Detailed study of the quark-antiquark flux tubes

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Motivation

- In previous works it was shown that fundamental flux tubes arise not only in meson systems but also in other systems
  - baryons
  - hybrids
  - tetraquarks [1]
  - pentaquarks [2]
- We have shown that higher representations flux tubes are just Casimir scaled fundamental ones[3]
- We will study
  - Fundamental flux tubes in a meson
  - F. flux tube interaction in a $QQ\overline{QQ}$ [4]

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[1] Phys.Rev. D84 (2011) 054508
[2] arXiv:1111.0342 [hep-lat]
[3] Phys.Lett. B710 (2012) 343-348
[4] arXiv:1204.5131 [hep-lat]

String model

- Nambu-Goto Action: $S = -\sigma \int d^2\Sigma$
- Arvis Potential:
  $V_n(R) = \sigma\sqrt{R^2 + \frac{2\pi}{\sigma}(n - \frac{D-2}{24})} = \sigma R + \frac{\pi}{R}(n - \frac{D-2}{24}) + \ldots$
- Width of the flux tube diverges logarithmicaly as $R \to \infty$ (roughening) [5]

$$w^2 \sim w_0^2 \log \frac{R}{R_0}$$

[5] M. Luscher (DESY), G. Munster, P. Weisz (Hamburg U.), Nucl.Phys. B180 (1981) 1

Chromo-fields

- The squared chromo-fields are computed by using:
  - $\langle E_i^2 \rangle = \langle P_{0i} \rangle - \frac{\langle WP_{\mathbf{0i}} \rangle}{\langle W \rangle}$
  - $\langle B_i^2 \rangle = \frac{\langle WP_{\mathbf{jk}} \rangle}{\langle W \rangle} - \langle P_{jk} \rangle$
- With $P_{\mu\nu} = 1 - \frac{1}{N_c} \mathrm{Tr}[U_\mu(\mathrm{s}) U_\nu(\mathrm{s} + \boldsymbol{\mu}) U_\mu^\dagger(\mathrm{s} + \boldsymbol{\nu}) U_\nu^\dagger(\mathrm{s})]$
- $\mathcal{L}$ and $\mathcal{H}$, defined the usual way:
  - $\mathcal{L} = \frac{1}{2}(E^2 - B^2)$
  - $\mathcal{H} = \frac{1}{2}(E^2 + B^2)$

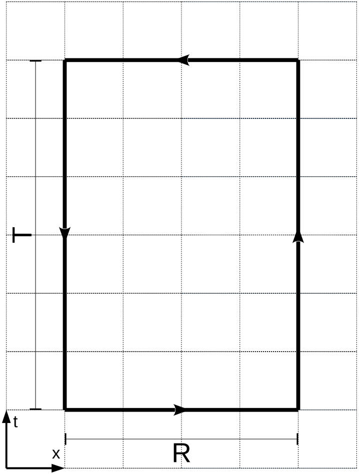
Wilson Loop

- Wilson Loop Operator

$$W(R,T) = \mathrm{Tr}[$$
$$U_\mu(0,0)\dots U_\mu(R-1,T)$$
$$U_4(R,0)\dots U_4(R,T-1)$$
$$U_\mu^\dagger(R-1,T)\dots U_\mu^\dagger(0,T)$$
$$U_4^\dagger(0,T-1)\dots U_4^\dagger(0,0)$$
$$]$$

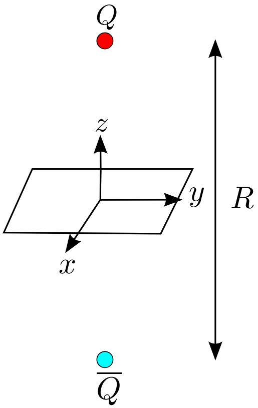
- Energy levels

$$\langle W(R,T)\rangle = \sum_n |C_n|^2 e^{-V_n T}$$



Chromo-fields

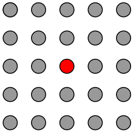
- We compute the chromo-fields in the mediator plane between the quark and the antiquark
- 1100 pure gauge 32^4 configurations with $\beta = 6.0$ are used



