Transport Coefficients of the Quark-Gluon Plasma

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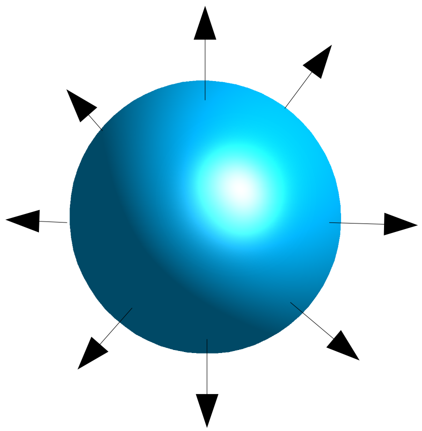
Motivation – Bulk and Shear Viscosities

$$T^{\mu\nu} = T^{\mu\nu}_{(0)}(\epsilon, P; u^\mu) + T^{\mu\nu}_{(1)}(\zeta, \eta; u^\mu, \partial_\nu u^\mu)$$

particle-antiparticle symmetric systems or systems w/o conserved charge number density

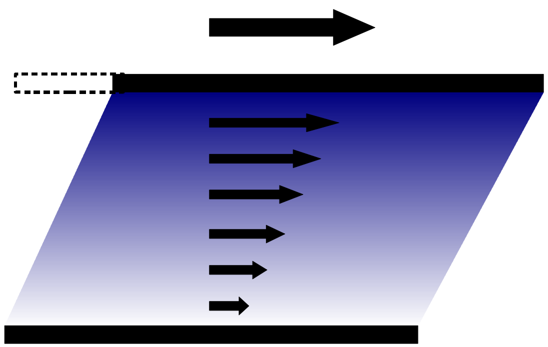
➡ $\partial_\mu T^{\mu\nu} = 0$

ζ : **bulk** (volume) **viscosity**



change in volume **BUT** constant shape/form

η : **shear viscosity**



change in shape **BUT** constant volume

➡ application of ideal hydrodynamics modelling heavy-ion collisions at RHIC and LHC suggests at most small dissipative effects; viscous calculations confirm this

Shear Viscosity of the QGP?

pitch: $\eta \sim 2.3 \cdot 10^8 \, Pa \cdot s$

honey: $\eta \sim 2 - 3 \, Pa \cdot s$

water: $\eta \sim 10^{-3} Pa \cdot s$

QGP: even more liquid

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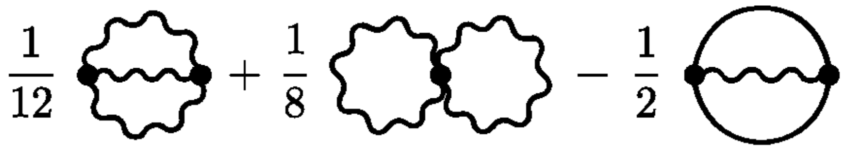
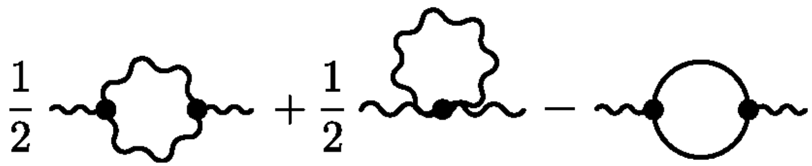
Is this necessarily true?

QPM based on Φ- functional approach to QCD:

$$\frac{\Omega[D,S]}{T} = \frac{1}{2}\mathrm{Tr}\left[\ln D^{-1} - \Pi D\right] - \mathrm{Tr}\left[\ln S^{-1} - \Sigma S\right] + \Phi[D,S]$$

$\Phi =$ 

$\Pi =$  , $\Pi = 2\frac{\delta\Phi}{\delta D}$

$\Sigma =$  , $\Sigma = -\frac{\delta\Phi}{\delta S}$

modell for equilibrium thermodynamics ► corresponding energy-momentum tensor:

$$T^{\mu\nu}_{(0)}(T) = \sum_i \int \frac{d^3\vec{p}}{(2\pi)^3 E_i(T)} p^\mu p^\nu f_i^{(0)} + g^{\mu\nu} B[\{\Pi_j(T)\}]$$

for excitations with medium-modified
dispersion relations (thermal mass) $E_i^2(T) = \vec{p}^{\,2} + \Pi_i(T)$

Effective Kinetic Theory

- self-consistent generalization of $T^{\mu\nu}_{(0)}$ to non-equilibrium systems:

  space-time dependence of $T(x)$ implies $E = E(x)$
  is a functional of the distribution function $f(x,p)$

