Nucleon spectroscopy on the lattice

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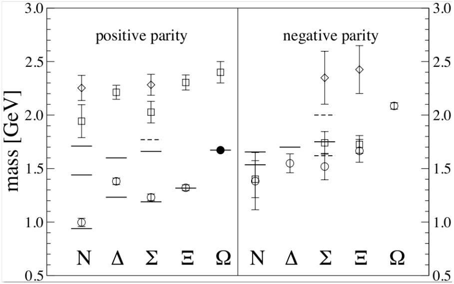
Overview

- **What?** Investigate the πN scattering using lattice spectroscopy.
- **Why?** Study nucleon states with positive and negative parity, with particular interest in the Roper resonance
- **How?** The distillation method and the phase shift analysis represent fundamental tools in the realization of the project.

QCD Spectrum on the lattice



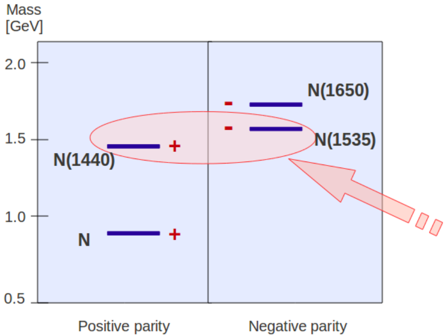
Engel et al. arXiv:1010.2366

Lattice QCD successfully estimates the ground states of the baryon spectrum

but

excited states still represent an outstanding challenge.

Roper resonance

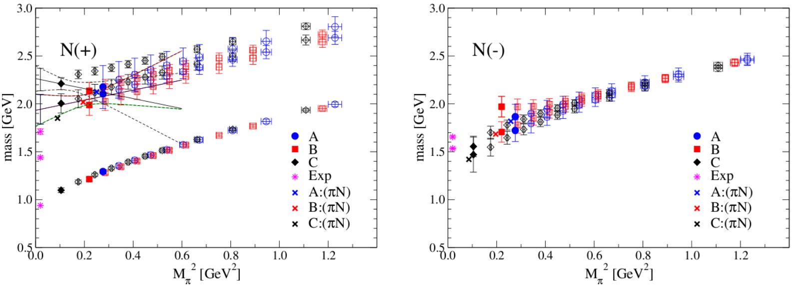


$N(1440)P_{11}$ is the lightest excitation of the Nucleon.

A constituent quark model based on $SU(6)$ symmetry predicts the spectrum of the nucleon to be arranged into successive bands of positive and negative parity.

Roper Resonance on the lattice

Lattice simulations have problems in reproducing the mass reversal order observed in nature.



Why?

$N\pi$ scattering state

The Roper excitation is compatible with the energy level of the scattering state $N\pi$.

It might be necessary to include pion-nucleon interpolators.

- Disconnected diagrams involving backtracking quark loops are needed.

Distillation method

- The study of a 2-particle scattering state requires special treatment:

Phase shift analysis

Mass Spectroscopy on the lattice: Ingredients

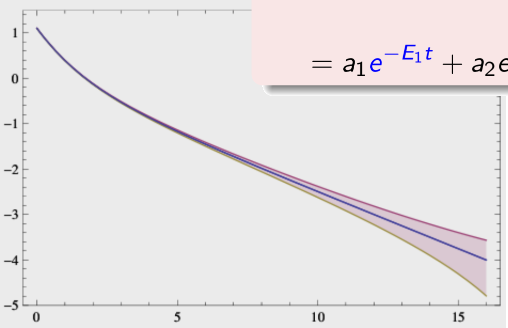
- Action $S = S_{Gauge} + S_{fermion}$
- Gauge configurations with Boltzmann distribution e^{-S}
- Observable for the estimation of the masses of the QCD spectrum: The hadron correlator function.

