The non-forward BFKL equation, Infrared Effects and Hard Diffraction

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Outline

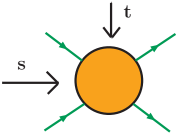
1. The non-forward BFKL equation in a generic color representation
   - Iterative expression for Monte Carlo
   - Running coupling and infrared effects
2. Numerical analysis of the gluon Green's function
3. Conclusions

Motivation

- Phenomenological study of diffractive processes
  - Mueller-Tang jets: forward-backward jets with a large rapidity gap
  - $W/Z/\gamma*$ + backward jet with rapidity gaps [M. Hentschinski, C.S.]
  - Diffractive DIS at LeHC
  - ...
- Running needed when considering higher order corrections
- Monte Carlo code ready

The low x limit: Regge theory

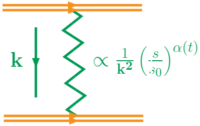
Regge (high energy) limit:



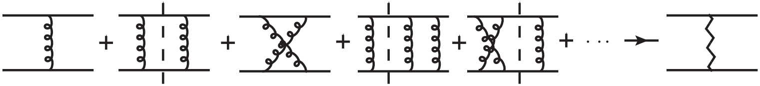
$$s >> |t|\,,\; Q^2 >> \Lambda^2_{\mathrm{QCD}}$$
$$\alpha_s(Q^2) << 1$$
$$x \simeq |t|/s << 1$$

Regge th. prediction: $\sigma(s) \sim s^{\alpha(0)-1}$

Reggeization

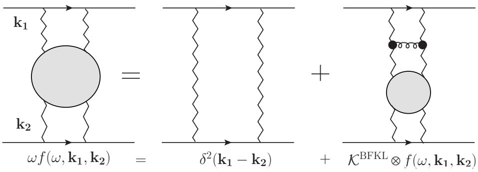


QCD and gluon reggeization: due to bootstrap condition



Non-forward BFKL evolution equation

- BFKL equation: Mellin transform of the imaginary part of the amplitude:



Non-forward:

- $t = -\mathbf{q}^2 \neq 0$
- diffractive
- hard scale: $|t| >> \Lambda^2_{QCD}$

Non-forward BFKL eq. in the singlet:

$$[\omega - \epsilon(\mathbf{k}_1) - \epsilon(\mathbf{k}_1 - q)] f_\omega(\mathbf{k}_1, \mathbf{k}_2; \mathbf{q}) = \delta^{(2)}(\mathbf{k}_1 - \mathbf{k}_2)$$
$$-\frac{\bar{\alpha}_s}{2\pi}\int d^2\mathbf{l}\left[\frac{\mathbf{q}^2}{(\mathbf{l}-\mathbf{q})^2\mathbf{k}_1^2} - \frac{1}{(\mathbf{l}-\mathbf{k}_1)^2}\left(1 + \frac{(\mathbf{k}_1-\mathbf{q})^2\mathbf{l}^2}{(\mathbf{l}-\mathbf{q})^2\mathbf{k}_1^2}\right)\right] f_\omega(\mathbf{l}, \mathbf{k}_2; \mathbf{q})$$

Non-forward BFKL evolution equation

BFKL equation in generic color projection $c_{\mathcal{R}}$:

$$\left\{\omega + (c_{\mathcal{R}} - 1)\frac{\bar{\alpha}_s}{2}\left[\frac{2}{\epsilon} - \log\left(\frac{\mathbf{q}_1^2}{\mu^2}\right) - \log\left(\frac{\mathbf{q}_1'^2}{\mu^2}\right)\right]\right.$$

$$\left. + c_{\mathcal{R}}\frac{\bar{\alpha}_s}{2}\left[\log\left(\frac{\mathbf{q}_1^2}{\lambda^2}\right) + \log\left(\frac{\mathbf{q}_1'^2}{\lambda^2}\right)\right]\right\}\mathcal{G}_\omega(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}) = \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2)$$