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- to assure **basic relations**:
  - $\partial_\mu T^{\mu\nu}(x) = 0$
  - $\delta\langle T^{00}\rangle / \delta f(x,p) = E$ (Fermi liquids)
  - in thermal equilibrium: $\epsilon + P = T\frac{\partial P}{\partial T}$

  one generalizes (in case of a one-component system) to

$$\boxed{T^{\mu\nu}(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} p^\mu p^\nu f(x,p) + g^{\mu\nu} B[\Pi(x)]}$$

kinetic term **potential term**

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$T^{\mu\nu}$ closely related to **effective kinetic equation of Boltzmann-Vlasov type** for the single-particle distribution function $f(x,p)$: $(\mathcal{L} + \mathcal{V})f = \mathcal{C}[f]$

- above conditions satisfied **if** $\frac{\partial B}{\partial \Pi} = -\frac{1}{2}\int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} f(x,p)$ related to form of Vlasov-term

Bulk and Shear Viscosity Coefficients

for quasiparticle systems

$$(\mathcal{L} + \mathcal{V})f = \mathcal{C}[f]$$

Bulk and Shear Viscosities

- decompose $T^{\mu\nu}$ and compare w/ definition: $T^{\mu\nu}_{(1)} = \zeta\, \Delta^{\mu\nu} \partial_\alpha u^\alpha + \eta\, S^{\mu\nu}{}_{\alpha\beta} \partial^\alpha u^\beta$

bulk viscosity:

$$\zeta = \frac{1}{T} \int \frac{d^3\vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} \times \left\{ \left[\left(\frac{pu}{T}\right)^2 - \frac{1}{2T} \frac{\partial \Pi}{\partial T} \right] T^2 v_s^2 + \frac{1}{3} [p^2 - (pu)^2] \right\}^2$$

shear viscosity:

$$\eta = \frac{1}{15T} \int \frac{d^3\vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} [p^2 - (pu)^2]^2$$

cf. Chakraborty, Kapusta (2010)
& MB, Kämpfer, Redlich (2009,'10,'11)

Bulk and Shear Viscosities

differences: Excitations with constant vs. thermal mass

bulk viscosity:

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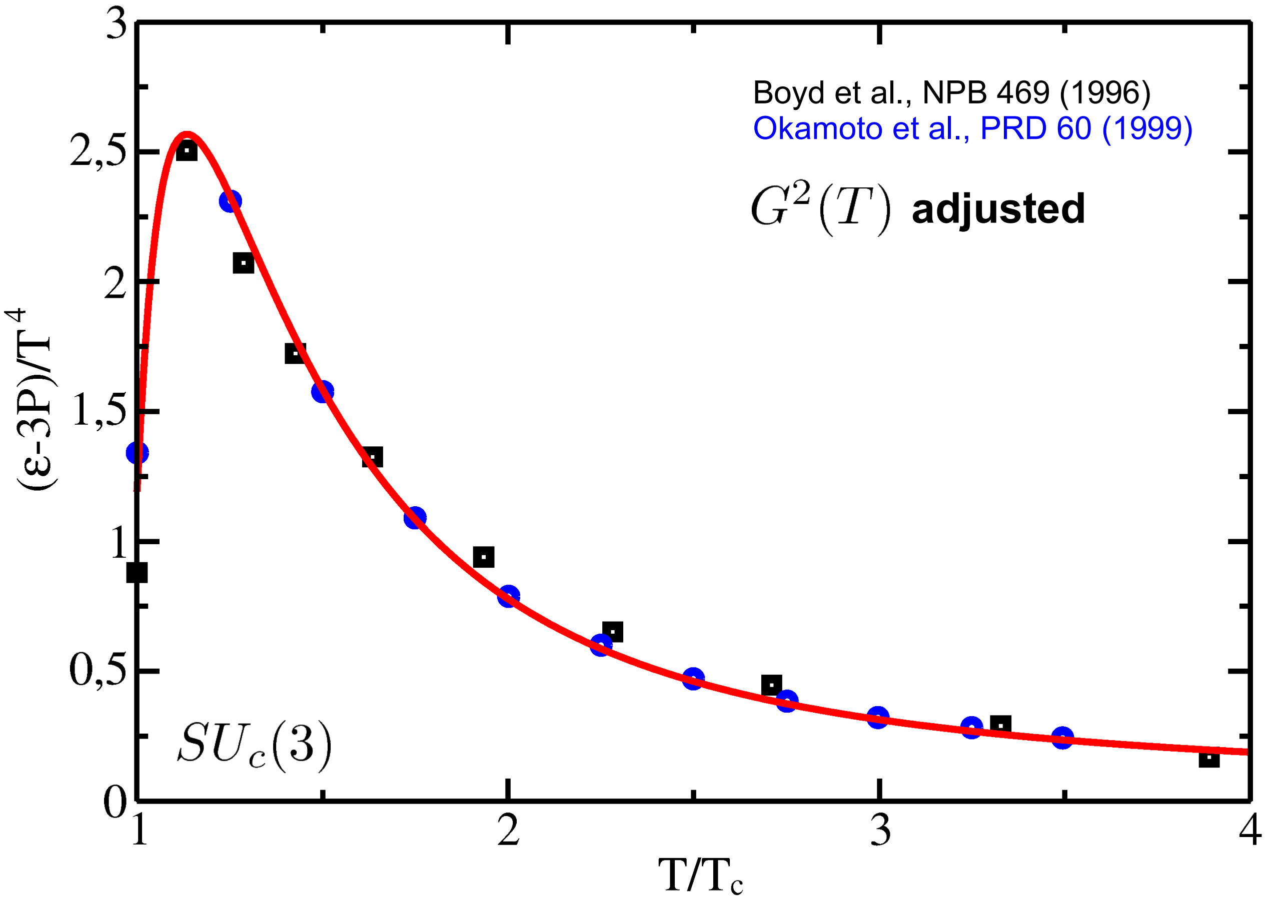
cf. Gavin (1985)

