

CuBA

The Boltzmann Approach for Many Parton Scattering written with CUDA



CFTP, Departamento de Física
Instituto Superior Técnico
Lisboa

Pedro Bicudo
Nuno Cardoso
Janni Bouras
Ulrike Eilhauer

Motivation

- Simulate heavy ion collision, particularly gluon plasma
- Cpu code BAMPS was developed by C. Greiner and Z. Xu from Frankfurt.
- CuBA is physically equivalent to BAMPS, but recoded in CUDA
- The code is benchmarked with the relativistic Riemann Problem
- Comparison to the original BAMPS

Considerations

Number of particles per cell > 10

Cell size:

$$\lambda = \frac{1}{n\sigma} = \frac{L^3}{N_{num}\sigma}$$

Monte Carlo:

$$P_{22} = v_{rel} \frac{\sigma}{r_{test}} \frac{\Delta t}{L^3} \quad v_{rel} = \frac{s}{2E_1 E_2}$$

Relativity effects

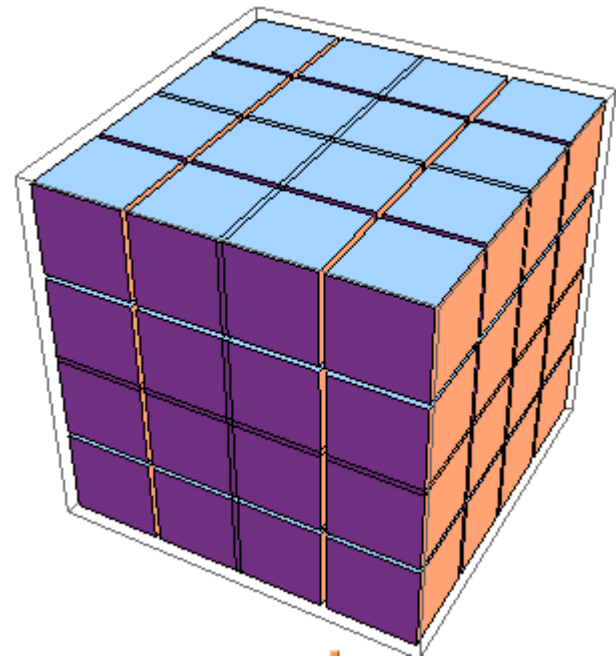


Figure1: Dividing 3D-Space into cells

The code in CUDA

- Advantages of using CUDA:
 - CUDA has a fast shared memory region that can be shared amongst threads.
 - Fast accesses to and from memory of the GPU
 - Calculations and data is GPU
 - Try to minimize transfers
- Memory accommodates states of all particles corresponding to each particle:
 - Cell number and particle ID
 - 4-vector position
 - 4-vector momentum

The code in CUDA

- CUDA logic:
 - Each block has until 1024 threads and each block has to be lower than $1024 \times 1024 \times 64$
 - Each block has 256 threads
 - Each grid has about $65535 \times 65535 \times 65535$ blocks for Fermi and $65535 \times 65535 \times 1$ for older architectures

