2 + \Lambda_{QCD}^2}\right)^{2\kappa} \theta(\Lambda - \mathrm{p})$$

[1] – O. Oliveira, P.J. Silva, PRD 79 (2009) 031501



- Form factors as a function of p. The figure includes typical form factors used in previous studies

	m_q [MeV]	M_q [MeV]	$-\langle\bar{q}q\rangle^{1/3}$ [MeV]	$G\Lambda^2$	$\Gamma_{\pi\gamma\gamma}$ [eV]
$\kappa = 0.50$	4.187	360.5	271.4	6.441	5.44
$\kappa = 0.529$	4.205	383.6	271.1	7.491	7.79

- A collection of parameters which reproduce the experimental M_π and f_π for a cut-off $\Lambda = \Lambda_{QCD} = 800$ MeV
- The results differ essentially on the value of the decay width $\Gamma_{\pi\to\gamma\gamma}$ ($\Gamma^{exp}_{\pi\to\gamma\gamma} = 7.78(56)$ eV)

It is important to point out that this model is consistent with the Gell-Mann–Oakes–Renner relation (GMOR)

$$M_\pi^2 f_\pi^2 = -2m_q \langle \bar{q}q \rangle_0$$

preserving chiral low-energy theorems and current algebra relations.

- GMOR value for the current quark mass at $\kappa = 0.5$:

$$m_q^{GMOR} = -\frac{M_\pi^2 f_\pi^2}{2\langle \bar{q}q \rangle_0} = 4.159\text{MeV}$$

differs less then 1% of the calculated value $m_q = 4.187$ MeV.

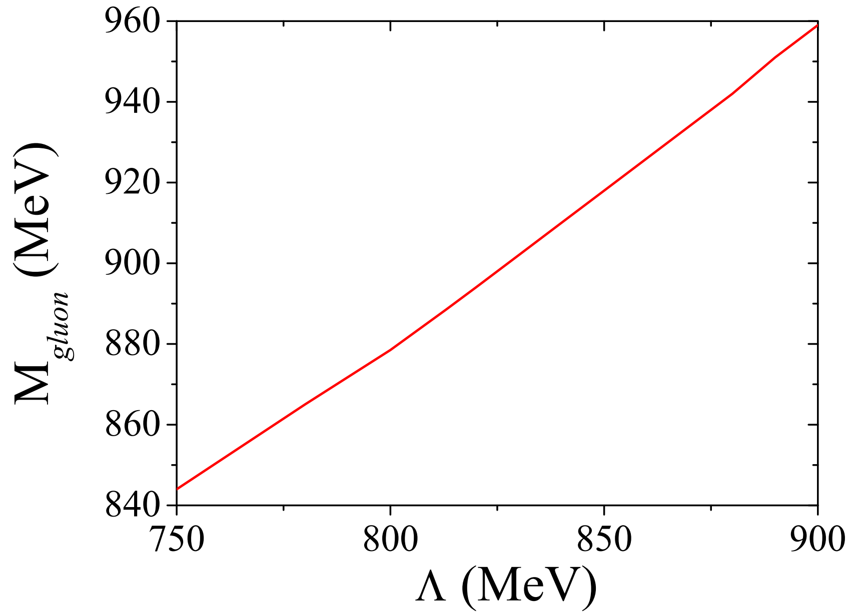
$\kappa = 0.5$ (decoupling type of propagator):

$$D(p^2) = \frac{Z}{p^2 + M_{gluon}^2}$$

Λ [MeV]	M_{gluon} [MeV]	m_q [MeV]	M_q [MeV]	$-\langle\bar{q}q\rangle^{1/3}$ [MeV]	$G\Lambda^2$
750	843.8	4.5	460.9	264.2	6.03
800	878.7	4.2	409.8	271.6	5.39
813	888.5	4.1	400.0	273.5	5.27
850	917.7	3.9	373.3	278.9	4.96
900	959.1	3.6	345.7	286.3	4.66

$\kappa = 0.5$ (decoupling type of propagator):

$$D(p^2) = \frac{Z}{p^2 + M_{gluon}^2}$$



$M_{gluon}(\Lambda)$ required to reproduce the experimental $\Gamma_{\pi\to\gamma\gamma}$

Outlook Part II

- Generalization of the results to non-zero temperature (this requires modeling the gluon propagator by a functional form compatible with both Dyson-Schwinger and lattice QCD results);
- Investigate the meson properties at finite temperature as probes for the chiral symmetry restoration;
- Study how dynamical fermions change the gluon propagator at finite temperature, and how our nonlocal model can accommodate these changes.