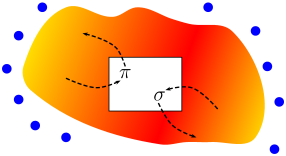
The QCD phase diagram in NχFD

Marlene Nahrgang

SUBATECH, Nantes & FIAS, Frankfurt



Excited QCD 2012, Peniche

Subatech

FIAS Frankfurt Institute for Advanced Studies

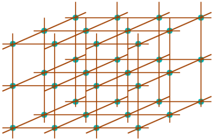
$$\mathcal{L} = \overline{q}^{\alpha}(i\gamma^{\mu}D_{\mu,\alpha\beta} - m_q\delta_{\alpha\beta})q^{\beta} - \frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu}_{a}$$

How to study the QCD phase diagram...

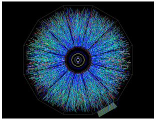
... be brave and solve

$$Z(T,\mu_B) = \int \mathcal{D}(A,q,q^\dagger)\mathrm{e}^{-S^E_{\mathrm{QCD}}}$$

ab initio and nonperturbatively,



... be strong and collide heavy ions at ultrarelativistic energies,



... be creative and study effective models of QCD.

$$\mathcal{L}_{\mathrm{eff}}$$

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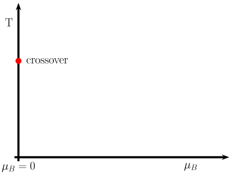

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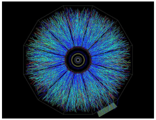

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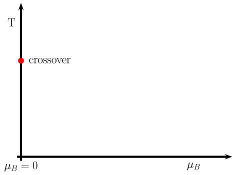

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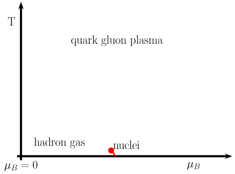

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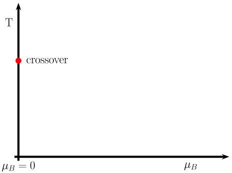

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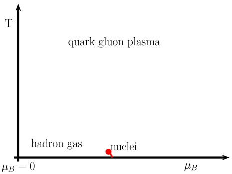

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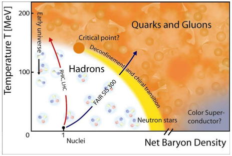
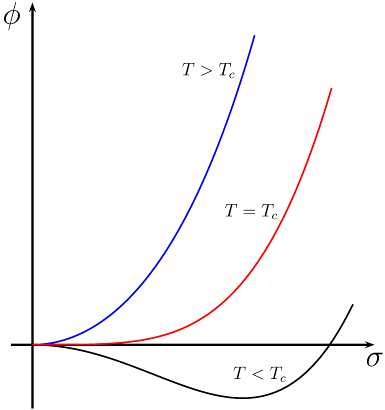


Critical point

- $m_\sigma^2 = \frac{\partial^2 V}{\partial \sigma^2} \to 0$
- correlation length diverges
  $\xi = \frac{1}{m_\sigma} \to \infty$
- universality classes
  for QCD: $\mathcal{O}(4)$ Ising model in 3d $\Rightarrow \langle \sigma^2 \rangle \propto \xi^2$
- renormalization group
- critical opalescence

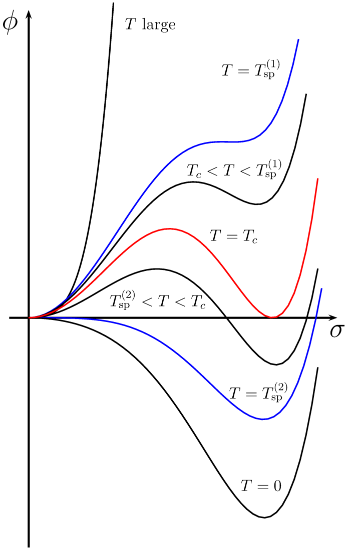


$\Rightarrow$ Large event-by-event fluctuations in thermal systems!

