# Study of compact U(1) flux tubes in lattice gauge theory using GPU's

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May 10, 2012

# Outline

- Introduction
- Methods
- Results
- Final remarks

# U(1) gauge theory in the lattice



- Feynman path integral

$$\mathcal{Z} = \int \mathcal{D}A_\mu \; e^{-S_E[A_\mu]} \qquad S_E = \int d^4x \; \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- Operator average $$\langle O \rangle = \frac{\int \mathcal{D}A_\mu \; O[A_\mu] \; e^{-S[A_\mu]}}{\int \mathcal{D}A_\mu \; e^{-S[A_\mu]}}$$

- Discretized action of a pure gauge theory

$$P_{\mu\nu}(x) = 1 - \mathrm{Re}\;\mathrm{Tr}\; U_\mu(x)\; U_\nu(x+\hat{\mu})\; U_\mu^\dagger(x+\hat{\nu})\; U_\nu^\dagger(x)$$

$$S_E = \beta \sum_x \sum_{\mu<\nu} P_{\mu\nu}(x) \text{ where } \beta = \tfrac{1}{g^2}$$

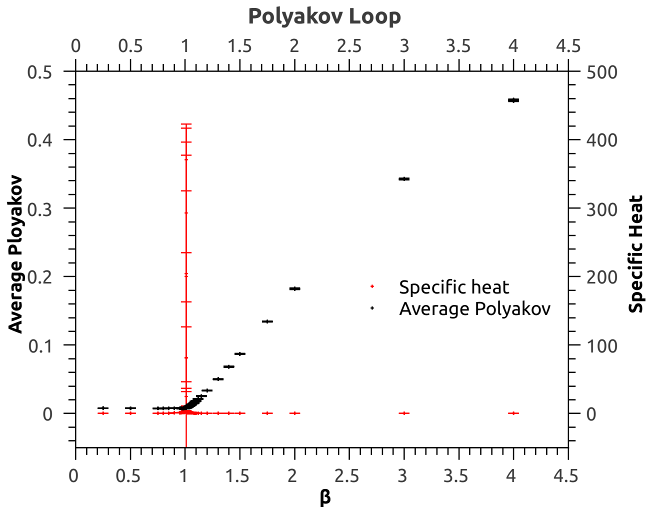
- Polyakov loop

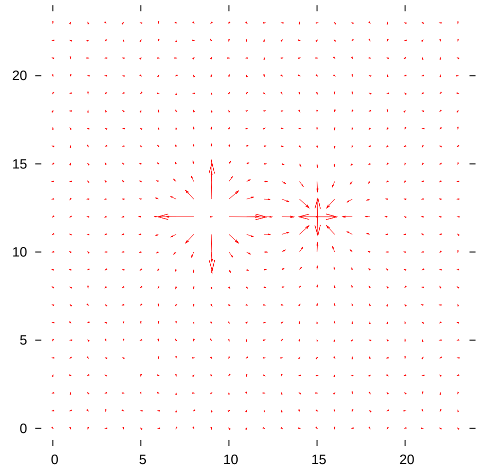
$$L(x) = \prod_{a=0}^{N_t-1} \mathrm{Tr}\; U_0(x + a\hat{t})$$

# 4D compact U(1)



- U(1) group elements $$U_\mu(x) = e^{i\theta_\mu(x)}$$
- 2 phases: confining and unconfining
- Unconfining phase $\longrightarrow$ usual QED



# Potential



- Polyakov loop correlations $\langle L(0)L^\dagger(r)\rangle \approx \exp(-N_t V(r))$