$$\Pi_g(T) = \frac{1}{2}T^2G^2(T) \text{ , where } \qquad G^2(T) = \frac{48\pi^2}{11N_c \log\left(\frac{\lambda(T-T_s)}{T_c}\right)^2}$$



→ maximum around $T/T_c \simeq 1.15$

Relaxation Time

- concentrate on SU(3): 2 ◄► 2 gluon-gluon scatterings

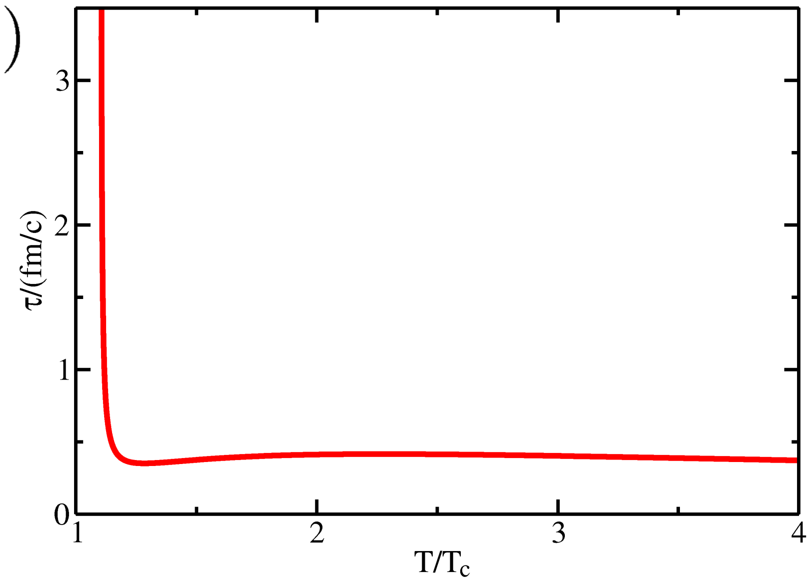
  parametrically

  $$\tau^{-1} \sim T G^4(T) \ln(a/G^2(T))$$

  based on perturbative considerations

  cross section depends crucially on ratio of maximum to minimum momentum transfer $\sim a$

