

# Next-to-leading and Resummed BFKL Evolution with Saturation Boundary

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We have simulated BFKL evolution using collinear resummation to address the instabilities arising from the NLL kernel, and a saturation boundary instead of the nonlinear term to tame the growth of the solution at low momenta. Our results establish that the saturation boundary alone does not cure the NLL instabilities, but with the resummation, it allows us to extract the rapidity dependence of the saturation scale and demonstrates some possibly important pre-asymptotic behavior.

## 1 BFKL Evolution

In the calculations of scattering amplitudes for strongly bound systems, one of the key components is the fundamental QCD dipole amplitude. When the rapidity  $Y = \ln \frac{1}{x}$  is large, this amplitude is governed by the BFKL equation [1, 2, 3], which is a linear equation that gives the evolution in rapidity of the dipole amplitude.

For purposes of this project [4], the BFKL equation can be written in the form

$$\omega F(\omega, \mathbf{k}) = F_0(\mathbf{k}) + \int \frac{d^2 \mathbf{k}'}{\pi^2} K(\mathbf{k}, \mathbf{k}') F(\omega, \mathbf{k}')$$

where  $\omega$  is the Mellin variable conjugate to  $x$ ,  $F(\omega, \mathbf{k})$  is a convolution of the gluon Green's function with one impact factor, and the kernel  $K(\mathbf{k}, \mathbf{k}')$  is known to NLL order.

Of particular interest in these analyses are the singularity structures of the kernel in double Mellin space: where are the poles in  $\gamma$ , the Mellin variable conjugate to  $\ln k^2$ , and what orders are they? For the leading term, we can write the singularity structure as

$$K_0 \sim \frac{1}{\gamma} + \frac{1}{1-\gamma}$$

(single poles at  $\gamma = 0, 1$ ), and for the NLL term,

$$K_1 \rightarrow \chi_1(\gamma) \sim -\frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3} - \frac{11/12}{\gamma^2} - \frac{11/12+b}{(1-\gamma)^2} + \mathcal{O}\left(\frac{1}{\gamma}\right)$$

This expression contains triple poles which arise from kinematical constraints, and double poles which come from the running of the strong coupling and from the nonsingular terms in the DGLAP splitting function. With the NLL term included, the BFKL evolution is unstable: it gives rise to negative or oscillating cross sections.

Fortunately, it is possible to cure these instabilities by using a resummation procedure to incorporate certain higher-order terms into the BFKL kernel. There are various procedures one can use; this project employs the renormalization-group improved kernel, scheme B from [5]. This scheme basically amounts to shifting the locations of the poles in the Mellin-space kernel, and once we no longer have the double and triple poles at 0 and 1, the instabilities in the evolution which lead to nonsensical cross sections are no longer present.

## 2 Traveling Waves and Saturation

Another problem with the BFKL evolution is that, at low momenta, the solution exhibits runaway growth, to the point where it violates the unitarity bound on the cross-section. To deal with this, one needs to account for gluon self-interactions by adding a negative nonlinear term. The rapidity-dependent threshold momentum at which the self-interactions become important is called the saturation scale,  $Q_s(Y)$ .

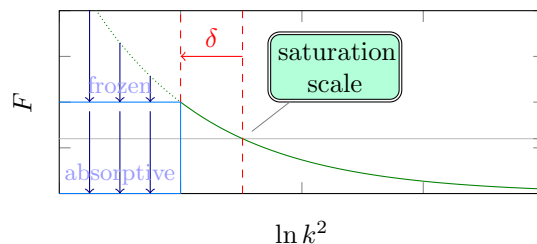
One can make an analogy between the BFKL equation with the nonlinear term and the general class of diffusive equations of the form

$$\partial_t u(x, t) = \mathbb{D}_x \cdot u(x, t) + f(u(x, t))$$

where  $\mathbb{D}_x$  is a differential operator in  $x$ . These equations admit solutions which depend only on the combination  $x - vt$ , thus containing an advancing wavefront, in the region where the nonlinear term is negligible. This wavefront manifests itself in BFKL evolution as geometric scaling: the cross section depends on  $k^2$  and  $Y$  in the combination  $\frac{k^2}{Q_s^2(Y)}$ . [6]

We can exploit this “flow” of the solution towards higher momenta to avoid solving the nonlinear equation directly, which would be very computationally intensive. Instead of actually including the nonlinear term, we use the pure linear equation with a saturation boundary: simply replace the normal BFKL evolution below some threshold momentum with some other evolution that keeps the solution well bounded. Imposing the boundary should ideally simulate the effect the nonlinear term has at  $k^2 \gtrsim Q_s^2(Y)$ , including preserving the wavefront, thereby allowing us to determine the form of  $Q_s(Y)$  without having to do the full nonlinear calculation.

It turns out that the rapidity dependence of the saturation scale is, to a large extent, independent of the exact nature of how this boundary is implemented. In this work we use two schemes, the frozen boundary and the absorptive boundary. In both cases, we begin by identifying the saturation momentum, up to an arbitrary constant factor, as the value  $Q_s$  satisfying  $F(Q_s) = c$ , where  $c$  is a fixed cutoff value. At each step of the simulation, we replace the region of the solution at  $k < Q_s - \delta$  with either 0 (for the absorptive boundary) or  $F(Q_s - \delta)$  (for the frozen boundary). We then run the evolution in rapidity for one step. The propagation of the wavefront increases the saturation scale, and we repeat the application of the boundary.