First order phase transitions

- two degenerate minima separated by a barrier
- latent heat
- phase coexistence
- supercooling effects in nonequilibrium situations
- nucleation
- spinodal decomposition

(I.N.Mishustin, PRL **82** (1999); Ph.Chomaz, M.Colonna, J.Randrup, Physics Reports **389** (2004))



$\Rightarrow$ (Large) fluctuations in single events in nonequilibrium situations!

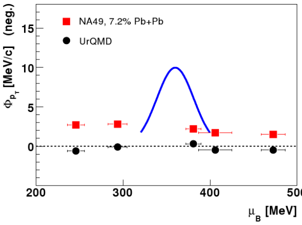
The critical point in heavy-ion collisions

$$\langle(\Delta\sigma)^2\rangle = \frac{T}{V}\xi^2$$

assuming a coupling of the order parameter to pions $g\sigma\pi\pi$ and protons $G\sigma\bar{p}p$

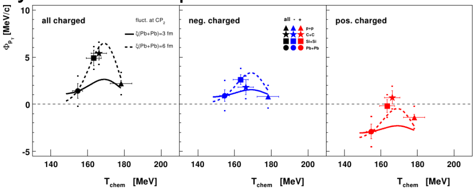
$$\frac{\langle(\delta N)^2\rangle}{N} \propto \xi^2 \propto \langle(\Delta\sigma)^2\rangle$$

(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRL **81** (1998), PRD **60** (1999))



(NA49 collaboration J. Phys. G **35** (2008))

system size dependence



(NA49 collaboration, Acta Phys. Pol. **41** (2010))

more systematically in NA61

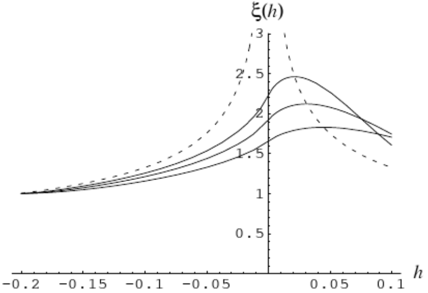
The critical point in dynamic systems

long relaxation times near a critical point $\Rightarrow$ critical slowing down
$\Rightarrow$ the system is driven out of equilibrium

$$\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)})$$

with $\Gamma(m_\sigma) = \frac{A}{\xi_0}(m_\sigma \xi_0)^z$
$z = 3$
(dynamic) critical exponent

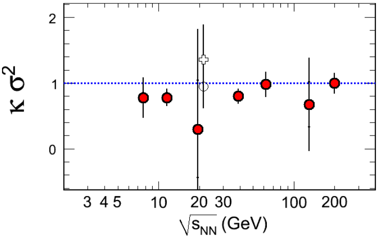
$\Rightarrow \xi \sim 1.5 - 2.5$ fm



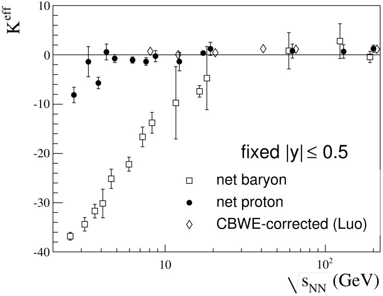
(B. Berdnikov and K. Rajagopal, PRD **61** (2000)); D.T.Son, M.Stephanov, PRD **70** (2004); M.Asakawa, C.Nonaka, Nucl. Phys. A774 (2006))

Higher moments and the kurtosis

Higher moments are more sensitive to the growth of the correlation length: $\langle(\delta N)^4\rangle / N \propto \xi^4$ (M. A. Stephanov, PRL **102** (2009))



(STAR collaboration, CPOD 2011)



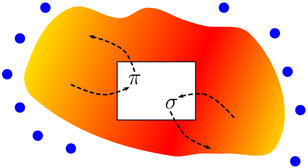
(MN, J.Phys.G **38** (2011))

- Kurtosis is negative at the critical point. (M. A. Stephanov, PRL **107** (2011))
- Negative kurtosis is also given by superposition many particle distributions, net-baryon number conservation...

