The Phase Diagram of Effective Theories for the QCD Center Degrees of Freedom

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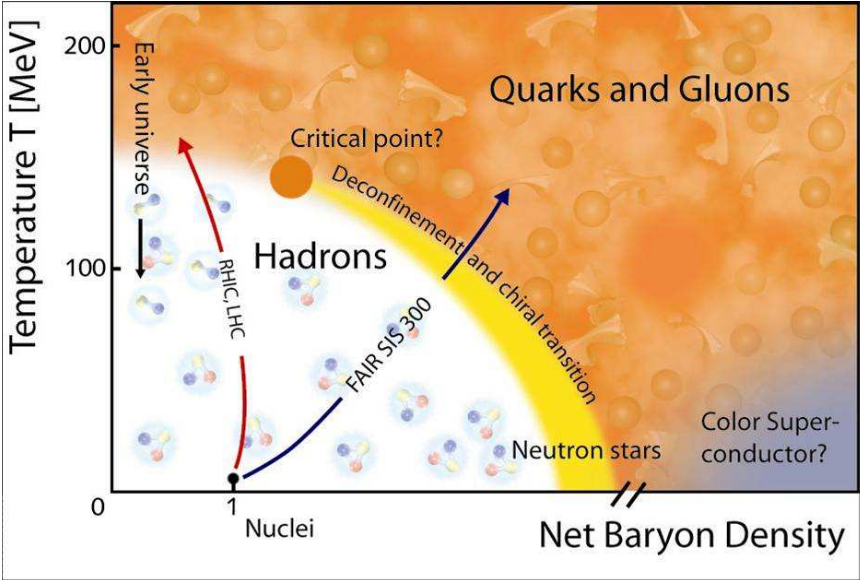
Excited QCD
Peniche, May 2012

Der Wissenschaftsfonds.

Something we would like to understand



Sign problem of LQCD at finite density

- Lattice QCD has problems with finite chemical potential μ.

$$Z = \int D[q, \overline{q}, A] e^{-S_G[A] - S_F[q, \overline{q}, A; \mu]}$$
$$= \int D[A] e^{-S_G[A]} det D[A; \mu]$$

- $det D[A; \mu]$ is complex for $\mu > 0$.
- $e^{-S_G[A]} det D[A; \mu]$ cannot be used as a probability in a Monte Carlo simulation.

Perspectives

- Only very little progress for Lattice QCD with chemical potential due to the sign problem.
- Some progress for QCD related effective theories.
- New ideas tried with effective theories. E.g. flux representation.

F. Karsch et al.(1984)

A. Patel,T. DeGrand,C. DeTar(1983)

Center symmetry and deconfinement

Center transformation:

$$z \in \mathbb{Z}_3 \;=\; \{\mathbb{1}, e^{i2\pi/3}\mathbb{1}, e^{-i2\pi/3}\mathbb{1}\}: \;\; U_0(x) \;\rightarrow\; zU_0(x)$$

Important symmetry:

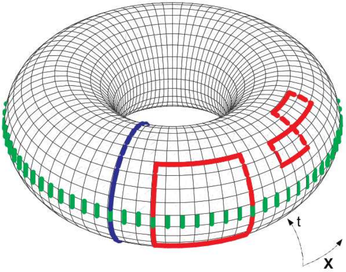
- Gauge action is invariant.
- Polyakov loop is transformed by an element of the center.

$$P \rightarrow zP$$

- P is a static source quark.

  $\langle P \rangle = 0 \;\rightarrow$ center sym. conserved.

  $\langle P \rangle \neq 0 \;\rightarrow$ center sym. broken.

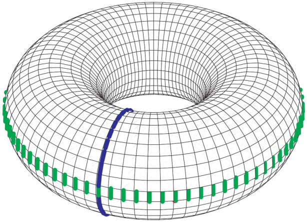


Center effective theory for QCD thermodynamics

$$S_{eff} = -\sum_x \left(\tau \sum_{\nu=1}^{3} \left[P(x)P(x+\hat{\nu})^\dagger + h.c. \right] \; + \; \kappa \left[e^{\mu} P(x) \; + \; e^{-\mu} P(x)^\dagger \right] \right)$$

- The degrees of freedom are traced Polyakov loops.