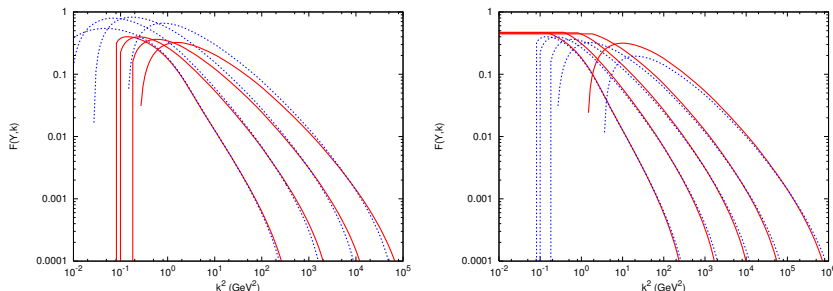


Figure 1: Solutions to the BFKL equation with the NLL kernel using a fixed coupling. On the left, the dotted lines show the solution computed without any saturation boundary at rapidities  $Y = 2, 6, 10, 14$ . The solid lines show the solution, computed with an absorptive boundary, at the same rapidities. The plot on the right compares the absorptive boundary (now the dotted lines) with the frozen boundary (solid lines), at  $Y = 2, 6, 10, 14, 20$ .

### 3 Results

The main objective of all this is to understand the interaction between the saturation boundary and the resummed kernel. We have analyzed the LL, NLL, and resummed kernels, solving the momentum-space BFKL equation with the saturation boundary in each case.

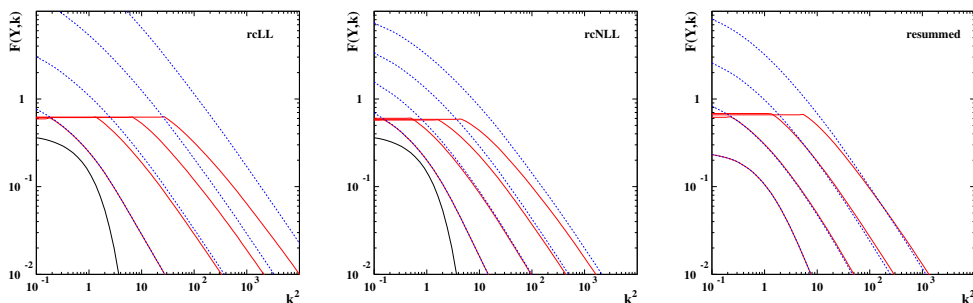


Figure 2: Solution to the BFKL equation with (solid) and without (dotted) the saturation boundary, here applied as the frozen boundary. From left to right, the kernels used are (1) LL with running coupling, (2) NLL with running coupling, and (3) collinear resummed.

Figure 1 shows the solution for  $F$  computed using the NLL kernel with fixed coupling. These results show evidence of the instabilities that occur at low momenta without the boundary, and we can also see the traveling wave behavior at larger momenta, which is only slightly affected by the application of the boundary in this case. However, at large values of  $Y$  as shown in the right hand plot, the solution eventually fails to reach sufficiently large values of  $F$  to satisfy  $F(Q_s) = c$ , meaning that we cannot continuously extract a saturation scale for  $Y \gtrsim 15$ . Evidently the saturation boundary is not enough to prevent the problems with BFKL evolution; we will need to incorporate higher-order terms, including and beyond the running coupling.

In figure 2, we have the solutions computed with the running coupling. These display the untamed growth at low momenta and, again, the traveling wave behavior at large momenta. However, in this case there is a noticeable difference in the propagation speed of the wavefront depending on which kernel is used and whether the saturation boundary is present.

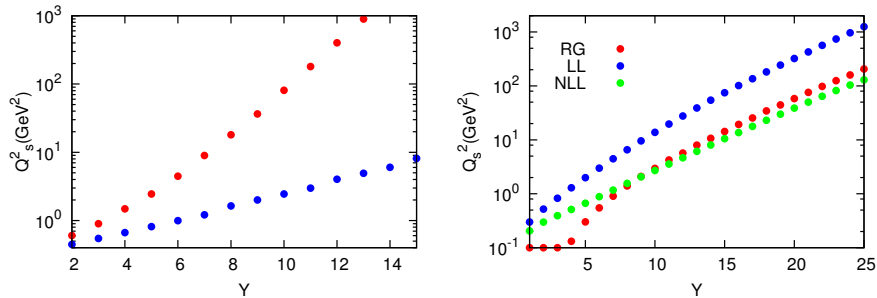


Figure 3: Evolution of the saturation scale with rapidity. On the left is fixed coupling, the upper dots being for the LL kernel and the lower dots for the NLL kernel, and on the right are the running coupling results. The rcLL kernel is the highest set of dots, rcNLL is the next one below it at the left edge of the plot, and the resummed kernel is the set of dots that starts out flat and rises sharply at  $Y \sim 4$ .

Finally, figure 3 shows the main result, the evolution of the saturation scale. With the fixed coupling, we see an essentially exponential growth in the saturation scale at LL, as predicted in [6, 7], which is suppressed by the NLL corrections. With the running coupling, the asymptotic trend in  $Q_s$  is roughly similar for all three kernels, which also agrees with predictions.

An interesting feature of the saturation scale dependence is that, with the resummed kernel, we see a characteristic plateau at low rapidity. This is related to a dip in the DGLAP splitting function described in [5]. The presence of this plateau indicates that the pre-asymptotic behavior of  $Q_s(Y)$  may have important consequences for future phenomenological studies.

## Acknowledgements

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