$$+ c_{\mathcal{R}}\int \frac{d^2\mathbf{k}}{\pi\mathbf{k}^2}\theta\left(\mathbf{k}^2 - \lambda^2\right)\frac{\bar{\alpha}_s}{2}\left[1 + \frac{\mathbf{q}_1'^2(\mathbf{q}_1 + \mathbf{k})^2 - \mathbf{q}^2\mathbf{k}^2}{(\mathbf{q}_1' + \mathbf{k})^2\mathbf{q}_1^2}\right]\mathcal{G}_\omega(\mathbf{q}_1 + \mathbf{k}, \mathbf{q}_2; \mathbf{q})$$

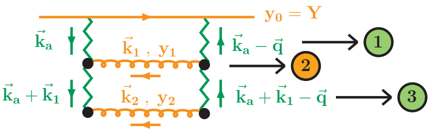
Notation:

$$\left\{\omega - \omega^{(\epsilon;\lambda)}(\mathbf{q}_1; \mathbf{q})\right\}\mathcal{G}_\omega(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q})$$

$$= \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2) + c_{\mathcal{R}}\int \frac{d^2\mathbf{k}}{\pi\mathbf{k}^2}\theta\left(\mathbf{k}^2 - \lambda^2\right)\xi(\mathbf{q}_1, \mathbf{k}; \mathbf{q})\mathcal{G}_\omega(\mathbf{q}_1 + \mathbf{k}, \mathbf{q}_2; \mathbf{q})$$

$$\mathcal{G}_\omega(\mathbf{l}, \mathbf{q}_2; \mathbf{q}) \equiv f_\omega(\mathbf{l}, \mathbf{q}_2; \mathbf{q})\,\frac{\mathbf{l}^2}{\mathbf{q}_1^2},$$

Iteration for Monte Carlo: 1-block structure



1. t-channel props.: $e^{-\frac{\bar{\alpha}_s}{2}\log\frac{\mathbf{k}_a^2}{\lambda^2}(Y-y_1)} \cdot e^{-\frac{\bar{\alpha}_s}{2}\log\frac{(\mathbf{k}_a-\mathbf{q})^2}{\lambda^2}(Y-y_1)}$
2. s-channel prop.: $\xi\,(\mathbf{k}_a, \mathbf{k}_1, \mathbf{q})$
3. t-channel props.: $e^{-\frac{\bar{\alpha}_s}{2}\log\frac{(\mathbf{k}_a+\mathbf{k}_1)^2}{\lambda^2}(y_1-y_2)} \cdot e^{-\frac{\bar{\alpha}_s}{2}\log\frac{(\mathbf{k}_a+\mathbf{k}_1-\mathbf{q})^2}{\lambda^2}(y_1-y_2)}$

Phase space:

- Ordering in rapidity: $\int_0^Y dy_1 \int_0^{y_1} dy_2$
- s-channel propagators: $\int \frac{d^2\mathbf{k}_1}{\pi\mathbf{k}_1^2}\theta(\mathbf{k}_1^2-\lambda^2) \times \int \frac{d^2\mathbf{k}_2}{\pi\mathbf{k}_2^2}\theta(\mathbf{k}_2^2-\lambda^2)$

Iteration for Monte Carlo

Mellin transform:

$$\mathcal{F}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) = \int \frac{d\omega}{2\pi i} e^{\omega \mathrm{Y}} \mathcal{G}_\omega(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}),$$

