ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

FONDAZIONE BRUNO KESSLER

# Recent results on the IR sector of QCD


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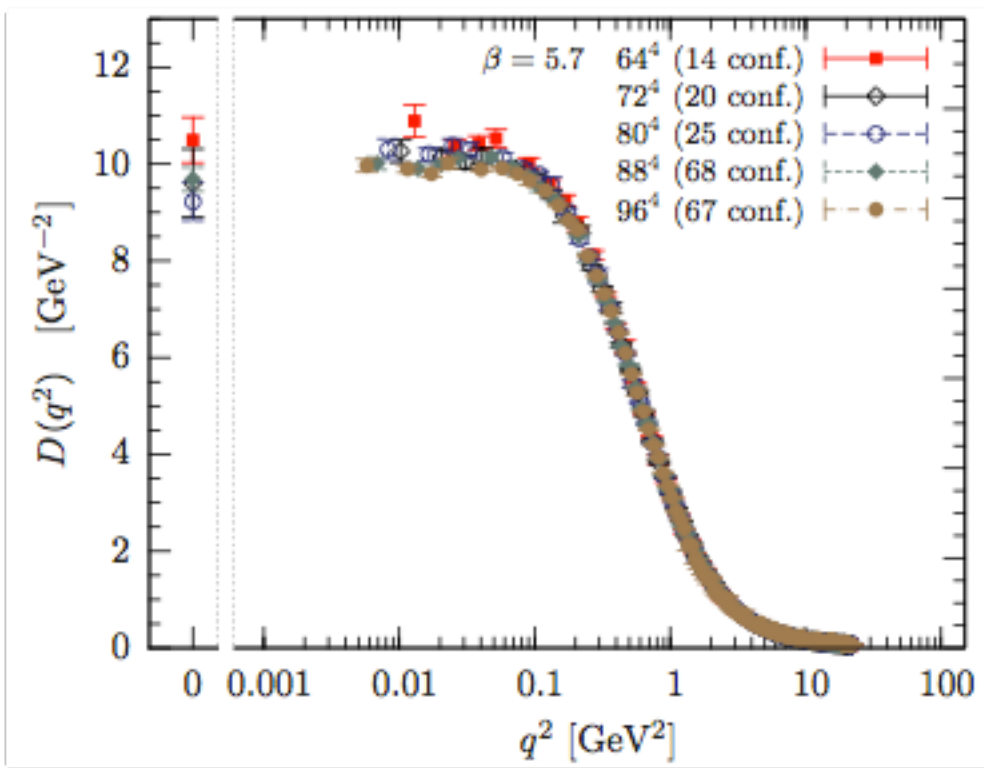
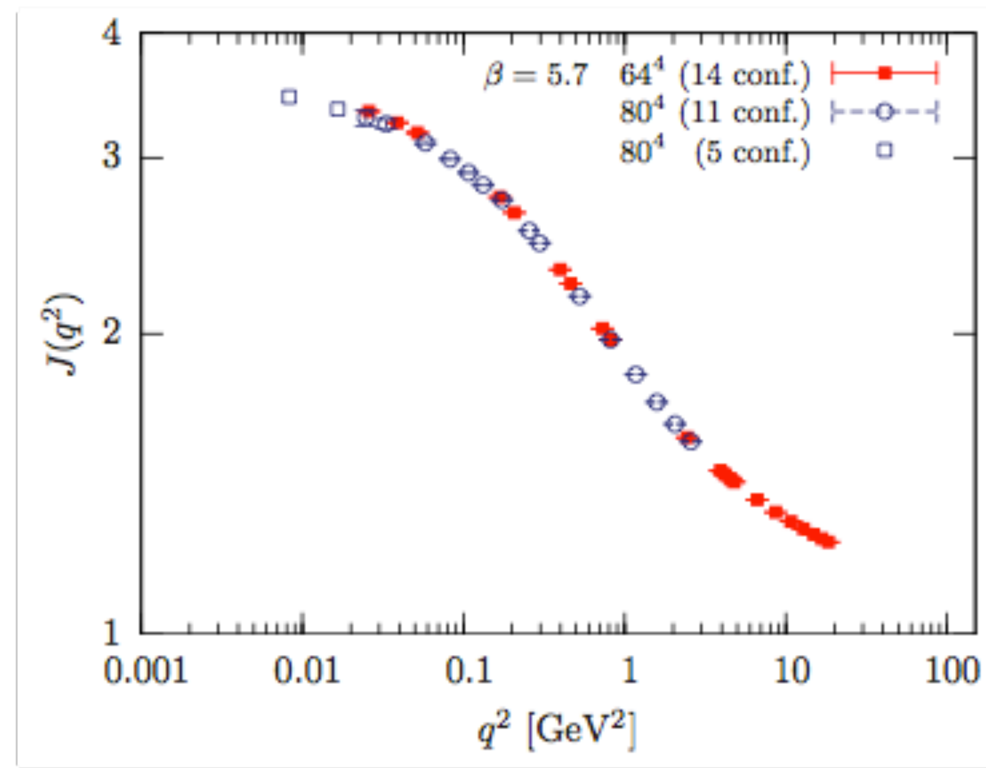
eQCD - Peniche

06|05|12-12|05|12



$\beta = 5.7$
$64^4$ (14 conf.)
$72^4$ (20 conf.)
$80^4$ (25 conf.)
$88^4$ (68 conf.)
$96^4$ (67 conf.)
$D(q^2)$ [GeV$^{-2}$]
$q^2$ [GeV$^2$]

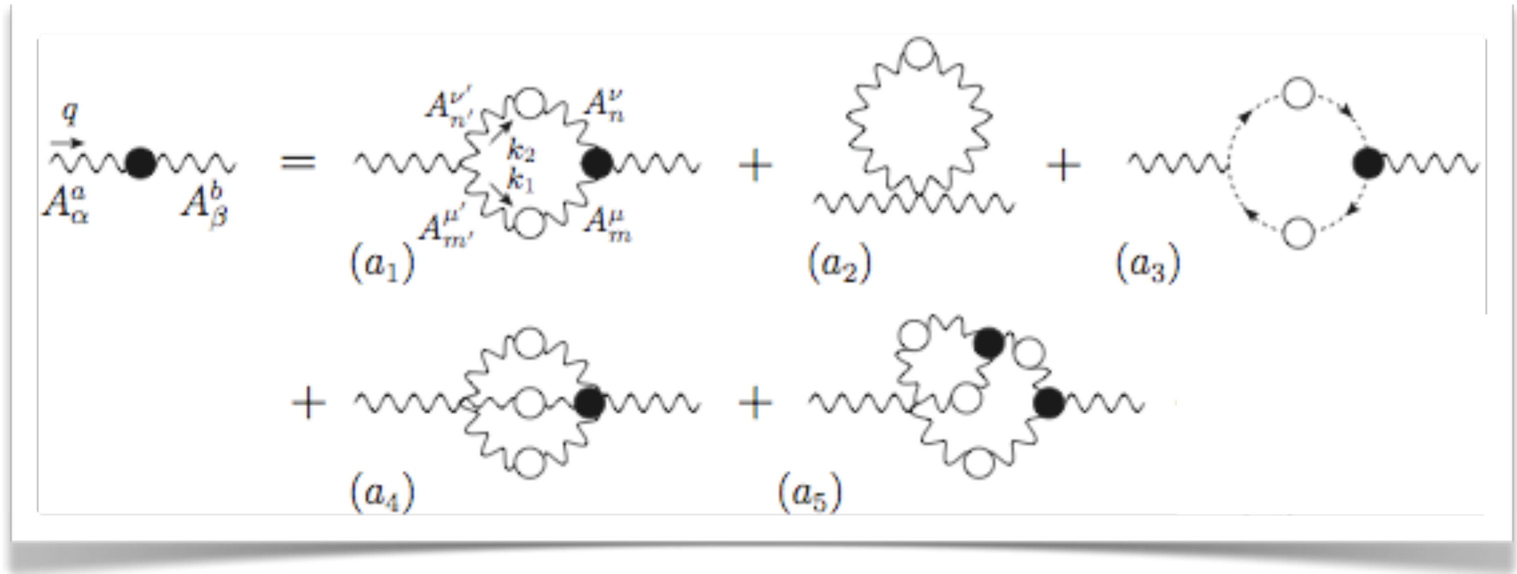
$\beta = 5.7$
$64^4$ (14 conf.)
$80^4$ (11 conf.)
$80^4$ (5 conf.)
$J(q^2)$
$q^2$ [GeV$^2$]

# PT-BFM: a primer

# (problems with) conventional formalism

- Schwinger-Dyson eqs: way of **treating purely non-perturbative phenomena** (e.g., mass gap generation)
  - Infinite system of **coupled non-linear integral equations**
    - captures the **full quantum e.o.m.**
    - expansion about the free-field vev, but finally no reference to it
  - Require a **truncation scheme**
    - gauge and renormalization group invariance should be respected

- **Gluon propagator**
  - BRST demands $q^\alpha \sum_{i=1}^{5} (a_i)_{\alpha\beta} = 0$
    - very difficult diagrammatic verification
    - **cannot truncate in any obvious way**




- Retaining $(a_1)$ and $(a_2)$ only is not correct even at one loop
  - $q^\alpha \Pi_{\alpha\beta}(q)|_{(a_1)+(a_2)} \neq 0$
- Adding $(a_3)$ is not sufficient for a full analysis; beyond one loop
  - $q^\alpha \Pi_{\alpha\beta}(q)|_{(a_1)+(a_2)+(a_3)} \neq 0$

# (problems with) conventional formalism

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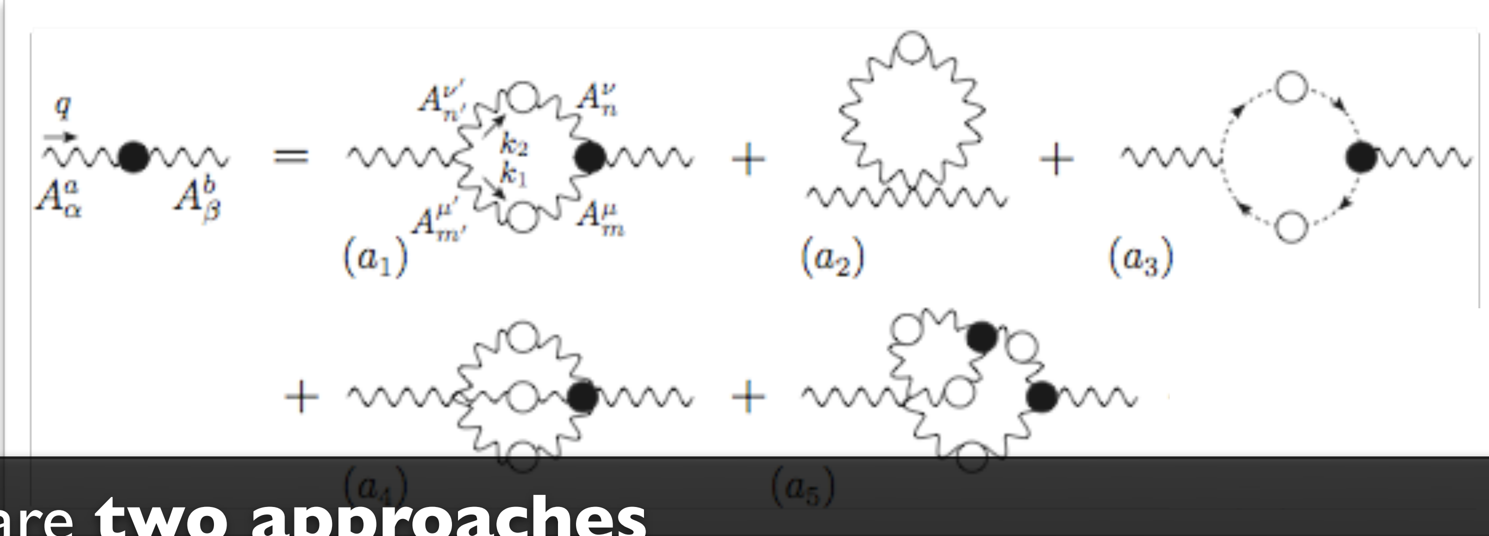



There are **two approaches**

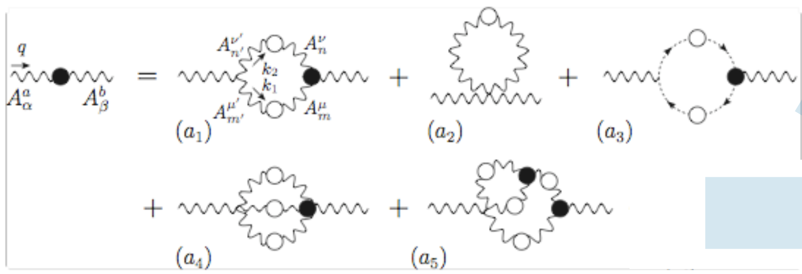
conventional ("plug & pray")
- **Check transversality of the answer** at the end of the calculation
- **Approximate gauge-invariance** (might be even lost in intermediate steps)
- **Results** possibly **plagued by gauge artifacts**

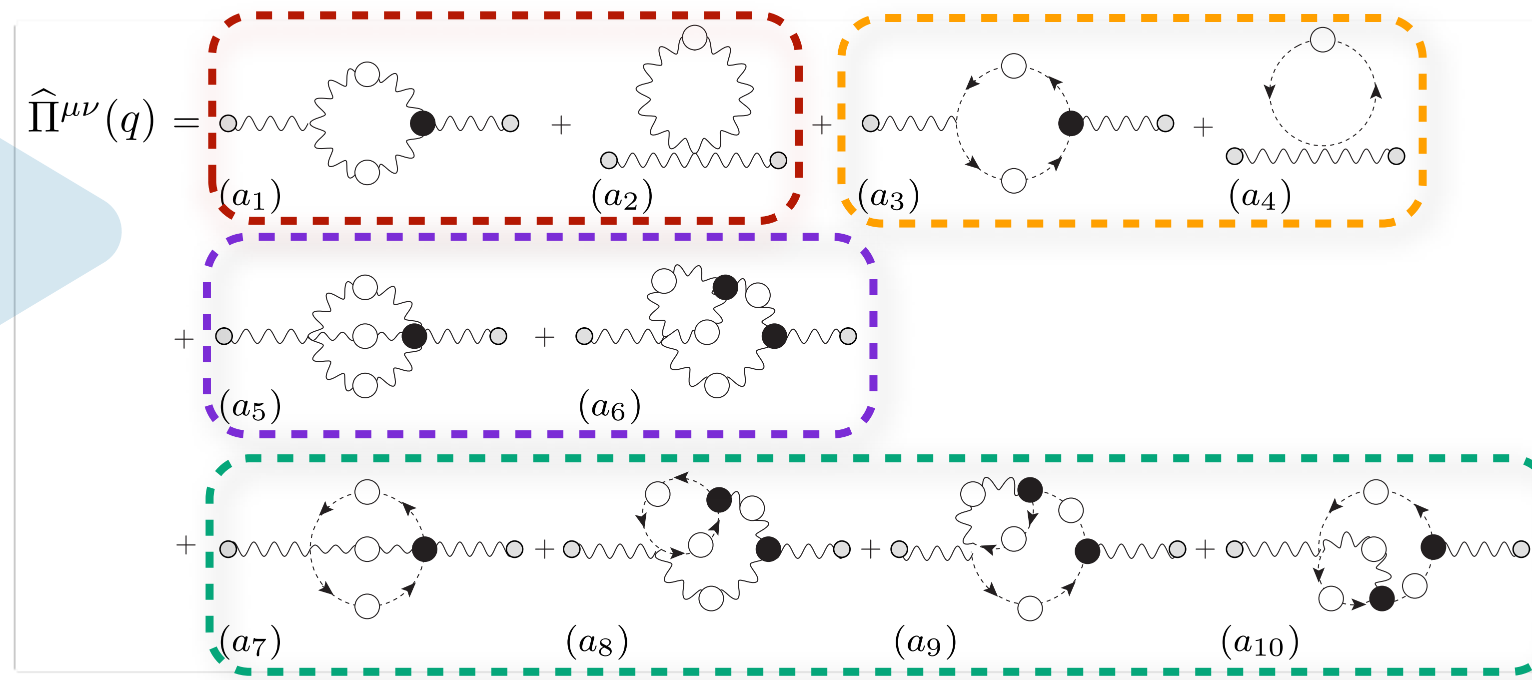
PT-BFM scheme
- **Resum the equation** into a new one with better truncation properties
- **Gauge invariance exactly preserved** at each step
- **Results** are **fully gauge invariance**

D. B. & J. Papavassiliou, Phys. Rev. D77, 061702(R) (2008)
D. B. & J. Papavassiliou, JHEP 11, 063 (2008)

# PT-BFM resummed Schwinger-Dyson series

- Apply the pinch technique to the Schwinger-Dyson equation of the gluon propagator




- graphs made out of **new vertices**, (inside **conventional props**)
- new vertices corresponds to **BFM vertices**
  - **external gluons** dynamically converted into **background gluons**

- New Schwinger-Dyson equation has a **special structure**
  - **Subgroups** (one-/two-loop dressed gluon/ghost) are **individually transverse**