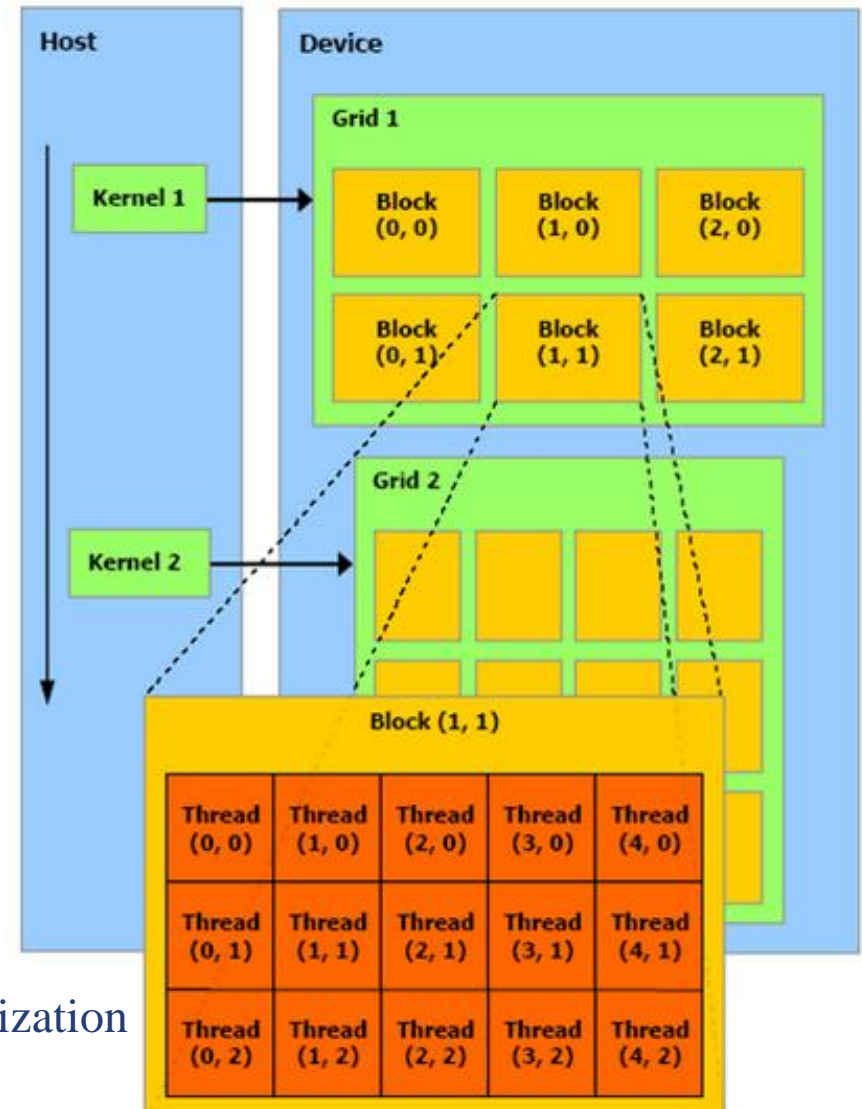