Hadron Correlation Function

The hadron correlation function in the Euclidean is defined as

$$C_{ij}(t) = \langle \chi_i(t)\chi_j^\dagger(0)\rangle = \sum_n \langle 0|\chi_i|n\rangle e^{-E_n t}\langle n|\chi_j^\dagger|0\rangle =$$

$$= a_1 e^{-E_1 t} + a_2 e^{-E_2 t} + a_3 e^{-E_3 t} + \ldots + \text{noise}$$



$$m_i^2 = E_i^2 - \mathbf{p}^2$$

Compute the Correlation Function

The interpolators:

$$\chi_B(x) = \epsilon_{abc}\,\Gamma_A\, q_a(x)\, q_b(x)\, \Gamma_B\, q_c(x) \qquad \chi_M(x) = q_a(x)\,\Gamma\,\bar{q}_a(x)$$

The correlator involves terms like

$$C(x,y) = \langle\, D\;\, q_a(x)\, q_b(x)\, \underbrace{q_c(x)\,\bar{q}_e(y)}_{M^{-1}(x,y)}\, \bar{q}_f(y)\, \bar{q}_g(y)\rangle.$$

It is required to solve $N_s^3 \times N_t \times N_c$ equations like

$$M(x,y)\phi(y) = \eta(x) \qquad \rightarrow \qquad \phi(x) = M^{-1}(x,y)\eta(y)$$

Determining and storing all the elements of M^{-1} is not possible!

Quark Propagator

- Point-to-all method: compute the propagator for one localized source at given time slice to all the lattice. It works for correlators of single hadron operators concerning connected diagrams.

- We want to study the Roper resonance using multi-hadron operators. Disconnected diagrams are involved.

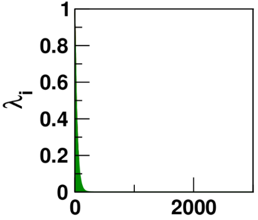
- To evaluate backtracking loops we need to consider many sources on each time slice: all-to-all propagator.

$N_S^3 \times N_c$ inversions are needed: too expensive!

Solution: Distillation Method

Smeared sources + Cut measurement costs

Smearing the quarks with a very low rank operator written in terms of eigenvectors of the 3D Laplacian



$$S(x,x') = \sum_{i=1}^{N_V} v_i(x) v_i^\dagger(x')$$

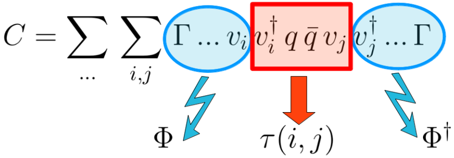
$$u(x)\,\Gamma_1\,u(x)\,\Gamma_2\,d(x) \longmapsto S(x,x')u(x')\,\Gamma_1\,S(x,x')u(x')\,\Gamma_2\,S(x,x')d(x')$$

Distillation

After the distillation

$$q(x) \longmapsto S(x,x^{'})q(x^{'}) = \sum_{i=1}^{N_V} v_i(x)v_i^{\dagger}(x^{'})q(x^{'})$$

the correlation function becomes



$$\phi_{i,j,k}(t) = \Gamma \, v_i(t)v_j(t)v_k(t)$$

N_v^3 matrix

$$\tau_{i,i'}(t,t_0) = v_i^{\dagger}(t)M^{-1}(t,t_0)v_{i'}(t_0)$$

The "perambulator": $(N_d N_v)^2$ matrix

Distillation

The Nucleon two point function on each time slice

$$C(t_1,t_0) = < \mathrm{Tr}\,[\,\Gamma\,\Gamma^\dagger\, M^{-1}_{n,n'}(t_1,t_0)\, M^{-1}_{m,m'}(t_1,t_0)\, M^{-1}_{l,l'}(t_1,t_0)\,] >$$

after the distillation treatment becomes

$$< \mathrm{Tr}\,[\,\phi_{ijk}(t_1)\,\tau_{ii'}(t_1,t_0)\,\tau_{jj'}(t_1,t_0)\,\tau_{kk'}(t_1,t_0)\,\phi^\dagger_{i'j'k'}(t_0)\,] >$$

$$\tau_{ij} = v_i^\dagger(x) M^{-1} v_j(y)$$

$N_v(N_T N_d)$ inversions

instead of

$$M^{-1}(x,y)$$

$N_S^3 N_c (N_T N_d)$ inversions

Simulation setting

- Fermion action: Wilson Clover action with 2 degenate flavours.
- Configurations: 280
- Lattice size: $16^3 \times 32$ $(a = 0.12\,\mathrm{fm})$
- Pion masses: 266 MeV