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**Problem**

Not a genuine Schwinger-Dyson equation (**mixes pinch technique** and **conventional** propagators)

- Express the **Schwinger-Dyson eq** in terms of a background-quantum identity
  - $$\Delta^{-1}(q^2)[1+G(q^2)]^2 P_{\mu\nu}(q) = q^2 P_{\mu\nu}(q) \sum_{i=0}^{10} (a_i)_{\mu\nu}$$
    $$\widehat{\Delta}(q^2) = \left[1+G(q^2)\right]^{-2} \Delta(q^2)$$
  - In $4d$ the function $G$ is directly related to the inverse of the ghost dressing function

$$F^{-1}(q^2) \approx 1 + G(q^2)$$

# BQQ vertex and Schwinger mechanism

J. S. Schwinger, Phys. Rev. 125, 397 (1962)
J. S. Schwinger, Phys. Rev. 128, 2425 (1962)

**Dyson resum**

$$\Delta(q^2) = \frac{1}{q^2\left[1+\Pi(q^2)\right]}$$

**Idea**

**If** $\Pi(q^2)$ **has a pole at** $q^2 = 0$ the vector meson is **massive** even though it is massless in the absence of interactions

- Requires **massless**, **longitudinally coupled** Goldstone like **poles** $1/q^2$
- **Occur dynamically** (even in the **absence** of canonical **scalar fields**) as **composite excitations** in a **strongly coupled** gauge theory

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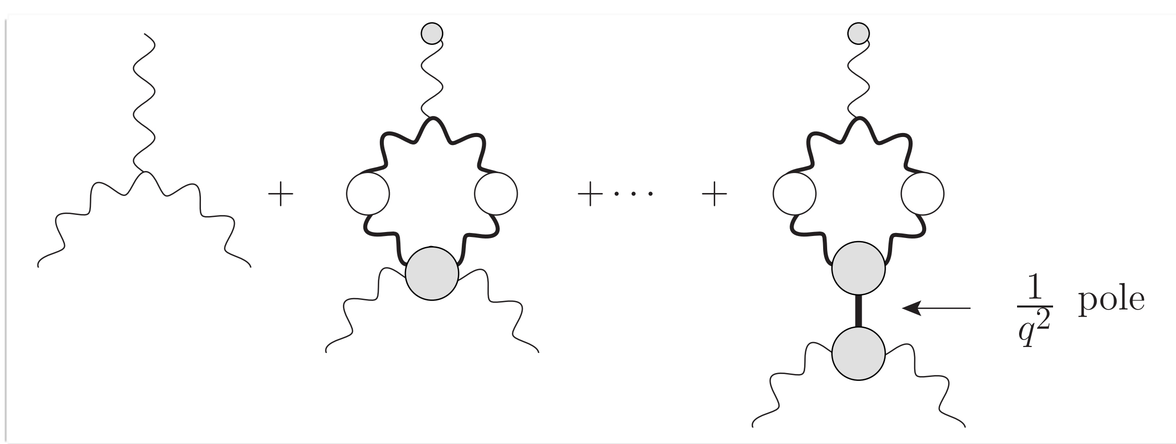
R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)
J. M. Cornwall and R. E. Norton, Phys. Rev. D8, 3338 (1973)
E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)

Dynamics enters through the **three-gluon vertex**

- **Longitudinally coupled massless poles**
  - **Not** a **kinematic singularity**, rather **bound states poles** non-perturbatively produced
  - **Do not appear** in the $S$ matrix of the theory ("eaten-up" by the gluons to become massive)
- **Instrumental for ensuring that**

$$\Delta^{-1}(0) > 0$$



A. Aguilar, D. B. & J. Papavassiliou, Phys. Rev. D84, 085026 (2011)

# PT-BFM
# DMG generation

- How does dynamical gluon mass generation work in practice?
  - **Assumes** the formation of a **longitudinally coupled massless poles** that...
  - ...will **modify the vertex** of the theory...
  - ...which will lead to **massive type solutions** of the corresponding SDE

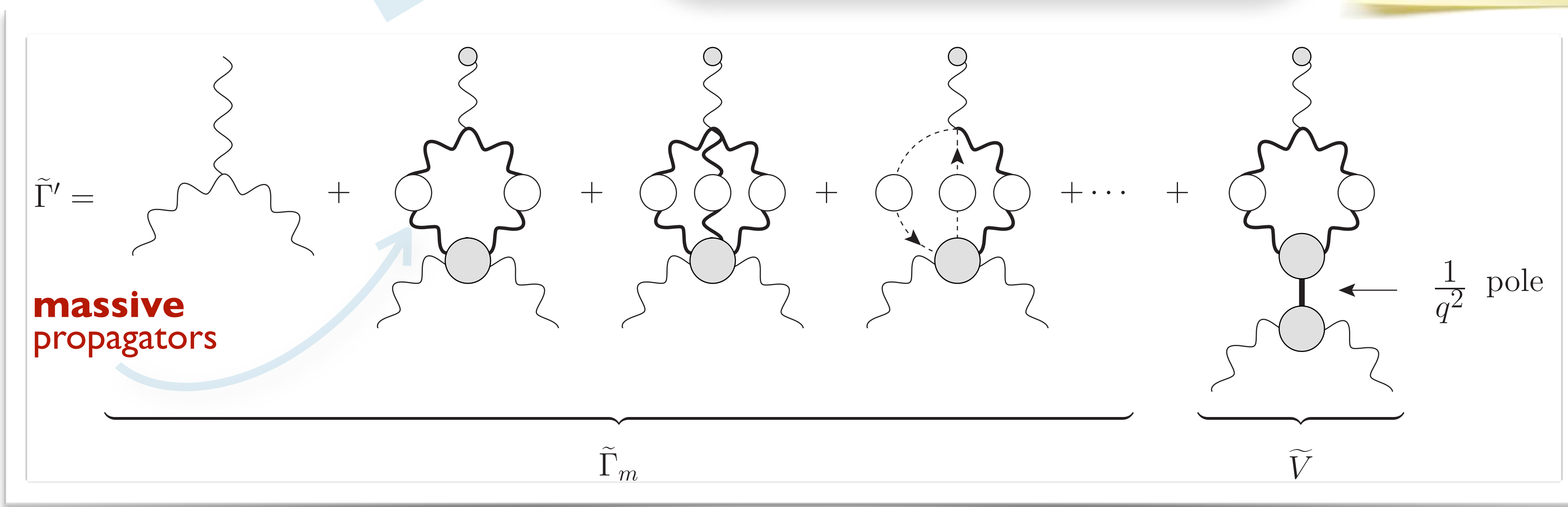
- Two levels
  - **Kinematical**

$$\Delta^{-1}(q^2) = q^2 J(q^2) \longrightarrow \Delta_m^{-1}(q^2) = q^2 J_m(q^2) - m^2(q^2)$$
$$J(q^2) \sim \ln q^2 \longrightarrow J_m(q^2) \sim \ln(q^2 + m^2)$$
$$q^2 J_m(q^2) \xrightarrow{q^2 \to 0} 0$$

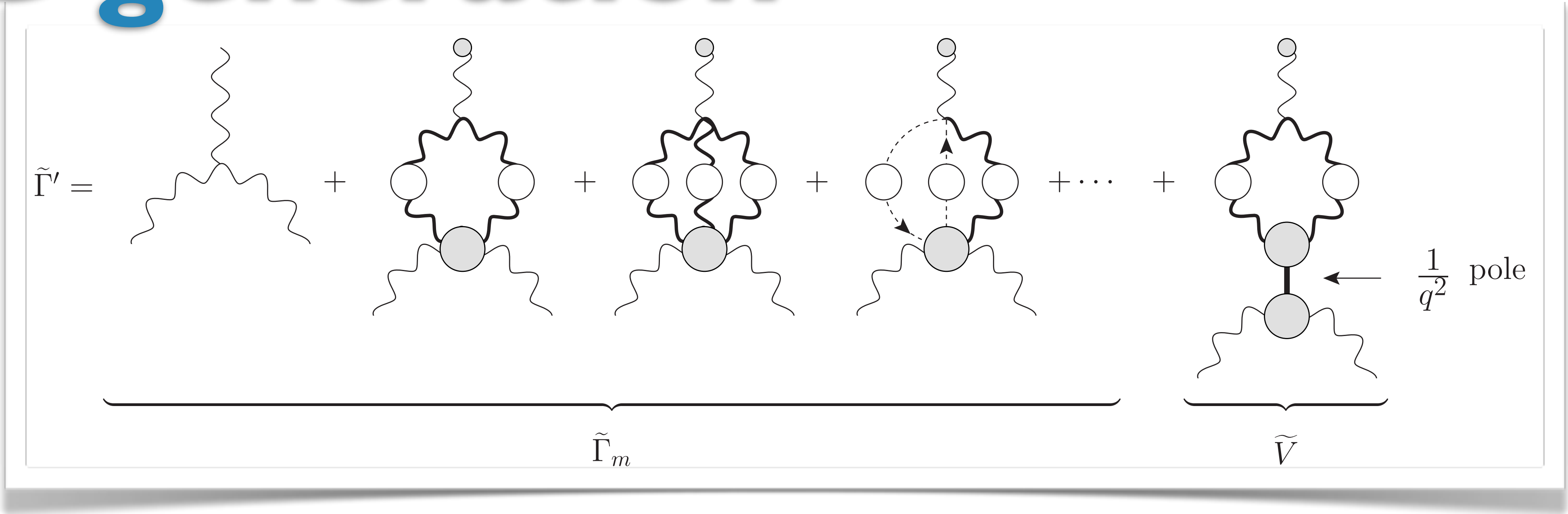
  - **Dynamical**

$$\widetilde{\Gamma} \longrightarrow \widetilde{\Gamma}' = \widetilde{\Gamma}_m + \widetilde{V}$$

$V$ is **totally longitudinally coupled** ($PPPV = 0$)



# PT-BFM
# DMG generation




- $\widetilde{\Gamma}_m$ satisfies the **same identities** as $\widetilde{\Gamma}$ with the replacement $J \longrightarrow J_m$

$$q_\alpha \widetilde{\Gamma}_m^{\alpha\mu\nu}(q,r,p) = p^2 J_m(p^2) P^{\mu\nu}(p) - r^2 J_m(r^2) P^{\mu\nu}(r)$$

- $\widetilde{\Gamma}'$ satisfies the **same identities** as $\widetilde{\Gamma}$ with the replacement $\Delta \longrightarrow \Delta_m$

  - $$q^\alpha \widetilde{\Gamma}'_{\alpha\mu\nu}(q,r,p) = p^2[J_m(p^2)P_{\mu\nu}(p) - m^2(p^2)] - [r^2 J_m(r^2) - m^2(r^2)]P_{\mu\nu}(r)$$

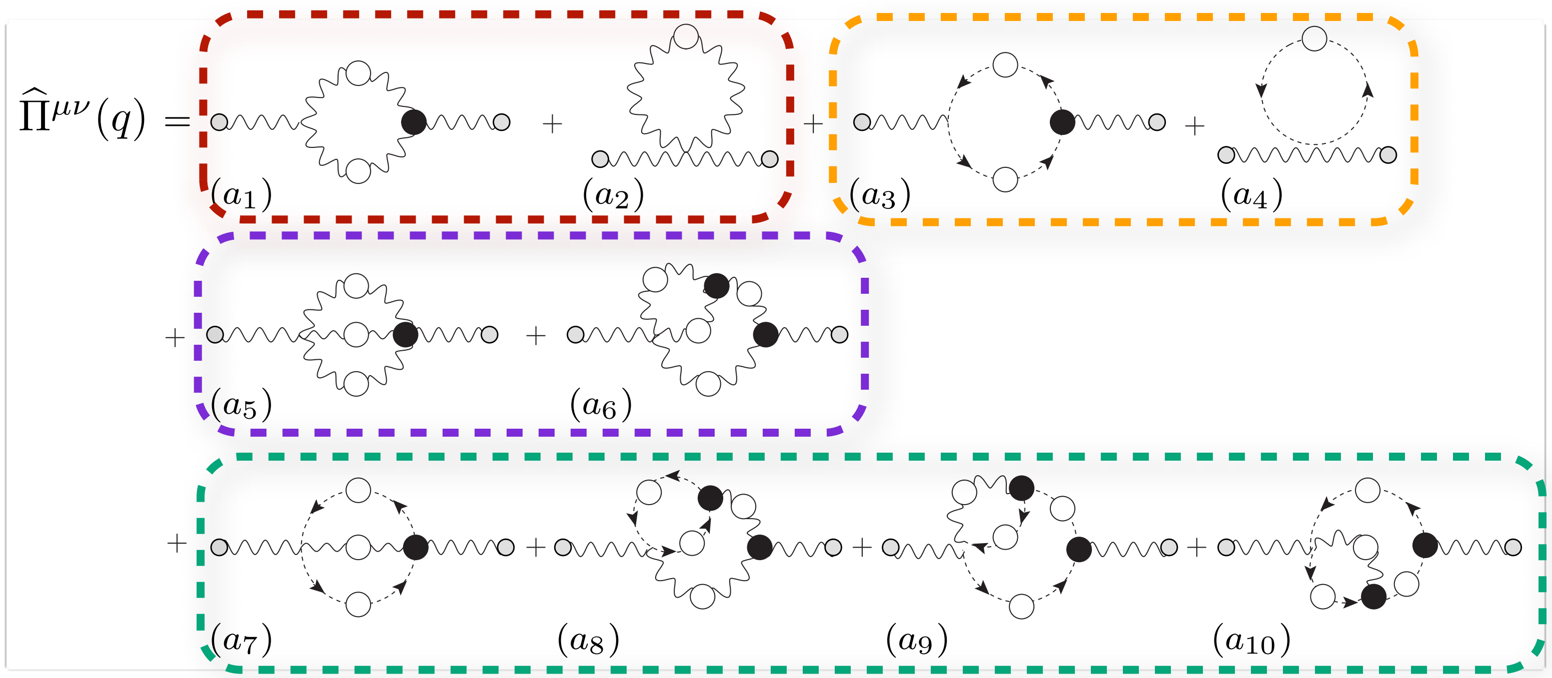
---

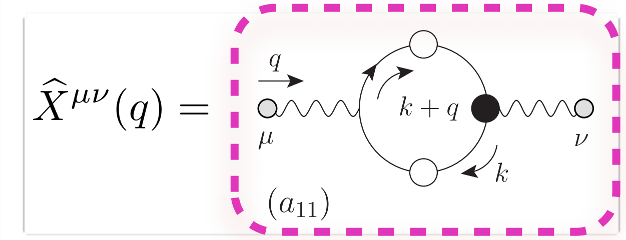
- The $V$ and $\widetilde{V}$ vertices can be **explicitly determined** by **exploiting** the **total longitudinality** condition $PPPV = PPP\widetilde{V} = 0$ **and** the **STIs/WI** they satisfy
  - **Not needed** (in the Landau gauge) at the **one-loop** dressed level but **fundamental** at the **two-loop dressed** level

# Unquenching the SDEs

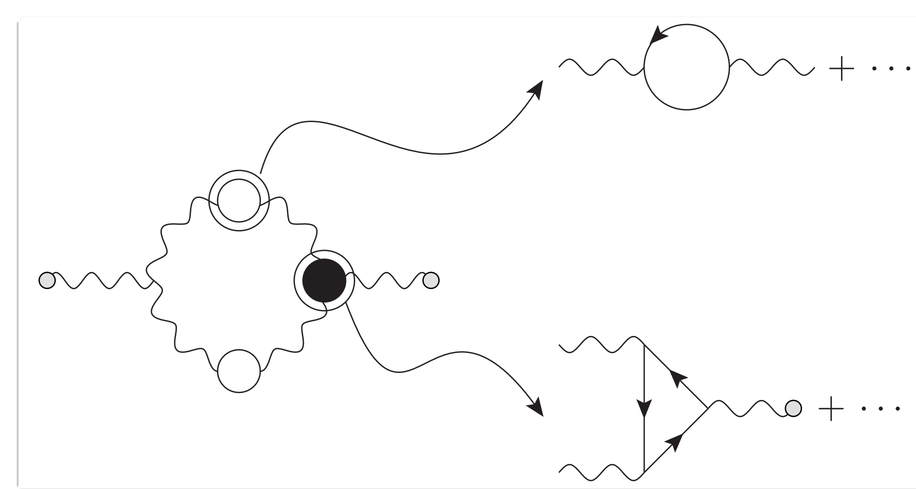
A. Aguilar, D. B. & J. Papavassiliou, 1204.3868 [hep-ph]

# adding quarks to the SDE




- **Gauge invariant subset** (also in in the conventional formulation!)
- **PT-BFM vertex** satisfies a **linear WI**
  - **Many Abelian Ansatze** available in the market (*e.g.*, Ball-Chiu, Curtis-Pennington)