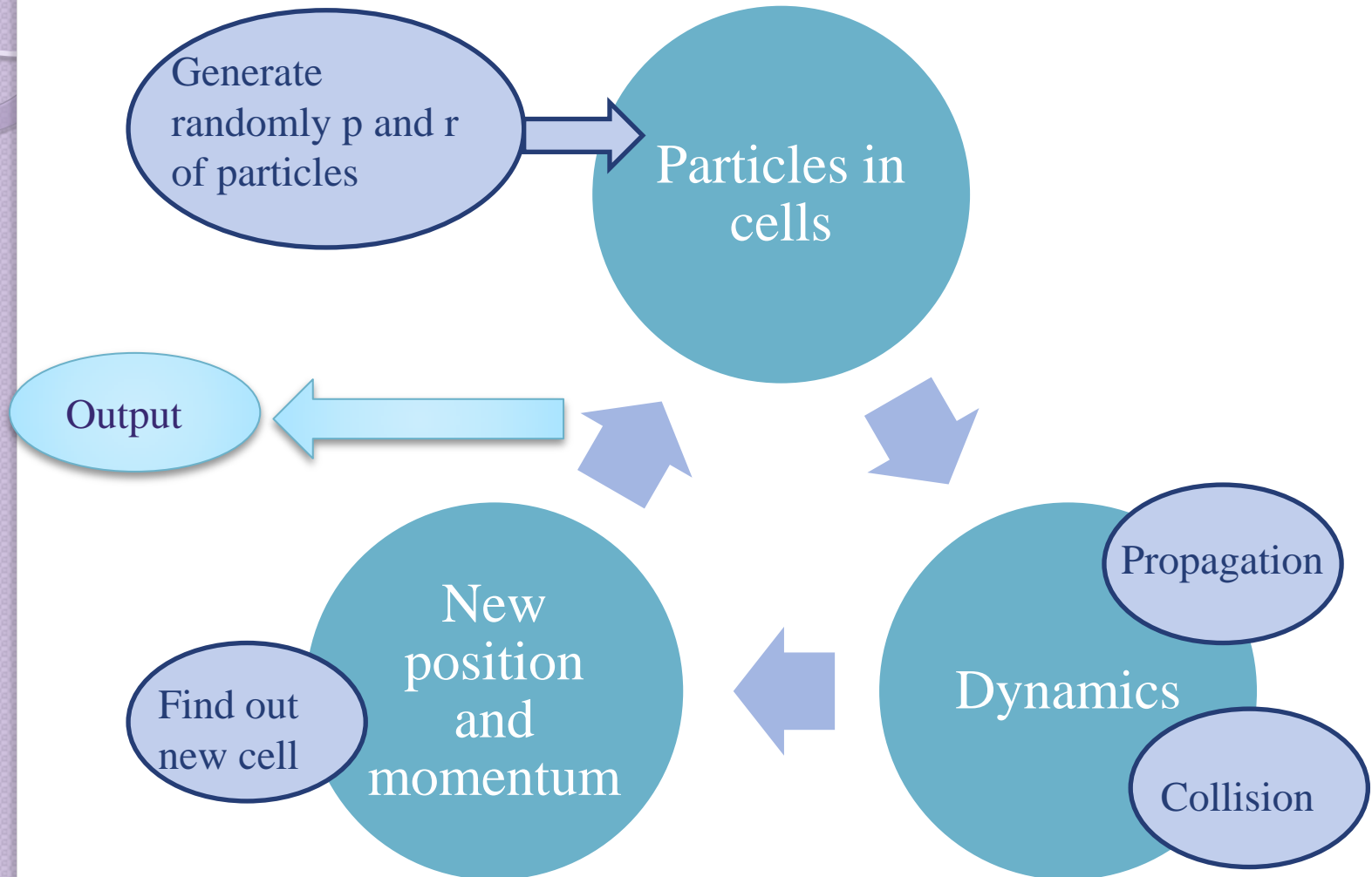