  $P(x) = TrL(x)$ with $L(x) \; \in \; SU(3)$



- P is a static source quark.

  $T < T_c : \langle P \rangle = 0 \; \rightarrow$ quarks confined

  $T > T_c : \langle P \rangle \neq 0 \; \rightarrow$ quarks deconfined

Nearest neighbor interaction term

$$\sum_{x} \tau \sum_{\nu=1}^{3} \left[P(x) P(x+\hat{\nu})^{\dagger} + h.c. \right]$$

- Gauge action $\rightarrow$ nearest neighbor interaction for $P(x)$.
- From strong coupling expansion of the gluonic action.
- Temperature $\rightarrow \tau$ (increases with T).
- Spontaneous center symmetry breaking $\rightarrow$ deconfinement transition.

Magnetic term

$$\sum_x \kappa \left[e^{\mu} P(x) + e^{-\mu} P(x)^{\dagger} \right]$$

- Fermion action $\rightarrow$ magnetic term.
- From hopping expansion (large mass) of the fermionic action.
- Quark mass $\rightarrow \kappa$ (decreases with m_q).
- It contains center symmetry breaking terms and chemical potential.
- Sign problem.

Flux representation - 1

- Effective center model still has complex action $\Rightarrow$ new variables!
- We map the partition sum to a flux representation:
  - Expand interaction term:

$$e^{P(x)P(x+\hat{\nu})^\dagger} \rightarrow \sum_{l_{x,\nu}} \frac{\left[P(x)P(x+\hat{\nu})^\dagger\right]^{l_{x,\nu}}}{l_{x,\nu}!}$$

  - Expand magnetic term:

$$e^{P(x)} \rightarrow \sum_{s_x} \frac{P(x)^{s_x}}{s_x!}$$

  - Integrate out $SU(3)$ variables $\rightarrow$ new degrees of freedom:
    - ★ Dimers $l_{x,\nu}$, live on the links (x,ν)
    - ★ Monomers s_x, live on the sites x

C. Gattringer, Nucl. Phys. B **850** (2011) 242

Y.D., C. Gattringer, arXiv:1204.6074 [hep-lat]

Flux representation - 2

- For the neighbor interaction term:

$$e^{\tau[P(x)P(x+\hat{\nu})^\dagger + P(x)^\dagger P(x+\hat{\nu})]} =$$

$$\sum_{l_{x,\nu}=0}^{+\infty} \frac{\tau^{l_{x,\nu}}}{l_{x,\nu}!} \left[P(x)P(x+\hat{\nu})^\dagger\right]^{l_{x,\nu}} \sum_{\bar{l}_{x,\nu}=0}^{+\infty} \frac{\tau^{\bar{l}_{x,\nu}}}{\bar{l}_{x,\nu}!} \left[P(x)^\dagger P(x+\hat{\nu})\right]^{\bar{l}_{x,\nu}}$$

- The magnetic term:

$$e^{\kappa e^{\mu} P(x) + \kappa e^{-\mu} P(x)^\dagger} =$$

$$\sum_{s_x=0}^{+\infty} \frac{(\kappa e^{\mu})^{s_x}}{s_x!} P(x)^{s_x} \sum_{\bar{s}_x=0}^{+\infty} \frac{(\kappa e^{-\mu})^{\bar{s}_x}}{\bar{s}_x!} P(x)^{\dagger\ \bar{s}_x}$$

- New variables:
  - **dimers** : $l_{x,\nu}, \bar{l}_{x,\nu} \in [0, +\infty)$ on the link (x, ν).
  - **monomers**: $s_x, \bar{s}_x \in [0, +\infty)$ on the site x.