Iterative expression:

$$\mathcal{F}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) = \exp\left\{\omega^{(\epsilon;\lambda)}(\mathbf{q}_1; \mathbf{q})\mathrm{Y}\right\}\left\{\delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2)\right.$$

$$+ \sum_{n=1}^{\infty} \prod_{i=1}^{n} c_{\mathcal{R}} \int \frac{d^2\mathbf{k}_i}{\pi \mathbf{k}_i^2} \theta(\mathbf{k}_i^2 - \lambda^2)\, \xi\left(\mathbf{q}_1 + \sum_{l=1}^{i-1} \mathbf{k}_l, \mathbf{k}_i; \mathbf{q}\right) \delta^{(2)}\left(\mathbf{q}_1 + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{q}_2\right)$$

$$\left.\times \int_0^{y_{i-1}} dy_i \exp\left\{\left[\omega^{(\epsilon;\lambda)}\left(\mathbf{q}_1 + \sum_{l=1}^{i} \mathbf{k}_l; \mathbf{q}\right) - \omega^{(\epsilon;\lambda)}\left(\mathbf{q}_1 + \sum_{l=1}^{i-1} \mathbf{k}_l; \mathbf{q}\right)\right] y_i\right\}\right\}$$

Independent of λ, ϵ.

Iteration for Monte Carlo

In the forward limit and color singlet, $c_R = 1$:

$$\mathcal{F}(\mathbf{q}_1, \mathbf{q}_2; \mathrm{Y}) = \frac{1}{\pi\sqrt{\mathbf{q}_1^2 \mathbf{q}_2^2}} \sum_{n=-\infty}^{\infty} \int \frac{d\omega}{2\pi i} \int \frac{d\gamma}{2\pi i} \left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)^{\gamma-\frac{1}{2}} \frac{e^{\omega \mathrm{Y} + i n \theta}}{\omega - \bar{\alpha}_s \chi_n(\gamma)}$$

We numerically studied the finite part:

$$\mathcal{H}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) = \left(\frac{\lambda^2}{\sqrt{\mathbf{q}_1^2 \mathbf{q}_1'^2}}\right)^{c_\mathcal{R} \bar{\alpha}_s \mathrm{Y}} \Bigg\{ \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2)$$

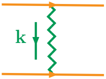
$$+ \sum_{n=1}^{\infty} \prod_{i=1}^{n} c_\mathcal{R} \int \frac{d^2\mathbf{k}_i}{\pi \mathbf{k}_i^2} \theta(\mathbf{k}_i^2 - \lambda^2) \frac{\bar{\alpha}_s}{2} \left(1 + \frac{\left(\mathbf{q}_1' + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2 (\mathbf{q}_1 + \sum_{l=1}^{i} \mathbf{k}_l)^2 - \mathbf{q}^2 \mathbf{k}_i^2}{(\mathbf{q}_1' + \sum_{l=1}^{i} \mathbf{k}_l)^2 \left(\mathbf{q}_1 + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2}\right)$$

$$\times \int_0^{y_{i-1}} \mathrm{d}y_i \left(\frac{\left(\mathbf{q}_1 + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2 \left(\mathbf{q}_1' + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2}{\left(\mathbf{q}_1 + \sum_{l=1}^{i} \mathbf{k}_l\right)^2 \left(\mathbf{q}_1' + \sum_{l=1}^{i} \mathbf{k}_l\right)^2}\right)^{\frac{\bar{\alpha}_s}{2} y_i} \delta^{(2)}\left(\mathbf{q}_1 + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{q}_2\right) \Bigg\}$$

Running coupling & IR effects

Bootstrap & running

Fixed coupling:

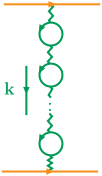


Propagator: $\frac{\bar{\alpha}_s(\mu^2)}{\mathbf{k}^2}\left(\frac{s}{s_0}\right)^{\epsilon(-\mathbf{k}^2)}$

Trajectory: $\epsilon(-\mathbf{k}^2) = -\frac{\bar{\alpha}_s(\mu^2)}{4\pi}\int \frac{d^2\mathbf{k}'}{\mathbf{k}'^2}\frac{\mathbf{k}^2}{(\mathbf{k}-\mathbf{k}')^2}$

Impose bootstrap condition to trajectory $\Longleftrightarrow$ $\mathcal{K}^{\mathrm{LO}}_{\mathrm{BFKL}}(\mathbf{q},\mathbf{k},\mathbf{k}')$

Running coupling: by insertion of a gluon chain.



