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Is the $X(3872)$ a molecule?

Susana Coito

CFIF, Instituto Superior Técnico, Lisboa

Supervisors: George Rupp, Eef van Beveren

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I. The mysterious $X(3872)$

Experimental features:

- The existencence of the $X(3872)$ is very well established.

- It was discovered by the Belle Collaboration, in 2003, in the decay $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$.

- It is a "charmonium-like state".

- It has two possible J^{PC} assignments, 1^{++} or 2^{-+}.

- Mass: $m = 3871.57 \pm 0.25$ MeV$/c^2$, $\Gamma < 2.3$ MeV$/c^2$. (PDG(2010))

- Hadronic decay modes: $\rho^0 J/\psi$, $\omega J/\psi$ and $D^0 D^{*0}$.

Theoretical enigma:

◇ The $X(3872)$ does not fit in the conventional models for $q\bar{q}$ mesons.

◇ It lies very close to the $D^0 D^{*0}$ threshold.

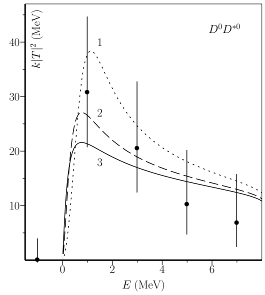
◇ It has an isospin violating and two OZI-supressed hadronic decays.

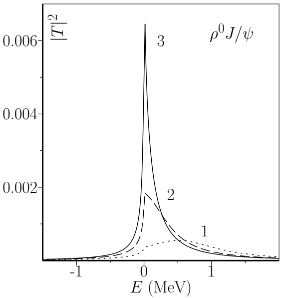
◇ Angular momentum and parity are very hard to determine experimentally.

◇ This enhancement is an actual challenge to the theorists, who try to predict and describe it through a panoplly of approaches, namelly, molecules.

Our Motivation:

In a previous study of the $X(3872)$ (Eur. Phys. J. C (2011) 71:1762), we employed the Resonance Spectrum Expansion formalism, with nine coupled-channels, including the $\rho^0 J/\psi$ and the $\omega J/psi$. We got very optimistic results:





• In the view of this results, which are encouraged to believe that no molecular (or other exotic) description is needed, so fast, to describe this state.

• Also, we are motivated by the work of Eric Braaten and Meng Lu (PRD 76, 094028 (2007)) which considering the $X(3872)$ as a $D^0 - D^{*0}$ molecule, with 1^{++}, conclude it fits to a bound state below the $D^0 D^{*0}$.

• However, the analysis of the wave function probabilities should give us a description of the dominant modes. This, in principle, is realized by a two-channel Schrödinger potential model.

• Then, the goal of this work, still in progress, is to study the probabilities of the two-component wave-function within a $c\bar{c} - D^0 D^{*0}$. If the probablibily of the $c\bar{c}$ component does not vanish near the threshold, then the $X(3872)$ is not a molecule.

II.A simple two-channels model for hadronic resonances

Let us consider a $q\bar{q} - MM$ system. q-quark, M-meson.
The $q\bar{q}$ state is confined through an harmonic-oscilator (H.O.) potential. The MM final state is composed of two free mesons.

Then, we write the radial Schrödinger equation:

$$\begin{pmatrix} h_c & V \\ V & h_f \end{pmatrix} \begin{pmatrix} u_c \\ u_f \end{pmatrix} = E \begin{pmatrix} u_c \\ u_f \end{pmatrix}$$

With the following hamiltonians:

$$h_c = \frac{1}{2\mu_c}\left(-\frac{d^2}{dr^2} + \frac{l_c(l_c+1)}{r^2} \right) + \frac{1}{2}\mu_c \omega^2 r^2 + m_q + m_{\bar{q}}$$

$$h_f = \frac{1}{2\mu_f}\left(-\frac{d^2}{dr^2} + \frac{l_f(l_f+1)}{r^2} \right) + M_1 + M_2$$