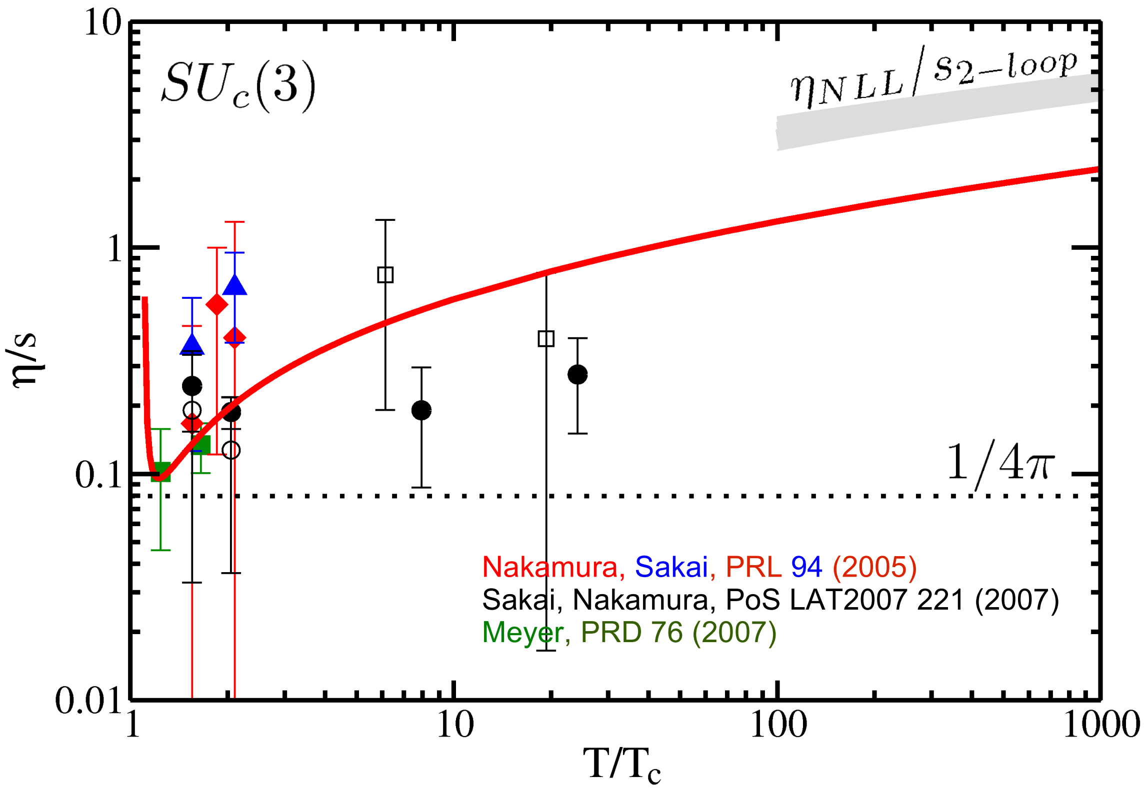
  cf. Heiselberg (1993)



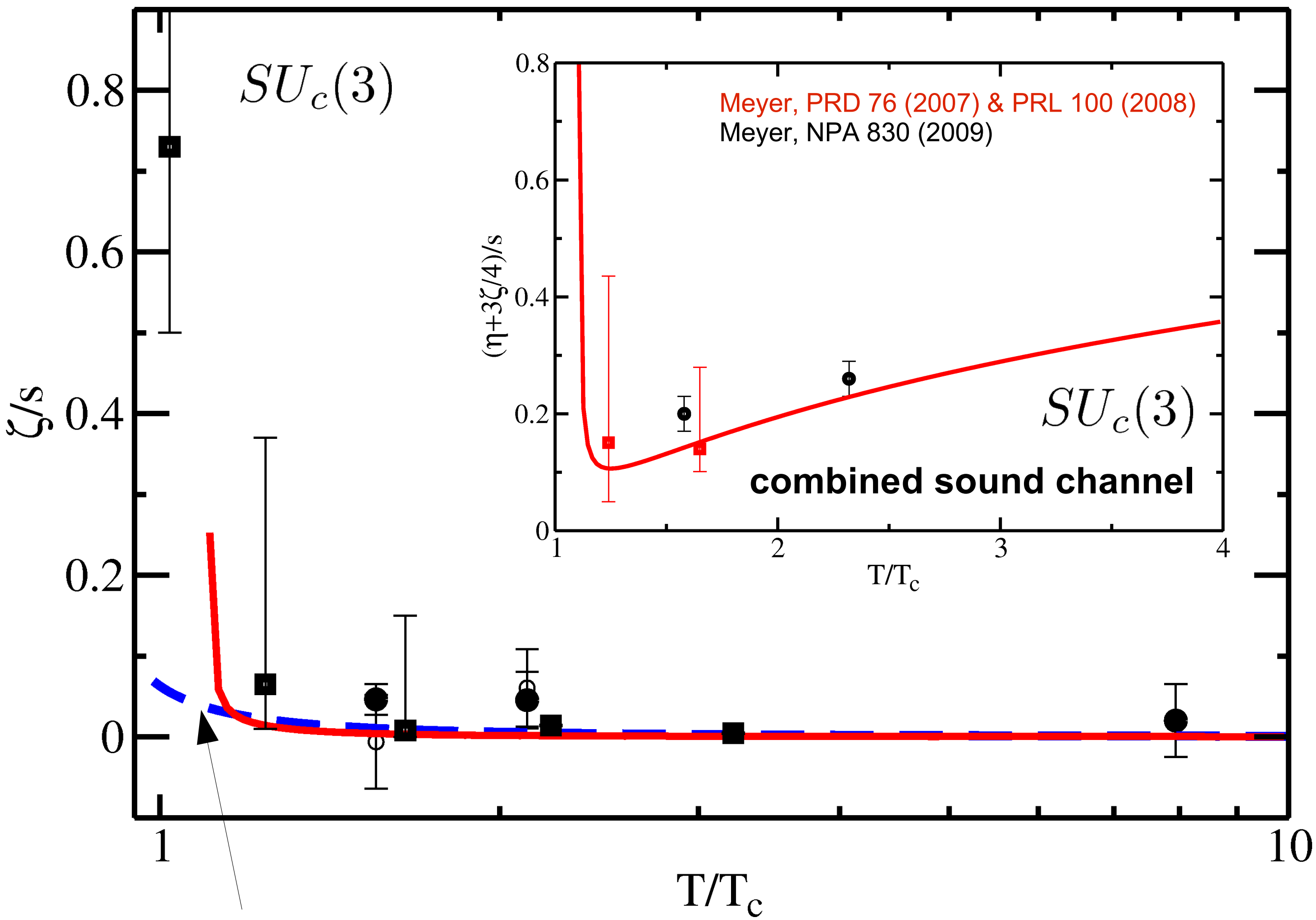
- parametric dependencies of pQCD results for ζ and η on coupling and temperature reproduced at large T

