Scalar and Axial-Vector Mesons in a Three-Flavour Sigma Model

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and PhD Thesis of D. Parganlija (2012)]

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Introduction: Definitions and Experimental Data

- **Mesons: quark-antiquark states**
- **Quantum numbers: J^{PC}**

Total Spin **Parity** **Charge Conjugation**

- **Scalar mesons: $J^{PC} = 0^{++}$ [σ or $f_0(600)$, $a_0(980)$, $a_0(1450)$…]**
- **Pseudoscalar mesons: $J^{PC} = 0^{-+}$ [*π*, *K*, *η*, *η´*…]**
- **Vector mesons: $J^{PC} = 1^{--}$ [*ρ*, *K**, *ω*, *φ*(1020)…]**
- **Axial-Vector mesons: $J^{PC} = 1^{++}$ [$a_1(1260)$, $f_1(1285)$, $K_1(1270)$, $K_1(1400)$…]**

Motivation:
PDG Data on $J^{PC} = 0^{++}$ Mesons

$\bar{n}n \propto \bar{u}u + \bar{d}d \quad \bar{s}s$

- Six states up to 1.8 GeV (isoscalars)

State	Mass [MeV]	Width [MeV]
$f_0(600)$	400 - 1200	600 - 1000
$f_0(980)$	980 ± 10	40 - 100
$f_0(1370)$	1200 - 1500	200 - 500
$f_0(1500)$	1505 ± 6	109 ± 7
$f_0(1710)$	1720 ± 6	135 ± 8
$f_0(1790)$	1790^{+40}_{-30}	270^{+60}_{-30}

$\bar{q}\bar{q}qq$ Glueball meson - meson boundstate

Motivation: Reasons to Consider Mesons

- **Mesons: hadronic states with integer spin**
- **More scalar mesons than predicted by quark-antiquark picture → Classification needed**

  Look for tetraquarks, glueballs…
- **Walecka Model: Nucleon-nucleon interaction via *σ* meson**
- **Restoration of chiral invariance and decofinement ↔ Degeneration of chiral partners *π* and *σ* → *σ* has to be a quarkonium**
- **Identify the scalar quarkonia → Need a model with scalar and other states**

An Effective Approach: Linear Sigma Model

- Implements features of QCD:
  - $SU(N_\mathrm{f})_L$ x $SU(N_\mathrm{f})_R$ Chiral Symmetry
  - Explicit and Spontaneous Chiral Symmetry Breaking; Chiral $U(1)_A$ Anomaly
- Vacuum calculations → calculations at $T \neq 0$
- Chiral-Partners degeneration above T_C → order parameter for restoration of chiral symmetry

- The model here: $N_\mathrm{f} = 3$ (mesons with *u*, *d*, *s* quarks) in scalar, pseudoscalar, vector and axial-vector channels

→ extended Linear Sigma Model - eLSM

Vacuum spectroscopy of quark-antiquark states

Resonances I

- **Pseudoscalars**

$$P=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

η, η'

- **Vectors**

$$V_\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_{N\mu}+\rho_\mu^0}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & \frac{\omega_{N\mu}-\rho_\mu^0}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \omega_{S\mu} \end{pmatrix}$$

$\omega_N \equiv \omega(782) = \bar{n}n$

$\omega_S \equiv \varphi(1020) = \bar{s}s$

$\rho \equiv \rho(770)$

$K^* \equiv K^*(892)$

Resonances II

- **Axial-Vectors**

$$A_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N\mu}+a^0_{1\mu}}{\sqrt{2}} & a^+_{1\mu} & K^+_{1\mu} \\ a^-_{1\mu} & \frac{f_{1N\mu}-a^0_{1\mu}}{\sqrt{2}} & K^0_{1\mu} \\ K^-_{1\mu} & \bar{K}^0_{1\mu} & f_{1S\mu} \end{pmatrix}$$

$f_{1N} \equiv f_1(1285) = \bar{n}n$

$a_1 \equiv a_1(1260)$

$f_{1S} \equiv f_1(1420) = \bar{s}s$

$$K_1 \equiv \begin{cases} K_1(1270) \\ K_1(1400) \end{cases}$$

- **Scalars**

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N+a^0_0}{\sqrt{2}} & a^+_0 & K^+_S \\ a^-_0 & \frac{\sigma_N-a^0_0}{\sqrt{2}} & K^0_S \\ K^-_S & \bar{K}^0_S & \sigma_S \end{pmatrix}$$

$$\begin{cases} \sigma_N \equiv \bar{n}n \\ \sigma_S \equiv \bar{s}s \end{cases} \rightarrow \begin{cases} f_0^L \equiv \text{predominantly } \bar{n}n \\ f_0^H \equiv \text{predominantly } \bar{s}s \end{cases}$$

$$a_0 = \begin{cases} a_0(980) \\ a_0(1450) \end{cases} \qquad K_S = \begin{cases} K_0^*(800)\ /\ \kappa \\ K_0^*(1430) \end{cases}$$

The Lagrangian I

- **Scalars and Pseudoscalars**

$$\mathcal{L}_{SP} = \mathrm{Tr}[(D^{\mu}\Phi)^{\dagger}(D^{\mu}\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^{\dagger}\Phi) - \lambda_1[\mathrm{Tr}(\Phi^{\dagger}\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^{\dagger}\Phi)^2$$
$$+ \mathrm{Tr}\,[H(\Phi+\Phi^{\dagger})] + c[(\det\Phi + \det\Phi^{\dagger})^2 - 4\det(\Phi\Phi^{\dagger})]$$

Explicit Symmetry Breaking **Chiral Anomaly**

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \qquad P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \qquad \Phi = S + iP$$

$$D_{\mu}\Phi = \partial_{\mu}\Phi + ig_1(\Phi R_{\mu} - L_{\mu}\Phi)$$

The Lagrangian II

- **Vectors and Axial-Vectors**

$$\mathcal{L}_{\mathrm{VA}} = -\frac{1}{4}\mathrm{Tr}\,(L_{\mu\nu}^{2} + R_{\mu\nu}^{2}) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_{\mu}^{2} + R_{\mu}^{2})\right]$$
$$-2ig_2(\mathrm{Tr}\{L_{\mu\nu}[L^{\mu}, L^{\nu}]\} + \mathrm{Tr}\{R_{\mu\nu}[R^{\mu}, R^{\nu}]\})$$

$$L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu$$
$$R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu$$

$$\Delta \triangleq \begin{pmatrix} \delta_n(m_{u,d}^2) & & \\ & \delta_n(m_{u,d}^2) & \\ & & \delta_s(m_s^2) \end{pmatrix}$$

$$V_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_{N\mu} + \rho_\mu^0}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & \frac{\omega_{N\mu} - \rho_\mu^0}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \omega_{S\mu} \end{pmatrix} \quad A_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N\mu} + a_{1\mu}^0}{\sqrt{2}} & a_{1\mu}^+ & K_{1\mu}^+ \\ a_{1\mu}^- & \frac{f_{1N\mu} - a_{1\mu}^0}{\sqrt{2}} & K_{1\mu}^0 \\ K_{1\mu}^- & \bar{K}_{1\mu}^0 & f_{1S\mu} \end{pmatrix}$$

$$L_\mu = V_\mu + A_\mu$$
$$R_\mu = V_\mu - A_\mu$$

Sigma Model Lagrangian with Vector and Axial-Vector Mesons ($N_f = 3$)