- Potential $V(r) = -\frac{1}{N_t}\log(\langle L(0)L^\dagger(r)\rangle)$

- Plaquette expansion

$$P_{\mu\nu}(x) = U_\mu(x)\, U_\nu(x+\hat{\mu})\, U^\dagger_\mu(x+\hat{\nu})\, U^\dagger_\nu(x) = \tag{1}$$

$$= e^{iag[A_\mu(x+\frac{\hat{\mu}}{2})+A_\nu(x+\hat{\mu}+\frac{\hat{\nu}}{2})-A_\mu(x+\hat{\nu}+\frac{\hat{\mu}}{2})-A_\nu(x+\frac{\hat{\nu}}{2})]} = \tag{2}$$

$$= e^{iag[A_\mu(r-\frac{\hat{\nu}}{2})+A_\nu(r+\frac{\hat{\mu}}{2})-A_\mu(r+\frac{\hat{\nu}}{2})-A_\nu(r-\frac{\hat{\mu}}{2})]}$$

# Electric field operator



Noting that $A_\mu(r - \frac{\hat{\nu}}{2}) = A_\mu(r) + \partial_\nu A_\mu \left(-\frac{a}{2}\right) + \mathcal{O}(a^2)$

$$
\begin{aligned}
& iag[A_\mu(r - \frac{\hat{\nu}}{2}) - A_\mu(r + \frac{\hat{\nu}}{2}) + A_\nu(r + \frac{\hat{\mu}}{2}) - A_\nu(r - \frac{\hat{\mu}}{2})] = \\
= \quad & ia^2 g F_{\mu\nu} + i\mathcal{O}(a^3)
\end{aligned}
$$

$$P_{\mu\nu}(x) = e^{ia^2 g F_{\mu\nu} + i\mathcal{O}(a^3)} = 1 + ia^2 g F_{\mu\nu} - \frac{a^4 g^2}{2} F_{\mu\nu}^2 + i\mathcal{O}(a^3) + \mathcal{O}(a^5)$$

- Electric field operator $\quad a^2 F_{\mu\nu} = \sqrt{\beta}\ \mathrm{Im} P_{\mu\nu} + \mathcal{O}(a^3)$

# Flux tube



- Electric field average in presence of Polyakov loops

$$\langle E \rangle_{PP} = \frac{\langle E P(0) P^\dagger(R) \rangle}{\langle P(0) P^\dagger(R) \rangle} - \langle E \rangle = \frac{\langle E P(0) P^\dagger(R) \rangle}{\langle P(0) P^\dagger(R) \rangle}$$

- We can use an ansatz to extract the shape of the profile

$$\langle E \rangle_{PP} = A \exp(-x^2/s) \frac{1+B\exp(-x^2/s)}{1+D\exp(-x^2/s)} + K$$