Propagator: $\frac{\bar{\alpha}_s(\mathbf{k}^2)}{\mathbf{k}^2}\left(\frac{s}{s_0}\right)^{\tilde{\epsilon}(-\mathbf{k}^2)}$

Trajectory: $\tilde{\epsilon}(-\mathbf{k}^2) = -\frac{1}{4\pi}\int \frac{d^2\mathbf{k}'}{\eta(\mathbf{k}')}\frac{\eta(\mathbf{k})}{\eta(\mathbf{k}-\mathbf{k}')}$ with $\eta(\mathbf{k}) \equiv \frac{\mathbf{k}^2}{\bar{\alpha}_s(\mathbf{k}^2)}$

Take $\tilde{\epsilon}(-\mathbf{k}^2)$ and insert it into bootstrap eq. $\Longrightarrow$ $\mathcal{K}^{\mathrm{running}}_{\mathrm{BFKL}}(\mathbf{q},\mathbf{k},\mathbf{k}')$

[Braun hep-ph/9408261; Levin hep-ph/9412345; Kovchegov, Weigert hep-ph/0609090]

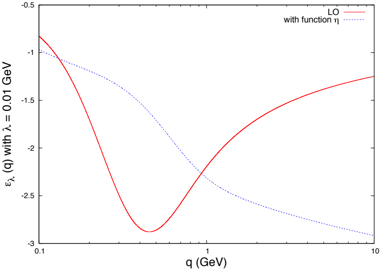
The trajectory

LO trajectory:

$$\epsilon_\lambda^{\rm LO}(\mathbf{q}^2) = -\bar\alpha_s(\mathbf{q}^2)\,\mathbf{q}^2 \int \frac{d^2\mathbf{k}^2}{2\pi\,\mathbf{k}^2}\,\frac{\theta(\mathbf{k}^2-\lambda^2)}{\mathbf{k}^2+(\mathbf{k}+\mathbf{q})^2} \simeq -\frac{\bar\alpha_s(\mathbf{q}^2)}{2}\ln\frac{\mathbf{q}^2}{\lambda^2}$$

Trajectory with running coupling:

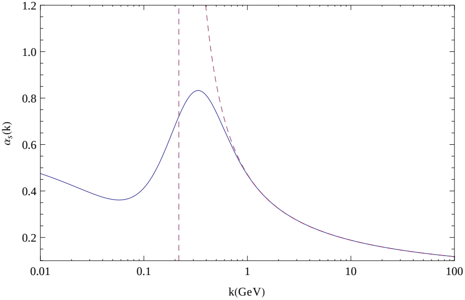
$$\epsilon_\lambda^{\rm run}(\mathbf{q}^2) \quad = \quad -\frac{\mathbf{q}^2}{\bar\alpha_s(\mathbf{q}^2)}\int \frac{d^2\mathbf{k}}{2\pi\,\mathbf{k}^2}\,\frac{\bar\alpha_s(\mathbf{k}^2)\,\bar\alpha_s(\mathbf{k}+\mathbf{q})}{\mathbf{k}^2+(\mathbf{k}+\mathbf{q})^2}\,\theta(\mathbf{k}^2-\lambda^2)$$



A model for the running

Running coupling analytical in the infrared and compatible with power corrections to jet observables:

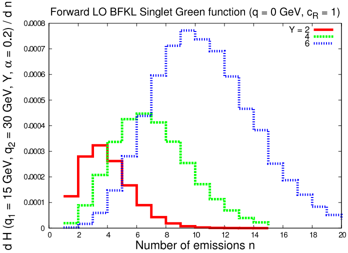
$$\bar{\alpha}_s\left(\mathbf{k}^2\right) = \frac{4N_c}{\beta_0}\left(\frac{1}{\ln\frac{\mathbf{k}^2}{\Lambda^2_{QCD}}} + 125\frac{\left(\Lambda^2_{QCD} + 4\mathbf{k}^2\right)}{\left(\Lambda^2_{QCD} - \mathbf{k}^2\right)\left(4 + \frac{\mathbf{k}^2}{\Lambda^2_{QCD}}\right)^4}\right)$$