Extended Multihit

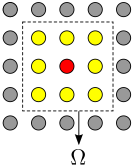
- In the multihit method we replace each temporal link by its thermal average, with it's first neighbours fixed

$$U_4 \rightarrow \overline{U}_4 = \frac{\int dU_4 \; U_4 \; e^{\beta \mathrm{Tr}[U_4 F^\dagger]}}{\int dU_4 \; e^{\beta \mathrm{Tr}[U_4 F^\dagger]}}$$



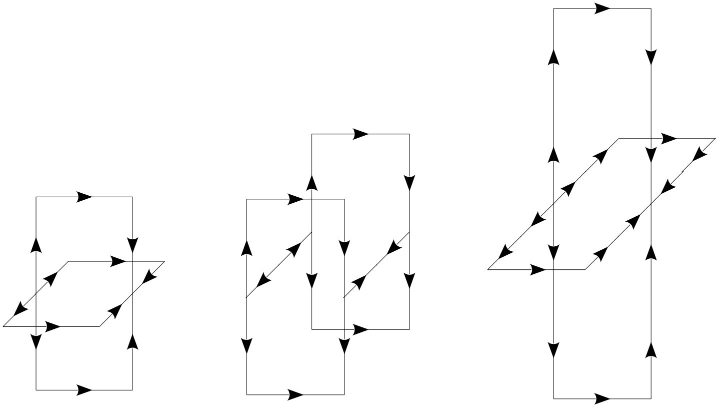
- We generalize this method by instead replacing each link by it's thermal average with the first N neighbhours fixed

$$U_4 \rightarrow \overline{U}_4 = \frac{\int [\mathcal{D}U]_\Omega \; U_4 \; e^{\beta \sum_{\mu s} \mathrm{Tr}[U_\mu(s) F^\dagger_\mu(s)]}}{\int [\mathcal{D}U]_\Omega \; e^{\beta \sum_{\mu s} \mathrm{Tr}[U_\mu(s) F^\dagger_\mu]}}$$



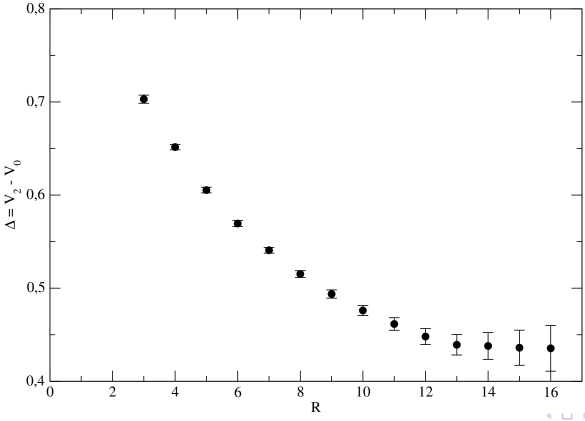
- We use extended spatial smearing:

$$U_i \to \mathcal{P}_{SU(3)}\left[U_i + w_1 \sum_j S^1_{ij} + w_2 \sum_j S^2_{ij} + w_3 \sum_j S^3_{ij}\right]$$