- **More (Pseudo)scalar – (Axial-)Vector Interactions**

$$\mathcal{L}_{\text{INT.}} = \frac{h_1}{2}\,\text{Tr}\,(\Phi^\dagger\Phi)\,\text{Tr}\,(L_\mu^2 + R_\mu^2) + h_2\,\text{Tr}\,[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\,\text{Tr}\,(\Phi R_\mu \Phi^\dagger L^\mu)$$

$$\mathcal{L} = \mathcal{L}_{\text{SP}} + \mathcal{L}_{\text{VA}} + \mathcal{L}_{\text{INT.}}$$

- **Perform Spontaneous Symmetry Breaking (SSB):** $\sigma_N \to \sigma_N + \phi_N,\ \sigma_S \to \sigma_S + \phi_S$
- **18 parameters, 10 independent, none free → fixed via fit of masses and decay widths/amplitudes**

Possible Assignments

- **Isospin 1**

$$a_0 = \begin{cases} a_0(980) \\ a_0(1450) \end{cases}$$

- **Isospin ½**

$$K_S = \begin{cases} K_0^*(800)\ /\ \kappa \\ K_0^*(1430) \end{cases}$$

- **Isospin 0 (Isoscalars)**

$$\begin{cases} \sigma_N \equiv \bar{n}n \\ \sigma_S \equiv \bar{s}s \end{cases} \rightarrow \begin{cases} f_0^L \equiv \text{predominantly } \bar{n}n \\ f_0^H \equiv \text{predominantly } \bar{s}s \end{cases}$$

$f_0(600)$ $f_0(980)$ $f_0(1370)$

$f_0(1500)$ $f_0(1710)$

Check all possibilities

Best Fit

Observable	Fit [MeV]	Experiment [MeV]
f_π	92.5	92.4 ± 0.9
f_K	109.6	$155.5/\sqrt{2} \pm 1.1$
m_π	139.0	138 ± 1.4
m_K	503.9	495.6 ± 5.0
m_η	526.5	547.9 ± 5.5
$m_{\eta'}$	967.7	957.8 ± 9.6
m_ρ	767.2	775.5 ± 7.8
$m_{K^\star}$	899.9	893.8 ± 8.9
m_φ	1014.0	1019.5 ± 1.02
m_{a_1}	1178.9	1230 ± 40
m_{K_1}	1296.4	1272 ± 12.7
$m_{f_1(1420)}$	1405.1	1426.4 ± 14.3
m_{a_0}	1441.7	1474 ± 74
$m_{K_0^\star}$	1536.5	1425 ± 71
$m_{f_0^L}$	1214.1	1350 ± 150
$m_{f_0^H}$	1584.1	1720 ± 86

Observable	Fit [MeV]	Experiment [MeV]
$\Gamma_{\rho\to\pi\pi}$	166.5	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.3	46.2 ± 2.3
$\Gamma_{a_1\to\rho\pi}$	737	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.650	0.640 ± 0.250
$\Gamma_{f_1(1420)\to K^\star K}$	43.3	43.9 ± 2.2
$\Gamma_{f_0^L\to\pi\pi}$	122.3	250 ± 100
$\Gamma_{f_0^L\to KK}$	125.7	150 ± 100
$\Gamma_{f_0^H\to\pi\pi}$	31.3	29.3 ± 6.5
$\Gamma_{f_0^H\to KK}$	141.6	71.4 ± 29.1

$\begin{cases} f_0(1370) \text{ predominantly } \bar{n}n \\ f_0(1710) \text{ predominantly } \bar{s}s \end{cases}$

$a_1(1260)/\,K_1(1270)$ $\bar{q}q$ states

$m_\rho \leftrightarrow$ Gluon Condensate

+Quark Condensate;

Gluon Condensate dominant

$\eta - \eta'$ mixing angle $\sim 45°$

What We Did Not Find

- **No fit** with $f_0(600)$ and $f_0(980)$ as $\bar{q}q$ states
- **No fit** with $K_0^*(800)$ as $\bar{q}q$ state
- **No** reasonable **fit** with $f_0(600)$ and $f_0(1370)$ as $\bar{q}q$ states

  $\rightarrow m_{K_0^*} \sim 1.1$ GeV or $m_{a_0} \sim 1.2$ GeV

$$\begin{cases} m_{K_0^*(800)\,/\,\kappa} = (676 \pm 40)\ \text{MeV} \\ m_{K_0^*(1430)} = (1425 \pm 50)\ \text{MeV} \end{cases} \quad \begin{cases} m_{a_0(980)} = (980 \pm 20)\ \text{MeV} \\ m_{a_0(1450)} = (1474 \pm 19)\ \text{MeV} \end{cases}$$

Thus: scalar $\bar{q}q$ states above 1 GeV

$\rightarrow f_0(1370)$ predominantly $\bar{n}n$

$\rightarrow f_0(1710)$ predominantly $\bar{s}s$

Summary

- Linear Sigma Model with $N_f = 3$ and vector and axial-vector mesons – eLSM
- Predominantly $\bar{q}q$ scalar states above 1 GeV: $f_0(1370)$, $f_0(1710)$, $a_0(1450)$, $K_0^*(1430)$
- Axial-Vectors a_1 and K_1 seen as $\bar{q}q$ states

Summary: Results on $J^{PC} = 0^{++}$ Mesons

State	Mass [MeV]	Width [MeV]
$f_0(600)$ tetraquark?	400 - 1200	600 - 1000
$f_0(980)$ tetraquark?	980 ± 10	40 - 100
$f_0(1370)$ predominantly $\bar{n}n$	1200 - 1500	200 - 500
$f_0(1500)$ predominantly glueball	1505 ± 6	109 ± 7
$f_0(1710)$ predominantly $\bar{s}s$	1720 ± 6	135 ± 8

[S. Janowski, D. Parganlija, F. Giacosa and D. H. Rischke, PR D 84 (2011) 054007]

Axial-vectors (a_1, K_1): $\bar{q}q$

Outlook

- **Lagrangian With Three Flavours + Glueball + Tetraquarks**
- **Mixing in the Scalar Sector: Quarkonia, Tetraquarks and Glueball**
- **Four Flavours**
- **Extension to Non-Zero Temperatures and Densities**
- **Include Tensor, Pseudotensor Mesons, Baryons (*Nucleons*)**

Spare Slides

Quantum Chromodynamics (QCD)