Figure2: CUDA grid organization

How to make the code work



How to make the code work

- Movement of particles:
 - Propagation
 - Collisions
 - Relativistic two body kinematics
 - With the wall
- Find out new cell

Propagation

Propagation is interesting for the particles without collision and after collision occurred.

$$x \rightarrow x + v_x \Delta t = x + c^2 \frac{p_x}{E} \Delta t$$

$$y \rightarrow y + v_y \Delta t = y + c^2 \frac{p_y}{E} \Delta t$$

$$z \rightarrow z + v_z \Delta t = z + c^2 \frac{p_z}{E} \Delta t$$

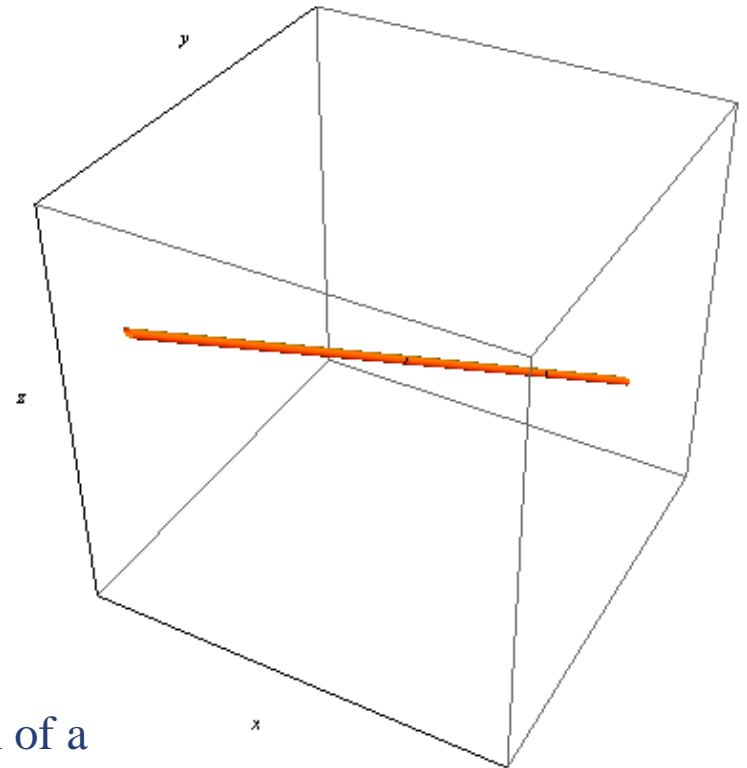


Figure3: Propagation of a particle in space

Relativistic Collision Kernel

S-wave scattering

Assume energy-
independent Cross section

Boost to the
center of mass
frame

Random
generation of
the momentum
direction

Boost back to
the plasma
frame

Relativistic Collision Kernel

1. Boost to the center of mass

$$\mathbf{p}'_1 + \mathbf{p}'_2 = 0 \quad \beta = \frac{V}{c} = c \frac{p_{1\parallel} + p_{2\parallel}}{E_1 + E_2} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

Getting:

$$E_1 \rightarrow E_1' = \gamma(E_1 - \beta c p_{1\parallel})$$
$$c p_{1\parallel} \rightarrow c p_{1\parallel}' = \gamma(-\beta E_1 + c p_{1\parallel})$$
$$c \mathbf{p}_{1\perp} \rightarrow c \mathbf{p}_{1\perp}' = c \mathbf{p}_{1\perp}$$

2. Random generation of the momentum direction

$$p_1''_x = \frac{1}{c} \sqrt{s/4 - M^2 c^4} \cos \phi \sqrt{1 - \omega^2}$$
$$p_1''_y = \frac{1}{c} \sqrt{s/4 - M^2 c^4} \sin \phi \sqrt{1 - \omega^2}$$
$$p_1''_z = \frac{1}{c} \sqrt{s/4 - M^2 c^4} \omega$$

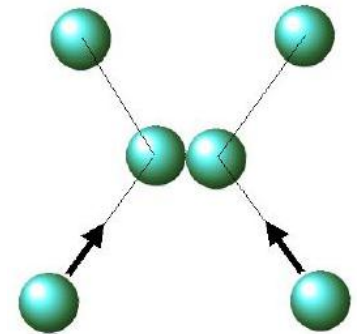


Figure4: Colliding particles

Relativistic Collision Kernel

3. Boost back to plasma frame

Using again the Lorentz transformation

$$\begin{aligned}E_1'' = E_1' &\rightarrow E_1''' = \gamma(E_1' + \beta cp_{1\parallel}'') \\cp_{1\parallel}'' &\rightarrow cp_{1\parallel}''' = \gamma(+\beta E_1' + cp_{1\parallel}'') \\cp_{1\perp}'' &\rightarrow cp_{1\perp}''' = cp_{1\perp}''\end{aligned}$$

$$\mathbf{p}_1''' = p_{1\parallel}''' \hat{P} + \mathbf{p}_{1\perp}'''$$

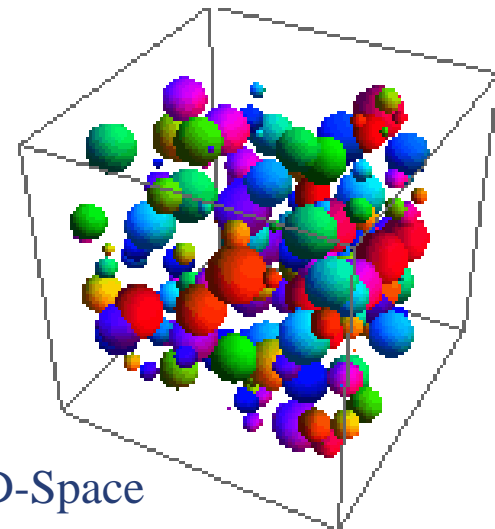


Figure5: Particles in 3D-Space

Collisions with the wall

We have six walls, so we have six 'ifs':

if $x > x_f$

if $x < x_b$

if $y > y_r$

if $y < y_l$

if $z > z_u$

if $z < z_d$

For any of these conditions we reverse the perpendicular momentum:

$$px = -px$$

$$px = -px$$

$$py = -py$$

$$py = -py$$

$$pz = -pz$$

$$pz = -pz$$

And the position:

$$x = -x + 2x_f$$

$$x = -x + 2x_b$$

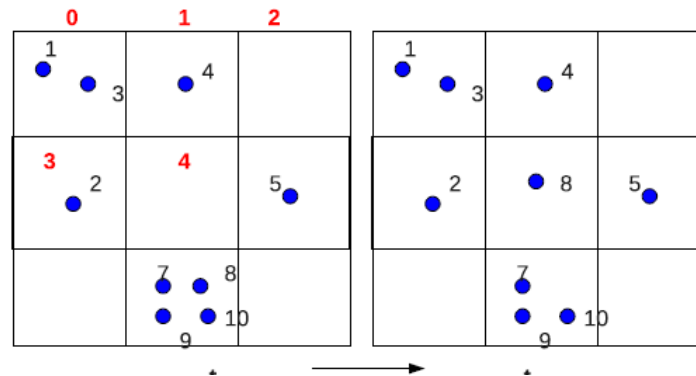
$$y = -y + 2y_r$$

$$y = -y + 2y_l$$

$$z = -z + 2z_u$$

$$z = -z + 2z_d$$

Find out new cell

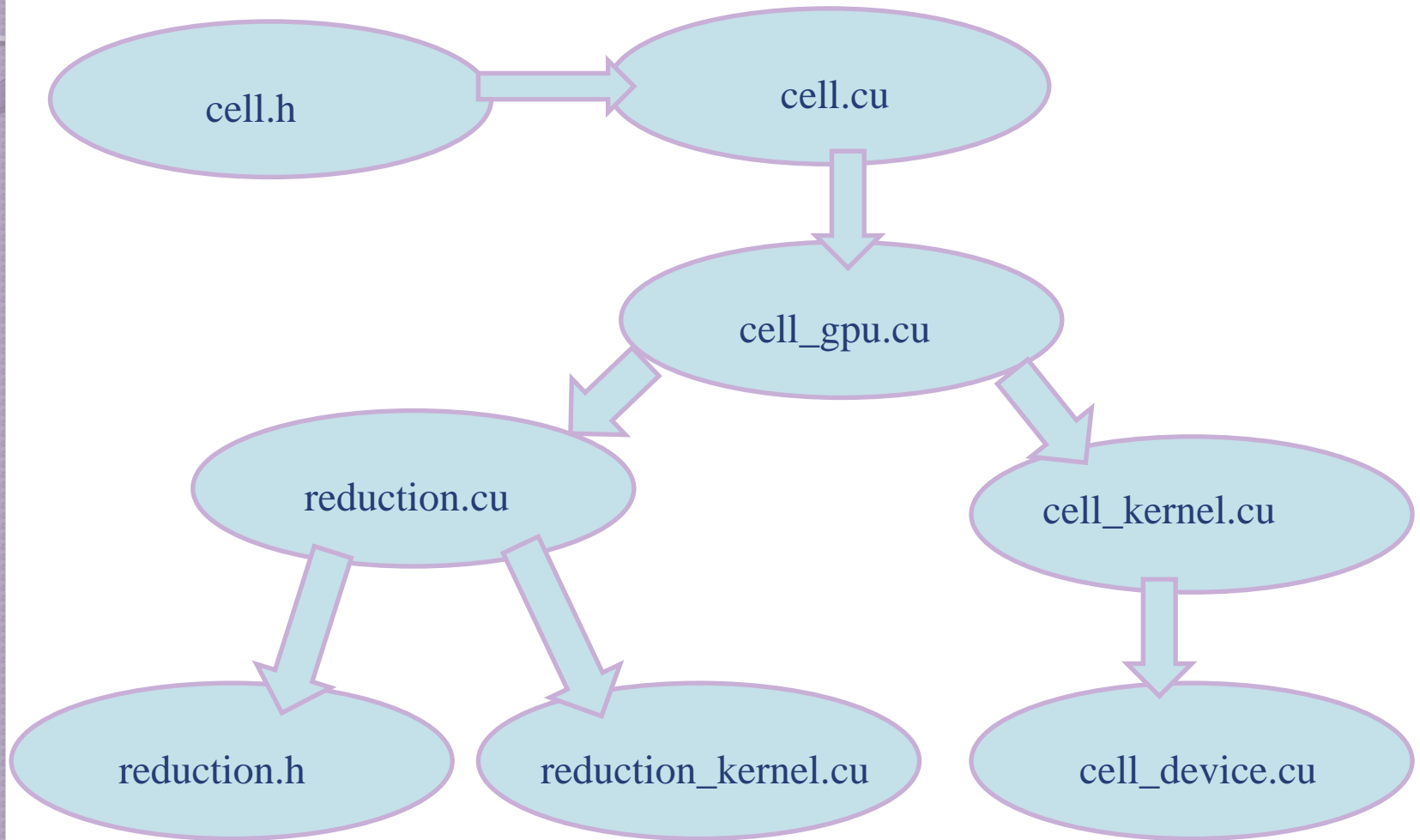


t_0	array index:	0	1	2	3	4	5	6	7	8
	.y (particle id)	1	3	4	2	5	7	8	9	10
	.x (cell id)	0	0	1	3	5	7	7	7	7
	Cell id = array index:	0	1	2	3	4	5	6	7	8
	cell starts on array 2	0	2	-1	3	-1	4	-1	5	-1

t_1	array index:	0	1	2	3	4	5	6	7	8
	.y (particle id)	1	3	4	2	8	5	7	9	10
	.x (cell id)	0	0	1	3	4	5	7	7	7
	Cell id = array index:	0	1	2	3	4	5	6	7	8
	cell starts on array 2	0	2	-1	3	4	5	-1	6	-1

Figure6: Scheme particles in a cell

Our code structure



Results

- Testing the code with the Riemann Problem
- Divide the system with a barrier, assuming special initial conditions
 - Massless Boltzmann gas
 - $T_{\text{left}} = 0.4\text{GeV}$ and $T_{\text{right}} = 0.2\text{GeV}$
 - Notice x is lower than Δt
- Objective: Comparison BAMPS and CuBA:
Compatibility and Time

Results

- Initial conditions:

```
:: cuBA ::
```

```
Number of CUDA devices: 2
    0 : GeForce GTX 580
    1 : GeForce GTX 580
Device supporting CUDA compute capability: 2.0
Double precision is supported.
```

```
> Using Double Precision.
```

```
----- Initial Conditions -----
Boxlength in X:      32.500000 GeV^-1
Boxlength in Y:      32.500000 GeV^-1
Boxlength in Z:      32.500000 GeV^-1
Volume of Box:       34328.125000 GeV^-3
CellLength in X:     0.084635 GeV^-1
CellLength in Y:     32.500000 GeV^-1
CellLength in Z:     32.500000 GeV^-1
Volume of cell:      89.396156 GeV^3
Total cells per side: 384 x 1 x 1
Total cells:         384
Initial particles left: 232927
Initial particles right: 29115
Total particles:     262042
TestParticles:       1
Temperature left:    0.400000
Temperature right:   0.200000
Number of iterations: 500
Time step:           0.010000 GeV^-1/c
Total time:          5.000000 GeV^-1/c
```