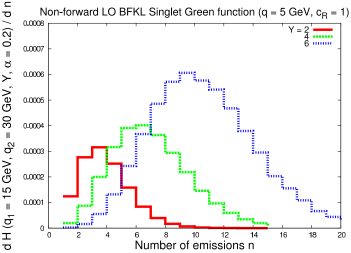


[B. Webber (1998) hep-ph/9805484]

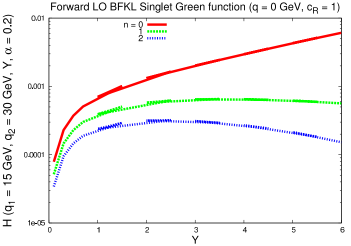
Numerical analysis

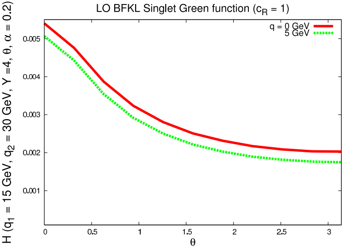
Convergence of the sum

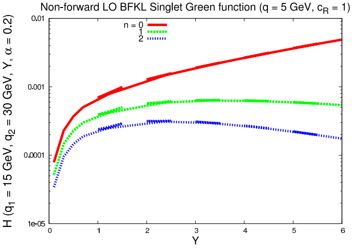


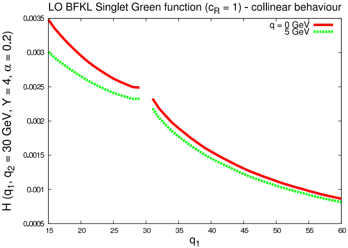


Gluon Green's function





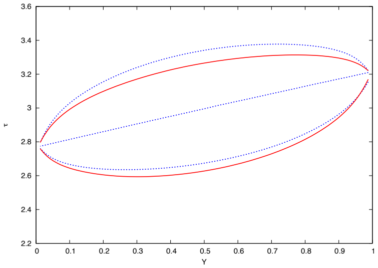




Diffusion

$$\tau = \log\left(\left(\mathbf{k}_a + \sum_{i=1} n\mathbf{k}_i\right)^2\right)$$

$$\sigma_1^2(Y') = \frac{2\int_{\langle\tau\rangle(Y')}^{\infty} d\tau(\tau - \langle\tau\rangle_{Y'})^2 f(\mathbf{k}_a, \mathbf{k}_b, \mathbf{q}; Y)}{\int_0^{\infty} d\tau f(\mathbf{k}_a, \mathbf{k}_b, \mathbf{q}; Y))} \ , \quad \sigma_2^2(Y') = \frac{2\int_0^{\langle\tau\rangle(Y')} d\tau \ \ idem}{\int_0^{\infty} d\tau f(\mathbf{k}_a, \mathbf{k}_b, \mathbf{q}; Y))}$$



- Blue: fixed coupling
- Red: running coupling: (shift to the IR)
- IR finite model for running
- $k_a = 5\text{GeV}$, $k_b = 4\text{GeV}$
- $Y = 1$, $\lambda = 0.5$

Comments and conclusions

- The insertion of the running coupling...
  - provides smooth transition between the hard and soft Pomeron
  - leads to the appearance of renormalons $\Rightarrow$ sensitivity to the infrared
- The model used for the running coupling...
  - is analytic at the Landau pole, freezing in the IR
  - compatible with power corrections to jet observables
- Forward and non-forward BFKL eq. solved numerically using an iterative expression for the running case
- We are ready to implement this formalism for the study of LHC-like processes