- **QCD Lagrangian**

$$\mathrm{L}_{QCD} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

- **Symmetries of the QCD Lagrangian**

Local $SU(3)_c$ Colour Symmetry

Global Chiral $U(N_f)$x $U(N_f)$ Symmetry

CPT Symmetry

Z Symmetry

Trace Symmetry

Chiral Symmetry of QCD

- Left-handed and right-handed quarks:

$$q_f = q_{f\,L} + q_{f\,R};\; q_{f\,L,R} = P_{L,R}\, q_f$$

- Chirality Projection Operators

$$P_{L,R} = \frac{1 \pm \gamma_5}{2}$$

- Transform quark fields

$$q_{f\,L} \to q'_{f\,L} = U_L q_{f\,L} = e^{-i\alpha_L^j t^j} q_{f\,L},\; j = 0,\ldots,N_f^2 - 1$$

$$q_{f\,R} \to q'_{f\,R} = U_R q_{f\,R} = e^{-i\alpha_R^j t^j} q_{f\,R}$$

- Quark part of the QCD Lagrangian:

$$\mathcal{L}_{QCD}\Big|_{quarks} = \bar{q}_{f\,L} i\not{D} q_{f\,L} + \bar{q}_{f\,R} i\not{D} q_{f\,R} - \bar{q}_{f\,L} m_f q_{f\,R} - \bar{q}_{f\,R} m_f q_{f\,L}$$

invariant
Chiral Symmetry

Explicit Breaking of
the Chiral Symmetry

Chiral Currents

- Noether Theorem: $U(N_f)_R \longmapsto R^\mu$
  $U(N_f)_L \longmapsto L^\mu$
- Vector current $V^\mu = (L^\mu + R^\mu)/2$
- Axial-vector current $A^\mu = (L^\mu - R^\mu)/2$
- Vector transformation of $\mathcal{L}_{QCD}\big|_{\text{quarks}}$

$$q_f \to q_f' = U_V q_f = e^{-i\sum_{j=0}^{N_f^2-1}\alpha_V^j t^j} q_f \xrightarrow{\text{cons. current}} \boxed{\rho^{\mu j} = \bar{q}_f \gamma^\mu t^j q_f}$$

ρ(770)-like

$j = \{1,2,3\}$

- Axial-vector Transformation of $\mathcal{L}_{QCD}\big|_{\text{quarks}}$

$$q_f \to q_f' = U_A q_f = e^{-i\sum_{j=0}^{N_f^2-1}\alpha_A^j \gamma^5 t^j} q_f \xrightarrow{\text{cons. current}} \boxed{a_1^{\mu j} = \bar{q}_f \gamma^\mu \gamma^5 t^j q_f}$$

a_1(1260)-like

Spontaneous Breaking of Chiral Symmetry

- Transform the (axial-)vector fields

$$\vec{\rho}^{\mu} \xrightarrow{\text{Vector}} \vec{\rho}^{\mu} + \vec{\alpha}_V \times \vec{\rho}^{\mu}$$

$$\vec{a}_1^{\mu} \xrightarrow{\text{Vector}} \vec{a}_1^{\mu} + \vec{\alpha}_V \times \vec{a}_1^{\mu}$$

$$\vec{\rho}^{\mu} \xrightarrow{\text{Axial}} \vec{\rho}^{\mu} + \vec{\alpha}_A \times \vec{a}_1^{\mu}$$

$$\vec{a}_1^{\mu} \xrightarrow{\text{Axial}} \vec{a}_1^{\mu} - \vec{\alpha}_A \times \vec{\rho}^{\mu}$$

Theory:

ρ and a_1 should degenerate

Experiment:

$$m_{a_1} \cong 2 m_{\rho}$$

Spontaneous Breaking of the Chiral Symmetry (SSB) → Goldstone Bosons (pions, kaons…)

- **Chiral Anomaly**

$$L_{QCD}\Big|_{\text{quarks}} \xrightarrow{\text{Axial Singlet } e^{-i\alpha_A^0 \gamma_5 t^0}} \partial_{\mu} A_0^{\mu} \propto m_f \text{, classically}$$

$$\partial_{\mu} A_0^{\mu}\Big|_{m_f = 0} \propto N_f G_{\mu\nu}^{a} \tilde{G}_{a}^{\mu\nu} \text{, at quantum level}$$

Two K_1 Fields

$$A_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N,A} + a_1^0}{\sqrt{2}} & a_1^+ & K_{1,A}^+ \\ a_1^- & \frac{f_{1N,A} - a_1^0}{\sqrt{2}} & K_{1,A}^0 \\ K_{1,A}^- & \bar{K}_{1,A}^0 & f_{1S,A} \end{pmatrix}_\mu \qquad B_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N,B} + b_1^0}{\sqrt{2}} & b_1^+ & K_{1,B}^+ \\ b_1^- & \frac{f_{1N,B} - b_1^0}{\sqrt{2}} & K_{1,B}^0 \\ K_{1,B}^- & \bar{K}_{1,B}^0 & f_{1S,B} \end{pmatrix}_\mu$$

Induce $K_{1,A}$ - $K_{1,B}$ mixing via Tr(Δ[A_μ, B_μ])

$$\Delta \triangleq \begin{pmatrix} \delta_n(m_{u,d}^2) & & \\ & \delta_n(m_{u,d}^2) & \\ & & \delta_s(m_s^2) \end{pmatrix}$$

Burakovsky, Goldman (1998):

$\varphi_{K_1} \sim 37°$ $\quad m_{K_{1,A}} = 1322\,\text{MeV}$ $\quad m_{K_{1,B}} = 1356\,\text{MeV}$

$m_{K_{1(1270)}} = 1273\,\text{MeV}$ $\quad m_{K_{1(1400)}} = 1402\,\text{MeV}$

Sigma Model Lagrangian with Vector and Axial-Vector Mesons ($N_f = 3$)

- **Scalars and Pseudoscalars**

$$\mathcal{L}_{SP} = \mathrm{Tr}[(D^\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] + c_1[(\det\Phi + \det\Phi^\dagger)^2 - 4\det(\Phi\Phi^\dagger)]$$

Explicit Symmetry Breaking **Chiral Anomaly**

$\bar{n}n$

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \qquad P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \qquad \Phi = S + iP$$

$\bar{s}s$

$$D_\mu\Phi = \partial_\mu\Phi + ig_1(\Phi R_\mu - L_\mu\Phi)$$

Where are scalar $\bar{q}q$ states? Under 1 GeV? Above 1 GeV?