Flux representation - 3

- The partition function in the flux representation:

$$Z = \sum_{\{l,\bar{l}\}} \sum_{\{s,\bar{s}\}} \left(\prod_{x,\nu} \frac{\tau^{l_{x,\nu}+\bar{l}_{x,\nu}}}{l_{x,\nu}!\bar{l}_{x,\nu}!} \right) \left(\prod_{x} \frac{(\kappa e^{\mu})^{s_x} (\kappa e^{-\mu})^{\bar{s}_x}}{s_x!\bar{s}_x!} \right) \times$$
$$\left(\prod_{x} \int D[L] TrL(x)^{f(x)} \; TrL(x)^{\dagger \; \bar{f}(x)} \right)$$

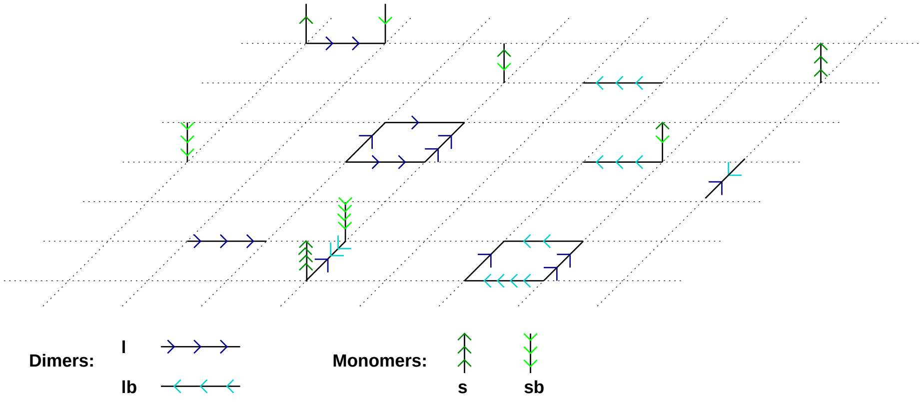
  - $f(x) \;=\; \sum_{\nu=1}^{3} [l_{x,\nu} + \bar{l}_{x-\hat{\nu},\nu}] \;+\; s_x$
  - $\bar{f}(x) \;=\; \sum_{\nu=1}^{3} [l_{x-\hat{\nu},\nu} + \bar{l}_{x,\nu}] \;+\; \bar{s}_x$

- The $SU(3)$ integrals:

$$\int D[L] \; TrL^{f} \; TrL^{\dagger \; \bar{f}} \;=\; \begin{cases} I_{f\bar{f}} & (f-\bar{f}) \; mod \; 3 = 0 \\ 0 & else \end{cases}$$

S. Uhlmann, R. Meinel, A. Wipf (2006); C. Gattringer (2011)

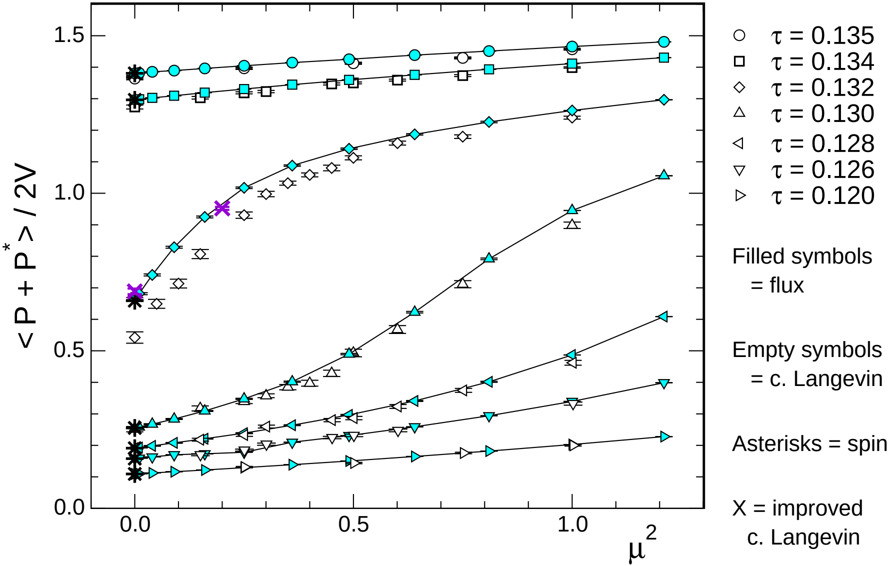
Graphical representation of configurations