The πN scattering

sink \ source	O_1	O_2
O_1	P → P	P π_0 → P N π_+ → P
O_2	P π_0 → P N π_+ → P	P π_0 → P π_0 N π_+ → N π_+ P π_0 → N π_+

- $O_1 = P = \chi_1,\ \chi_2,\ \chi_3$

$$\chi_i = \epsilon_{abc}\Gamma_1 u_a \{u_b^T \Gamma_2 d_c - d_b^T \Gamma_2 u_c\}$$

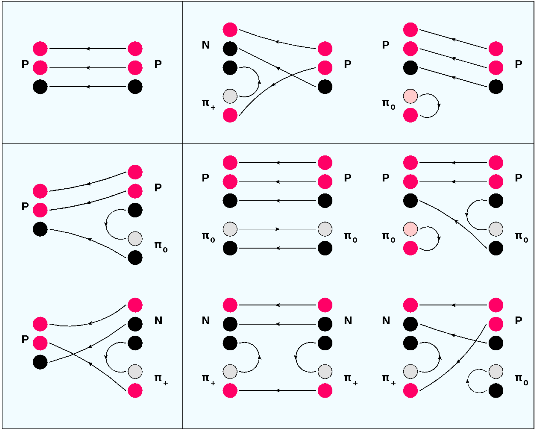
$$\chi_1 \ : \ (\mathbf{1},\ C\gamma_5)$$
$$\chi_2 \ : \ (\gamma_5,\ C)$$
$$\chi_3 \ : \ (i\mathbf{1},\ C\gamma_4\gamma_5)$$

- $O_2 = P\pi_0 + \sqrt{2}\, N\pi_+$

The π N scattering

2 terms

4 terms

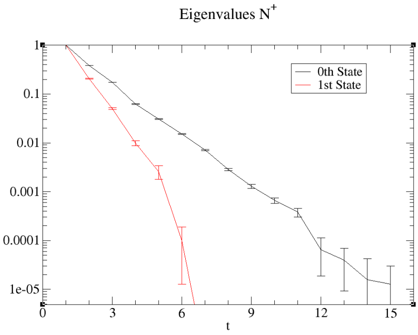


4 terms

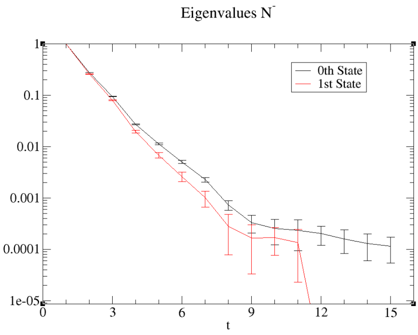
19 terms

Intermediate results: $N \to N$

60 configurations, 3 interpolators, 32 source and sink eigenvectors



positive parity



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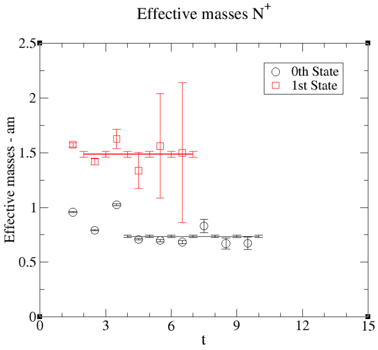

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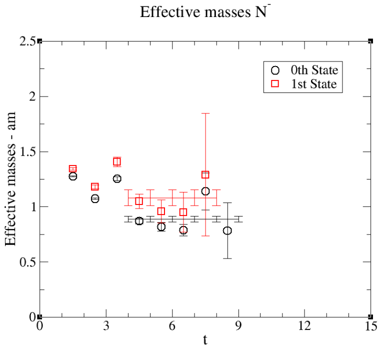