Sigma Model Lagrangian with Vector and Axial-Vector Mesons ($N_f = 3$)

- **Vectors and Axial-Vectors**

$$\mathcal{L}_{\mathrm{VA}} = -\frac{1}{4}\mathrm{Tr}\,(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right]$$

$$-2ig_2(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$

$$\Delta \equiv \begin{pmatrix} \delta_n(m_{u,d}^2) & & \\ & \delta_n(m_{u,d}^2) & \\ & & \delta_s(m_s^2) \end{pmatrix}$$

$$L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu$$

$$R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu$$

$$V_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_{N\mu} + \rho_\mu^0}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & \frac{\omega_{N\mu} - \rho_\mu^0}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \omega_{S\mu} \end{pmatrix} \quad A_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N\mu} + a_{1\mu}^0}{\sqrt{2}} & a_{1\mu}^+ & K_{1\mu}^+ \\ a_{1\mu}^- & \frac{f_{1N\mu} - a_{1\mu}^0}{\sqrt{2}} & K_{1\mu}^0 \\ K_{1\mu}^- & \bar{K}_{1\mu}^0 & f_{1S\mu} \end{pmatrix}$$

$$L_\mu = V_\mu + A_\mu$$

$$R_\mu = V_\mu - A_\mu$$

Motivation: QCD Features in an Effective Model

- **QCD Lagrangian**

$$\mathcal{L}_{QCD} = \bar{q}_f (i \not{D} - m_f) q_f \quad + \text{Gluons}$$

- **Chirality Projection Operators**

$$\mathcal{P}_{r,\,l} = \frac{1 \pm \gamma_5}{2}$$

$$\Downarrow$$

$$\mathcal{L}_{QCD} = i\,\bar{q}_{r,\,f} \not{D}\, q_{r,\,f} + i\,\bar{q}_{l,\,f} \not{D}\, q_{l,\,f} - \bar{q}_{r,\,f} m_f q_{l,\,f} - \bar{q}_{l,\,f} m_f q_{r,\,f}$$

Motivation:
QCD Features in an Effective Model

- **Global Unitary Transformations**

$$q_{r,\,f} \longrightarrow q'_{r,\,f} = U_r\, q_{r,\,f} := e^{-i\alpha_r^a t_a} q_{r,\,f}$$

$$q_{l,\,f} \longrightarrow q'_{l,\,f} = U_l\, q_{l,\,f} := e^{-i\alpha_l^a t_a} q_{l,\,f}$$

$$\downarrow$$

invariant | **not invariant**

$$\mathcal{L}_{QCD} = i\,\bar{q}_{r,\,f} \not{D}\, q_{r,\,f} + i\,\bar{q}_{l,\,f} \not{D}\, q_{l,\,f} - \bar{q}_{r,\,f} m_f q_{l,\,f} - \bar{q}_{l,\,f} m_f q_{r,\,f}$$

Chiral Symmetry | **Explicit Symmetry Breaking**

Spontaneously Broken in Vacuum

In addition: Chiral $U(1)_A$ Anomaly

Motivation: Structure of Scalar Mesons

- **Spontaneous Breaking of Chiral Symmetry → Goldstone Bosons ($N_f = 2 \rightarrow \pi$)**
- **Restoration of Chiral Invariance and Deconfinement ↔ Degeneration of Chiral Partners (π/σ)**

  $f_0(600)$, „sigma“ **$f_0(1370)$**
- **Nature of scalar mesons**
- **Scalar $q\bar{q}$ states under 1 GeV → $f_0(600)$, $a_0(980)$ – not preferred by $N_f = 2$ results**
- **Scalar $q\bar{q}$ states above 1 GeV → $f_0(1370)$, $a_0(1450)$ – preferred by $N_f = 2$ results**

[Parganlija, Giacosa, Rischke in Phys. Rev. D 82: 054024, 2010; arXiv: 1003.4934]

Calculating the Parameters

- Shift the (axial-)vector fields:

$$f_{1N}^{\mu} \to f_{1N}^{\mu} + w_{f_{1N}} \partial^{\mu} \eta_N \quad f_{1S}^{\mu} \to f_{1S}^{\mu} + w_{f_{1S}} \partial^{\mu} \eta_S$$

$$\vec{a}_1^{\mu} \to \vec{a}_1^{\mu} + w_{a_1} \partial^{\mu} \vec{\pi} \quad K_1^{\mu} \to K_1^{\mu} + w_{K_1} \partial^{\mu} K \quad K^{*\mu} \to K^{*\mu} + w_{K^*} \partial^{\mu} K_S$$

- Canonically normalise pseudoscalars and K_S:

$$\eta_{N,S} \to Z_{\eta_{N,S}} \eta_{N,S} \quad \vec{\pi} \to Z_{\pi} \vec{\pi} \quad K \to Z_K K \quad K_S \to Z_{K_S} K_S$$

- Perform a fit of all parameters except g_2 (fixed via $\rho \to \pi\pi$)
- 9 parameters, none free $\to$ fixed via masses

$$m_{\pi}, m_K \quad m_{\eta}, m_{\eta'} \leftrightarrow m_{\eta_N}, m_{\eta_S} \quad m_{\rho}, m_{K^*}$$

$$m_{\omega_S} \equiv m_{\varphi(1020)}, m_{f_{1S}} \equiv m_{f_1(1420)}$$

$$m_{a_1} \equiv m_{a_1(1260)}, m_{K_1} \equiv m_{K_1(1270)}$$

$$m_{a_0} \equiv m_{a_0(1450)}, m_{K_S} \equiv m_{K_0^*(1430)}$$

[Parganlija, Giacosa, Rischke in Phys. Rev. D 82: 054024, 2010; arXiv: 1003.4934]

Preliminary: no fit with $m_{a_0} < 1\,\text{GeV}, m_{K_S} < 1\,\text{GeV}$

Other Results

- $\eta - \eta'$ mixing angle $\theta_\eta = 43.9°$ ↔ KLOE Collaboration: $\theta_\eta = 41.4° \pm 0.5°$
- Rho meson mass has two contributions:

$$m_\rho^2 = m_1^2 + \frac{\phi_N^2}{2}[h_1 + h_2 + h_3] + \frac{h_1}{2}\phi_S^2$$