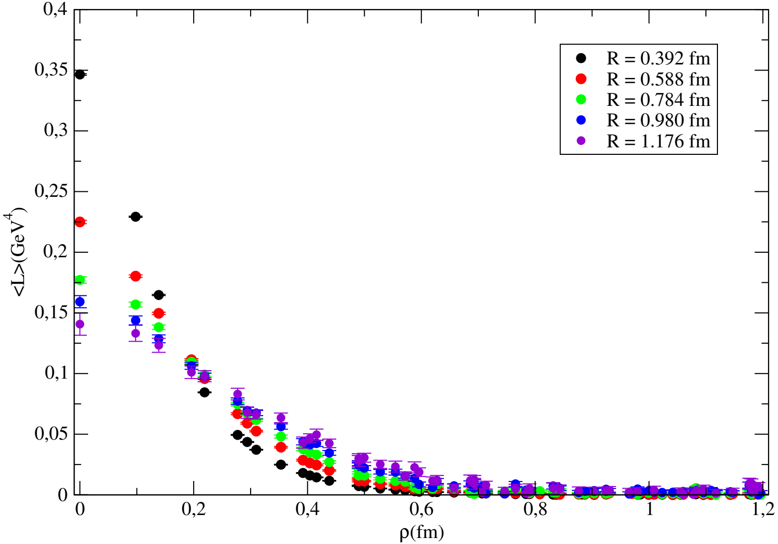


Computing Δ

- The plaquette correlators were fitted to the expression $a + b\, e^{-\Delta T}$, where $\Delta = V_1 - V_0$
- To compute $\Delta = V_1 - V_0$, we use a variational basis of different levels of APE smearing: $\langle W_{ij} \rangle = \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle$.

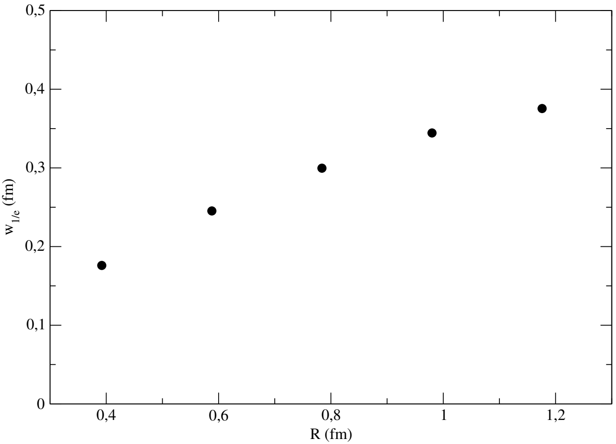


Lagrangian Density



Tube Flux Width

- We estimate the width by considering the distance where $\mathcal{L}$ falls to $1/e$ of the central value



Conclusion

- The Flux tube roughening has been observed from $R \sim 0.4\,fm$ to $R \sim 1.2\,fm$.
- We are working on pure gauge 32^4 configurations with $\beta = 5.8$. This will allow us to test larger distances.

$QQ\overline{QQ}$ system

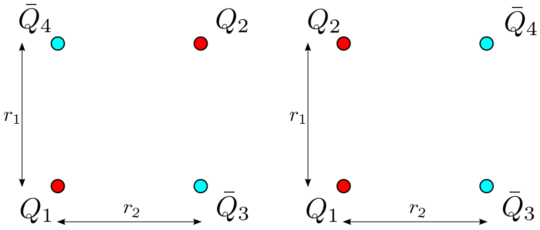
- Consider a system of two quarks Q_1 Q_2 and two antiquarks $\overline{Q_3}$ $\overline{Q_4}$
  - Meson-Meson Scattering
  - Tetraquarks

$QQ\overline{QQ}$ system

- Two linearly independent color singlets are possible
  - Two meson states:
    - $|I\rangle = \frac{1}{3}|Q_i Q_j \overline{Q}_i \overline{Q}_j\rangle$
    - $|II\rangle = \frac{1}{3}|Q_i Q_j \overline{Q}_j \overline{Q}_i\rangle$
  - Antisymmetric and symmetric color states: Consider a system of two quarks Q_1 Q_2 and two antiquarks $\overline{Q}_3$ $\overline{Q}_4$
    - $|A\rangle = \frac{\sqrt{3}}{2}(|I\rangle - |II\rangle)$
    - $|S\rangle = \sqrt{\frac{3}{8}}(|I\rangle + |II\rangle)$
- Ground state can be $|I\rangle$, $|II\rangle$ or $|A\rangle$
- Static Potential: $V_{FF} = \min(V_I, V_{II}, V_T)$

