Phenomenology of Dilaton in a Chiral Linear Sigma Model with Vector Mesons

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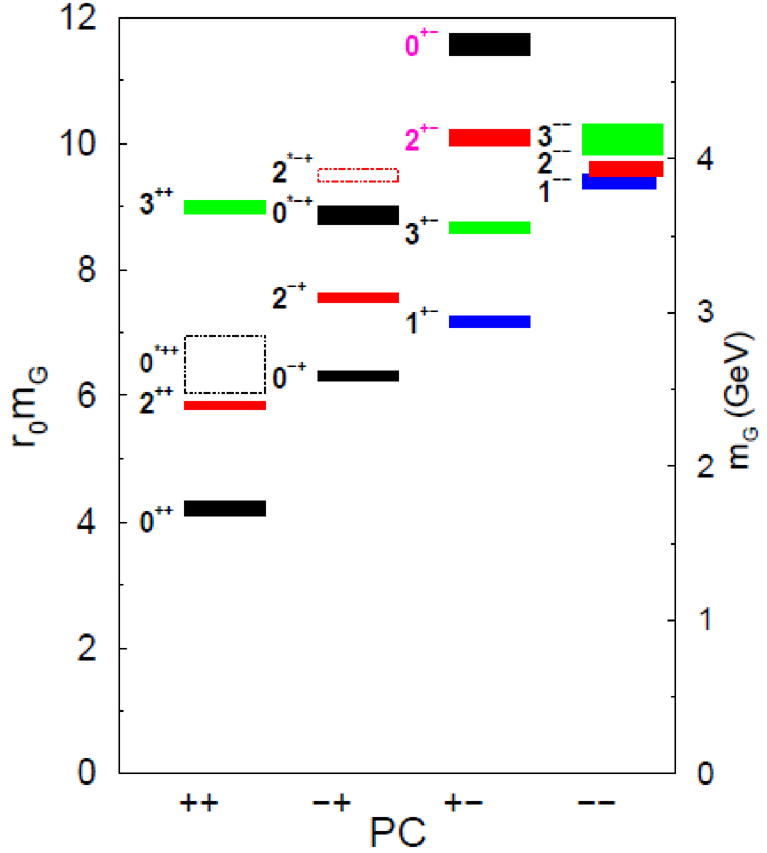
Motivation

Lattice QCD calculations predict
a scalar-isoscalar glueball with
a mass 1.4 -1.8 GeV

[C. Morningstar and M. J. Peardon, AIP Conf. Proc. 688, 220 (2004) [arXiv:nucl-th/0309068]]

[Y. Chen *et al*.**,** ***Phys. Rev.*** **D 73, 014516 (2006)]**



This glueball mixing with the scalar mesons → consider 2 scalar-isoscalar states:

1 glueball $G \equiv gg$ and 1 quarkonium $\bar{q}q \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ $J^{PC} = 0^{++}$

Motivation

Consider 2 scenarios:

$G \cong f_0(1500)$ and $\bar{q}q \cong f_0(1370)$

$G \cong f_0(1710)$ and $\bar{q}q \cong f_0(1370)$

But also tested:

$G \cong f_0(1500)$ and $\bar{q}q \cong f_0(600)$

$G \cong f_0(1710)$ and $\bar{q}q \cong f_0(600)$

The Effective Model

Dilatation:

$$x^\mu \longrightarrow x'^\mu = \lambda^{-1} x^\mu \qquad \varphi(x) \longrightarrow \varphi'(x) = \lambda^d \varphi(\lambda x)$$

$d = 1$ **for bosons**

$d = \frac{3}{2}$ **for fermions**

Trace of the energy-momentum tensor of the YM Lagrangian: $(T_{\mathrm{YM}})^\mu_\mu = \frac{\beta(g)}{4g} G^a_{\mu\nu} G^{\mu\nu}_a \neq 0$

Trace anomaly: gluon condensate

$$\left\langle T^\mu_{YM,\mu} \right\rangle = -\frac{11 N_c}{48} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu}_a \right\rangle = -\frac{11 N_c}{48} C^4 \neq 0$$

$\Rightarrow$ breaks dilatation invariance

QCD sum rules and QCD lattice

$$C^4 \simeq (300 - 600 \text{ MeV})^4$$

[M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* B 147,385 (1979)]

[A. Samsonov, arXiv:hep-ph/0407199]

[J. Kripfganz, Phys. Lett. B 101, 169 (1981)]

[A. Di Giacomo, H. Panagopoulos and E. Vicari, Nucl. Phys. B 338, 294 (1990)]

$\Rightarrow$ Effective Lagrangian:

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right)$$

$$\partial_\mu J^\mu_{dil} = T^\mu_{dil,\,\mu} = -\frac{1}{4} m_G^2 \Lambda^2$$

$$\Rightarrow \Lambda = \sqrt{11}\, C^2/(2 m_G)$$

[A. A. Migdal and M. A. Shifman, Phys. Lett. B 114, 445 (1982)]

[H. Gomm, P. Jain, R. Johnson and J. Schechter, Phys. Rev. D 33, 801 (1986)]

The Effective Model

Full effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_{dil} + \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi) - m_0^2\left(\frac{G}{G_0}\right)^2\Phi^\dagger\Phi - \lambda_2(\Phi^\dagger\Phi)^2\right]$$
$$- \lambda_1(\mathrm{Tr}\left[\Phi^\dagger\Phi\right])^2 + c[\det(\Phi^\dagger) + \det(\Phi)] + \mathrm{Tr}\left[H\left(\Phi^\dagger + \Phi\right)\right]$$
$$- \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \frac{m_1^2}{2}\left(\frac{G}{G_0}\right)^2\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right]$$
$$+ \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu] + h_2\mathrm{Tr}[\Phi^\dagger L_\mu L^\mu\Phi + \Phi R_\mu R^\mu\Phi^\dagger]$$
$$+ 2h_3\mathrm{Tr}[\Phi R_\mu\Phi^\dagger L^\mu]$$

Symmetries:

$$U(N_f)_L \times U(N_f)_R$$

$$\Phi \longrightarrow \Phi' = U_L\Phi U_R^\dagger$$
$$L^\mu \longrightarrow L^{\mu\prime} = U_L L^\mu U_L^\dagger$$
$$R^\mu \longrightarrow R^{\mu\prime} = U_R R^\mu U_R^\dagger$$

$$x^\mu \longrightarrow x'^\mu = \lambda^{-1}x^\mu$$

[D. Parganlija, F. Giacosa and D. Rischke, Phys. Rev. D 82, 054024 (2010)]

[S. Janowski, D. Parganlija, F. Giacosa and D. Rischke, Phys. Rev. D 84, 054007 (2011)]

Scalars and pseudoscalars:

$$\Phi = (\sigma + i\eta_N)\,t^0 + (\vec{a}_0 + i\vec{\pi})\cdot\vec{t}$$

Vectors and axial-vectors:

$$L^\mu = (\omega^\mu + f_1^\mu)\,t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu)\cdot\vec{t}$$
$$R^\mu = (\omega^\mu - f_1^\mu)\,t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu)\cdot\vec{t}$$

$$N_f = 2$$

Covariant derivative: $D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu)$

The Effective Model

Shifting $\begin{matrix} \sigma \to \phi + \sigma \\ G \to G_0 + G \end{matrix}$ and expanding the potential around the minimum to 2nd order:

$$V(\sigma, G) = V(\phi, G_0) + \frac{1}{2} M_\sigma^2 \sigma^2 + \frac{1}{2} M_G^2 G^2 + 2\, m_0^2 \, \frac{\phi}{G_0} \sigma G$$

mixing parameter

The physical fields σ' and G' :

$$\begin{pmatrix} \sigma' \\ G' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma \\ G \end{pmatrix} \equiv \begin{pmatrix} f_0(600) \text{ or } f_0(1370) \\ f_0(1500) \text{ or } f_0(1710) \end{pmatrix}$$

Mixing angle:

$$\theta = \frac{1}{2} \arctan \left[-4 \, \frac{\phi}{G_0} \, \frac{m_0^2}{M_G^2 - M_\sigma^2} \right]$$

Physical masses:

$$M_{\sigma'}^2 = M_\sigma^2 \cos^2\theta + M_G^2 \sin^2\theta + 2\, m_0^2 \, \frac{\phi}{G_0} \sin(2\theta)$$

$$M_{G'}^2 = M_G^2 \cos^2\theta + M_\sigma^2 \sin^2\theta - 2\, m_0^2 \, \frac{\phi}{G_0} \sin(2\theta)$$

The Effective Model

Determination of the 10 free parameters of the effective model:

$$m_0,\ \lambda_1,\ \lambda_2,\ m_1,\ g_1,\ c,\ h,\ \tilde{h} = h_1 + h_2 + h_3,\ m_G,\ \Lambda = \sqrt{11}\, C^2/(2m_G)$$

We use:

$m_\pi = 139.57\ MeV$ $m_\rho = 775.49\ MeV$	**[K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010)]**
$m_{\eta_N} = 716\ MeV$	**[F. Giacosa, arXiv:0712.0186 [hep-ph]]**
$m_{a_1} = 1050\ MeV$	**[M. Urban, M. Buballa and J. Wambach, Nucl. Phys. A 697, 338 (2002)]**
$Z = 1.67 \pm 0.2$	**[D. Parganlija, F. Giacosa and D. Rischke, Phys. Rev. D 82, 054024 (2010)]**
$f_\pi = 92.4\ MeV$	**[PDG, *Review of Particle Physics*, Eur. Phys. J. C 3 (1998), C 15 (2000)]**