## Highly non-linear propagation of the effect

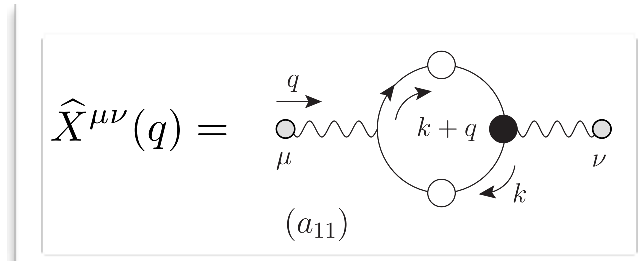
- The presence of **quarks** will also **affect** the **original** quenched **diagrams**
- Operating assumption: **non-linear effects** are **suppressed** wrt diagram $(a_{11})$

## Adding dynamical quarks gives

$$\Delta_Q^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\widehat{\Pi}^{Q}_{\mu\nu}(q) + i\widehat{X}_{\mu\nu}(q)}{[1+G_Q(q^2)]^2}$$

Suffix $Q$ indicates the effects of quarks on quenched quantities

# adding quarks to the SDE




- Non-perturbative calculation of the term

$$\widehat{X}^{\mu\nu}(q) = -g^2 d_f \int_k \mathrm{Tr}\left[\gamma^\mu S(k) \widehat{\Gamma}^\nu(k+q,-k,-q) S(k+q)\right]$$

$$S^{-1}(k) = -i\left[A(k^2)\not{k} - B(k^2)\right] = -iA(k^2)\left[\not{k} - \mathcal{M}(k^2)\right]$$

$A$ and $B$ are **obtained** from **solving** the **quark gap equation**

$$q^\nu \widehat{\Gamma}_\nu(k+q,-k,q) = S^{-1}(k+q) - S^{-1}(-k)$$

**valid** for both BC and CP vertices

- In the $q \to 0$ limit

$$\widehat{X}(0) = -\frac{2g^2}{d-1}\int_k \frac{1}{A^2(k^2-\mathcal{M}^2)^2}\left\{A\left[(2-d)k^2 + d\mathcal{M}^2\right] + 2A'k^2\left(k^2+\mathcal{M}^2\right) - 4k^2 B'\mathcal{M}\right\}$$

- Use the seagull identity

$$\int_k k^2 \frac{\partial f(k^2)}{\partial k^2} + \frac{d}{2}\int_k \Delta(k) = 0$$

**keeps** the **gluon massless** in the **absence** of a **DMG mechanism**

$f(k^2) = \left[A(k^2)\left(k^2 - \mathcal{M}^2\right)\right]^{-1}$

$$\widehat{X}(0) = 0$$

The **quark loop does not contribute** to the **gluon mass!**

A. Aguilar, D. B. & J. Papavassiliou, 1204.3868 [hep-ph]

# adding quarks to the SDE

- So what happens when quarks are present?

$$\Delta^{-1}(q^2) = q^2 J(q^2) - m^2(q^2) \longrightarrow \Delta_Q^{-1}(q^2) = q^2 J_Q(q^2) - m_Q^2(q^2)$$

- $\widehat{X}(q^2)$ does not affect the value of the mass and therefore contributes to $J_Q(q^2)$

$$q^2 J_Q(q^2) = q^2 J(q^2) + i\frac{\widehat{X}(q^2)}{1+G(q^2)}$$

- However

$$\lambda^2 = m_Q^2(0) - m^2(0) \neq 0$$

First-principle **determination** would **require** the **knowledge** of the **mass equation**

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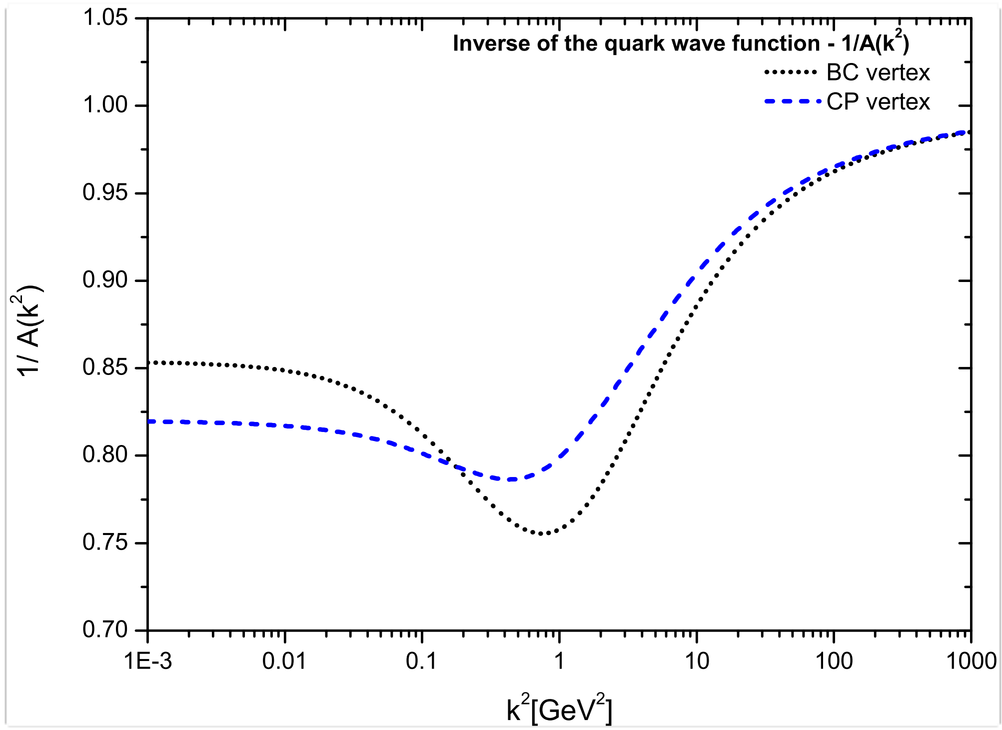
- Put everything together to write

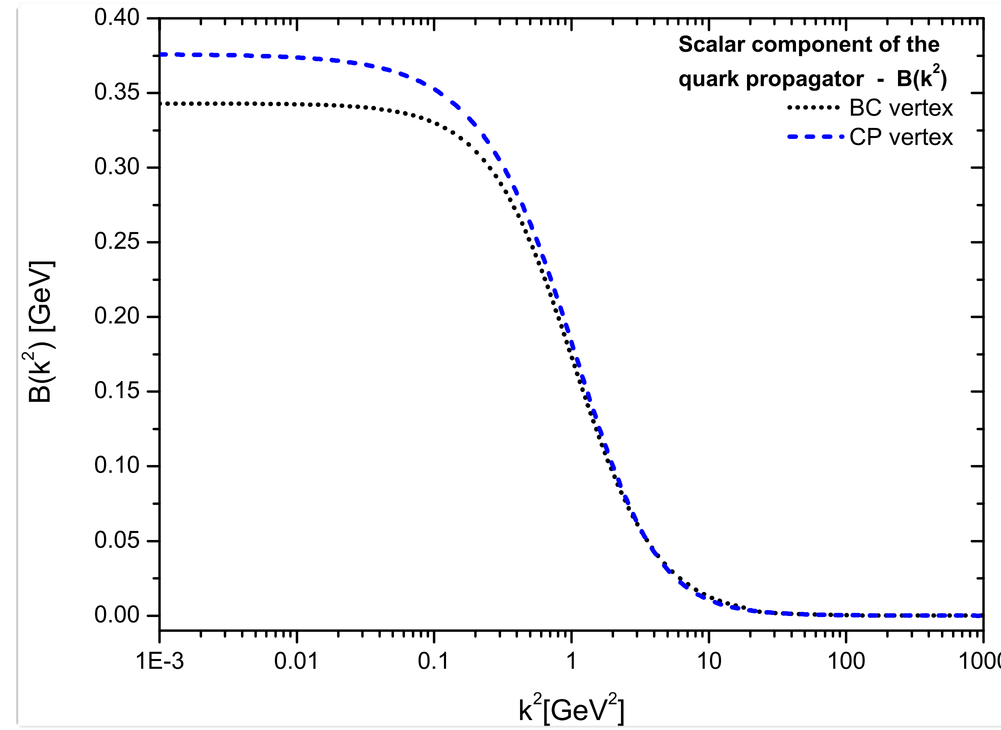
$$\Delta_Q(q^2) \simeq \frac{\Delta(q^2)}{1+\left\{i\widehat{X}(q^2)\left[1+G(q^2)\right]^{-2} - \lambda^2\right\}\Delta(q^2)}$$

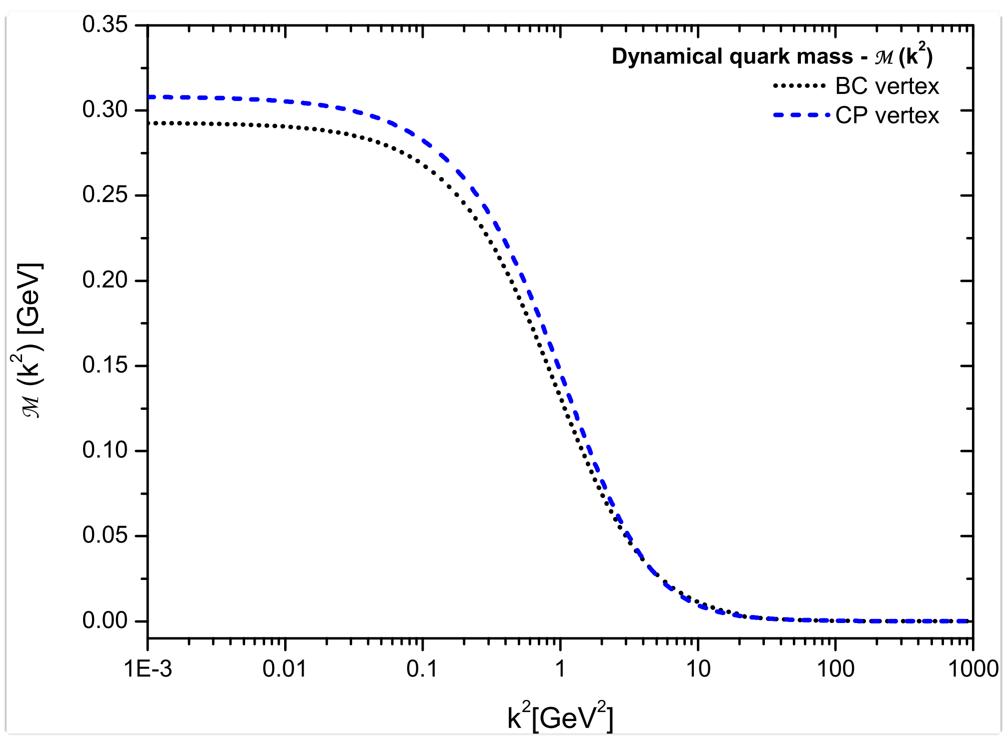
Use for $\Delta$ and $F$,$G$ the **quenched lattice data**

