The QCD phase diagram in $N_{\chi}FD$

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Excited QCD 2012, Peniche
\[ \mathcal{L} = \overline{q}^\alpha (i \gamma^\mu D_{\mu,\alpha\beta} - m_q \delta_{\alpha\beta}) q^\beta - \frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a} \]
How to study the QCD phase diagram...

... be brave and solve

\[ Z(T, \mu_B) = \int \mathcal{D}(A, q, q^\dagger) e^{-S_{\text{QCD}}^E} \]

ab initio and nonperturbatively,

... be strong and collide heavy ions at ultrarelativistic energies,

... be creative and study effective models of QCD.
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\[ \mathcal{L}_{\text{eff}} \]
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Critical point

\[ m_\sigma^2 = \frac{\partial^2 V}{\partial \sigma^2} \to 0 \]

- correlation length diverges
  \[ \xi = \frac{1}{m_\sigma} \to \infty \]
- universality classes
  for QCD: \( O(4) \) Ising model in 3d \( \Rightarrow \langle \sigma^2 \rangle \propto \xi^2 \)
- renormalization group
- critical opalescence

\[ \Rightarrow \text{Large event-by-event fluctuations in thermal systems!} \]
First order phase transitions

- two degenerate minima separated by a barrier
- latent heat
- phase coexistence
- supercooling effects in nonequilibrium situations
- nucleation
- spinodal decomposition

(I.N. Mishustin, PRL 82 (1999); Ph. Chomaz, M. Colonna, J. Randrup, Physics Reports 389 (2004))

⇒ (Large) fluctuations in single events in nonequilibrium situations!
The critical point in heavy-ion collisions

\[ \langle (\Delta \sigma)^2 \rangle = \frac{T}{V} \xi^2 \]

assuming a coupling of the order parameter to pions \( g(\sigma \pi \pi) \) and protons \( G(\sigma \bar{p}p) \)

\[ \frac{\langle (\delta N)^2 \rangle}{N} \propto \xi^2 \propto \langle (\Delta \sigma)^2 \rangle \]

(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRL 81 (1998), PRD 60 (1999))

system size dependence


more systematically in NA61
The critical point in dynamic systems

long relaxation times near a critical point $\Rightarrow$ critical slowing down $\Rightarrow$ the system is driven out of equilibrium

$$\frac{d}{dt} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{eq}(t)})$$

with $\Gamma(m_\sigma) = \frac{A}{\zeta_0} (m_\sigma \zeta_0)^z$

$z = 3$
(dynamic) critical exponent

$\Rightarrow \zeta \sim 1.5 - 2.5 \text{ fm}$

Higher moments and the kurtosis

Higher moments are more sensitive to the growth of the correlation length: \( \langle (\delta N)^4 \rangle / N \propto \xi^4 \) \(^{(M. A. Stephanov, PRL 102 (2009)}\)

Kurtosis is negative at the critical point. \(^{(M. A. Stephanov, PRL 107 (2011)}\)

- Negative kurtosis is also given by superposition many particle distributions, net-baryon number conservation...
Motivation

- Fluctuations have so far been investigated in static, equilibrated systems.
- However, systems created in heavy-ion collisions are finite in size and time and inhomogeneous.
- Necessary to propagate fluctuations explicitly!

- Nonequilibrium chiral fluid dynamics ($N\chi$FD):
  - phase transition model +
  - dissipation and noise +
  - fluid dynamics
Motivation

Two questions in finite, dynamic systems:

- Are nonequilibrium effects strong enough to develop signals of the first order phase transition?
- Do enhanced equilibrium fluctuations at the critical point survive?
The linear sigma model with constituent quarks

\[ \mathcal{L} = \overline{q} \left[ i \gamma^\mu \partial_\mu - g \left( \sigma + i \gamma_5 \tau \vec{\pi} \right) \right] q + \frac{1}{2} \left( \partial_\mu \sigma \right)^2 + \frac{1}{2} \left( \partial_\mu \vec{\pi} \right)^2 - U(\sigma, \vec{\pi}) \]

\[ U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4} \left( \sigma^2 + \vec{\pi}^2 - \nu^2 \right)^2 - h q \sigma - U_0 \]

\( g = 3.3 \): crossover at \( \mu = 0 \)

\( g = 5.5 \): first order pt at \( \mu = 0 \)

The effective potential at $\mu_B = 0$

$$\Omega_{\text{eff}} = -\frac{T}{V} \ln Z_{\text{th}} = -d_q T \int \frac{d^3p}{(2\pi)^3} \ln \left( 1 + \exp \left( -\frac{E}{T} \right) \right) + U(\sigma, \vec{\pi})$$

with dynamically generated quark masses $E = \sqrt{p^2 + g^2 \sigma^2 + g^2 \vec{\pi}^2}$

For a qualitative study tune the strength of the phase transition via the coupling $g$. 
Nonequilibrium chiral fluid dynamics - N$\chi$FD

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \rho_s + \eta \partial_t \sigma = \zeta$$

- Fluid dynamic expansion of the quark fluid = heat bath, including energy-momentum exchange

$$\partial_\mu T_{q\mu \nu} = S_\nu = -\partial_\mu T_{\sigma \mu \nu}$$

- Nonequilibrium equation of state

$$\rho = \rho(e, \sigma)$$

⇒ Selfconsistent approach within the two-particle irreducible effective action!