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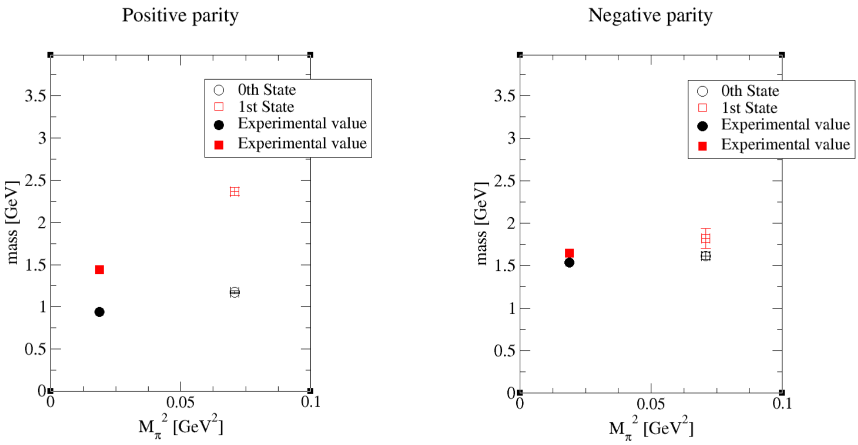
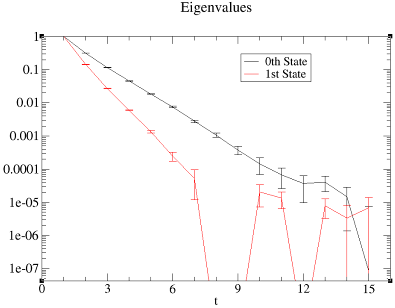


Intermediate results: $\Delta \rightarrow \Delta$

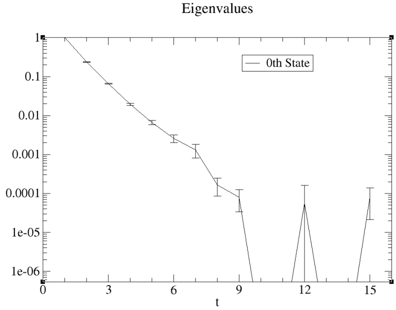
50 configurations, 1 interpolator, 32, 64, 96 eigenvectors

$$\Delta_i = \epsilon_{abc} u_a u_b^T C\gamma_i d_c$$

$$P^{3/2}_{\mu\nu}(\mathbf{0}) = \delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3}(\gamma_0\gamma_\mu\delta_{\mu\nu} + \delta_{\mu\nu}\gamma_\nu\gamma_0)$$



positive parity



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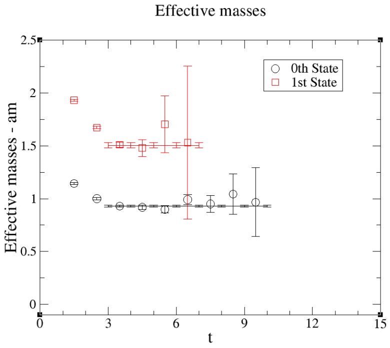

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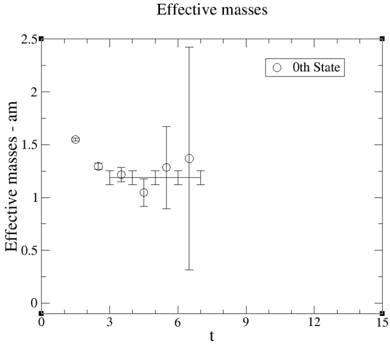

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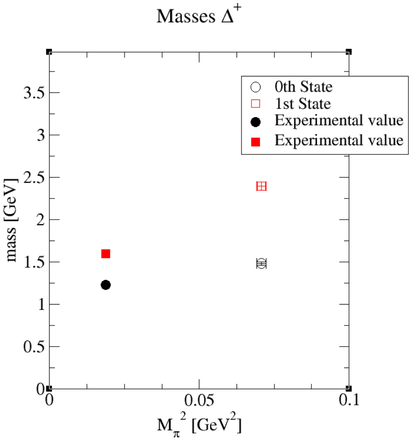
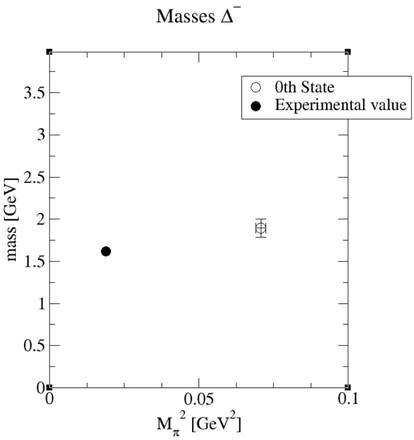




Two particle system

- The energy spectrum of an interacting system of particles is shifted from the system with two free particles.