Numerical analysis

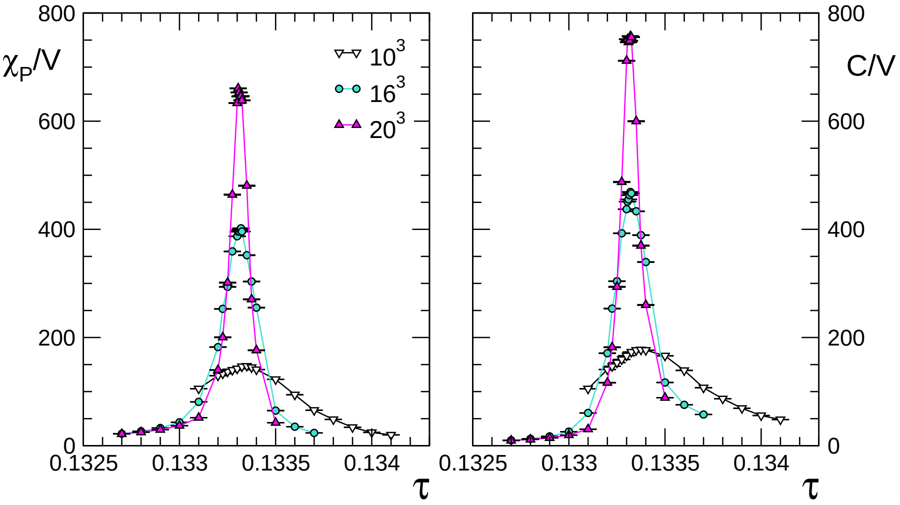
- Study the $\tau - \mu$ phase diagram.
- Identify the phase boundaries between confinement and deconfinement.
- Analyze the nature of the transitions.
- Location of the transition lines determined from the maxima of the Polyakov loop susceptibility χ_P and the heat capacity C.
- MC simulation: Metropolis algorithm of the flux representation.

Comparison with Complex Langevin



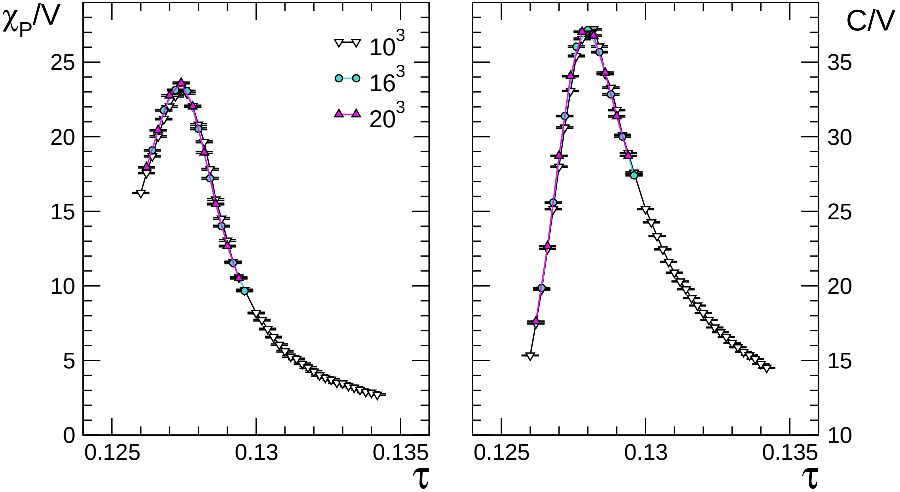
Determination of the phase boundaries

- Location of phase boundaries determined by the maxima of χ_P and C.
- Volume scaling and maxima at the same position $\longrightarrow$ first-order transition