Motivation

- Fluctuations have so far been investigated in static, equilibrated systems.
- However, systems created in heavy-ion collisions are finite in size and time and inhomogeneous.
- Necessary to propagate fluctuations explicitly!

- Nonequilibrium chiral fluid dynamics (NχFD):
  - phase transition model +
  - dissipation and noise +
  - fluid dynamics



Motivation

Two questions in finite, dynamic systems:

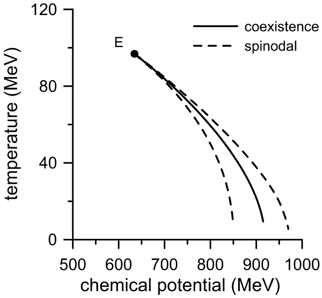
- Are nonequilibrium effects strong enough to develop signals of the first order phase transition?
- Do enhanced equilibrium fluctuations at the critical point survive?

The linear sigma model with constituent quarks

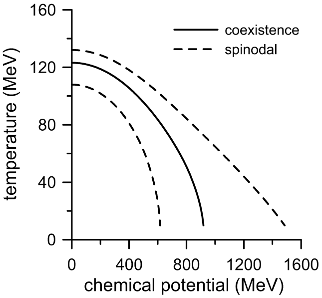
$$\mathcal{L} = \overline{q}\left[i\gamma^\mu\partial_\mu - g\left(\sigma + i\gamma_5\tau\vec{\pi}\right)\right]q + 1/2\left(\partial_\mu\sigma\right)^2 + 1/2\left(\partial_\mu\vec{\pi}\right)^2 - U\left(\sigma,\vec{\pi}\right)$$

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - h_q\sigma - U_0$$

$g = 3.3$: crossover at $\mu = 0$



$g = 5.5$: first order pt at $\mu = 0$

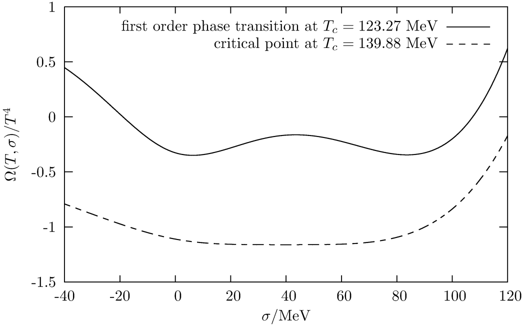


(O. Scavenius, A. Mocsy, I.N. Mishustin, D.H. Rischke, PRC **64** (2001); C.E. Aguiar, E.S. Fraga, T. Kodama, J.Phys.G **32** (2006))

The effective potential at $\mu_B = 0$

$$\Omega_{\mathrm{eff}} = -\frac{T}{V}\ln Z_{\mathrm{th}} = -d_q T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln\left(1 + \exp\left(-\frac{E}{T}\right)\right) + U(\sigma, \vec{\pi})$$

with dynamically generated quark masses $E = \sqrt{p^2 + g^2\sigma^2 + g^2\vec{\pi}^2}$



For a qualitative study tune the strength of the phase transition via the coupling g.

Nonequilibrium chiral fluid dynamics - NχFD

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Fluid dynamic expansion of the quark fluid = heat bath, including energy-momentum exchange

$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu}$$

- Nonequilibrium equation of state

$$p = p(e, \sigma)$$

$\Longrightarrow$ Selfconsistent approach within the two-particle irreducible effective action!