~ Gluon Condensate

Quark Condensates

We obtain $m_1 \propto 761\,\mathrm{MeV}$

- $K^* \to K\pi$

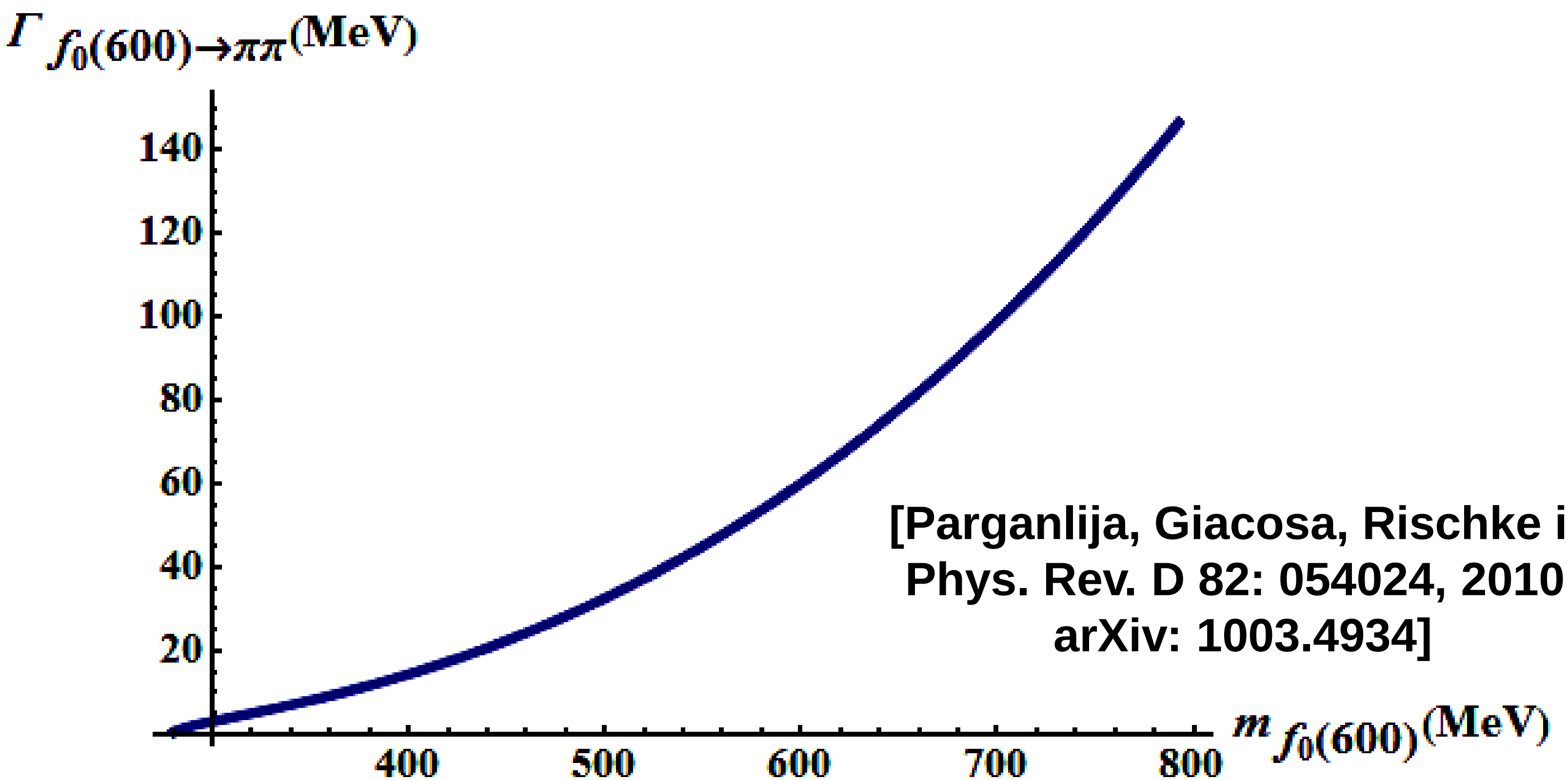
Data: 48.7 MeV Our value: 44.2 MeV

- $\varphi(1020) \to K^+ K^-$

Data: 2.08 MeV Our value: 2.33 MeV

Note: N_f = 2 Limit

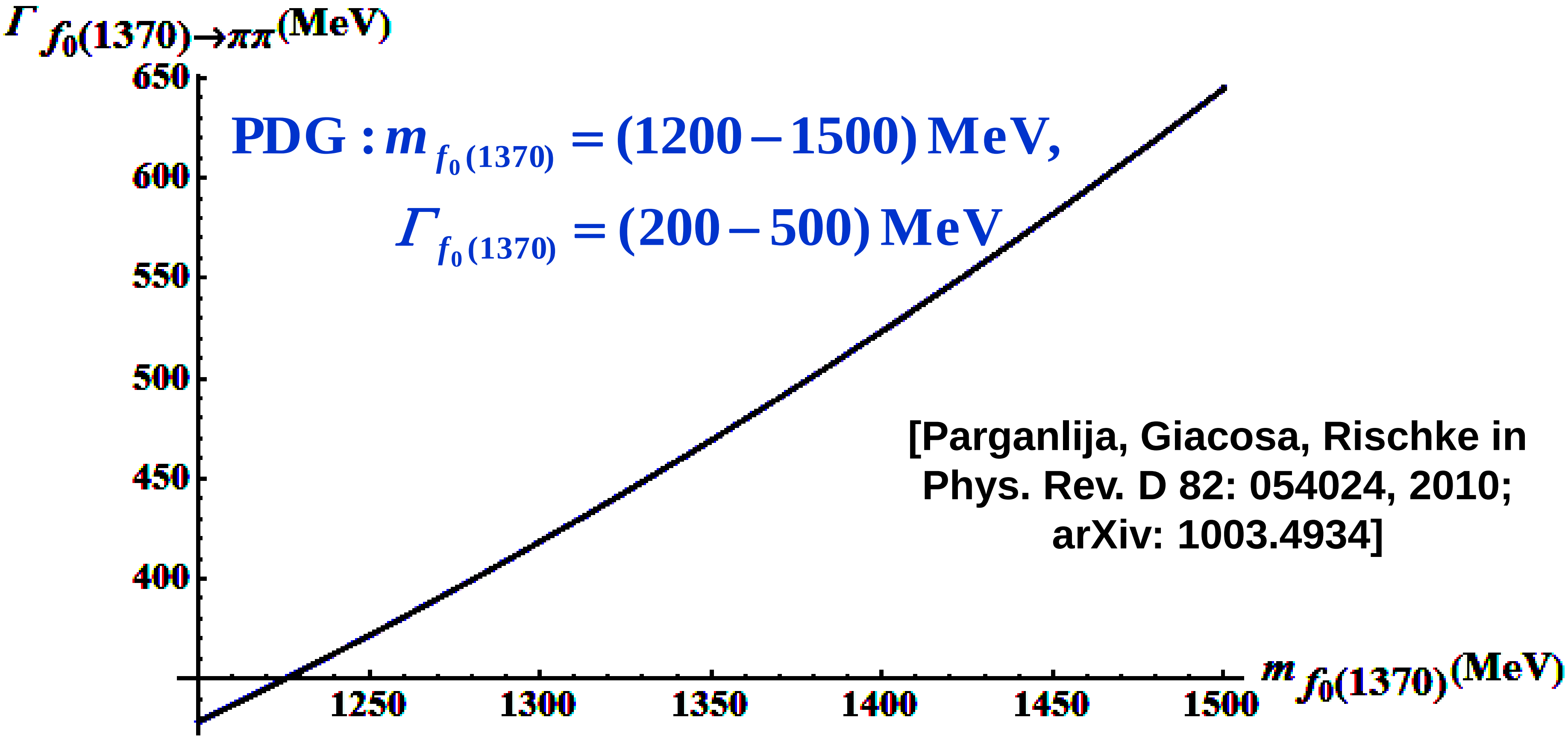
- **The $f_0(600)$ state not preferred to be quarkonium**



[Parganlija, Giacosa, Rischke in Phys. Rev. D 82: 054024, 2010; arXiv: 1003.4934]