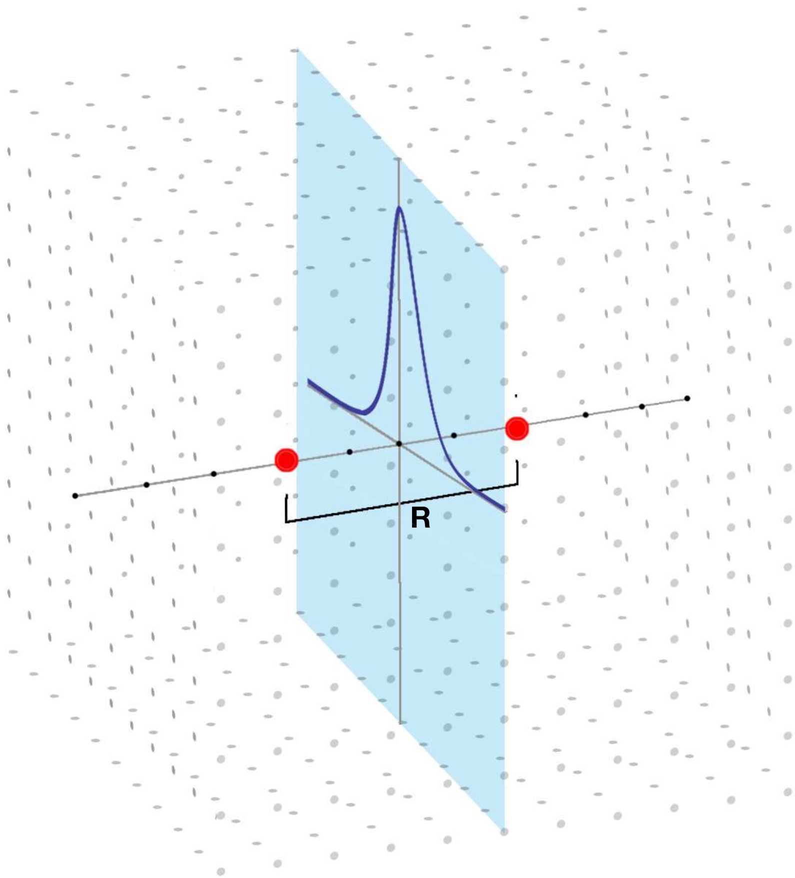
- and calculate its width

$$w_O^2 = \frac{\int dx \;\; x^2 \;\; O(x)}{\int dx \;\; O(x)}$$

# Flux tube



- Evaluation of the field in the middle plane



# Effective string model



- Effective bosonic string model
- Universal (doesn't depend on gauge group)
- Predicts
  - Flux tube between charges
  - Confining potential $V(r) \propto \sigma r - \frac{(d-2)\pi}{24r}$
  - Roughening of the flux tube
    - Low temperature $w_{lo}^2(r/2) \approx \frac{d-2}{2\pi\sigma} \log \frac{r}{r_0}$
    - High temperature $w_{lo}^2(r/2) \approx \frac{d-2}{2\pi\sigma} \log \frac{\beta}{4r_0} + \frac{d-2}{4\beta\sigma} r$

# Generation of configurations



- Metropolis + Overrelaxation

Typically 1 metropolis : 3 overrelaxation

- Lüscher-Weisz Multilevel algorithm (2 levels)
- Algorithm to get one multilevel configuration:

Update the whole lattice n times
i0$\times$
    Update j0$\times$ except time layers multiple of 4
        i1$\times$
            Update j1$\times$ except time layers multiple of 2
        Average operator in the sublattices multiple of 2
    Average the result in the sublattices multiple of 4
Average the result in the whole lattice