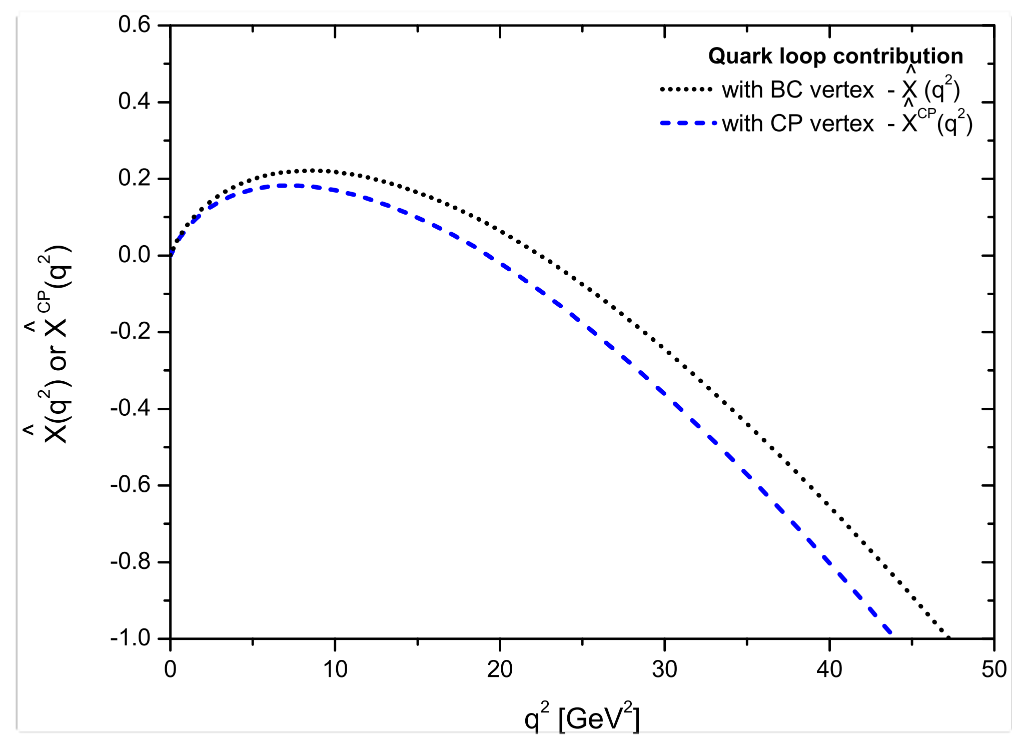
# adding quarks to the SDE

A. Aguilar, D. B. & J. Papavassiliou, 1204.3868 [hep-ph]



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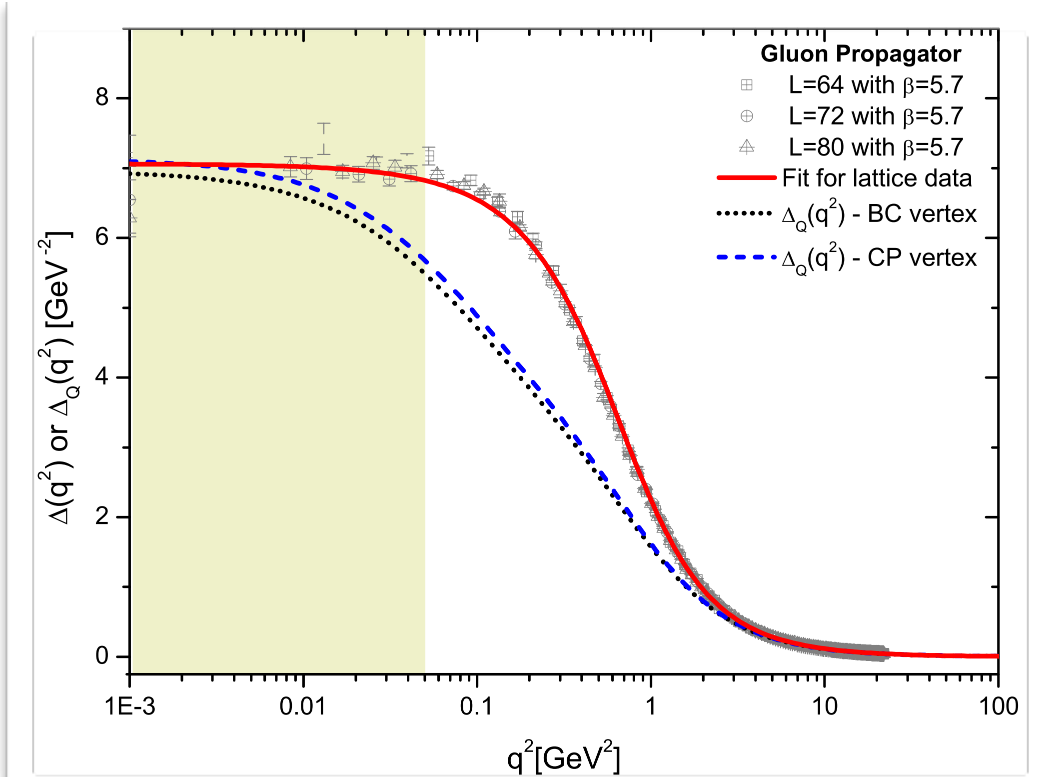
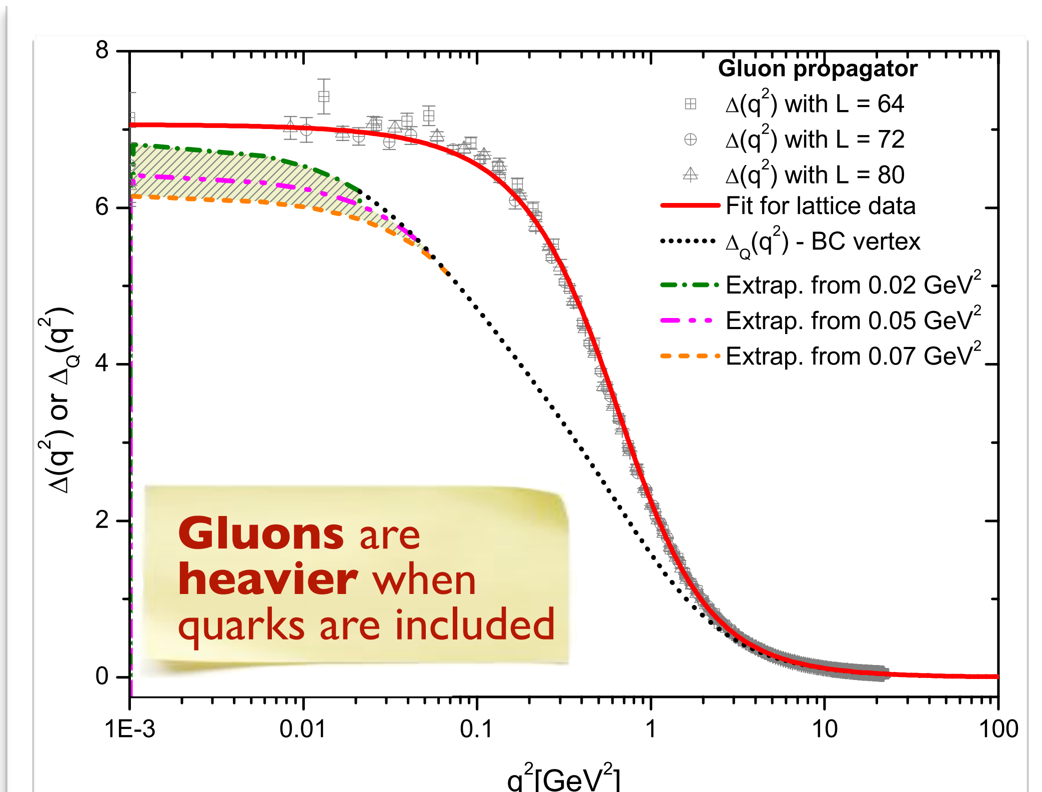
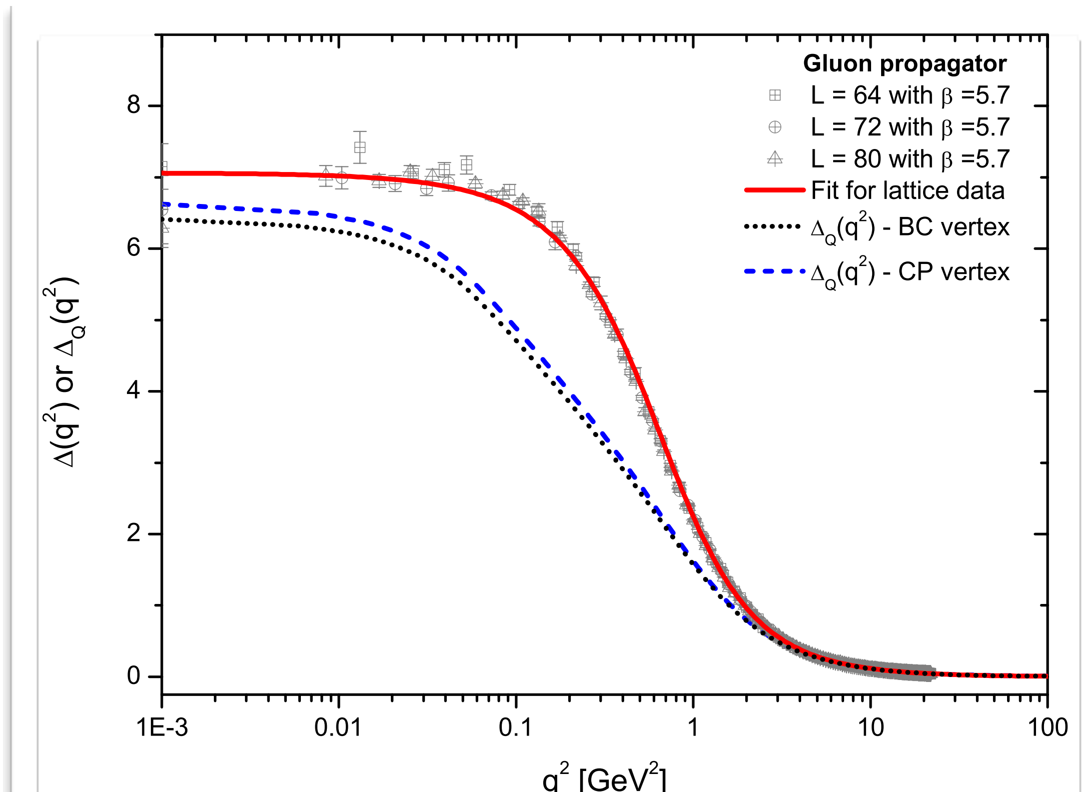



- **Minor difference** (~3%) between the BC and CP vertices
- **Suppression** of the "swelling" in the **intermediate momenta region**
- As anticipated one has the **same IR fixed** point as in the **quenched case**
  - **Use extrapolation** (cubic B-spline method) to account for our inability of determining $\lambda$




**Gluons** are **heavier** when quarks are included



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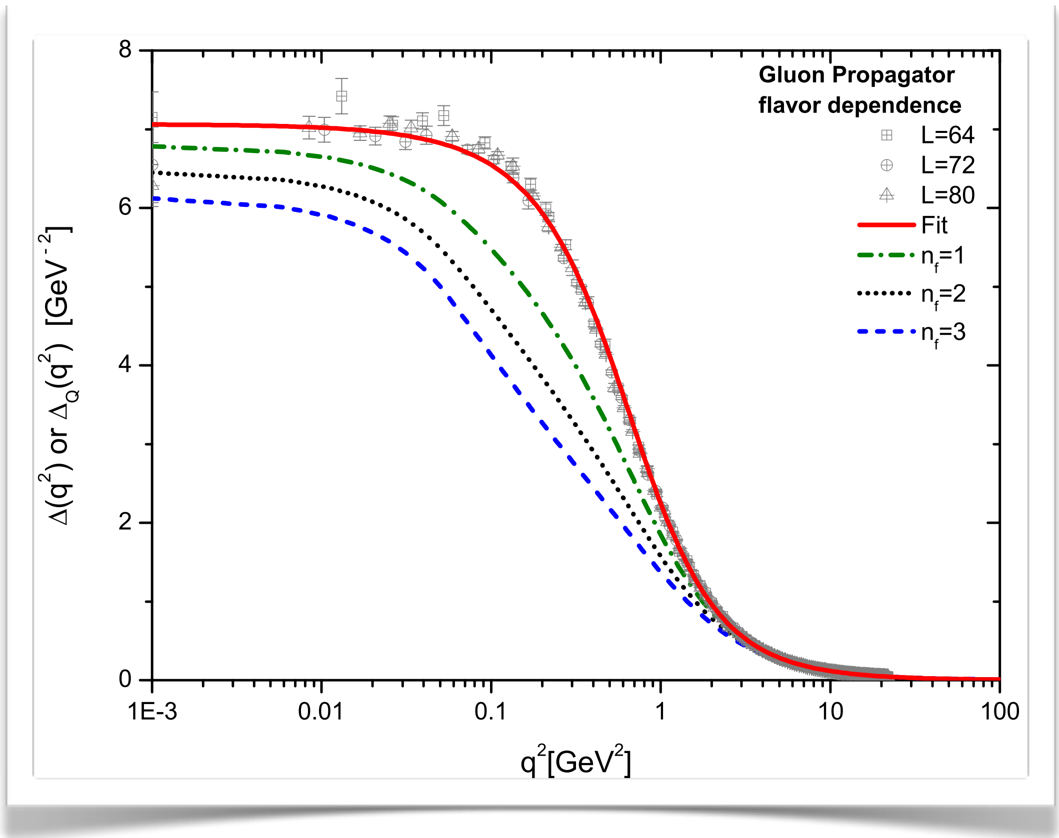
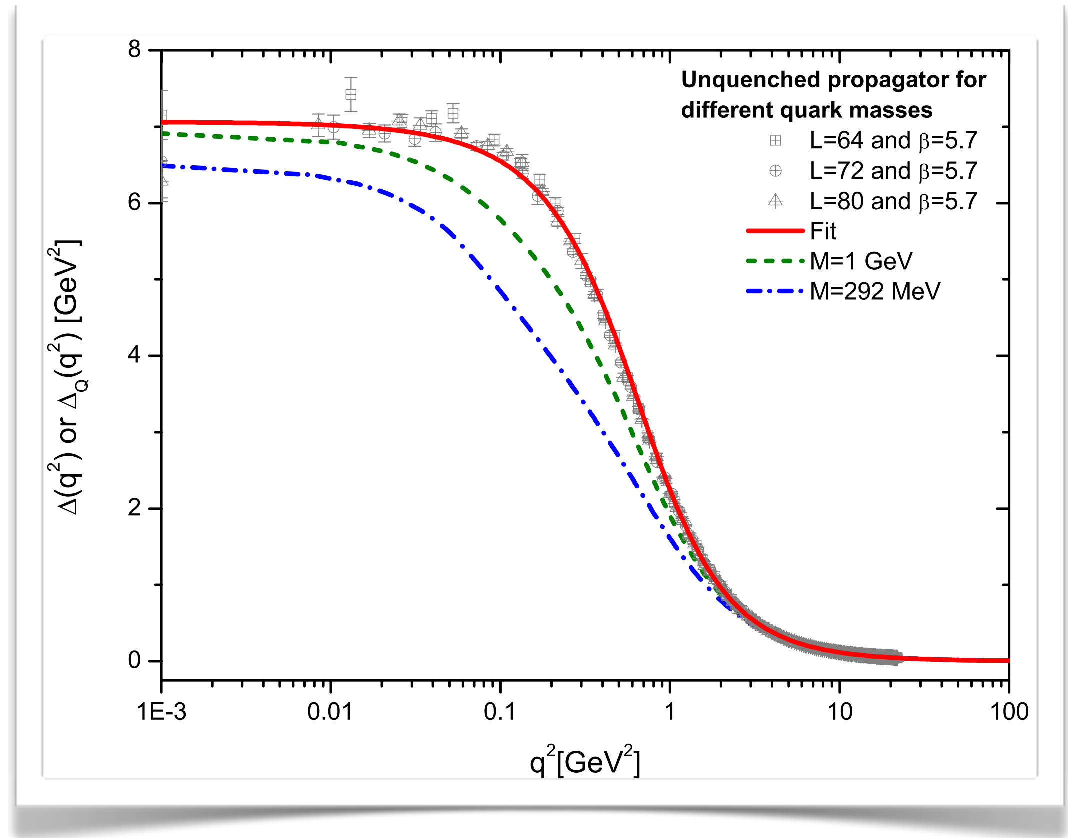
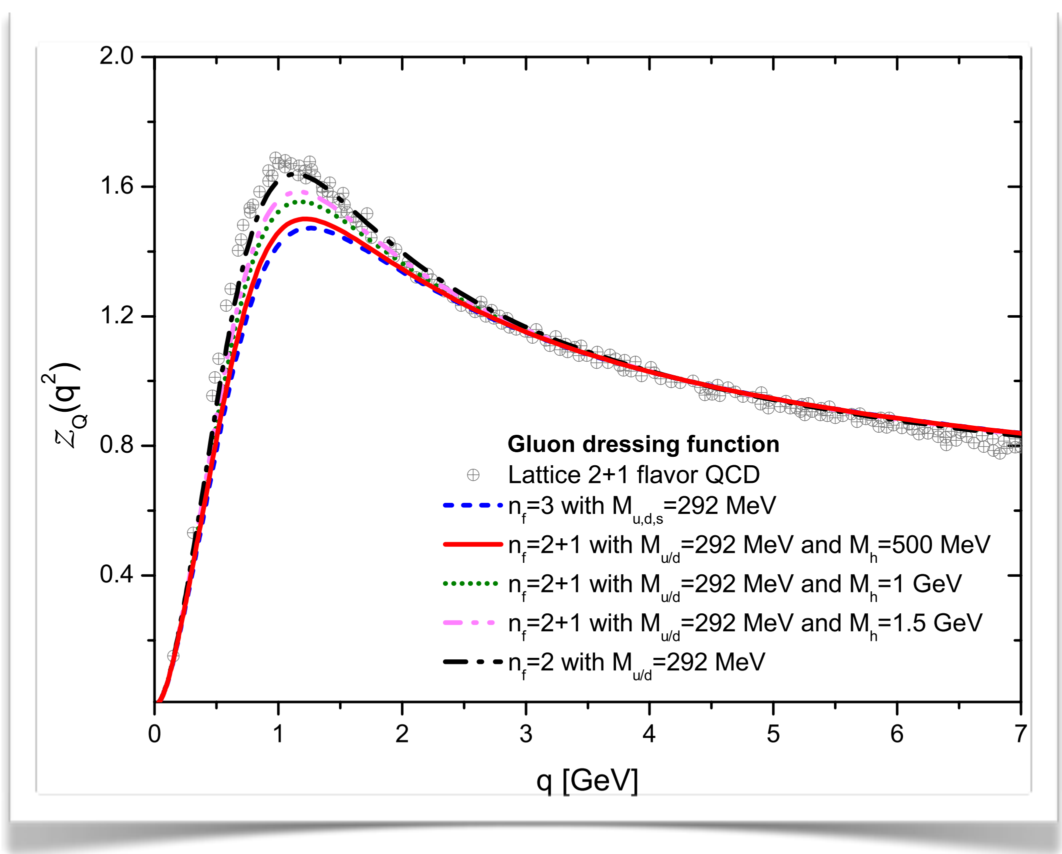