- The Roper resonance is a P wave of the πN system, relative momentum $\mathbf{p} \neq 0$ is needed.

$$\downarrow$$

The phase-shift analysis at different values of $\mathbf{p}$ gives precise information on the mass of the scattering resonance.

Non zero momentum

- $\mathbf{p} \neq 0$ involves relativistic kinematics

$$\cosh E_{CM} = \sum_{i=1}^{2} \cosh m_i - (1 - \cos p^*) = \gamma^{-1} E$$

- The relativistic distortion reduces the cubic symmetry O_h to that of prismatic dihedral groups

$$\mathbf{p} = (0,0,1) \quad m_1 \neq m_2 :$$

$$O_h \quad \rightarrow \quad C_{4v}$$

Phase shift analysis

The Luescher method connects the discrete spectrum in finite volume with the scattering phase shift in infinite volume

$$\det[e^{2i\delta}(M-i)-(M+i)]=0$$

$$M_{lm,l'm'}(q^2)$$

$$e^{2i\delta_l(p^*)}\delta_{ll'}\delta_{mm'}$$

$$q=\frac{Lp^*}{2\pi}$$

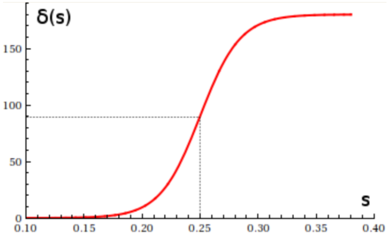
$N\pi$ system with $\mathbf{p} = (0,0,1)$

- The little group of symmetry on the lattice is C_{4v}
- The solutions of the scattering equation belong to reducible representations
  $\Gamma^0 = A_1$
  $\Gamma^1 = A_1 \oplus E$
  $\Gamma^2 = A_1 \oplus B_1 \oplus B_2 \oplus B_3$
- The $P-$wave phase shift is exstracted from irrep E:
  $O(p) = N(p)\pi(0)$

$$\tan \delta_1(p^*) = \frac{\pi^{\frac{3}{2}}\gamma q}{Z_{00}^d - \frac{1}{\sqrt{5}q^{-2}Z_{20}^d(1,q^2)}}$$

Phase shift analysis

The phase shift profile can be fitted against the Breit-Wigner form in the vicinity of a resonance

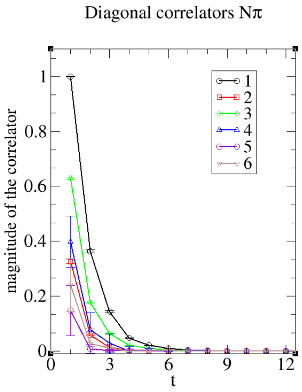


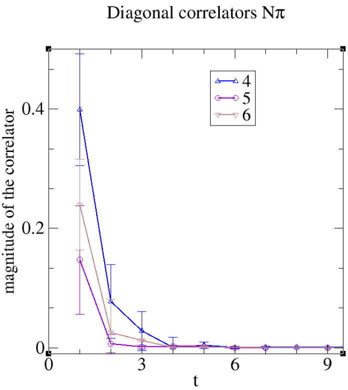
$$\sqrt{s}\,\Gamma(s)\cot\delta(s) = m_R^2 - s$$

to evaluate the mass and width of the resonance.

$N\pi$ correlators

Compare the magnitude of the correlators relative to 1-particle and 2-particle interpolators





Summary

- The mass reversal order of the Roper resonance is a not yet understood issue and the possibility of reproducing its mass in the lattice framework could be a considerable step forward.
- The attempt of obtaining the right mass pattern on the lattice failed. It might be necessary to investigate the pion-nucleon scattering.
- The distillation method represent a fundamental tool to deal with disconnected diagrams in a reasonable computer time.
- Thanks to the flexibility of the method, the same tool could be used for the study of other baryons of the QCD spectrum.