# Generation of configurations



- Multilevel

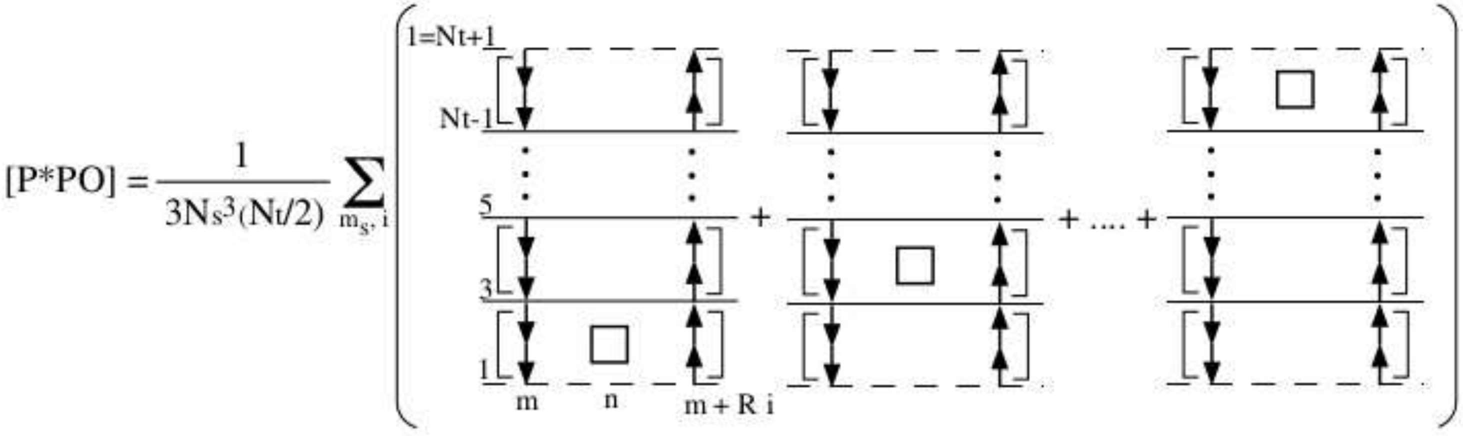
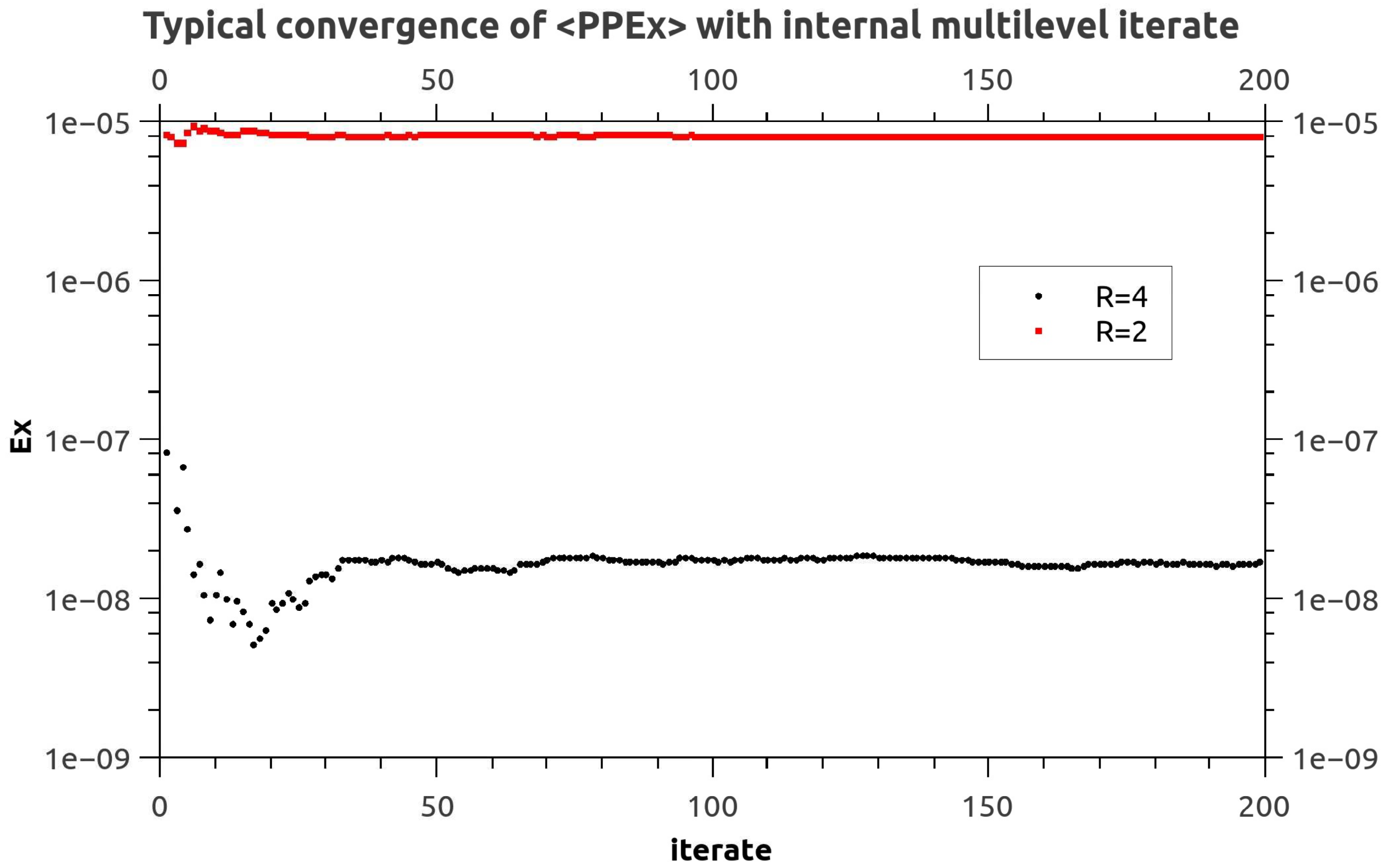