Quantitative Results – Specific Shear Viscosity

- behaviour close to T_c driven by τ



cf. MB, Kämpfer & Redlich (2009, 2011)



holographic QCD,
cf. Gürsoy et al. (2009)

cf. MB, Kämpfer & Redlich (2011)

Ratio of Bulk to Shear Viscosities

Bulk to Shear Viscosity Ratio

Big Theoretical Motivation: Viscosity coefficients in strongly interacting Quantum Field Theories can be deduced from Black Hole Physics

- Kovtun-Son-Starinets bound: $\eta/s \geq 1/(4\pi)$

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- similar *universal* bounds for other transport coefficients are unknown **BUT** in some special classes of theories with holographically dual supergravity description there exists a lower bound for the ratio

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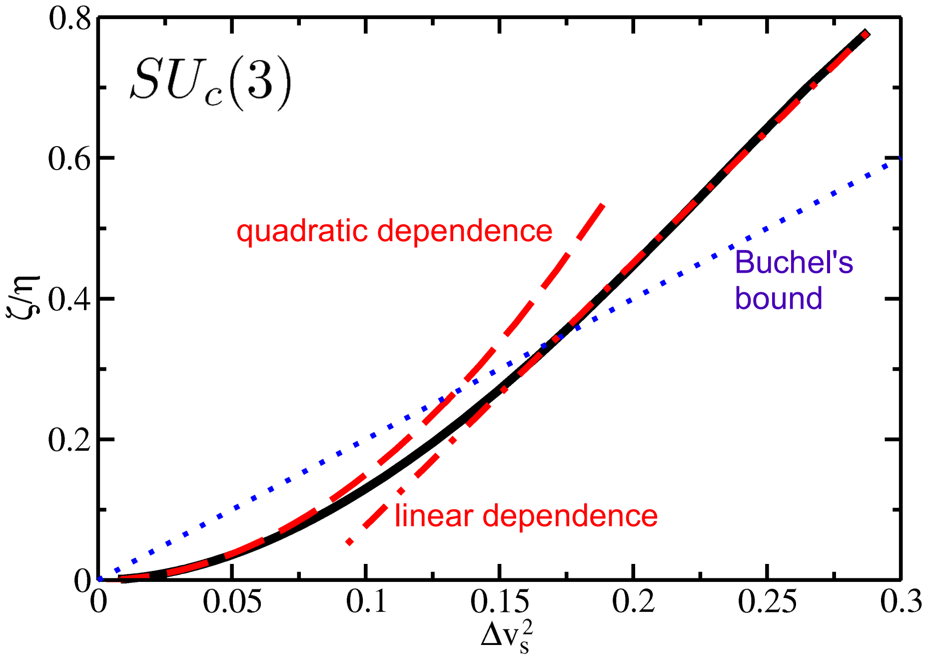
  Buchel bound: $(\zeta/\eta)_B \geq 2\left(\frac{1}{k} - v_s^2\right)$