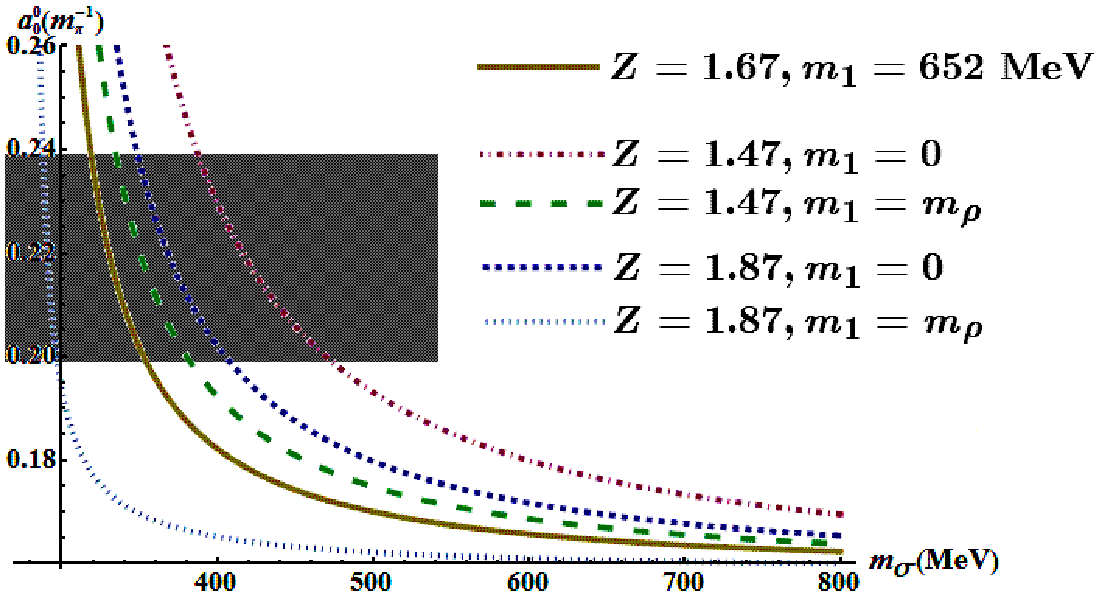
Note: $N_f = 2$ Limit

Experimental data favours $f_0(1370)$ as predominantly $\bar{q}q$



Scenario II (N_f =2): Scattering Lengths

- Scattering lengths saturated



- Additional scalars: tetraquarks, quasi-molecular states
- Glueball

Scenario II (N_f =2): Parameter Determination

- Masses: $m_\pi, m_\eta, m_{a_0}, m_\rho, m_{a_1}$
- Pion Decay Constant $f_\pi = \frac{\phi}{Z}$
- Five Parameters: Z, h_1, h_2, g_2, m_σ

$$\Gamma_{\rho\to\pi\pi} = (149.4 \pm 1.0)\,\mathrm{MeV} \Rightarrow g_2 = g_2(Z)$$

$$\Gamma_{a_0(1450)} = (265 \pm 13)\,\mathrm{MeV} \Rightarrow h_2 = h_2(Z)$$

$$h_1 \equiv 0\,(h_{2,3}\ \mathrm{small})$$

$$\Gamma_{a_1\to\pi\gamma}[Z] = (0.640 \pm 0.246)\,\mathrm{MeV} \to Z$$

$$m_\sigma \equiv m_{f_0(1370)}\ \mathrm{free}$$

Scenario I (N_f =2): Other Results

- $\Gamma_{\rho\to\pi\pi}[Z, g_2], \Gamma_{f_1\to a_0\pi}[Z, h_2]$ exact
- **Our Result**

$$\Gamma_{a_1\to\pi\gamma} = 0.640\ \mathrm{MeV}$$
$$a_0^0 = 0.218$$
$$a_0^2 = -0.0454$$
$$A_{a_0\to\eta\pi} = 3330\ \mathrm{MeV}$$
$$\Updownarrow$$
$$\eta - \eta' \text{ mixing angle} : 41.8^{+0.5}_{-0.2}\ \mathrm{deg}$$

[KLOE Collaboration, hep-ex/0612029v3]:

$$\eta - \eta' \text{ mixing angle} : 41.4 \pm 0.5\ \mathrm{deg}$$

Experimental Value

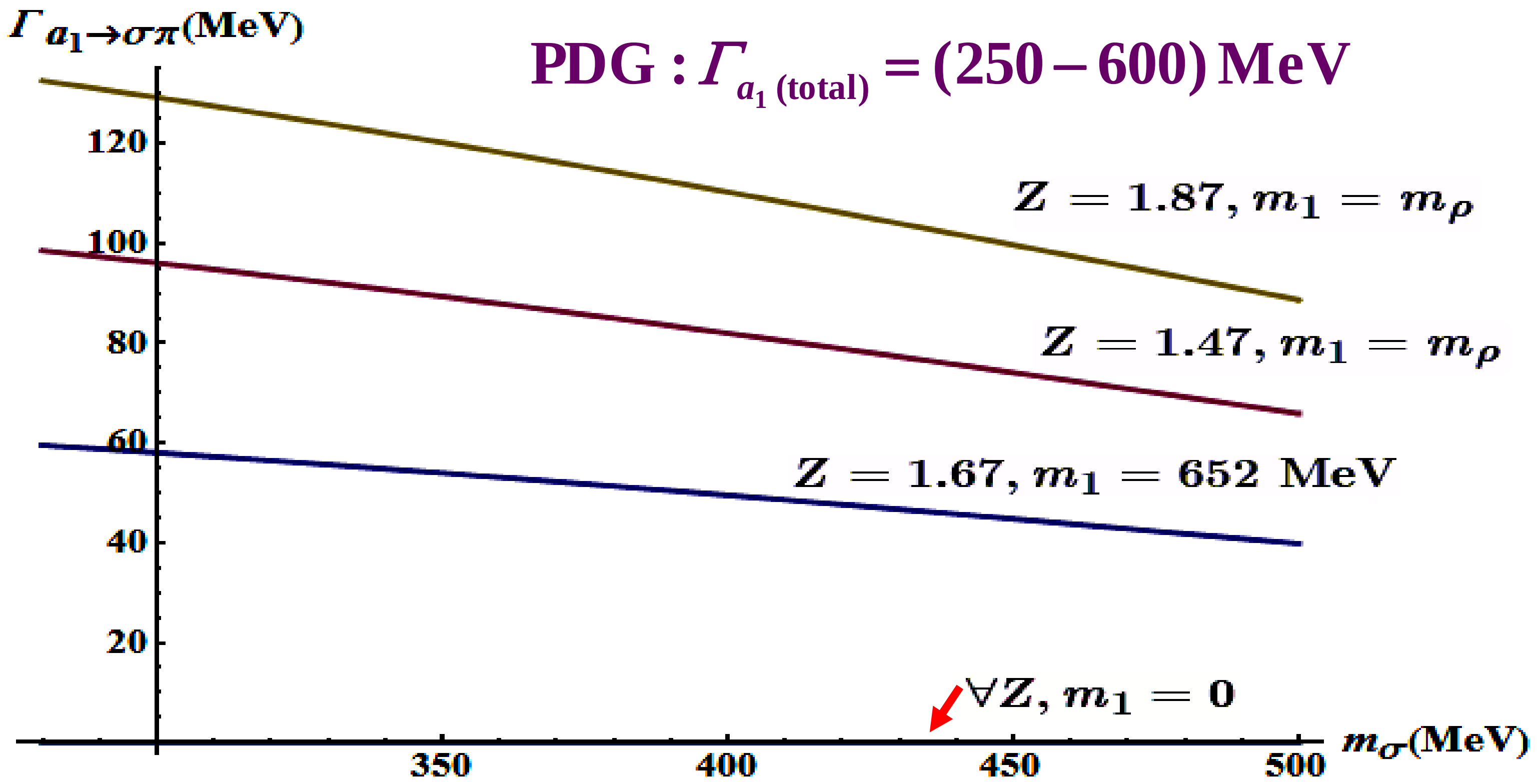
$$\Gamma_{a_1\to\pi\gamma} = 0.640\ \mathrm{MeV}$$
$$a_0^0 = 0.218\ (\mathrm{NA48/2})$$
$$a_0^2 = -0.0457\ (\mathrm{NA48/2})$$