Figure 1: [Y. Koma, M. Koma, P. Majumda, 2004]

- $T(m;R;i) = U_4^*(m)\ U_4(m + R\ \hat{i})$
- $O(m;n;R;i) = U_4^*(m)\ U_4(m + R\ \hat{i})\ \mathcal{O}(\backslash)$
- $TO^{(2)}(m;n;R;i) =$
  $[T(m;R;i)\ O(m+\hat{4};n;R;i) + T(m+\hat{4};R;i)\ O(m;n;R;i)]$

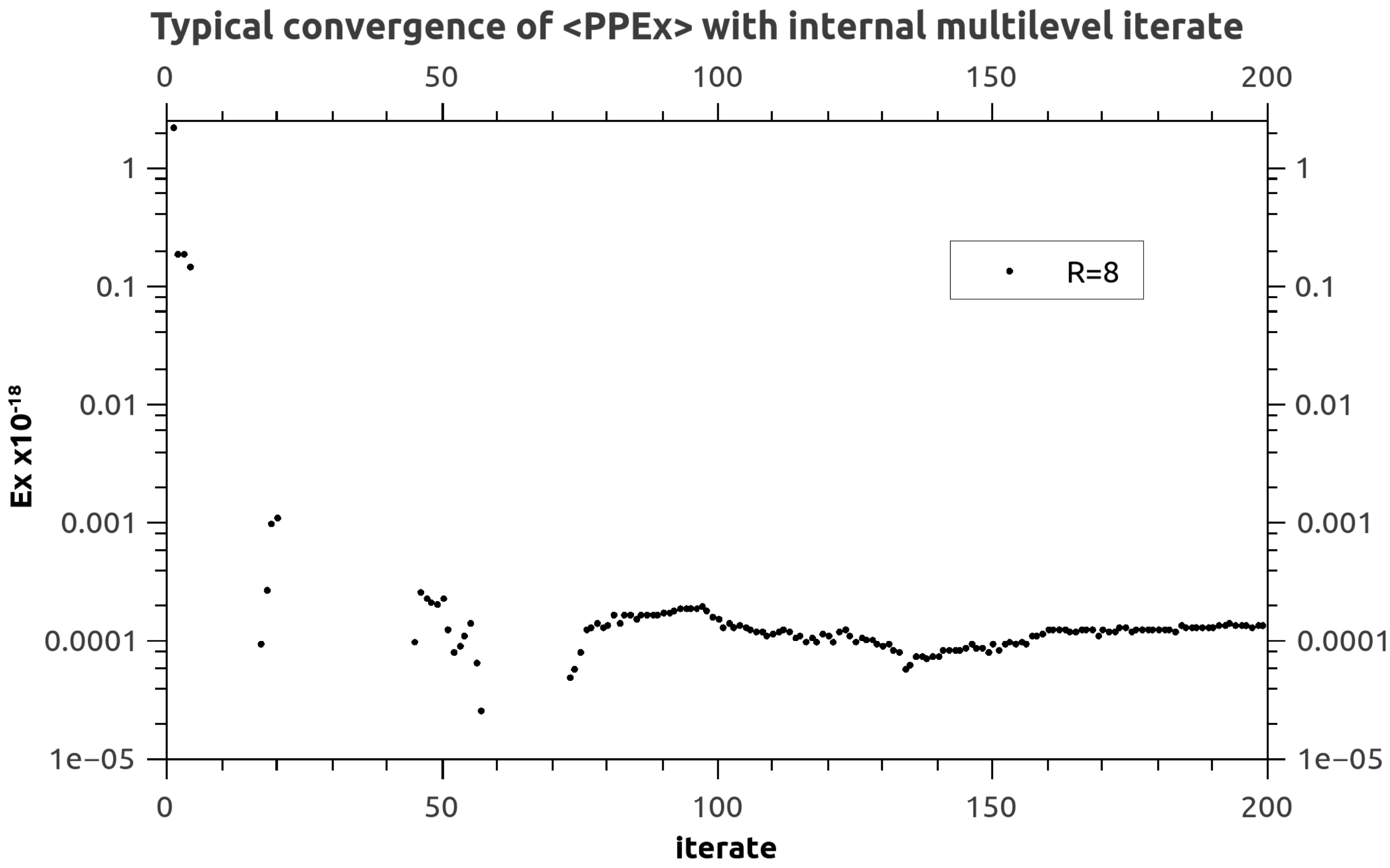
# Convergence with Multilevel iterate





# Convergence with Multilevel iterate





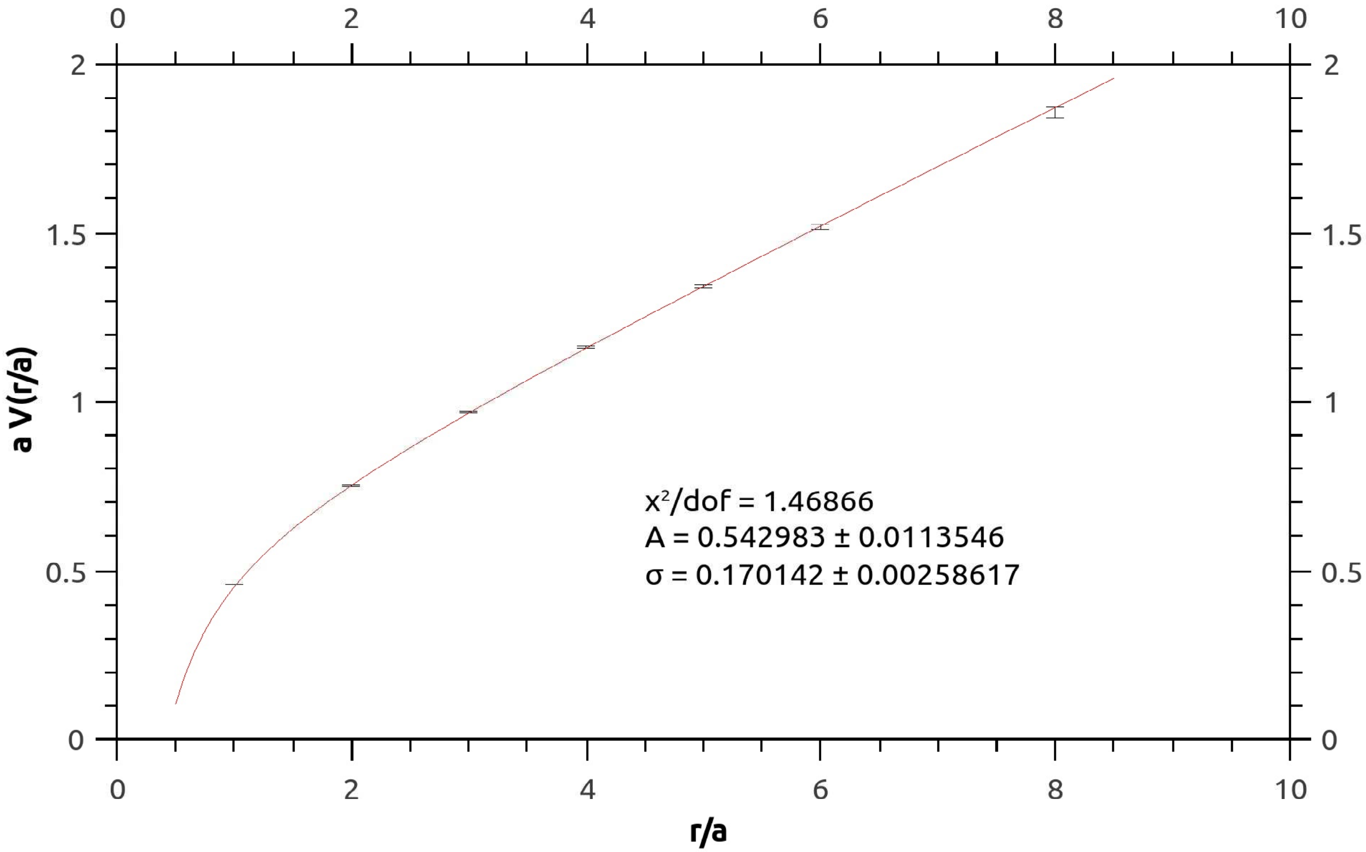
# CUDA



- NVIDIA GeForce GTX 580
  - Fermi architecture
  - 512 cores
  - 1536 or 3072 Mb