Fitting to experimental data yields the values of the remaining free parameters:

$$M_\sigma,\ m_G,\ m_1,\ C$$

Mass of the rho meson:

$$m_\rho^2 = m_1^2 + \phi^2\,(h_1 + h_2 + h_3)\,/2$$

Results

Favoured scenario: $\{\sigma', G'\} = \{f_0(1370),\ f_0(1500)\}$

Using the experimental data we obtain with the χ^2 method following parameters:

Quantity	Fit [MeV]	Experiment [MeV]
$M_{\sigma'}$	1191 ± 26	1200-1500
$M_{G'}$	1505 ± 6	1505 ± 6
$G' \to \pi\pi$	38 ± 5	38.04 ± 4.95
$G' \to \eta\eta$	5.3 ± 1.3	5.56 ± 1.34
$G' \to K\bar{K}$	9.3 ± 1.7	9.37 ± 1.69

Parameter	Fit [MeV]
C	699 ± 40
M_σ	1275 ± 30
m_G	1369 ± 26
m_1	809 ± 18

$\chi^2/\text{d.o.f.} = 0.29$

$\Rightarrow$ consistent with lattice QCD

The quarkonium-glueball mixing angle: $\theta = (29.7 \pm 3.6)^\circ \Rightarrow f_0(1500)\ [f_0(1370)]$ consists to 76% of glueball [quarkonium] and to 24% of quarkonium [glueball]

Consequences and predictions:

Decay Width	Our Value [MeV]	Experiment [MeV]
$G' \to \rho\rho \to 4\pi$	30	54.0 ± 7.1
$G' \to \eta\eta'$	0.6	2.1 ± 1.0
$\sigma' \to \pi\pi$	284 ± 43	-
$\sigma' \to \eta\eta$	72 ± 6	-
$\sigma' \to K\bar{K}$	4.6 ± 2.1	-
$\sigma' \to \rho\rho \to 4\pi$	0.09	-

Total decay width:

Experiment: $\Gamma_{f_0(1370)} = (200 - 500)\ MeV$

Our value: $\Gamma_{\sigma'} \simeq 360\ MeV$

Results

Not favoured scenario: $\{\sigma', G'\} = \{f_0(1370),\ f_0(1710)\}$

Using the experimental data we obtain with the χ^2 method following parameters:

Quantity	Fit [MeV]	Experiment [MeV]
$M_{\sigma'}$	1386 ± 134	1350 ± 150
$M_{G'}$	1720 ± 6	1720 ± 6
$G' \to \pi\pi$	29.7 ± 6.5	29.3 ± 6.5
$G' \to \eta\eta$	6.9 ± 5.8	34.3 ± 17.6
$G' \to K\bar{K}$	16 ± 14	71.4 ± 29.1
$\sigma' \to \pi\pi$	379 ± 147	250 ± 150

Parameter	Fit [MeV]
C	764 ± 256
M_σ	1516 ± 80
m_G	1531 ± 233
m_1	827 ± 36

$\chi^2/\text{d.o.f.} = 1.72$

The quarkonium-glueball mixing angle:

$\theta = (37.2 \pm 21.4)^\circ \Rightarrow$ unexpectedly large, reverse ordering is possible!

Large decay width, but experimentally not seen !

Decay Width	Our Value [MeV]	Experiment
$G' \to 4\pi$	115	-
$G' \to \eta\eta'$	16	-
$\sigma' \to \eta\eta$	153 ± 79	-
$\sigma' \to K\bar{K}$	$2.1^{+13.6}_{-2.1}$	-

Results

This scenarios also have been tested:

$$\{\sigma', G'\} = \{f_0(600),\ f_0(1500)\}$$

$$\{\sigma', G'\} = \{f_0(600),\ f_0(1710)\}$$

But in both scenarios the decay width is too small !

$$\Gamma_{\sigma' \to \pi\pi}$$

Conclusions and Outlook

- Global chiral invariant two-flavour linear sigma model with (axial-)vector mesons and dilaton d.o.f was presented
- The favoured scenario is, where the resonance $f_0(1500)$ is predominantly a glueball state and $f_0(1370)$ is predominantly a quark-antiquark state
- Other scenarios are not favoured
- The gluon condensate was determined and its value is in agreement with the lattice results
- Rho mass is mostly generated by dilaton

- Extension of the model by full inclusion of strangeness and inclusion of a nonet of tetraquark states as additional low-lying scalar states
- Investigation at nonzero temperature and density