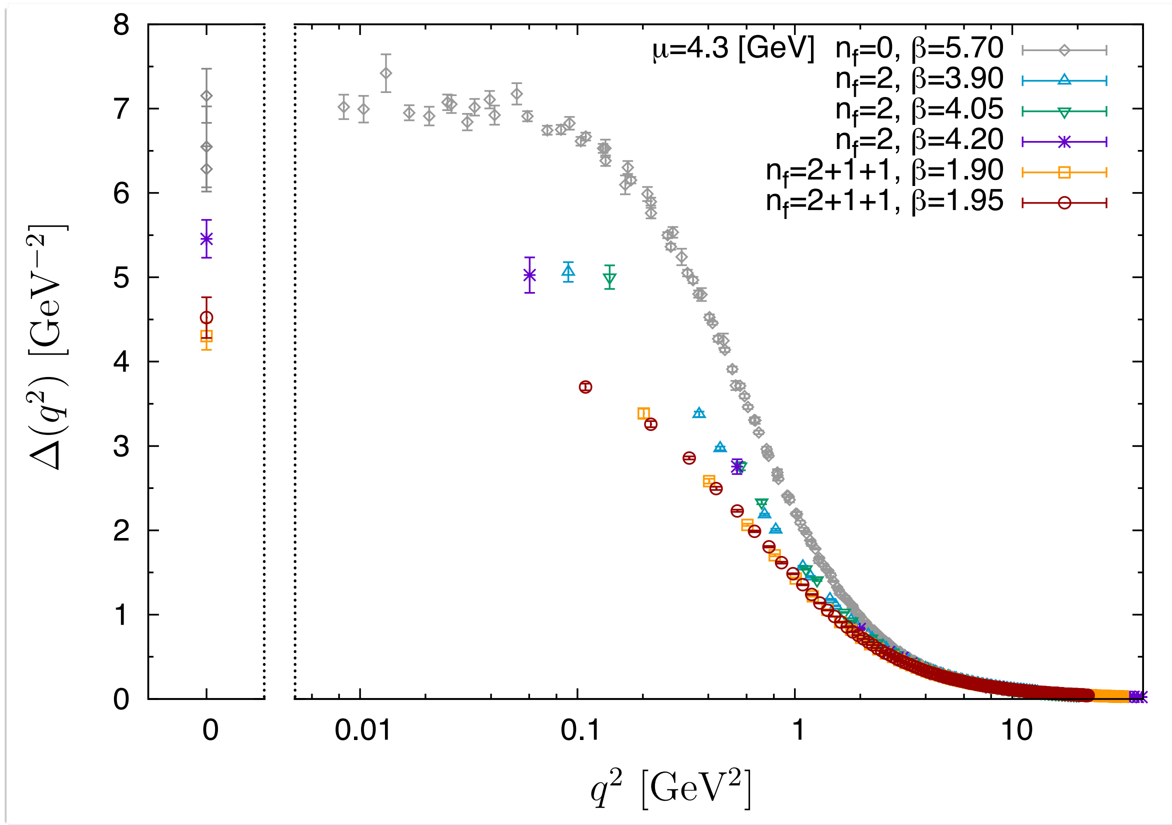
# Unquenching the lattice

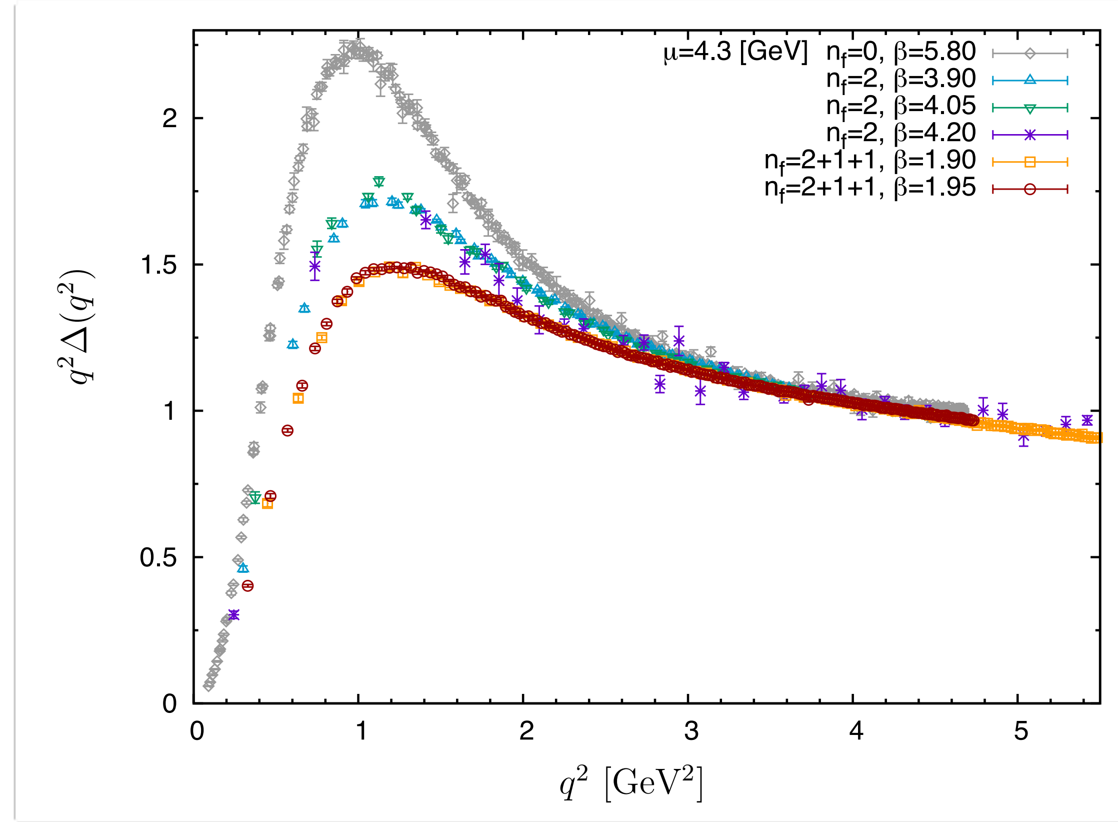
# adding
# quarks on the lattice

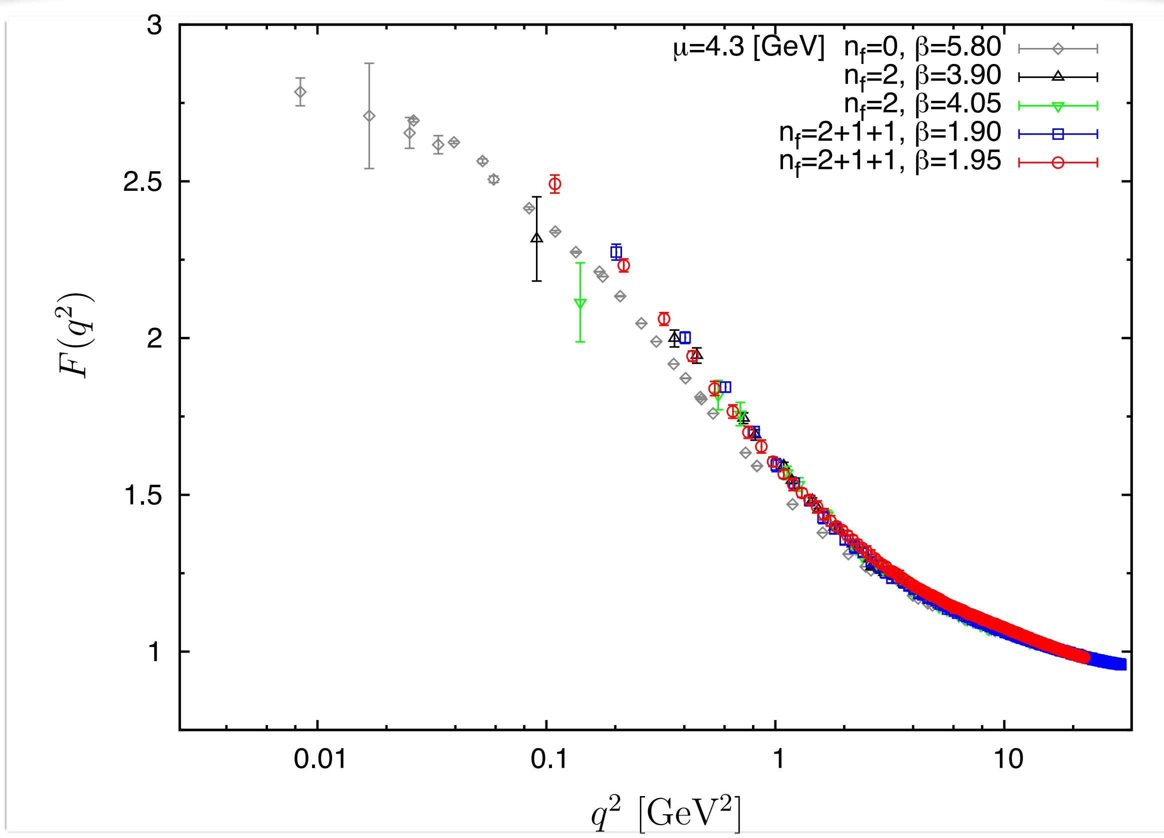
- No **systematic study** of the **IR sector** with **dynamical quarks**
  - 2007 data from the Adelaide (Bowman et al., Phys. Rev. D76, 094505) group but
    - Use staggared fermions configurations (MILC collaboration)
    - Consider only 2 light and 1 heavy quarks
    - Show only the gluon dressing function (no gluon propagator, no ghost)
    - You have to digitalize them...
- Use ETMC configurations projected to the Landau gauge
  - 2 light quarks and 2+1+1 configurations
  - Very **small current masses for up/down quarks 20-40 MeV**; strange 95 MeV, charm 1.51 GeV
  - Compensate for **O(4) breaking artifacts** (no cylindrical cut on data but H(4) extrapolation)
  - **Study** both the **gluon** and the **ghost IR sector**

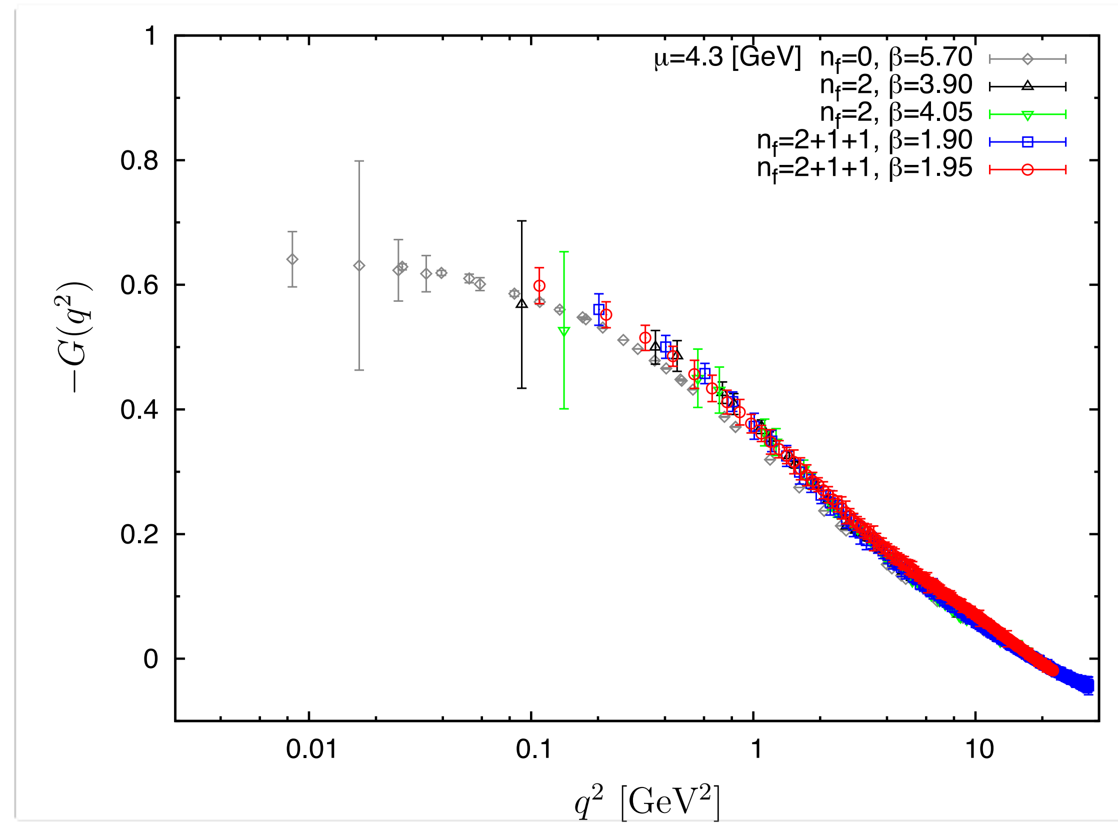
# (preliminary) results

M. Cristoforetti et al., in preparation



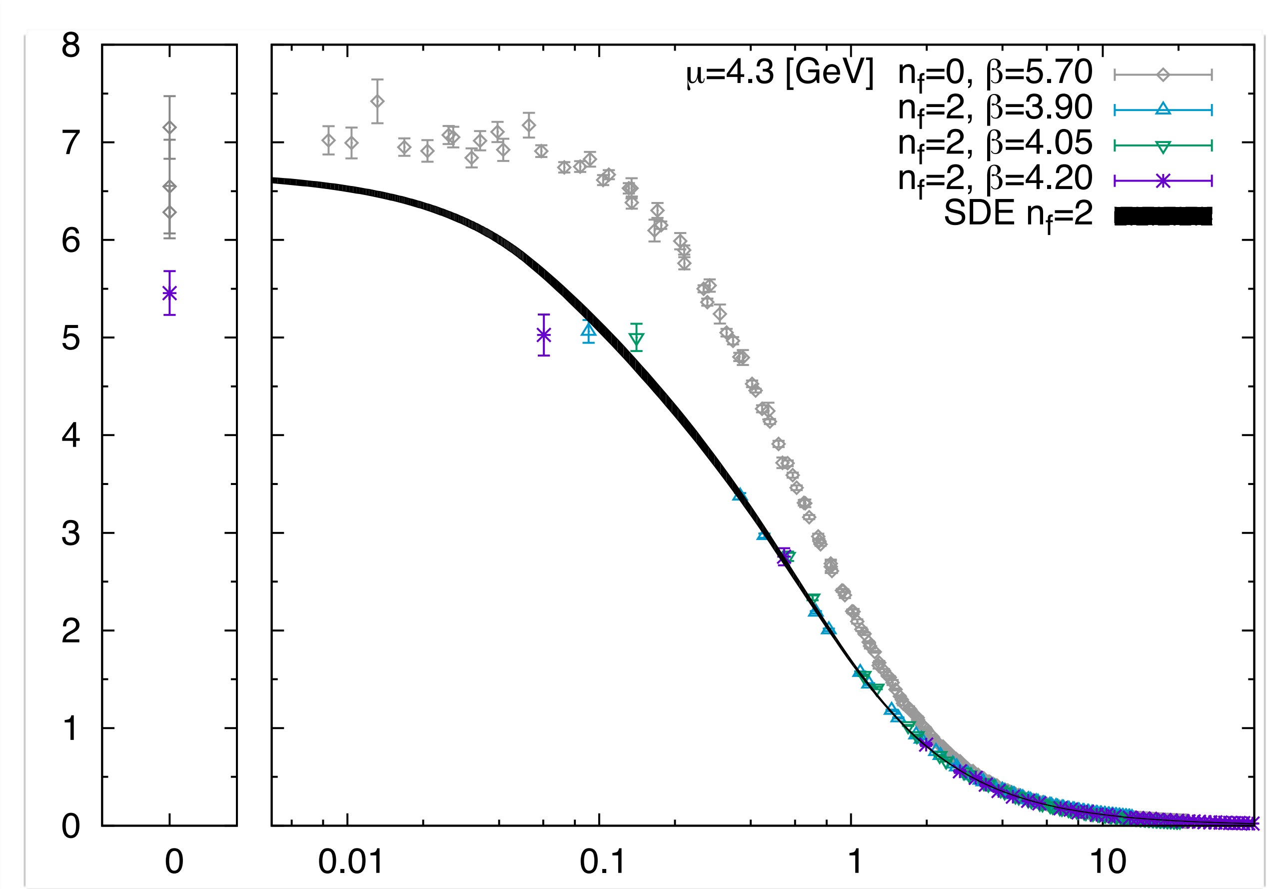



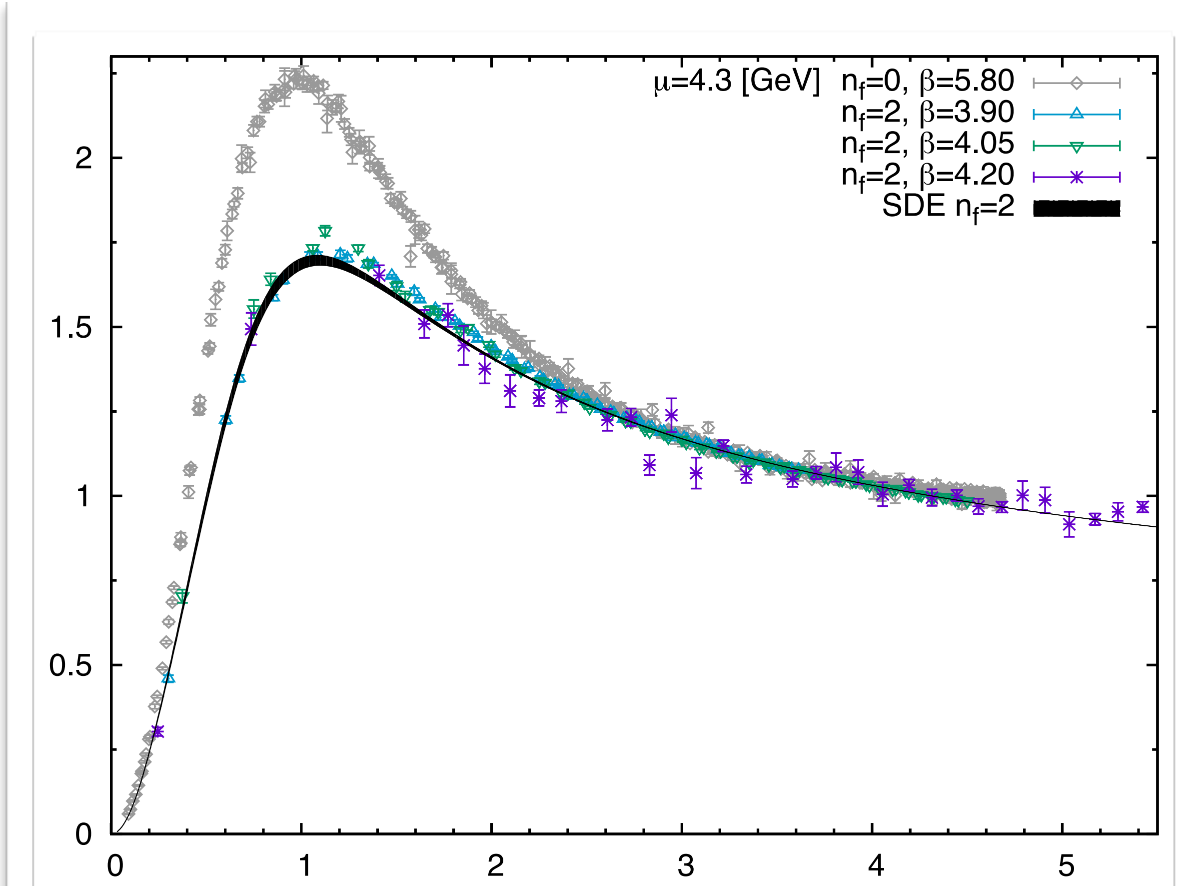



# (preliminary)
# SDEs comparison

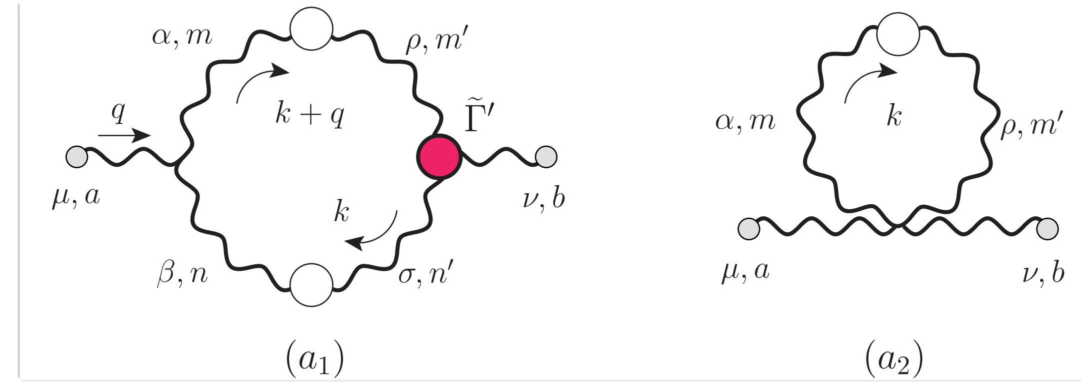
M. Cristoforetti et al., in preparation
A. Aguilar, D. B. & J. Papavassiliou, in preparation