$$A_{a_0\to\eta\pi} = 3330\ \mathrm{MeV}$$

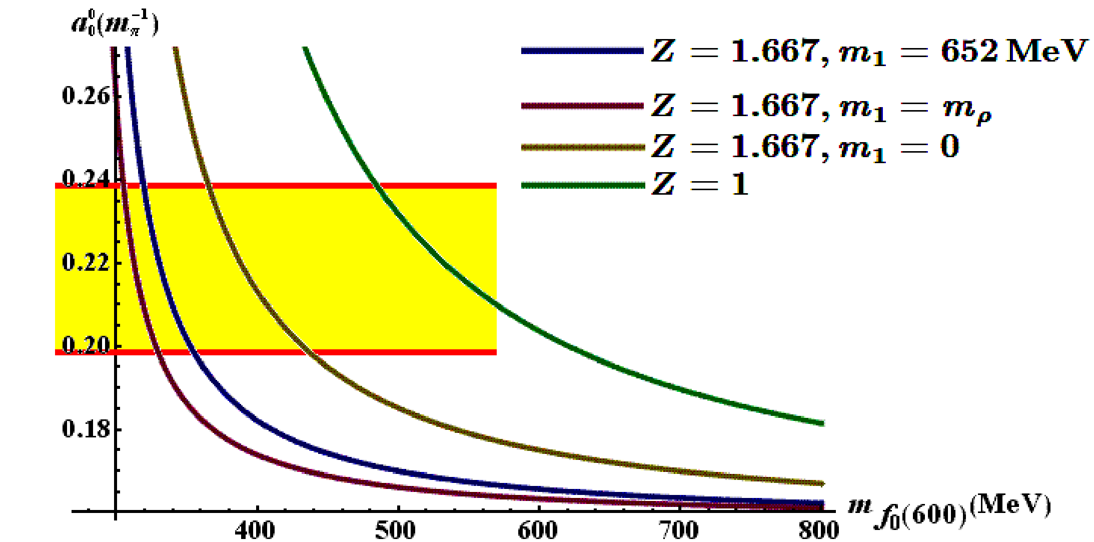
[D. V. Bugg *et al*., Phys. Rev. D 50, 4412 (1994)]

Scenario I (N_f =2): $a_1 \rightarrow \sigma\pi$ Decay

- $m_1 = 0 \rightarrow m_\rho$ generated from the quark condensate only; our result: $m_1 = 652$ MeV
- $a_1 \rightarrow \sigma\pi$



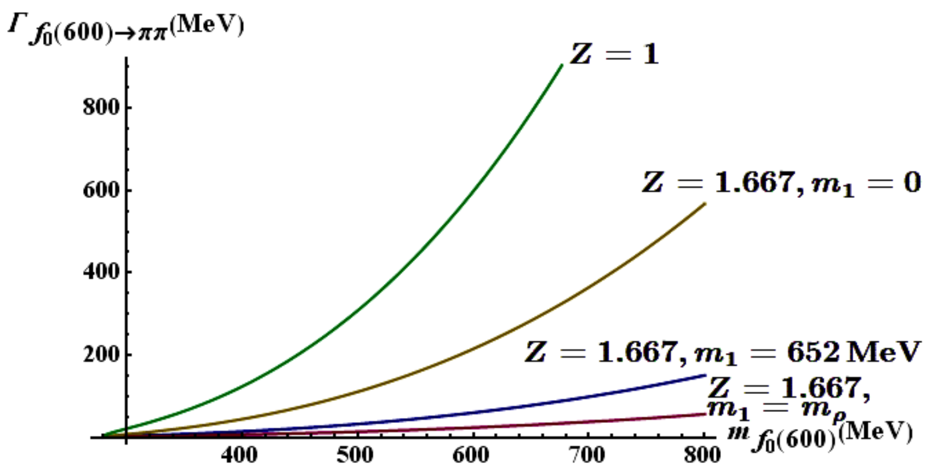
Comparison: the Model with and without Vectors and Axial-Vectors (N_f=2)