(MN, S. Leupold, C. Herold, M. Bleicher, PRC **84** (2011))

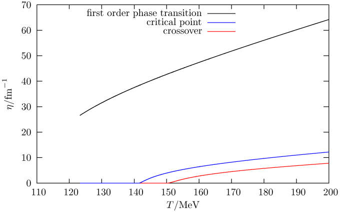
Semiclassical equation of motion for the sigma field

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

damping term η and noise ξ for $\mathbf{k} = 0$

$$\eta = g^2 \frac{d_q}{\pi} \left(1 - 2n_{\mathrm{F}}\left(\frac{m_\sigma}{2}\right)\right) \frac{\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{\frac{3}{2}}}{m_\sigma^2}$$

$$\langle \xi(t)\xi(t')\rangle = \frac{1}{V}\delta(t-t')m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)$$



below T_c damping by the interaction with the hard pion modes, apply $\eta = 2.2/\mathrm{fm}$

(T. S. Biro and C. Greiner, PRL **79** (1997))

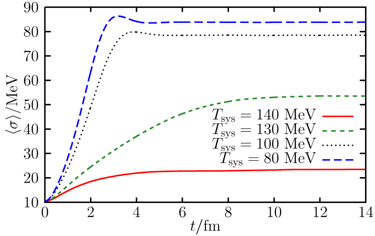
Evolution in a box

- nonexpanding, finite heat bath
- initialize the sigma field in equilibrium at $T > T_c$
- initialize the energy density at a $T_{\mathrm{sys}} < T_c$
- update sigma field on the grid according to the Langevin equation

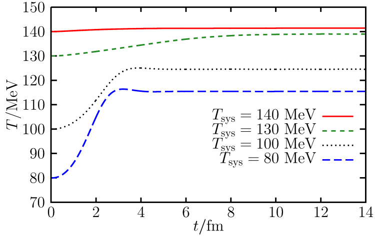
Equilibration for a heat bath with reheating

Critical point

relaxation of the σ field



temperature



$T_c = 139.8$ MeV

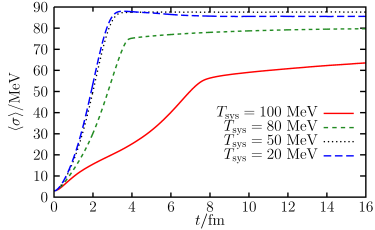
- During relaxation of the σ-field the temperature of the heat bath increases.
- Coupled dynamics equilibrate at a given T_{eq} and σ_{eq}.
- Green curves correspond to scenarios with T_{eq} near T_c. $\Rightarrow$ Critical slowing down!

(MN, S. Leupold, M. Bleicher, PLB **711** (2012))

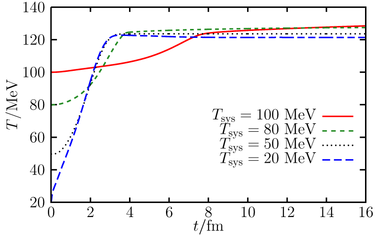
Equilibration for a heat bath with reheating

First order phase transition

relaxation of the σ field



temperature



$T_c = 123.3$ MeV

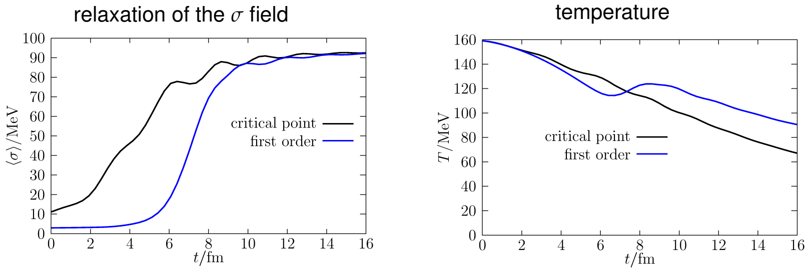
- Strong reheating during relaxation of the σ-field.
- Long (initial) relaxation times for T_{sys} close to the phase transition.
- Except for the scenario with $T_{\mathrm{sys}} = 20$ MeV the heat bath reheats to $T > T_c$.
- System gets trapped in metastable states.