# All-order mass eq.

# PT-BFM one-loop dressed mass equation




## Landau gauge mass equation (one-loop dressed)

- **Dynamical equation** derived as what **survives** in the $q \to 0$ limit
- **Seagull identity** can only happen in the $g_{\mu\nu}$ part
- **Sufficient** to look at **what survives the limit in the longitudinal terms** (keeping in mind that the answer must be transverse)

$$m^2(q^2) = -\frac{3g^2 C_A}{1+G(q^2)}\frac{1}{q^2}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} m^2(k^2)\Delta(k)\Delta((k+q)^2)\left[(k+q)^2 - k^2\right]$$

- The $q \to 0$ limit is particularly interesting

$m^2$ **cannot be** a **monotonically decreasing** function

$$m^2(0) = -\frac{3}{2}g^2 C_A F(0)\int_k m^2(k^2)[k^2\Delta^2(k^2)]'$$

must reverse sign and display a sufficiently deep negative region at intermediate momenta

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## This mass equation is **different** from the one that has appeared in PRD **84**, 085026

- Addresses a **very subtle issue** related to **taking the trace** and **completing the seagull identity** (resulting, rather ironically, in a breaking of transversality)
- The $q \to 0$ **limit** of the equation is however the **same**

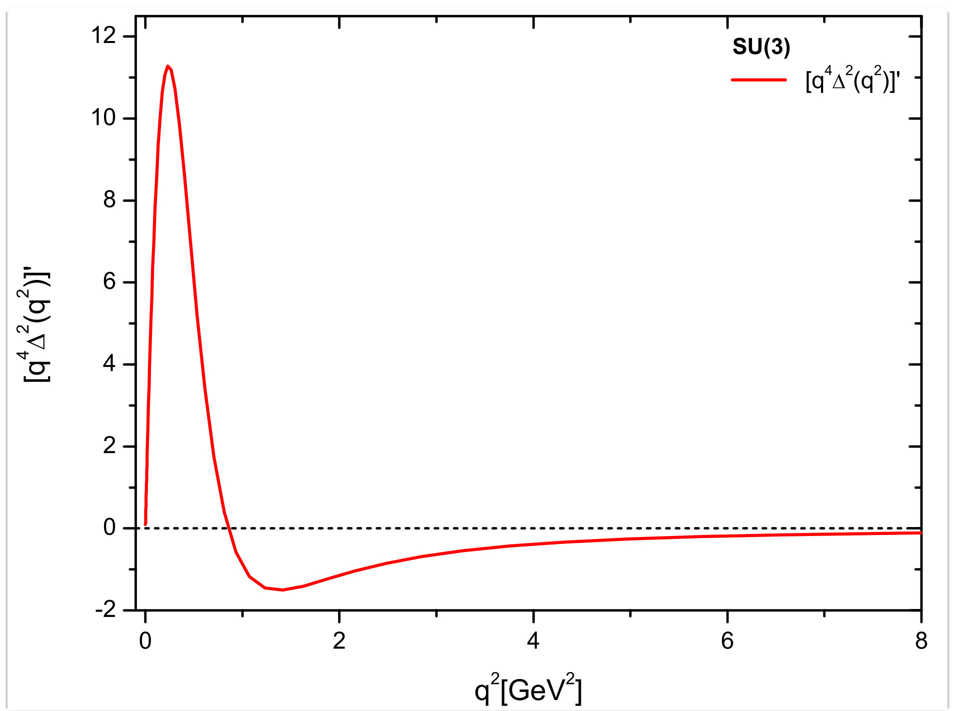
# PT-BFM one-loop dressed mass equation

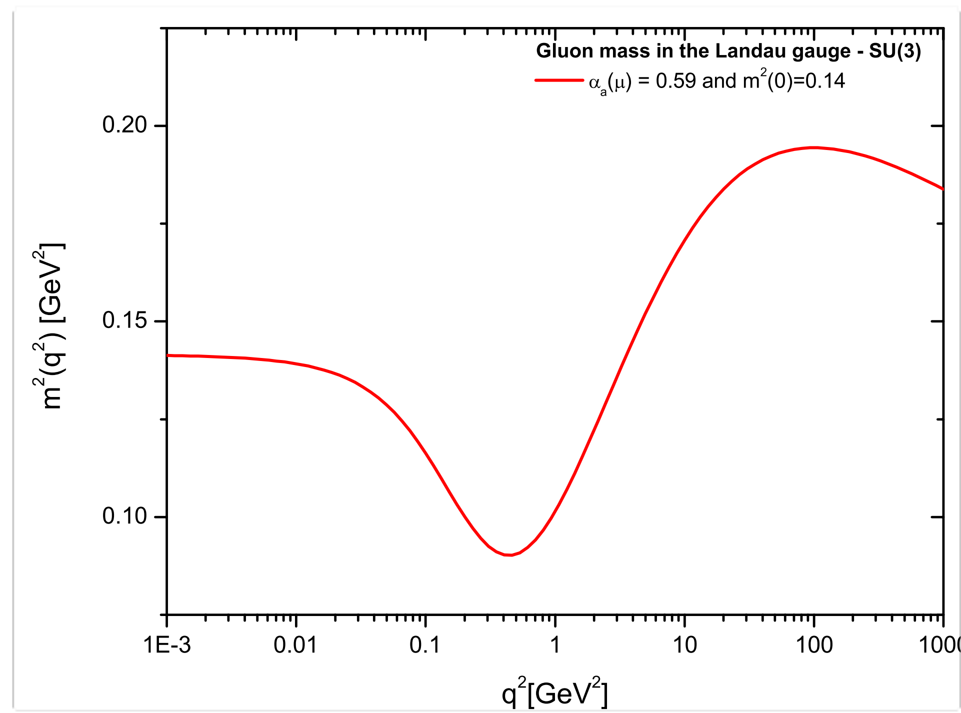
A. Aguilar, D. B. & J. Papavassiliou, Phys. Rev. D84, 085026 (2011)

- Within the standard angular approximation, the old equation yields

$$m^2(x) = m^2(0)\frac{F(x)}{F(0)} + \frac{\alpha_s C_A}{2\pi} F(x)\overline{R}(x)$$

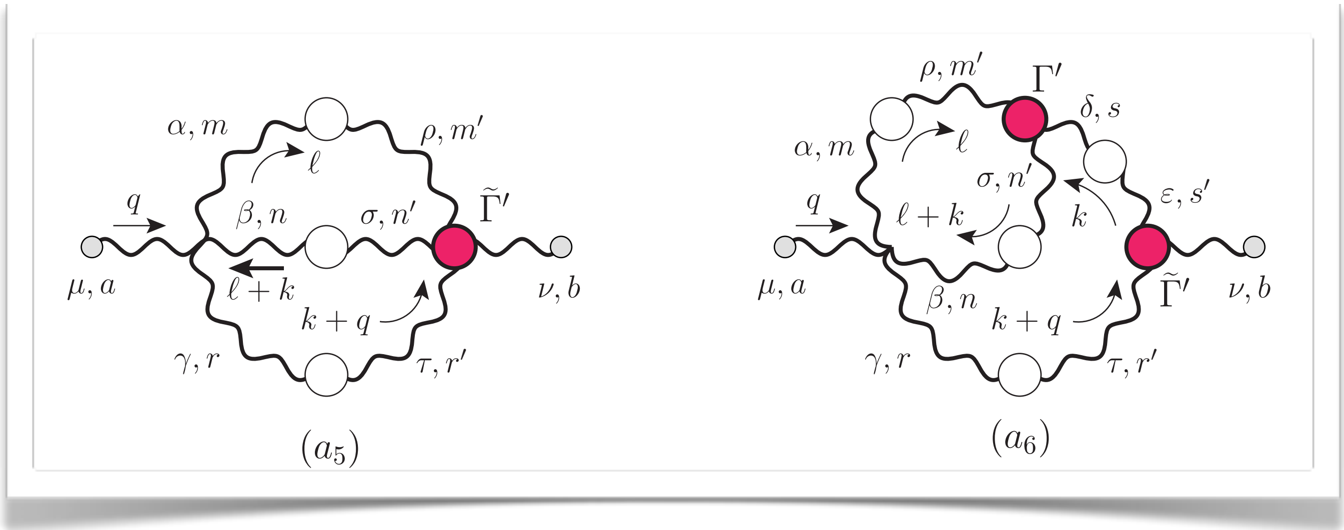
$$\begin{aligned}\overline{R}(x) &= \frac{1}{2}\int_0^x \mathrm{d}y\, y\, m^2(y)\left(1-\frac{y}{x}\right)\Delta^2(y) + \Delta(x)\int_0^x \mathrm{d}y\, y\left(y-\frac{x}{4}\right)\frac{m^2(x)-m^2(y)}{x-y}\Delta(y) \\ &- m^2(x)\, x^2\Delta^2(x) + \frac{3}{4}\int_0^x \mathrm{d}y\, m^2(y)[y^2\Delta^2(y)]'\end{aligned}$$



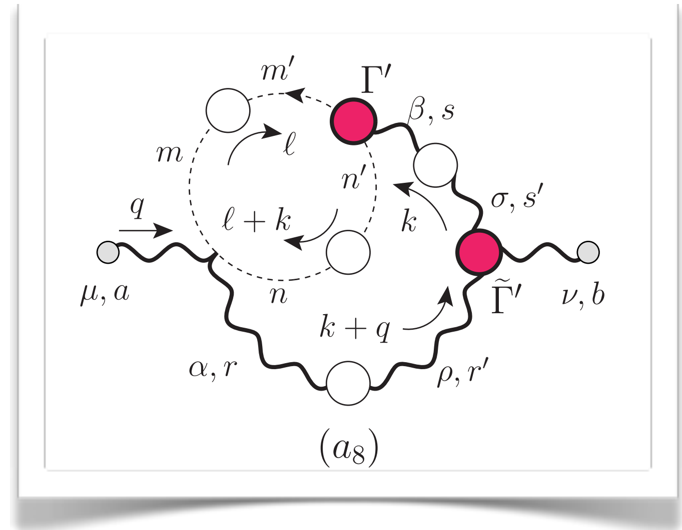



## PT-BFM

# two-loop dressed diagrams




$(a_5)$ $(a_6)$




$(a_8)$

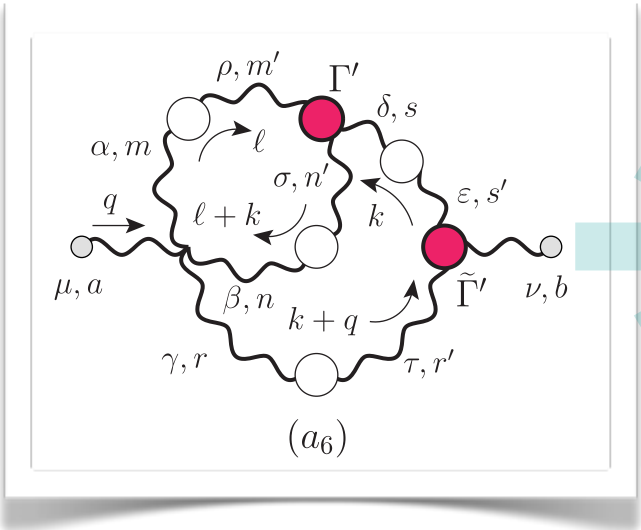
- We consider the **two-loop dressed** diagrams
  - If **ghosts** are **massless** these are the only contributions missing
- A **new ingredient** appears: $\widetilde{V}_4$ for the four-gluon vertex.
  - In principle **many new ghost Green's functions** appears due to the complicate STIs structure satisfied by the conventional four-gluon vertex
  - However in the **Landau gauge** we only need to know the contraction:

$$PPP\widetilde{V}_4 = \textit{linear combinations of } V_3$$

**no additional ghost Green's function @ 2 loops**

- It is therefore mandatory to explicitly determine the pole part of the three-gluon vertices $\widetilde{V}_3$ and $V_3$

# two-loop contribution to the mass equation




$$\frac{3}{2} i \int_k \frac{Y(k^2)}{q^2 k^2} \Delta(k)\Delta(k+q)(k \cdot q)[m^2(k) - m^2(k+q)]$$

- $$Y(k^2) = k^\alpha \int_\ell \Delta(\ell)\Delta(\ell+k) P_{\alpha\rho}(\ell) P_{\beta\sigma}(\ell+k) \mathbb{\Gamma}^{\sigma\rho\beta}(-\ell-k, \ell, k)$$

---

- Add this to the (one-loop) mass equation to get (Euclidean space)

$$m^2(q^2) = -\frac{g^2 C_A}{1+G(q^2)} \frac{d-1}{q^2} \int_k m^2(k)\Delta(k)\Delta(k+q)\left[(k+q)^2 - k^2\right] - \frac{g^4 C_A^2}{1+G(q^2)} \frac{3}{2q^2} \int_k \frac{Y(k^2)}{k^2} (k \cdot q)\Delta(k)\Delta(k+q)\left[m^2(k+q) - m^2(k)\right]$$

- Take the $q \to 0$ limit, use the seagull identity and introduce spherical coordinates

$$m^2(0) = -\frac{3C_A}{8\pi} \alpha_s F(0) \int_0^\infty \mathrm{d}y\, m^2(y) \left\{ \left[ 1 - \frac{1}{2} g^2 C_A \frac{Y(y)}{y} \right] y^2 \Delta^2(y) \right\}'$$

# two-loop contribution to the
# mass equation

- Calculate $Y$ to lowest order in perturbation theory

$$Y(k^2) = k_\alpha \int_\ell \frac{1}{\ell^2(\ell+k)^2} P^{\alpha\rho}(\ell) P^{\beta\sigma}(\ell+k) \Gamma^{(0)}_{\sigma\rho\beta}(-\ell-k, \ell, k)$$
$$= \frac{1}{(4\pi)^2} k^2 \left[ \frac{15}{4} \left( \frac{2}{\epsilon} \right) - \frac{15}{4} \left( \gamma_{\mathrm{E}} - \log 4\pi + \log \frac{k^2}{\mu^2} \right) + \frac{33}{12} \right]$$

- Renormalize subtractively

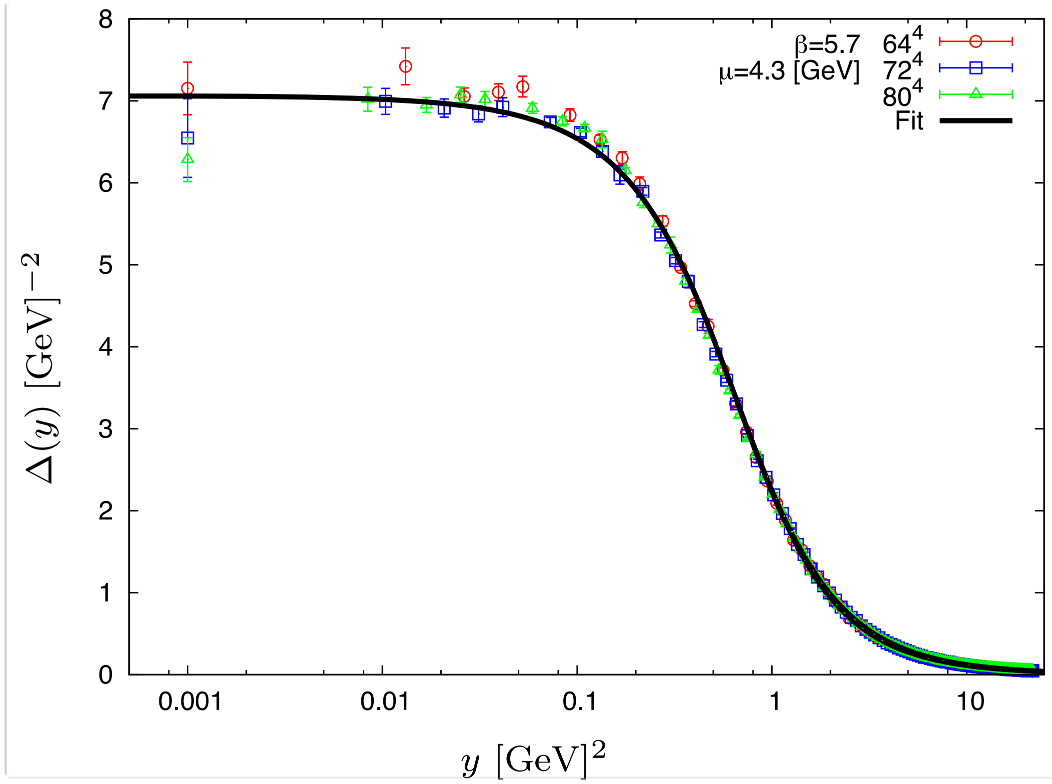
$$Y_{\mathrm{R}}(k^2) = -\frac{1}{(4\pi)^2} \frac{15}{4} k^2 \log \frac{k^2}{\mu^2}$$