At some 'string breaking' distance a, one can have a point transition from one state to the other. Then, this two-channels are coulpled, with strengh g, through a potential of the type:

$$V = \frac{g}{2\mu_c a}\delta(r - a)$$

The boundary conditions of this problem are:

$$u_c'(r \uparrow a) - u_c'(r \downarrow a) + \frac{\lambda}{a} u_f(a) = 0$$
$$u_f'(r \uparrow a) - u_f'(r \downarrow a) + \frac{\lambda \mu_f}{a \mu_c} u_c(a) = 0$$

$$u_c(r \uparrow a) = u_c(r \downarrow a)$$
$$u_f(r \uparrow a) = u_f(r \downarrow a)$$

A general solution for this problem is given by:

$$u_c(r) = \begin{cases} A_c F_c(r) & r < a \\ B_c G_c(r) & r > a \end{cases}$$

Where $F_c(r)$ is a function which vanishes at the origin and $G_c(r)$ is a function which dumps exponentially at infinity. If $z = \mu\omega r^2$ and $\nu = \frac{E-2m_c}{2\omega} - \frac{l_c+3/2}{2}$, they are defined by:

$$F(r) = \frac{1}{\Gamma(l+3/2)} z^{(l+1)/2} e^{-z/2} \phi(-\nu, l+3/2, z)$$

$$G(r) = -\frac{1}{2}\Gamma(-\nu) r z^{l/2} e^{-z/2} \psi(-\nu, l+3/2, z)$$

Now the *MM*, or final state solution:

$$u_f(r) = \begin{cases} A_f J_{l_f}(kr) & r < a \\ \\ B_f \left[J_{l_f}(kr) k^{2l_f+1} \cot\left(\delta_{l_f}(E)\right) - N_{l_f}(kr) \right] & r > a \end{cases}$$

With,

$$J_l(kr) = k^{-l} r j_l(kr)$$
$$N_l(kr) = k^{l+1} r n_l(kr)$$

From the boundary conditions, we get the relations:

$$\begin{cases} G_c'(r)F_c(a) - F_c'(a)G_c(a) = \frac{g}{a} J_{l_f}(ka)F_c(a)\frac{A_f}{B_c} \\ \\ J_{l_f}'(ka)N_{l_f}(ka) - J_{l_f}(ka)N_{l_f}'(ka) = \frac{g}{a}\frac{\mu_f}{\mu_c} J_{l_f}(ka)F_c(a)\frac{A_c}{B_f} \end{cases}$$

And also, one can verify that the Wronskian gives:

$$\begin{cases} W(F_c(a), G_c(a)) = \lim_{r\to a}[F_c(r)G_c'(r) - F_c'(r)G_c(r)] = 1 \\ \\ W(N_{l_f}(ka), J_{l_f}(ka)) = \lim_{r\to a}[N_{l_f}(kr)J_{l_f}'(kr) - N_{l_f}'(kr)J_{l_f}(kr)] = -1 \end{cases}$$

It follows that the partial amplitudes of the wave function relate as:

$$A_f B_f = -\frac{\mu_f}{\mu_c} A_c B_c$$

$$\frac{A_f}{B_f} = -\left[\frac{g^2}{a^2}\frac{\mu_f}{\mu_c} J_{l_f}^2(ka)F_c^2(a)\right]^{-1} \frac{B_c}{A_c}$$

Then, the cotangent comes:

$$\cot\left(\delta_{l_f}(E)\right) = -\left[g^2 \frac{\mu_f}{\mu_c} k j_{l_f}^2(ka) F_c(a) G_c(a)\right]^{-1} + \frac{n_{l_f}(ka)}{j_{l_f}(ka)}$$

Which relates to the scattering matrix through $S_{l_f}(E) = e^{2i\delta_{l_f}(E)}$. It comes, finally, an expression for S:

$$S_{l_f}(E) = \frac{1 - ig^2 \frac{\mu_f}{\mu_c} k h_{l_f}^{(2)}(ka) j_{l_f}(ka) F_c(a) G_c(a)}{1 + ig^2 \frac{\mu_f}{\mu_c} k h_{l_f}^{(1)}(ka) j_{l_f}(ka) F_c(a) G_c(a)}$$

The poles of the S-matrix are interpreted as resonances or bound states.