(MN, S. Leupold, C. Herold, M. Bleicher, PRC 84 (2011))
Semiclassical equation of motion for the sigma field

\[ \partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \rho_s + \eta \partial_t \sigma = \xi \]

damping term \( \eta \) and noise \( \xi \) for \( k = 0 \)

\[ \eta = g^2 \frac{d_\rho}{\pi} \left( 1 - 2n_F \left( \frac{m_\sigma}{2} \right) \right) \frac{\left( \frac{m_\sigma^2}{4} - m_q^2 \right)^3}{m_\sigma^2} \]

\[ \langle \xi(t)\xi(t') \rangle = \frac{1}{V} \delta(t-t') m_\sigma \eta \coth \left( \frac{m_\sigma}{2T} \right) \]

below \( T_c \) damping by the interaction with the hard pion modes, apply \( \eta = 2.2 / \text{fm} \)

(T. S. Biro and C. Greiner, PRL 79 (1997))
Evolution in a box

- nonexpanding, finite heat bath
- initialize the sigma field in equilibrium at $T > T_c$
- initialize the energy density at a $T_{\text{sys}} < T_c$
- update sigma field on the grid according to the Langevin equation
Equilibration for a heat bath with reheating

Critical point

**\( T_c = 139.8 \text{ MeV} \)**

- During relaxation of the \( \sigma \)-field the temperature of the heat bath increases.
- Coupled dynamics equilibrate at a given \( T_{eq} \) and \( \sigma_{eq} \).
- Green curves correspond to scenarios with \( T_{eq} \) near \( T_c \).

\( \Rightarrow \) Critical slowing down!

(MN, S. Leupold, M. Bleicher, PLB 711 (2012))
Equilibration for a heat bath with reheating

First order phase transition

\( T_c = 123.3 \text{ MeV} \)

- Strong reheating during relaxation of the \( \sigma \)-field.
- Long (initial) relaxation times for \( T_{\text{sys}} \) close to the phase transition.
- Except for the scenario with \( T_{\text{sys}} = 20 \text{ MeV} \) the heat bath reheats to \( T > T_c \).
- System gets trapped in metastable states.

(MN, S. Leupold, M. Bleicher, PLB 711 (2012))
Fluid dynamic expansion of the heat bath

- very simple initial conditions: almond-shaped initial temperature distribution, sigma field and energy density in equilibrium at $T(x)$
- 3+1d fluid dynamic expansion
- update sigma field on the grid according to the Langevin equation
- very good energy conservation
Reheating and supercooling

- oscillations at the critical point
- supercooling of the system at the first order phase transition
- reheating effect visible at the first order phase transition

Intensity of sigma fluctuations
in single events

\[ \frac{dN_\sigma}{d^3 k} = \frac{\left( \omega_k^2 |\sigma_k|^2 + |\partial_t \sigma_k|^2 \right)}{(2\pi)^3 2\omega_k} \]

\[ \omega_k = \sqrt{|k|^2 + m_\sigma^2} \]

\[ m_\sigma = \sqrt{\frac{\partial^2 V_{\text{eff}}}{\partial \sigma^2} |_{\sigma = \sigma_{\text{eq}}}} \]

Realistic initial conditions

initial conditions from the hybrid UrQMD+hydro approach (profiles from Pb+Pb at $E_{\text{lab}} = 40A$ GeV)

(H. Petersen et al. PRC 78 (2008))
Dynamic domain formation
First order phase transition

Sigma field fluctuations: \( \Delta \sigma = \sqrt{(\sigma - \sigma_{eq})^2} \)

- highly supercooled state at \( t = 4.0 \) fm

(MN, I. Mishustin in preparation)
Dynamic domain formation
First order phase transition

sigma field fluctuations: \( \Delta \sigma = \sqrt{(\sigma - \sigma_{eq})^2} \)

- highly supercooled state at \( t = 4.0 \) fm
- dynamic formation of domains at \( t = 5.6 \) fm

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Dynamic domain formation
First order phase transition

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- highly supercooled state at \( t = 4.0 \) fm
- dynamic formation of domains at \( t = 5.6 \) fm
- dynamic decay of domains at \( t = 7.2 \) fm

This could lead to non-statistical fluctuations in hadron multiplicities.

(MN, I.Mishustin in preparation)
Dynamic correlation length

Critical point

Correlation function: \( G(r) \propto \exp\left(-\frac{r}{\xi}\right) \)

Growth of \( \xi \) up to 1.5 – 2.5 fm in a dynamic model!

Very preliminary results, systematic study is presently carried out!
Dynamic enhancement of event-by-event fluctuations

Critical point

Enhancement of event-by-event fluctuations of the sigma field in a critical point scenario!

Very preliminary results, systematic study is presently carried out!
Summary

- nonequilibrium chiral fluid dynamics including damping and noise
- energy-momentum conservation by the back reaction on the heat bath
- effects of supercooling, reheating, critical slowing down
- dynamic formation and decay of domains at the first order phase transition
- dynamic enhancement of event-by-event-fluctuations of the sigma mode at the critical point
Face diagram of EQCD 2012