Geometries

- We use two different geometries: Parallel and anti-parallel



- 1121 Quenched $24^3 \times 48$ lattice configurations with $\beta = 6.2$ used
- APE Smearing applied in the spatial links
- Hypercubic blocking applied in the temporal links

- To obtain not only the ground state, but also the first excited state, we use a variational basis (with two operators)

$$\langle W_{ij} \rangle = \langle \mathcal{O}_i^\dagger(0)\mathcal{O}_j(t)\rangle$$

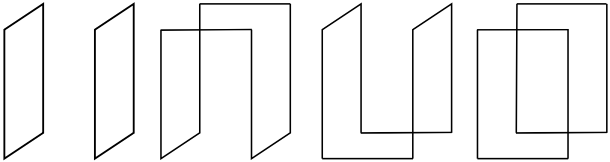
- To find the states, we solve the generalized eigensystem

$$\langle W_{ij}(t)\rangle \, c_n^j(t) = w_n \langle W_{ij}(0)\rangle \, c_n^j(t)$$

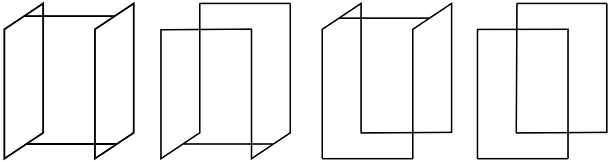
- The fields are calculated by $\langle P\rangle_n = \frac{\langle W_n P\rangle}{\langle W_n\rangle} - \langle P\rangle$ with $W_n = c_n^i W_{ij} c_n^j$

Variational Method

- This gives the Wilson Loops:
  - For the mesons-mesons transition:

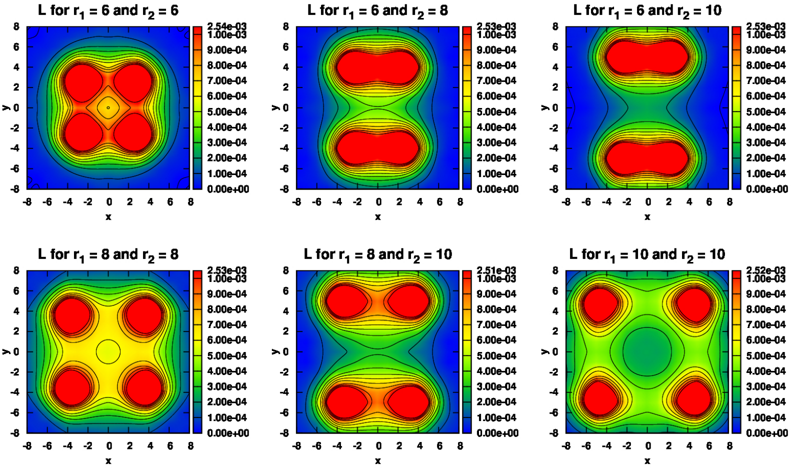


  - For the tetraquark-mesons transition:



Antiparallel geometry I

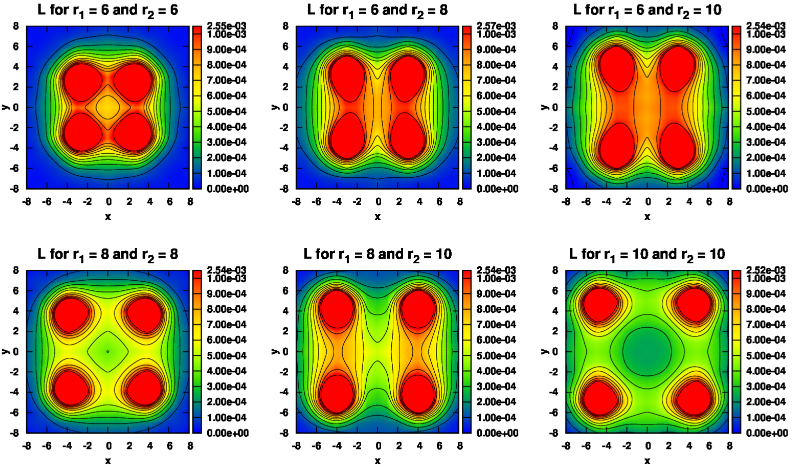
- $\mathcal{L}$ for the ground state



- For $r_1 = r_2$ the ground state is color symmetric

Antiparallel geometry II

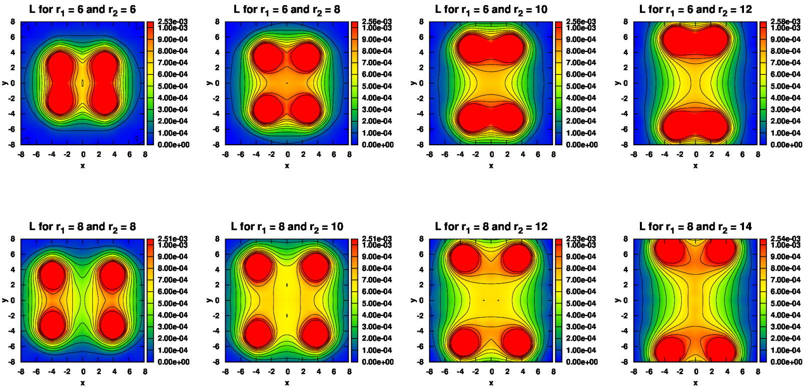
- $\mathcal{L}$ for the first excited state



- For $r_1 = r_2$ the first excited state is color anti-symmetric

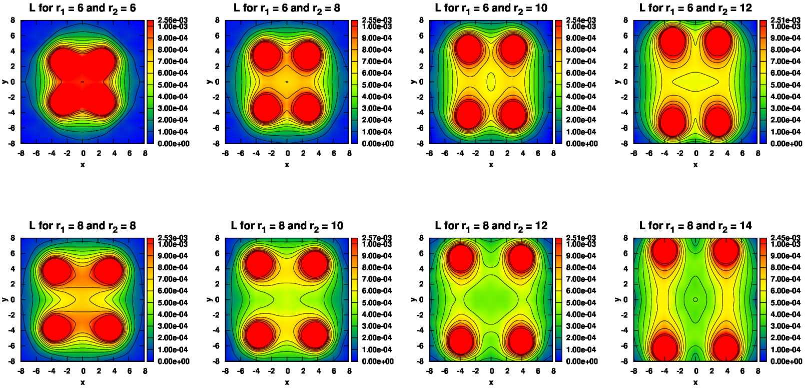
Parallel Geometry I

- $\mathcal{L}$ for the ground state



Parallel Geometry II

- $\mathcal{L}$ for the first excited state



Discussion/Conclusion

- The results for the ground state are the expected ones, and are consistent with the flip-flop ground state potential

$$V_{FF} = \min(V_I, V_{II}, V_T)$$

- We get a two meson state and a tetraquark (antisymmetric) color state where they are expected to be the ground states
- The first excited state are not so readily explainable.

Discussion/Conclusion

- Tube flux excitations effects should be negligible
- The First excited state should be orthogonal to the ground one
  - $|\overline{I}\rangle = -\frac{1}{\sqrt{3}}|S\rangle + \sqrt{\frac{2}{3}}|A\rangle$, orthogonal to $|I\rangle$
  - $|\overline{II}\rangle = -\frac{1}{\sqrt{3}}|S\rangle + \sqrt{\frac{2}{3}}|A\rangle$ orthogonal to $|II\rangle$
  - $|S\rangle$ orthogonal to $|A\rangle$

Discussion/Conclusion

- Comparing with Casimir scaling coefficients

$\vert\Psi\rangle$	C_{12}	C_{13}	C_{14}
$\vert\bar{I}\rangle$	$1/4$	$-1/8$	$7/8$
$\vert\overline{II}\rangle$	$1/4$	$7/8$	$-1/8$
$\vert S\rangle$	$-1/4$	$5/8$	$5/8$

- The results agree at least qualitatively