Setting the amplitudes:

$$\begin{cases} A_c = 1 \\ B_c = \frac{F_c(a)}{G_c(a)} \\ A_f = \frac{a}{g}\frac{1}{J_{l_f}(ka)G_c(a)} \\ B_f = -\frac{g}{a}\frac{\mu_f}{\mu_c}J_{l_f}(ka)F_c(a) \end{cases}$$

The radial wave functions $R(r)$, where $u(r) = rR(r)$, are fully determined.

$$R_c(r) = \begin{cases} \frac{F_c(r)}{r} & , r < a \\ \frac{F_c(a)}{G_c(a)}\frac{G_c(r)}{r} & , r > a \end{cases}$$

$$R_f(r) = \begin{cases} \frac{1}{g j_{l_f}(ka) G_c(a)} j_{l_f}(kr) & , r < a \\ \left[-g\frac{\mu_f}{\mu_c}k j_{l_f}(ka)F_c(a)\right]\left[j_{l_f}(kr)\cot\left(\delta_{l_f}(E)\right) - n_{l_f}(kr)\right] & , r > a \end{cases}$$

III. The $c\bar{c} - D^0 D^{*0}$ system

Now we apply this formalism to the case of the coupled $c\bar{c} - D^0 D^{*0}$. In the confined channel we have $c\bar{c}$, with $l_c = 1$ while in the final two-meson channel we have $D^0 D^{*0}$, where $l_f = 0$. To this system, we compute:

$$\mu_c = \frac{1}{2} m_c$$

$$\mu_f = \frac{E}{4} \left[1 - \left(\frac{m_{D^0}^2 - m_{D^{*0}}^2}{E} \right)^2 \right]$$

$$k = \frac{E}{2} \left\{ \left[1 - \left(\frac{m_{D^0} + m_{D^{*0}}}{E} \right)^2 \right] \left[1 - \left(\frac{m_{D^0} - m_{D^{*0}}}{E} \right)^2 \right] \right\}^{1/2}$$

The S-matrix poles are given by:

$$0 = 1 + ig^2 \frac{\mu_f}{\mu_c} k h^{(1)}_{l_f}(ka) j_{l_f}(ka) F_c(a) G_c(a)$$

$$\nu = \frac{E - 2m_c}{2\omega} - \frac{l_c + 3/2}{2}$$

The coupling g and 'string-breaking' distance a are left as free parameters. In the following table we summarize the employed parameter values:

Parameter	ω	m_c	m_{D^0}	$m_{D^{*0}}$	$m_{D^0} + m_{D^{*0}}$
Value (MeV)	190	1562	1864.84	2006.97	**3871.81**

For integer values of ν, the poles of the S-matrix shall be eigenvalues of the H.O., for which case the two channels decouple.

ν	0	1	2
E(MeV)	3599	3979	4359

Table: Eigenvalues of the H.O.

IV. Preliminary results

'String Breaking' **a**	Coupling **g**	Solution	Type
2.0	1.150	3871.81	virtual
2.0	1.153	3871.81	real
2.0	1.127	3871.56	virtual
2.0	1.177	3871.57	real
2.5	1.372	3871.81	virtual
2.5	1.377	3871.81	real
3.0	2.144	3871.81	virtual
3.0	2.150	3871.81	real
3.0	2.220	3871.57	virtual
3.0	2.081	3871.57	real

Figure: Dynamic pole trajectory. Virtual (green), real (red).

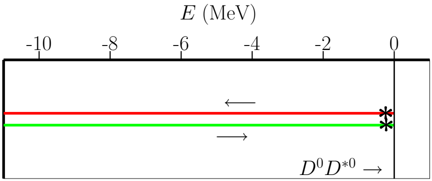


Figure: Dynamic pole trajectory. Virtual (green), real (red).