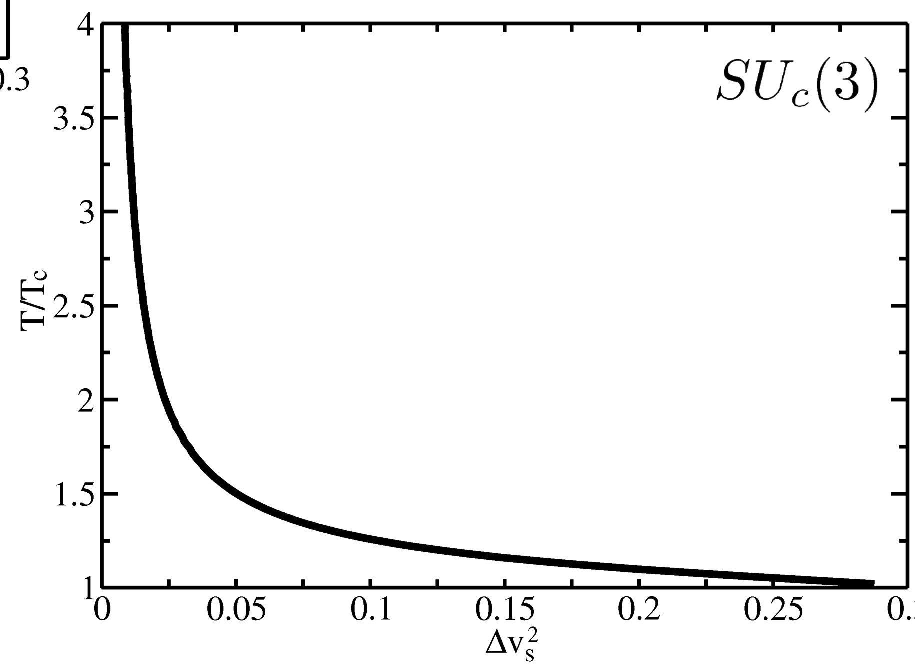
  - specific strongly coupled but nearly conformal theories (AdS/CFT) $\zeta/\eta \sim \Delta v_s^2 \equiv \left(\frac{1}{3} - v_s^2\right)$
  - for scalar theory or photons in hot fluid $\zeta/\eta = 15\left(\Delta v_s^2\right)^2$ parametrically correct also in pQCD (weak coupling)

- might expect that there is a gradual change from one behaviour to the other as a function of temperature



→ linear dependence on Δv_s^2
Buchel's bound satisfied
for $T \leq 1.15\, T_c$

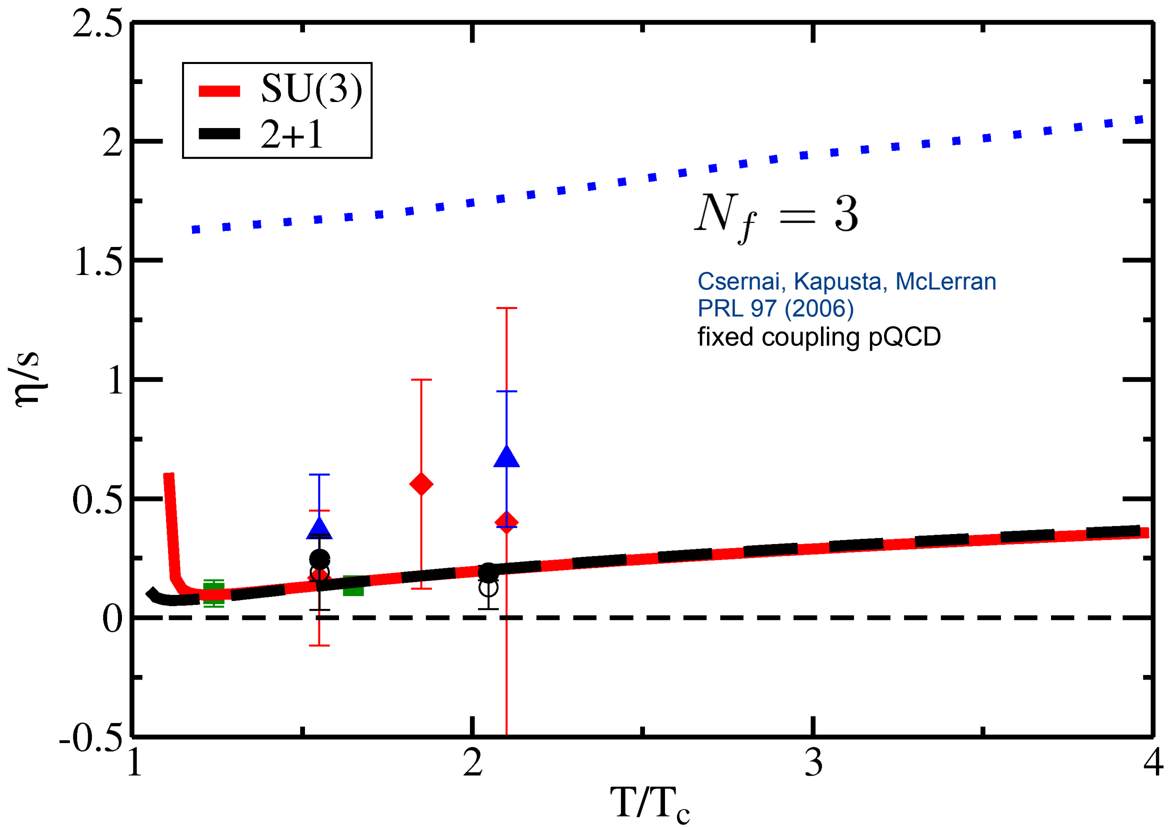
- for $T \to \infty$: $\Delta v_s^2 \to 0$
- for $T \to T_c$: $\Delta v_s^2 \to 1/3$