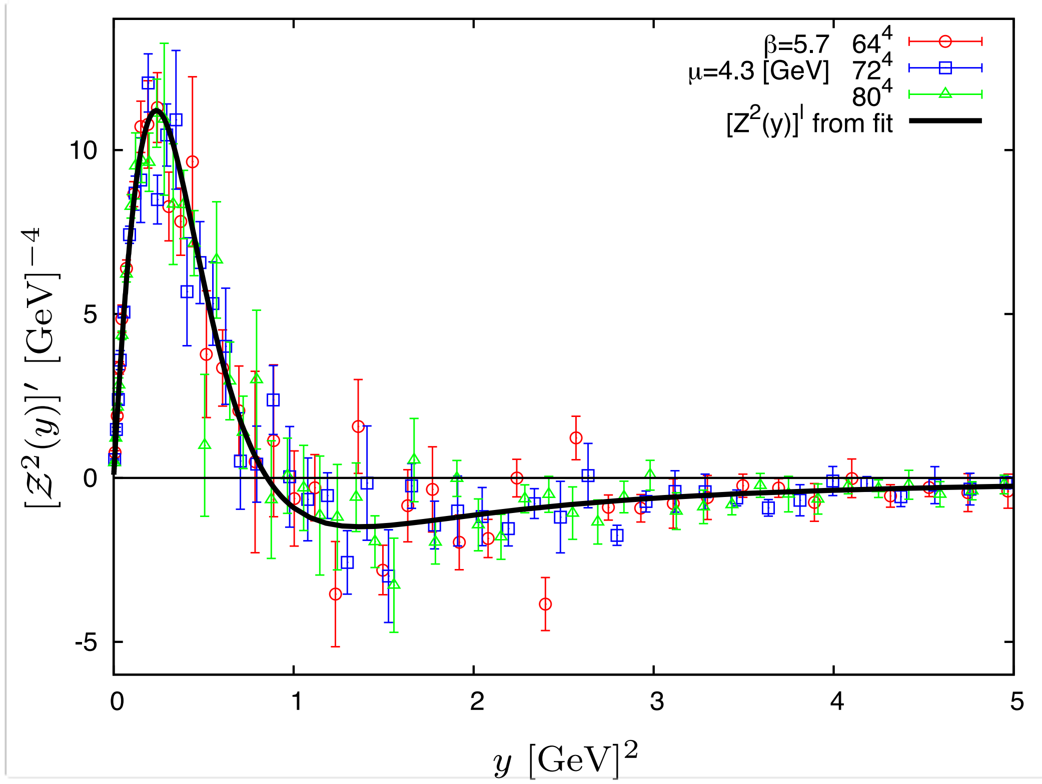
- Substitute to the mass equation to get the final equation

$$m^2(0) = -\frac{3}{8\pi} \alpha_s C_A F(0) \int_0^\infty \mathrm{d}y\, m^2(y) \underbrace{\left[ \left( 1 + \frac{15 C_A}{32\pi} \alpha_s \log \frac{y}{\mu^2} \right) \overbrace{\mathcal{Z}^2(y)}^{y^2 \Delta^2(y)} \right]'}_{\mathcal{K}(y)}$$

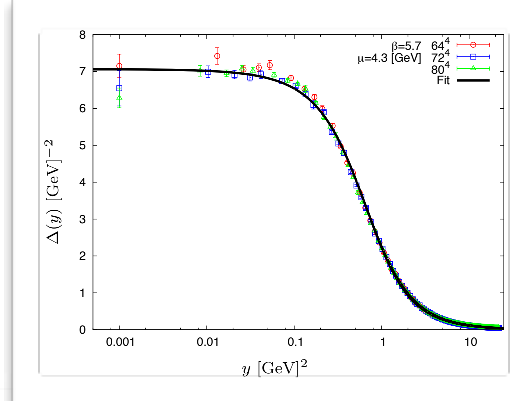
# (quenched) numerical analysis

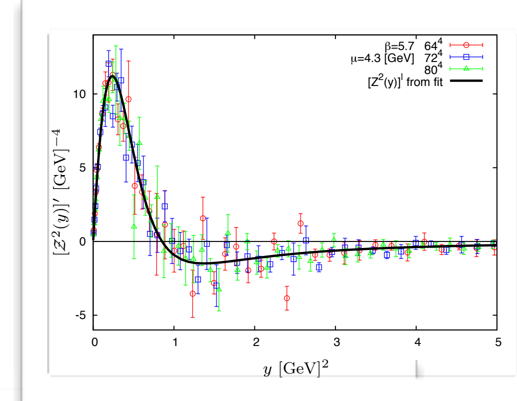
I. L. Bogolubsky et al., Phys. Lett. B676, 69 (2009)

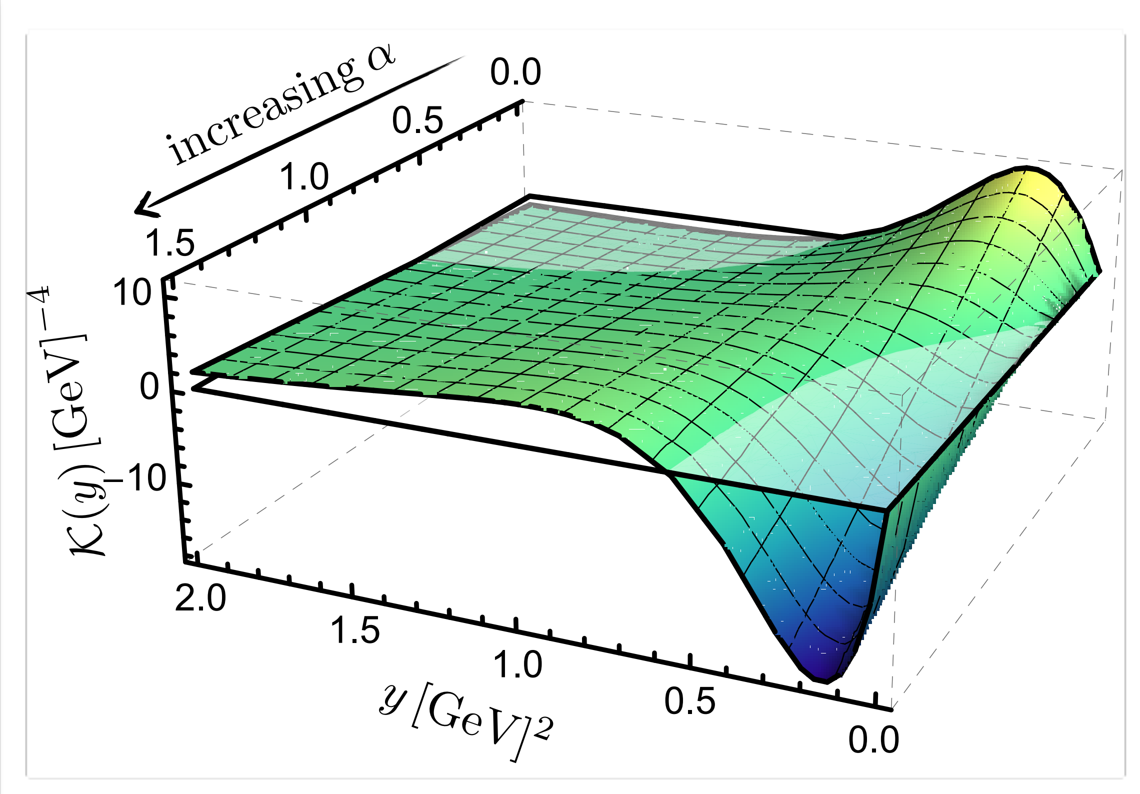


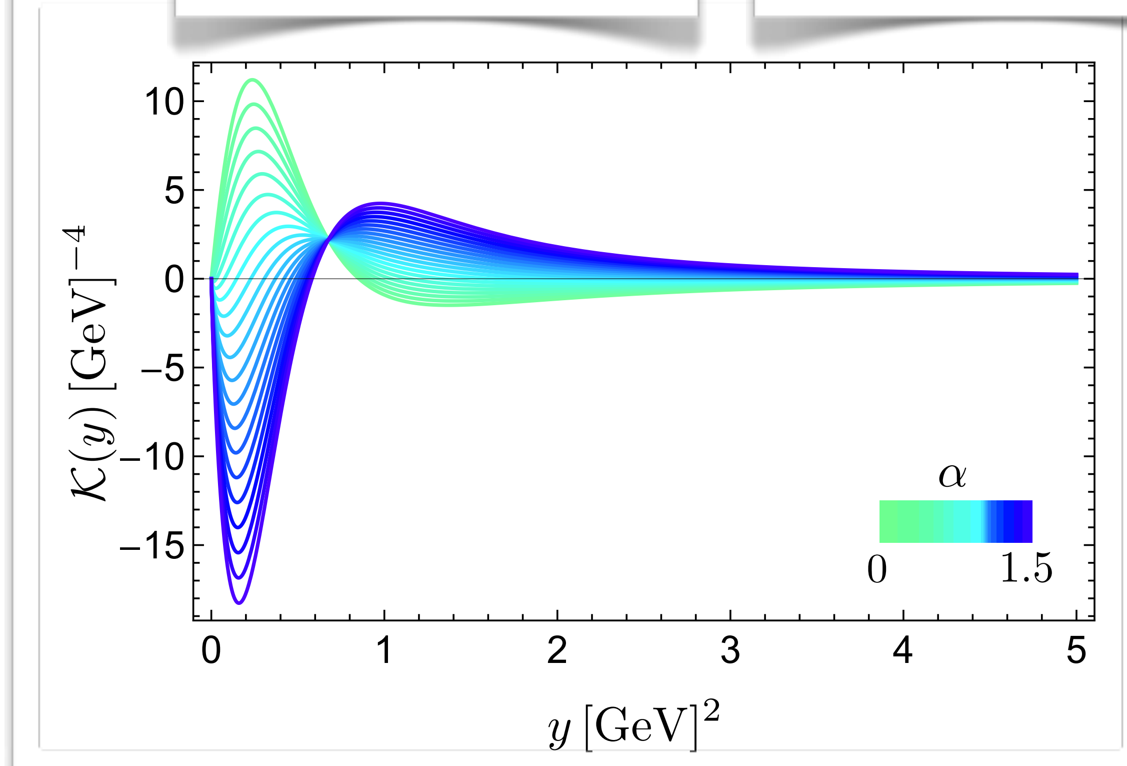



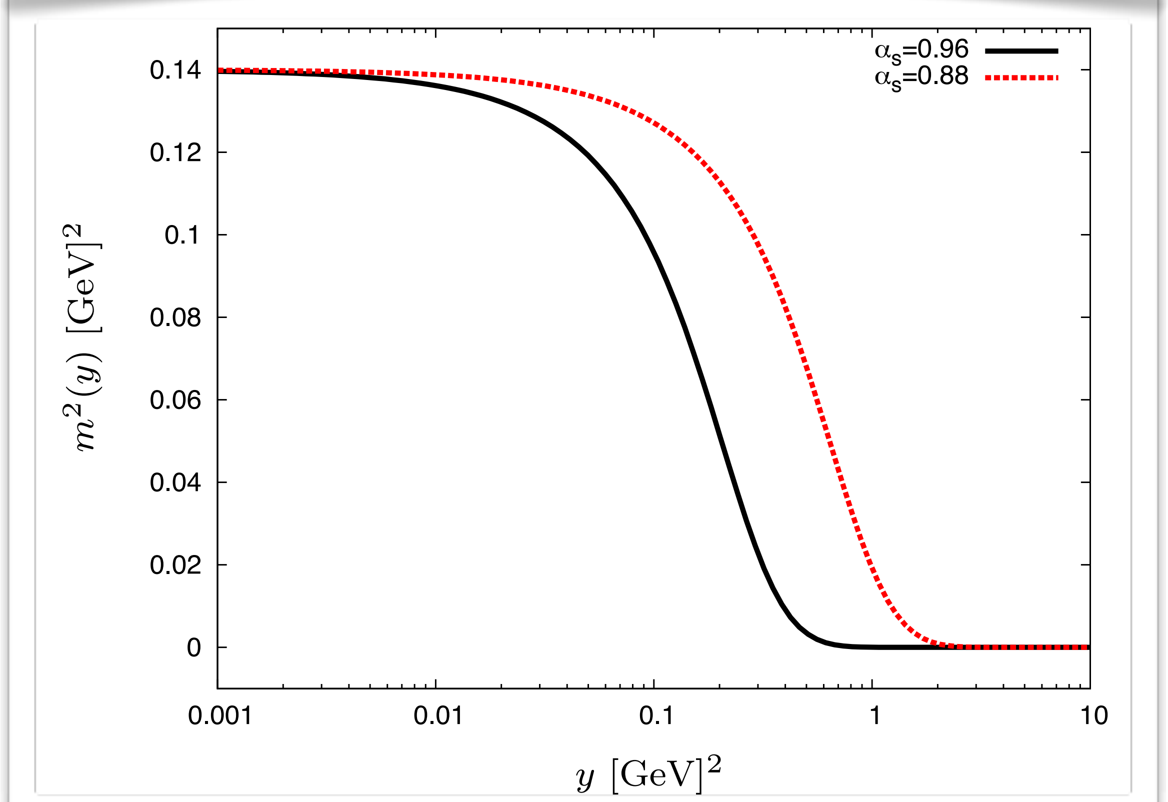
# (quenched)
# numerical analysis



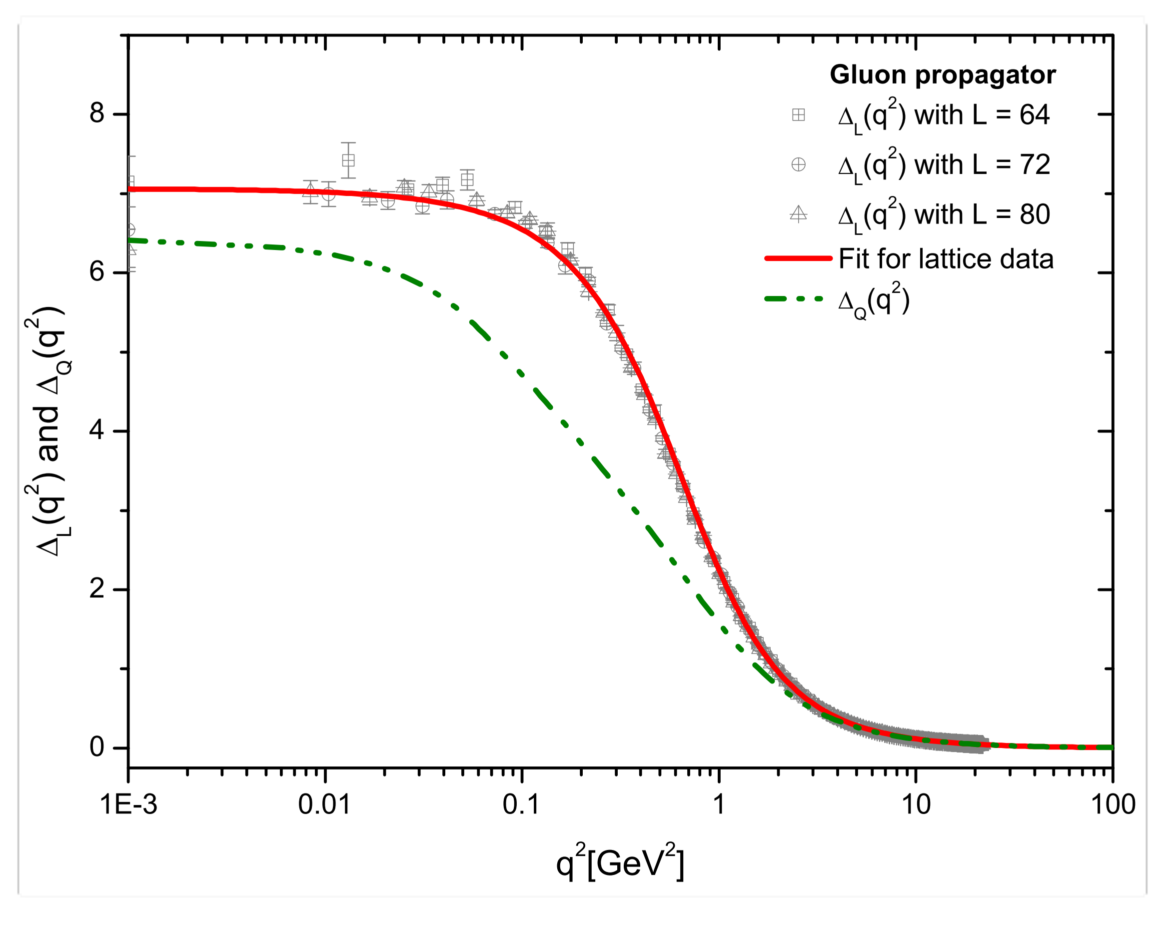

- **Solutions** of the **integral condition** for the quenched $SU(3)$ mass found in lattice simulations

$$m^2(0) \approx 0.14$$

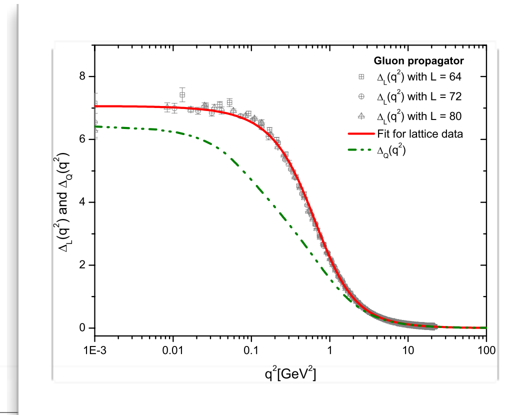
- Back to **monotonically decreasing masses**
- **Natural** notion of a **critical coupling**
  - QCD has to be "**strong enough**" **to dynamically generate a gluon mass**
  - For the unquenched lattice case we find