(MN, S. Leupold, M. Bleicher, PLB **711** (2012))

Fluid dynamic expansion of the heat bath

- very simple initial conditions: almond-shaped initial temperature distribution, sigma field and energy density in equilibrium at $T(x)$
- 3+1d fluid dynamic expansion
- update sigma field on the grid according to the Langevin equation
- very good energy conservation

Reheating and supercooling



- oscillations at the critical point
- supercooling of the system at the first order phase transition
- reheating effect visible at the first order phase transition

MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962

Intensity of sigma fluctuations

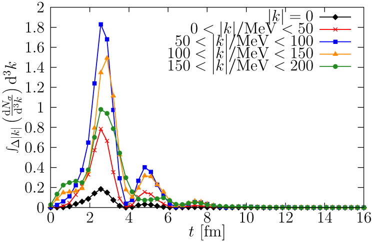
in single events

$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3k} = \frac{(\omega_k^2|\sigma_k|^2 + |\partial_t \sigma_k|^2)}{(2\pi)^3 2\omega_k}$$

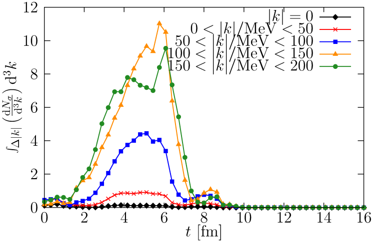
$$\omega_k = \sqrt{|k|^2 + m_\sigma^2}$$

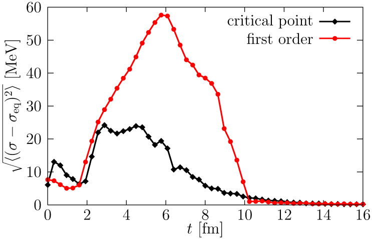
$$m_\sigma = \sqrt{\partial^2 V_{\mathrm{eff}} / \partial \sigma^2|_{\sigma=\sigma_{\mathrm{eq}}}}$$

critical point



first order phase transition





deviation from equilibrium

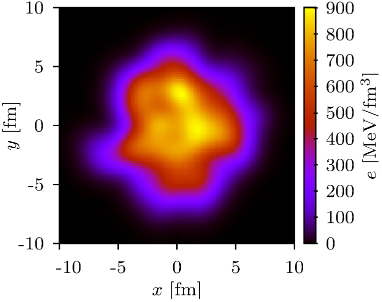
MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962

Realistic initial conditions

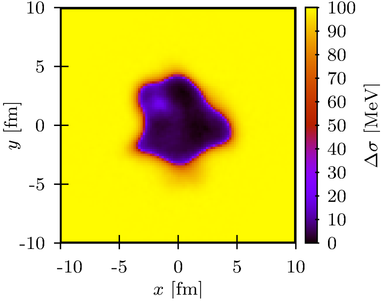
initial conditions from the hybrid UrQMD+hydro approach
(profiles from Pb+Pb at $E_{\mathrm{lab}} = 40A$ GeV)

(H.Petersen et al. PRC **78** (2008))

energy density e



sigma field σ



Dynamic domain formation

First order phase transition

sigma field fluctuations: $\Delta\sigma = \sqrt{(\sigma - \sigma_{\rm eq})^2}$

- highly supercooled state at $t = 4.0$ fm

(MN, I.Mishustin in preparation)

Dynamic domain formation

First order phase transition

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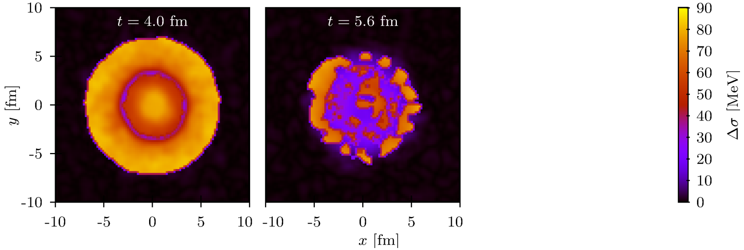