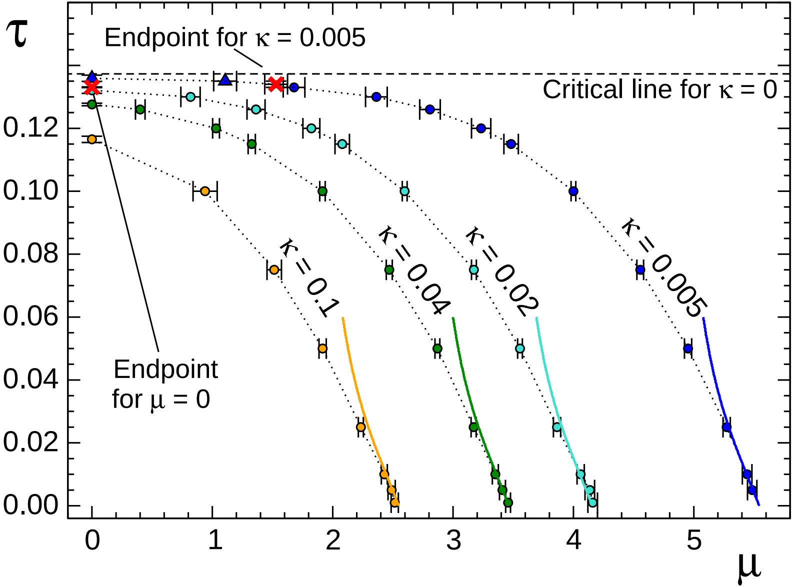


Determination of the phase boundaries

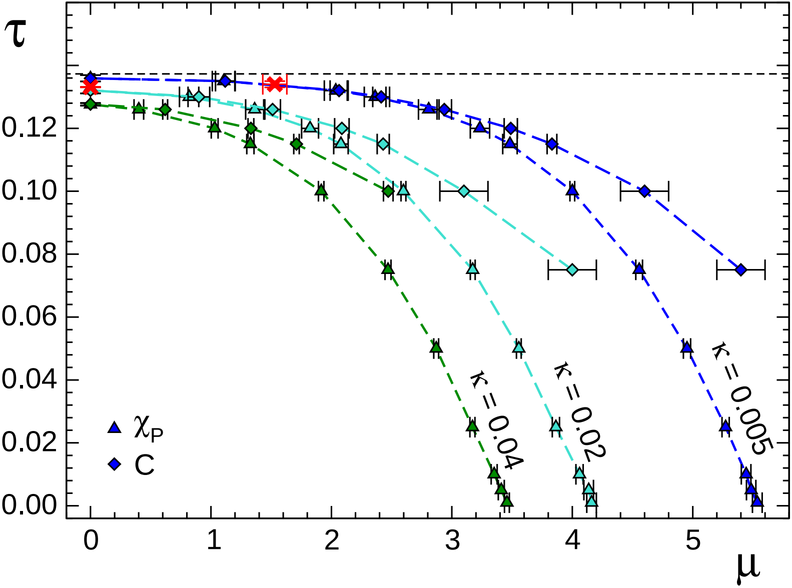
- Location of phase boundaries determined by the maxima of χ_P and C.
- No volume scaling and maxima at different positions $\longrightarrow$ crossover



Phase diagram from χ_P



Comparison of phase boundaries from C and χ_P



Summary

- We have studied an effective center theory of QCD with finite quark density at non-zero temperature.
- In the flux representation the model is free of the complex phase problem and can be simulated with MC techniques.
- When only center degrees of freedom are considered all transitions are of crossover type (unless $m_q >> 1$).
- We generated reference results at finite μ which can be used to test other approaches.

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Thank you for your attention!