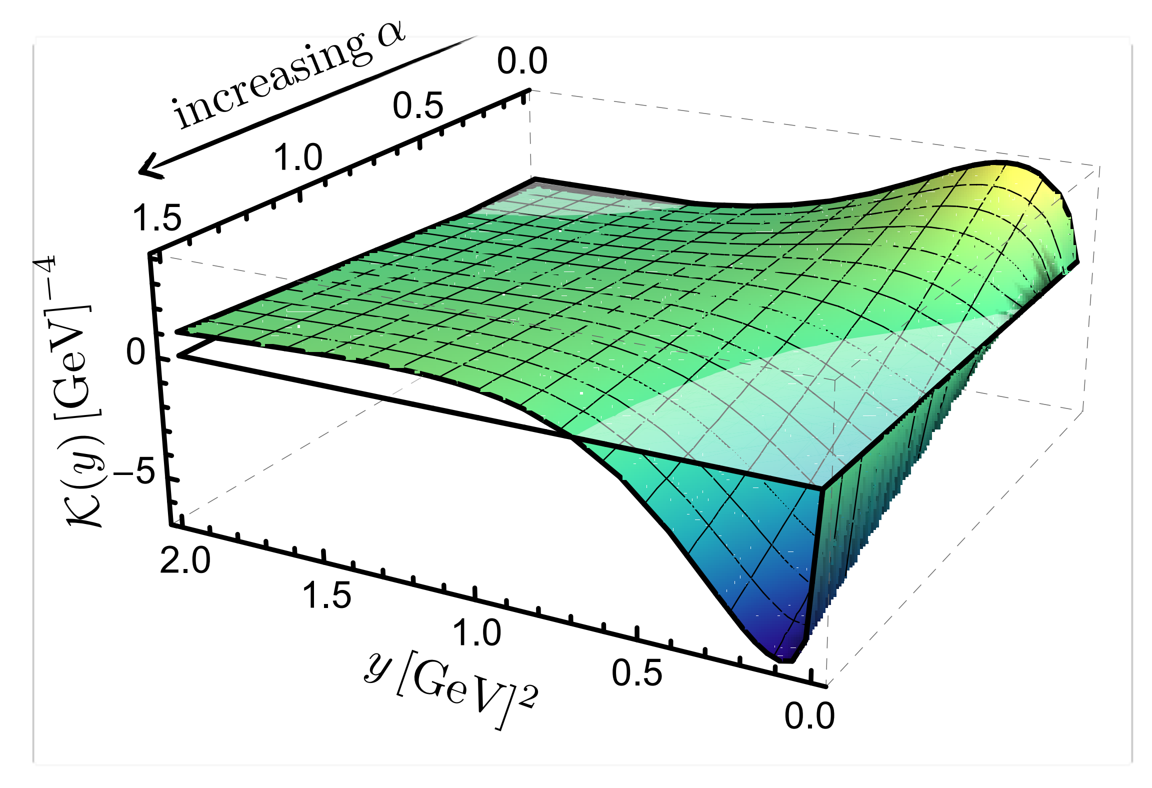
$$\overline{\alpha}_s \approx 0.83$$

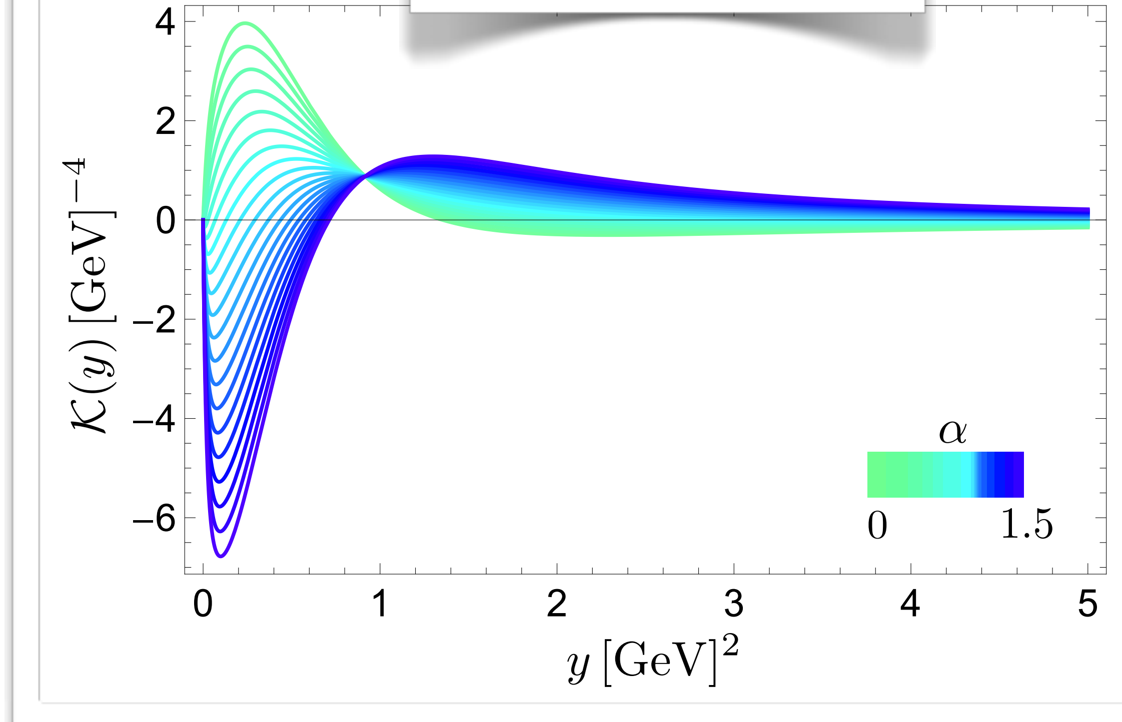
# (unquenched) numerical analysis

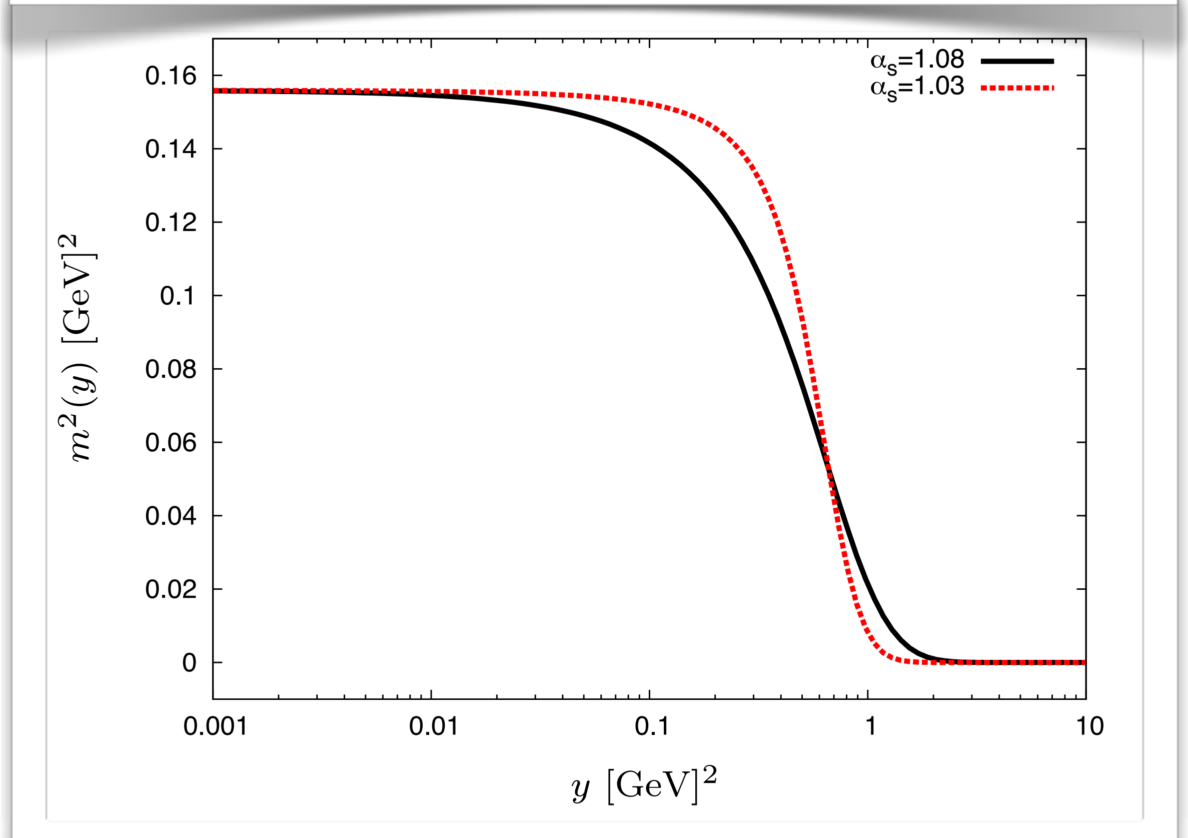


# (unquenched) numerical analysis



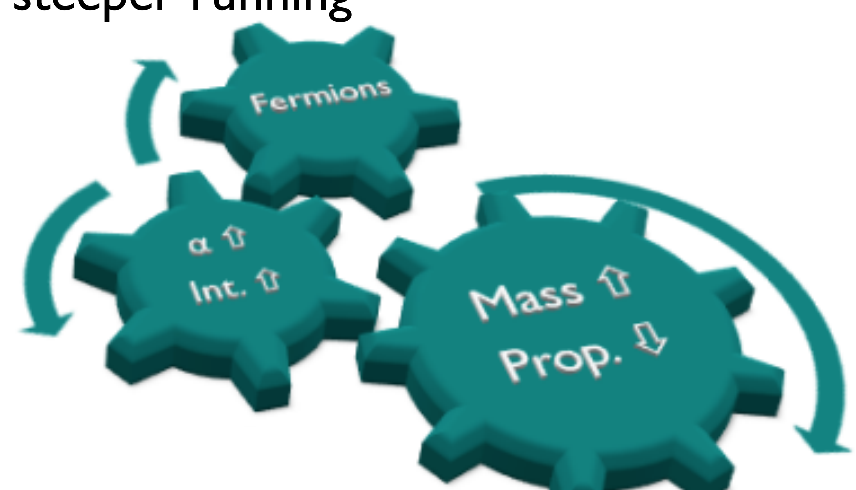




- **Solutions** of the **integral condition** for the unquenched $SU(3)$ mass found in *SDE studies* ($n_f$=2)

$$m^2(0) \approx 0.156$$

- Solutions requires a bigger (~20%) coupling and, possibly, a steeper running



# BFM on the lattice

# quantizing gauge theories in a **background**

- Consider **quantum fluctuations** around some **non-trivial background** $\widehat{A}$

$$A_\mu = \widehat{A}_\mu + Q_\mu$$

- One wishes to **carry out** the **path integral over** $Q$, *e.g.*, to captures the main features of topologically inequivalent sectors of the theory
  - Many examples in the literature:
    - **‘t Hooft computation** of the **one-loop effective action** in an **instanton** configuration
    - **Quantum corrections** around **static solutions (baryons)** in **chiral lagrangians**

- A second point of view is the BFM
  - Considers the **background field** as an **unspecified source**
  - Exploits **residual gauge invariance** of the (gauge fixed) theory to simplify the calculations (**background WI**)

Set to zero after taking the appropriate derivatives of the vertex functional, wrt to the background

# quantizing gauge theories in a background

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Set to zero after taking the appropriate derivatives of the vertex functional, wrt to the background

**Can we implement the BFM in a fashion suitable for non-perturbative analysis?**

# BFM as a canonical transformation

- Can one **control in a unique way** the **dependence** of the vertex functional (local and non-local) **on** the **background by symmetry arguments** only?
- The answer is a surprising **yes**!
  - The appropriate mathematical tool is a **canonical transformation**
    - Symmetry pattern **common to perturbation theory, lattice, non-perturbative analytical method**
    - Hope for **bridging** these **computations** in the **relevant matching regimes**
    - Very nice **analogy** with the theory of **finite canonical transformations** in **classical analytical mechanics**

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also A. Cucchieri & T. Mendez, 1204.0216 [hep-ph]

- **Dynamical ghosts** are **not needed**: we can **implement** the **BFM** in **non-perturbative lattice gauge** theory

$$\mathcal{F}[g] = -\int \mathrm{d}^4x \,\mathrm{Tr}\,(A^g_\mu - \widehat{A}_\mu)^2$$

$A^g_\mu$ is the gauge transform of the gauge field

# Conclusions & outlook

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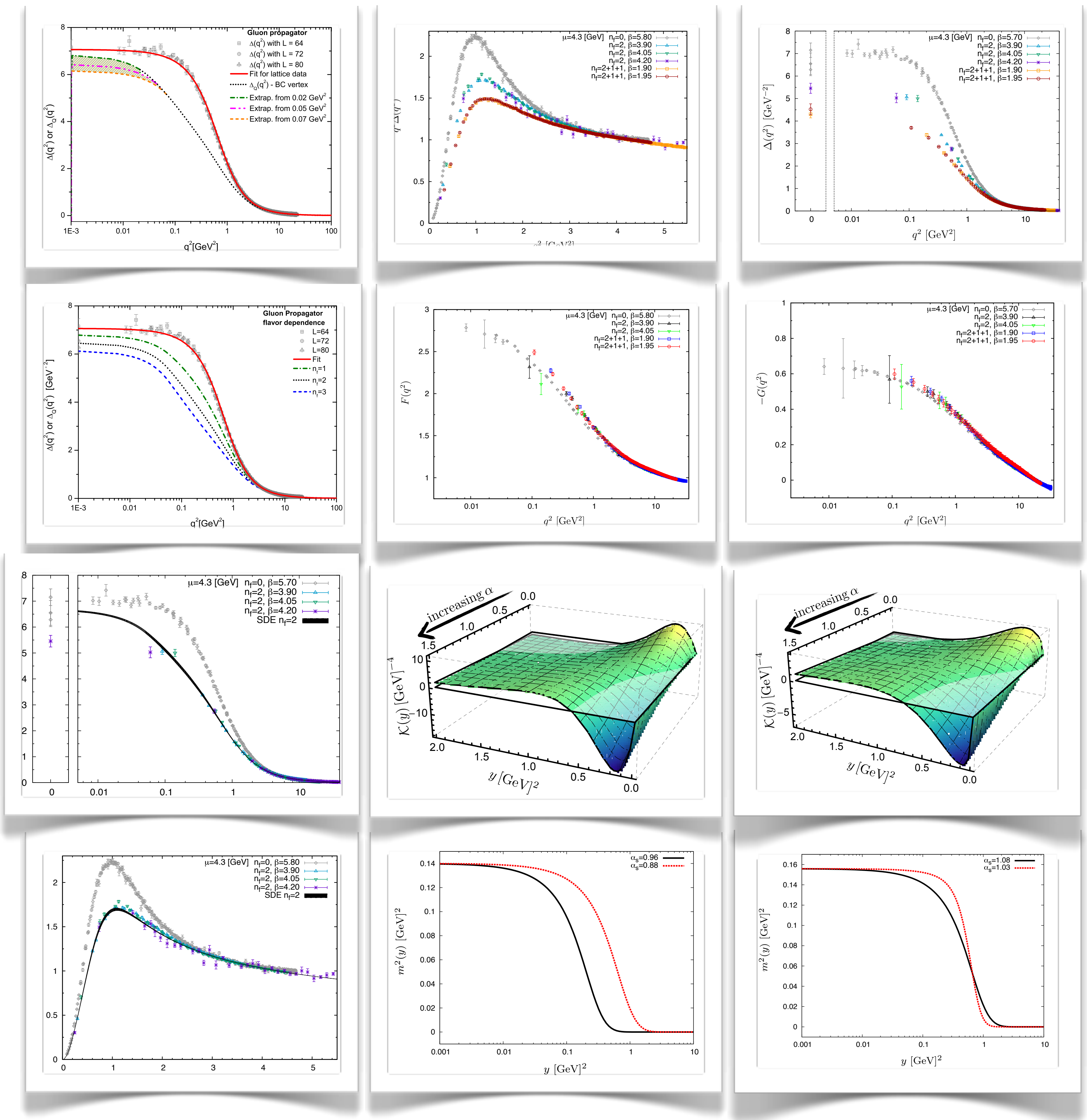


- **Lot of results...**
  - Unquenched SDEs and lattice simulation
  - All-order mass equation
  - General non-perturbative formulation of the BFM
  - BFM on the lattice
- **...much more to be done**
  - Finite temperature, chemical potential,...
  - Explore the new BFM formulation
    - Lattice simulations of PT Green's functions
    - 2PI formalism in the BFM
    - ...

# the end

thank**you**