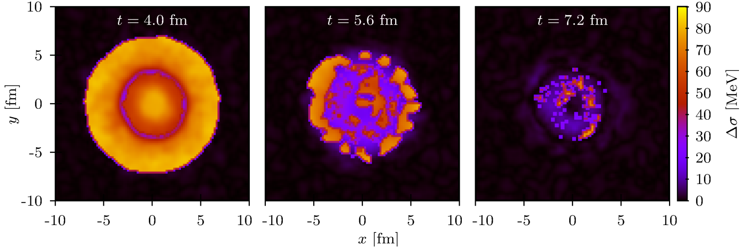
- highly supercooled state at $t = 4.0$ fm
- dynamic formation of domains at $t = 5.6$ fm

(MN, I.Mishustin in preparation)

Dynamic domain formation

First order phase transition

sigma field fluctuations: $\Delta\sigma = \sqrt{(\sigma - \sigma_{\text{eq}})^2}$



- highly supercooled state at $t = 4.0$ fm
- dynamic formation of domains at $t = 5.6$ fm
- dynamic decay of domains at $t = 7.2$ fm

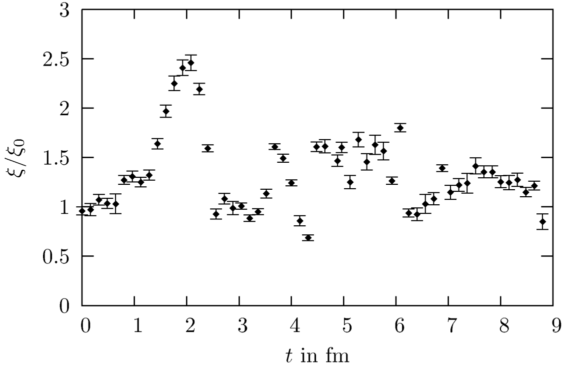
This could lead to non-statistical fluctuations in hadron multiplicities.

(MN, I.Mishustin in preparation)

Dynamic correlation length

Critical point

Correlation function: $G(r) \propto \exp(-r/\xi)$

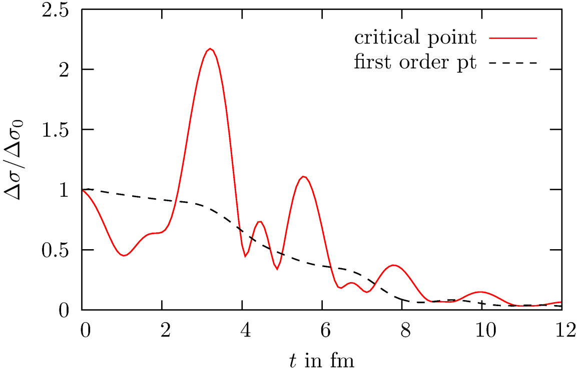


Growth of ξ up to $1.5 - 2.5$ fm in a dynamic model!

Very preliminary results, systematic study is presently carried out!

Dynamic enhancement of event-by-event fluctuations

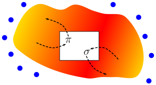
Critical point



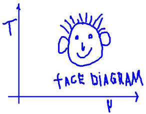
Enhancement of event-by-event fluctuations of the sigma field in a critical point scenario!

Very preliminary results, systematic study is presently carried out!

Summary



- nonequilibrium chiral fluid dynamics including damping and noise
- energy-momentum conservation by the back reaction on the heat bath
- effects of supercooling, reheating, critical slowing down
- dynamic formation and decay of domains at the first order phase transition
- dynamic enhancement of event-by-event-fluctuations of the sigma mode at the critical point



Face diagram of EQCD 2012