leading-order estimate:

$$\eta = \eta_g + \eta_q$$

τ_i inversely additive



→ **mild overall increase with T; still small at $3T_c$**

Jet Quenching Parameter

dynamical definition: $\hat{q}(\Lambda) \equiv \int_{q_\perp < \Lambda} d^2 q_\perp \frac{d\Gamma_{el}}{d^2 q_\perp} q_\perp^2$ cf. Arnold, Xiao (2008)

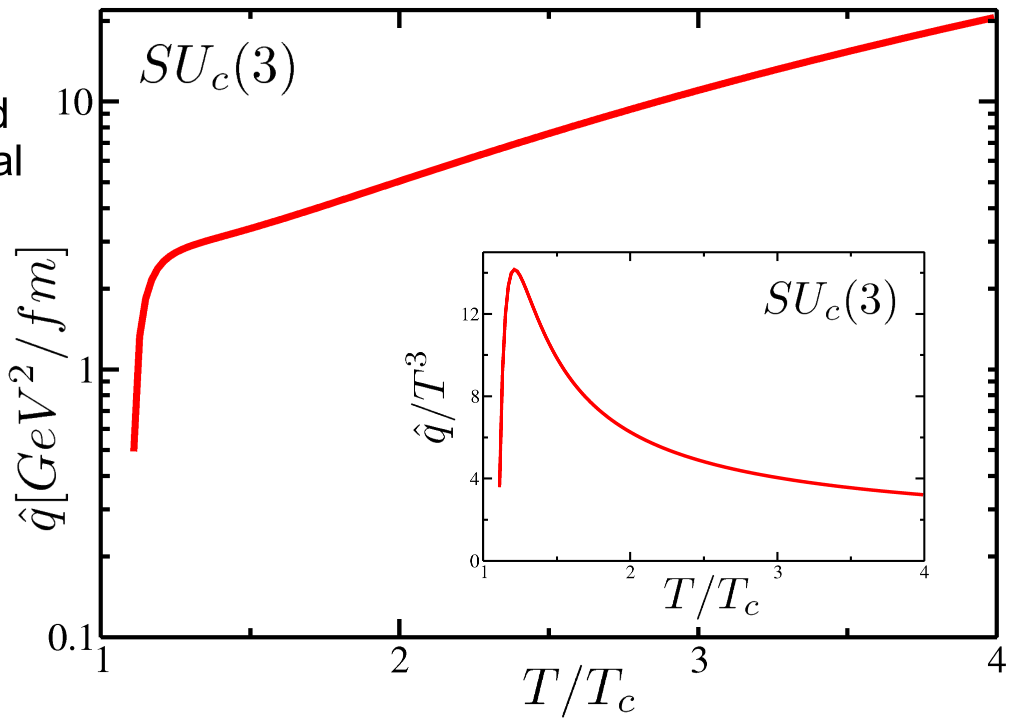
measures transverse momentum transfer squared per unit distance to an energetic parton

assumption: $\Lambda \sim T$

interaction between energetic parton and medium is of same structure and strength as interaction among thermal excitations