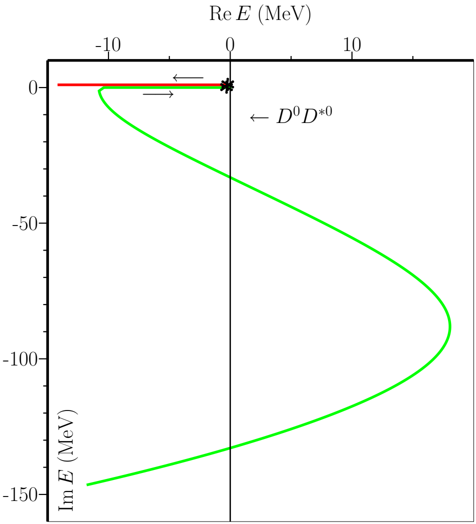


Figure: Trajectory of the H.O eigenvalue 3979 MeV (n=1) for increasing g.

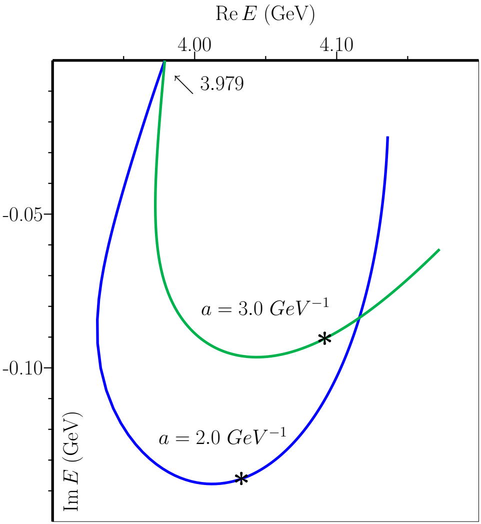


Figure: Radial wave-functions for $E = 3871.57$ MeV and $g = 1.177$

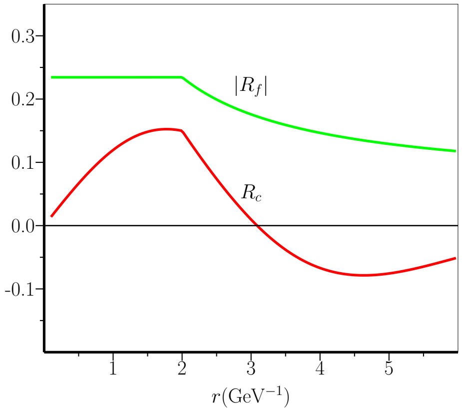
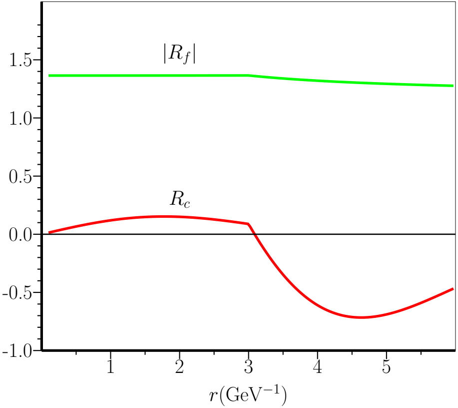


Figure: Radial wave-functions for $E = 3871.57$ MeV and $g = 2.081$



V. Conclusions

Although this work is still in progress, we can already conclude:

◇ The model properly describes the behavior of an S-wave decay channel bellow the threshold.

◇ The two-component wave function is stable to a range of more than 10 MeV bellow threshold.

◇ The pole above threshold goes to the $n = 1$ H.O. eigenvalue, as it should, in the case of the decoupling.

◇ The 3872 MeV pole is dynamically generated in the context of this very simplified model.

$\Rightarrow$ In order to know whether the mysterious $X(3872)$ is a molecule, we still need to numerically integrate the wave-function components, to study their probabilities!

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