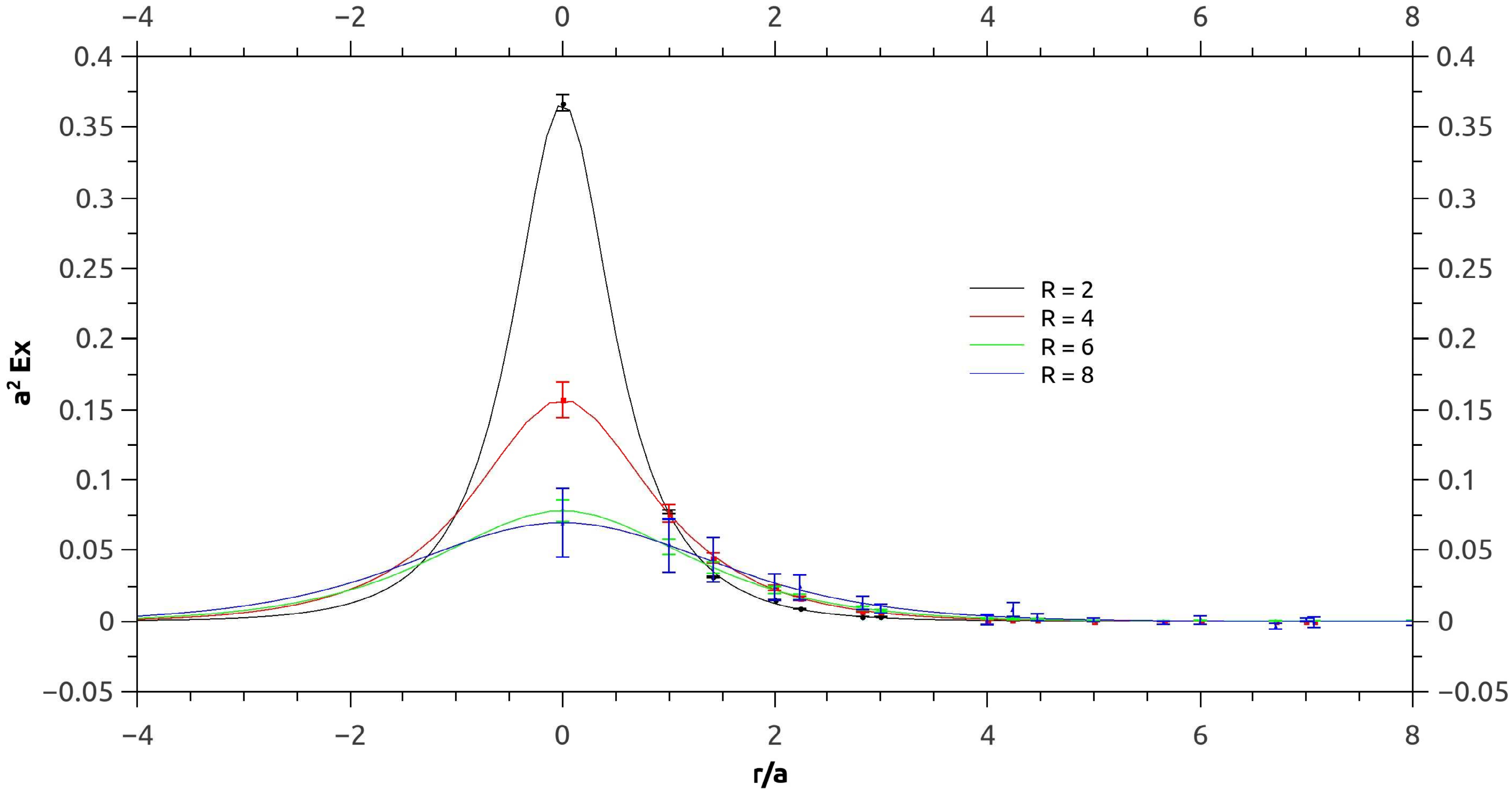
# Potential

- Potential $V(r) = A + \sigma x - \frac{\pi}{12x}$



# Flux tube profile

- Flux tube profile $\langle E \rangle_{PP} = A \exp(-x^2/s) \frac{1+B\exp(-x^2/s)}{1+D\exp(-x^2/s)} + K$

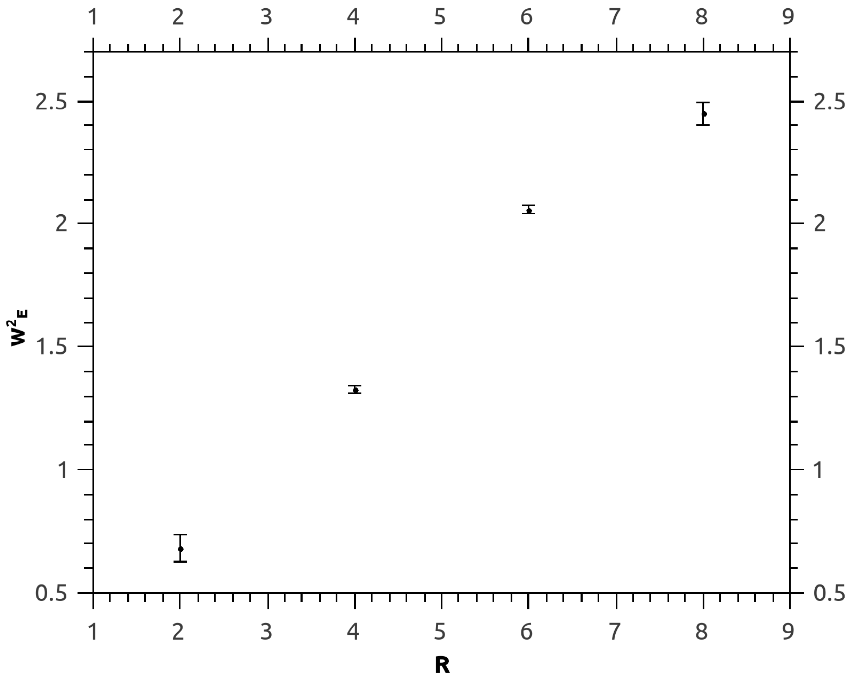


# Flux tube width



- Flux tube width

| R | $w^2$ |
|---|---|
| 2 | $0.682636 \pm 0.0550651$ |
| 4 | $1.32546 \pm 0.0154068$ |
| 6 | $2.05796 \pm 0.0177265$ |
| 8 | $2.44814 \pm 0.0460675$ |



# Final remarks

- Test the Effective bosonic string model validity in U(1)
- Caracterize the confinement in U(1), specially the roughening of the flux tube
- Simulations at 0 and finite temperature
- Increase our statistics
- Calculate $\langle E^2 \rangle$ and $\langle B^2 \rangle$ to compare with string model results
- Eventually compare our results with the results obtained using dual models of 4D compact U(1)