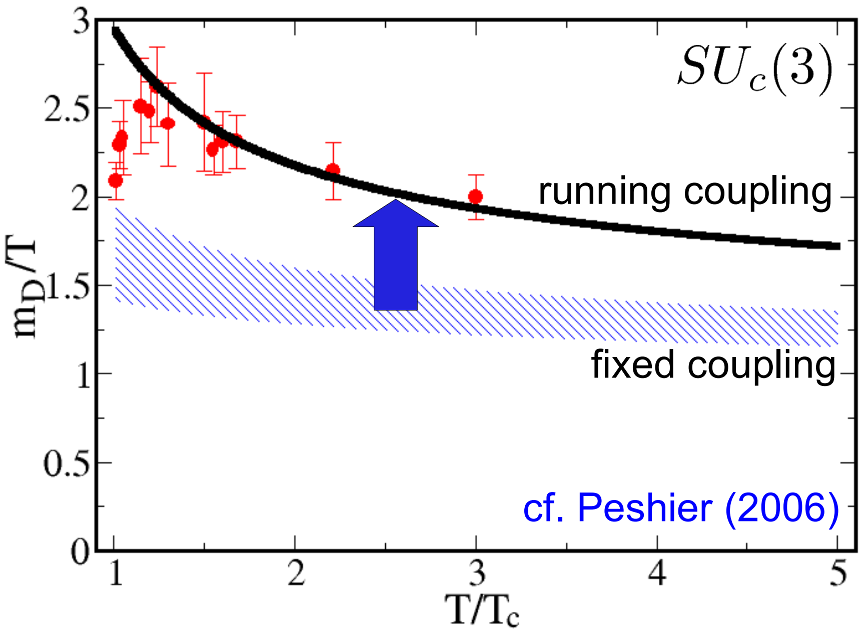
$\Rightarrow \frac{\hat{q}}{T^3} \sim \left(\frac{\eta}{s}\right)^{-1}$

cf. Majumder, Müller, Wang (2007)



➡ Take running of coupling into account in perturbative approach

with Marlene Nahrgang



Motivation: unique relation between η/s and $\hat{q}$?

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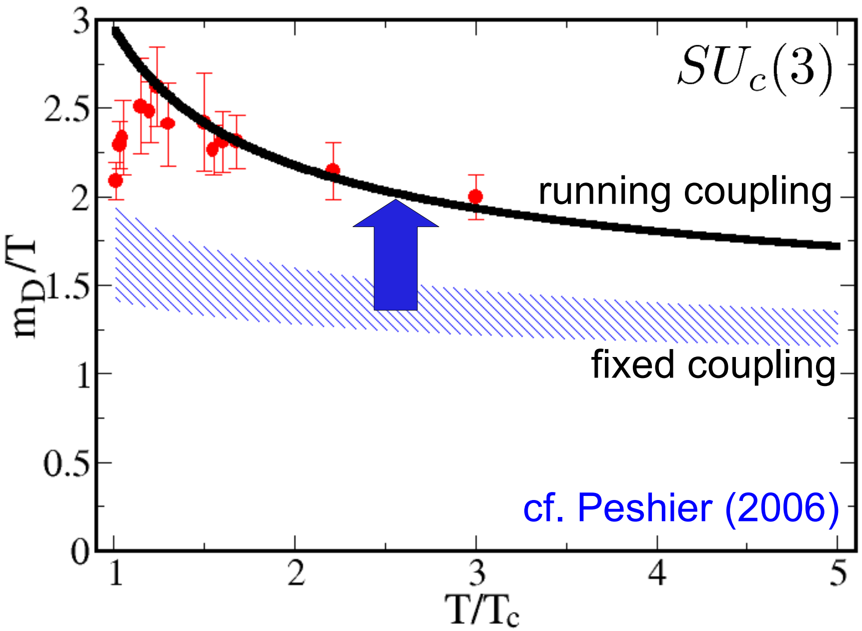

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Work in Progress!

Conclusions

- knowing QCD transport coefficients important for understanding the behaviour of strongly interacting matter observed in high-energy nuclear collisions

  → **picture: excitations with effective thermal mass**

  - inclusion of mean field term in energy-momentum tensor necessary for self-consistency of the approach
  - follows from kinetic equation of Boltzmann-Vlasov type

- influence of a medium-dependent effective mass in dispersion relation minor on shear but prominent in bulk viscosity

- fairly nice agreement w/ available lQCD data; specific shear viscosity as small as $1/4\pi$

- ratio of bulk to shear viscosities exhibits both quadratic and linear dependence on conformality measure; turning point located at the maximum in the scaled interaction measure

- jet quenching parameter pronounced near phase transition
  → influence on energy loss sensitive observables