Include vectors

↓

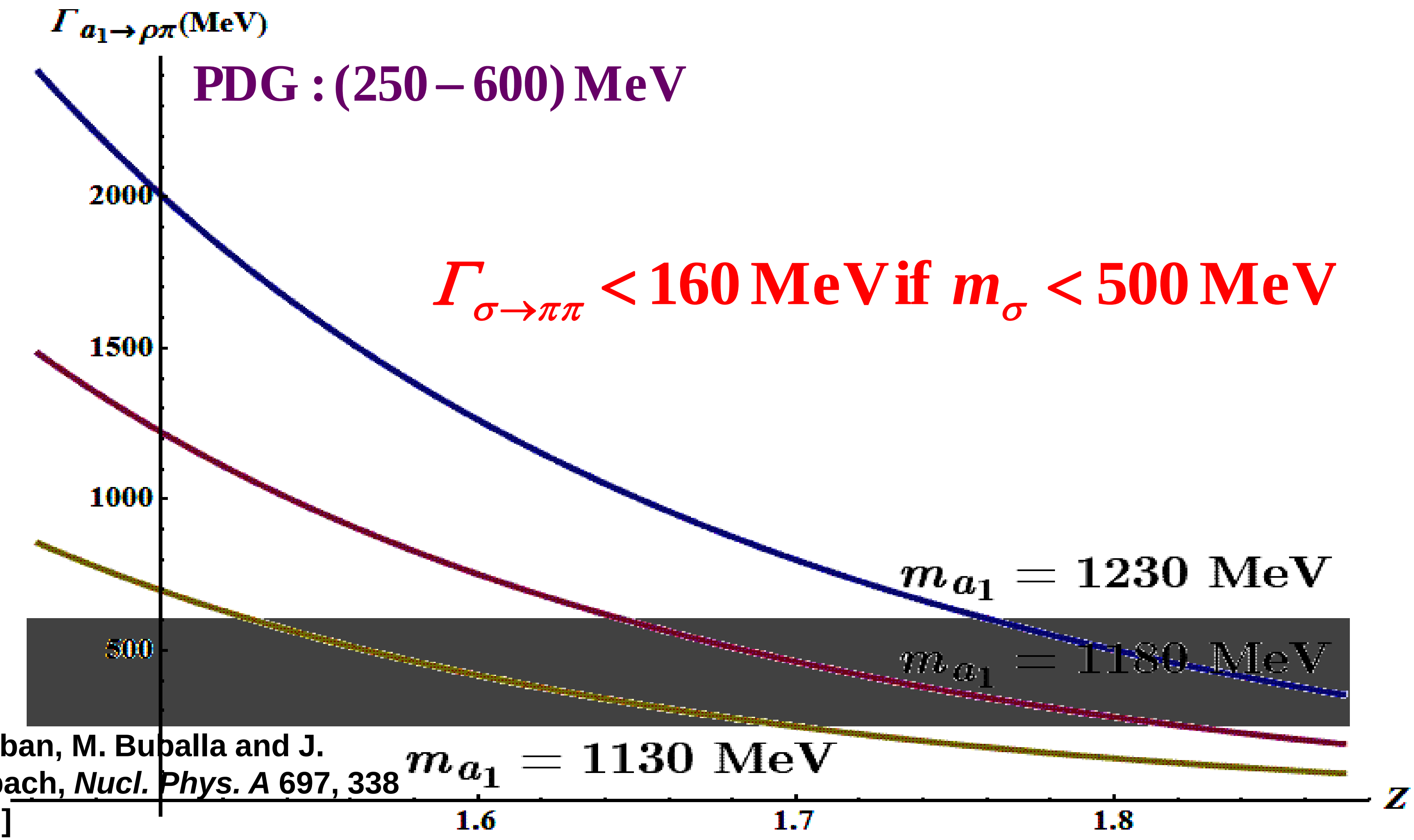
values decrease



Note: other observables (*ππ* scattering lengths, a_0(980)→*ηπ* decay amplitude, phenomonology of a_1, and others) are fine

[Parganlija, Giacosa, Rischke, Phys. Rev. D 82: 054024, 2010]

Scenario I (N_f =2): $a_1 \rightarrow \rho\pi$ Decay



[M. Urban, M. Buballa and J. Wambach, *Nucl. Phys. A* 697, 338 (2002)]

Scenario I (N_f =2) : Parameter Determination

- **Three Independent Parameters: Z, m_1, m_σ**

$$\Gamma_{a_1 \to \pi\gamma}[Z] = (0.640 \pm 0.246)\,\text{MeV} \;\rightarrow\; Z = 1.67 \pm 0.20$$

$$m_\rho^2 = m_1^2 + \frac{\phi^2}{2}[h_1 + h_2(Z) + h_3(Z)] \qquad m_1 = 652^{+123}_{-652}\,\text{MeV}$$

~ Gluon Condensate

Quark Condensate

[S. Janowski (Frankfurt U.), Diploma Thesis, 2010]

Isospin

$$m_\sigma \in [288, 477]\,\text{MeV}$$

$$a_0^0[Z, m_1, m_\sigma] = 0.218 \pm 0.020\,[m_\pi^{-1}]$$

[NA48/2 Collaboration, 2009]

Angular Momentum (*s* wave)

Lagrangian of a Linear Sigma Model with Vector and Axial-Vector Mesons (N_f =2)

- **Vectors and Axial-Vectors**

$$\mathcal{L}_{\mathrm{VA}} = -\frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \left(\frac{m_1^2}{2} + \Delta\right)\mathrm{Tr}[(L^{\mu})^2 + (R^{\mu})^2]$$
$$- 2ig_2(\mathrm{Tr}\{L_{\mu\nu}[L^{\mu}, L^{\nu}]\} + \mathrm{Tr}\{R_{\mu\nu}[R^{\mu}, R^{\nu}]\})$$
$$- 2g_3\{\mathrm{Tr}[(\partial_{\mu}L_{\nu} - ieA_{\mu}[t^3, L_{\nu}] + \partial_{\nu}L_{\mu} - ieA_{\nu}[t^3, L_{\mu}])\{L^{\mu}, L^{\nu}\}]$$
$$+ \mathrm{Tr}[(\partial_{\mu}R_{\nu} - ieA_{\mu}[t^3, R_{\nu}] + \partial_{\nu}R_{\mu} - ieA_{\nu}[t^3, R_{\mu}])\{R^{\mu}, R^{\nu}\}]\}$$

$$L^{\mu\nu} = \partial^{\mu}L^{\nu} - \partial^{\nu}L^{\mu} - (ieA^{\mu}[t^3, L^{\nu}] - ieA^{\nu}[t^3, L^{\mu}])$$
$$R^{\mu\nu} = \partial^{\mu}R^{\nu} - \partial^{\nu}R^{\mu} - (ieA^{\mu}[t^3, R^{\nu}] - ieA^{\nu}[t^3, R^{\mu}])$$

$$\Delta \;\hat{=}\; \begin{pmatrix} \delta_n(m_{u,d}^2) & & \\ & \delta_n(m_{u,d}^2) & \\ & & \delta_s(m_s^2) \end{pmatrix}$$

vectors

$$L^{\mu} = (\omega^{\mu} + f_1^{\mu})\, t^0 + (\vec{\rho}^{\mu} + \vec{a}_1^{\mu}) \cdot \vec{t}$$
$$R^{\mu} = (\omega^{\mu} - f_1^{\mu})\, t^0 + (\vec{\rho}^{\mu} - \vec{a}_1^{\mu}) \cdot \vec{t}$$

axialvectors

Lagrangian of a Linear Sigma Model with Vector and Axial-Vector Mesons ($N_f = 2$)

- **Scalars and Pseudoscalars**

$$\mathcal{L}_{SP} = \mathrm{Tr}[(D^\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$+\,\mathrm{Tr}\,[H(\Phi+\Phi^\dagger)] + c[(\det\Phi + \det\Phi^\dagger)^2 - 4\det(\Phi\Phi^\dagger)]$$

Explicit Symmetry Breaking

Chiral Anomaly

scalars

$$\Phi = (\sigma + i\eta)\,t^0 + (\vec{a}_0 + i\vec{\pi})\cdot\vec{t}$$

pseudoscalars

$$D^\mu\Phi = \partial^\mu\Phi + ig_1(\Phi R^\mu - L^\mu\Phi) - ieA^\mu[t^3,\Phi]$$

photon

$$\{\sigma, a_0\} \rightarrow \{f_0(600), a_0(980)\}\ \mathrm{or}\ \{f_0(1370), a_0(1450)\}$$

Where is the scalar $\bar{q}q$ state?