Results

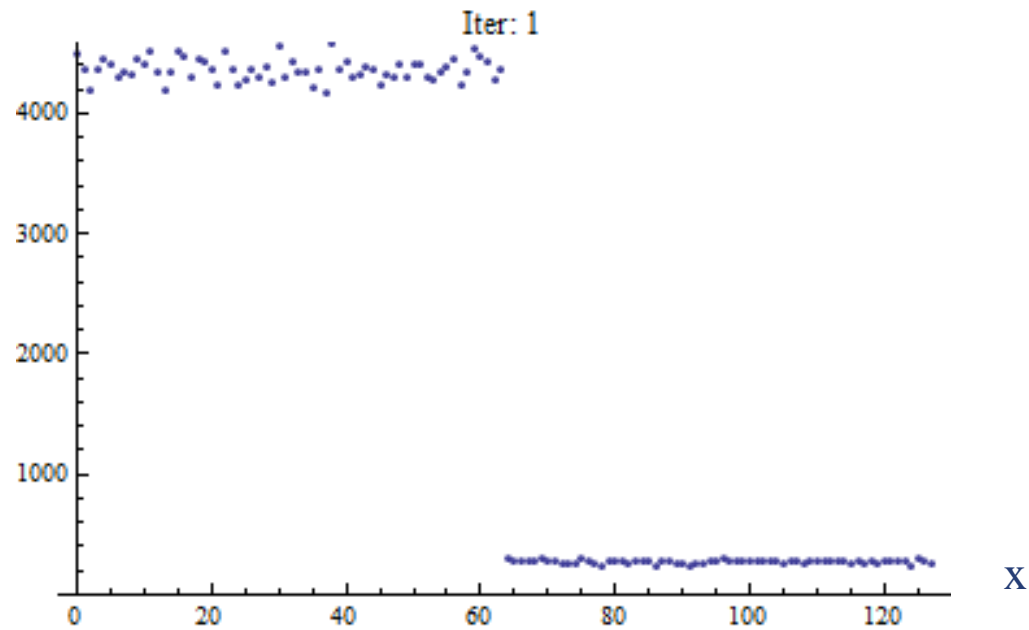
- Observing energy conservation:

```
writing file....  
<-> Iteration: 1  
-> Time: 1.0000e-02 fm/c  
  > Collision Kernel  
  > Propagate Kernel  
  > Sort Cells and Particles per cell  
  > Start/End Cell  
  > Calculate Energy and Mean velocity per cell  
  > Calculate Total Energy  
-> Total Energy: 2.96720997e+05 GeV  
  
writing file....  
<-> Iteration: 500  
-> Time: 5.0000e+00 fm/c  
  > Collision Kernel  
  > Propagate Kernel  
  > Sort Cells and Particles per cell  
  > Start/End Cell  
  > Calculate Energy and Mean velocity per cell  
  > Calculate Total Energy  
-> Total Energy: 2.96720997e+05 GeV  
.....
```

- Time: Total time: 33.888449 (s)
Time to initialize: 0.157593 (s)
Time spent in iterate: 33.730855 (s)

The Riemann Problem

Energy (GeV)



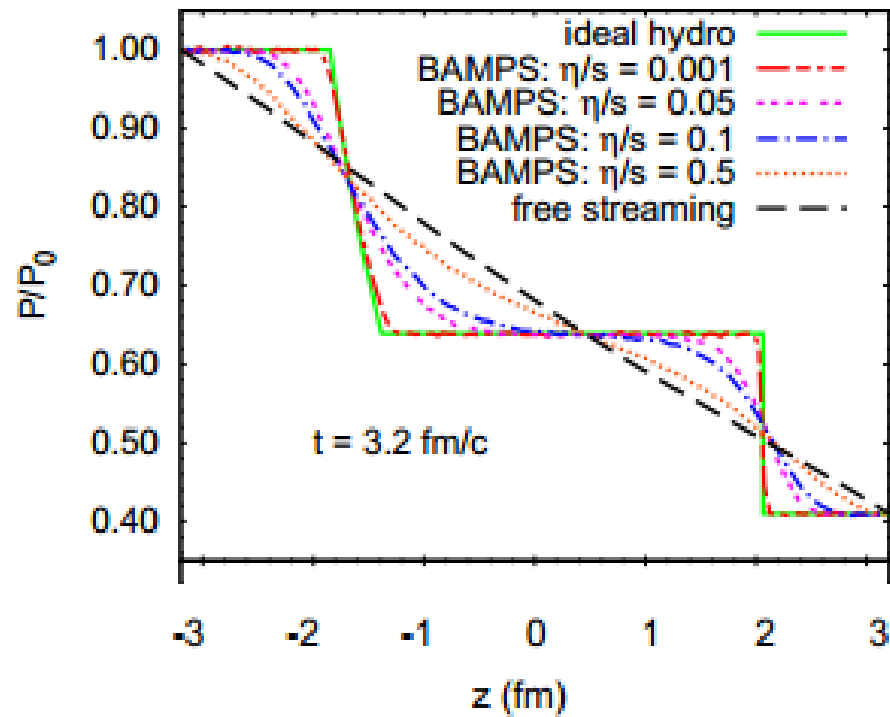
Video: Energy in function of y

Comparison BAMPS vs. CuBA

- Energy conservation is present
- Time : CuBA is x times faster than BAMPS
- Improving : CuBA is still in evolution
- Next step: More variables to compare

The Riemann Problem

BAMPS result:



THE END

