Dijet Production in QCD and $\mathcal{N} = 4$ SYM

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Outline

- $\mathcal{N} = 4$ SYM as a Testing Ground for QCD
- Dijet Production at Large Rapidity Separation
- Ratios of Angular Correlations and $\text{SL}(2, \mathbb{C})$ Invariance
- The Rôle of the Renormalization Prescription
- Similarity of QCD and $\mathcal{N} = 4$ SYM Predictions
- Conclusions
The Properties of $\mathcal{N} = 4$ SYM

Why $\mathcal{N} = 4$ SYM?

$\mathcal{N} = 4$ SYM: the maximally supersymmetric version of QCD

- Coincides with QCD at tree level and in leading $\ln s$
- Conformal invariant at the quantum level [Sohnius&West’81, Mandelstam’83]
- Great amount of symmetry PSU(2,2|4)
- Amazing simplicity of scattering amplitudes [Parke&Taylor’86, Witten’04, Bern,Dixon&Smirnov’05, Drummond&Henn’09, Arkani-Hamed et al.’10…]
- Expected to provide in the planar limit the first example of an integrable (exactly solvable) nontrivial QFT [Lipatov’94, Bena,Polchinski&Roiban’04,Beisert,Eden&Staudacher’07]
- Concrete realization of the AdS/CFT duality [Maldacena’98]

Observables allowing for comparison between QCD and $\mathcal{N} = 4$ SYM are most important
$\mathcal{N} = 4$ SYM as a Playground for QCD

- $\mathcal{N} = 4$ SYM with gauge group $\text{SU}(N_c)$ and QCD very similar at weak coupling in the Regge limit $s \gg -t$.
- $\mathcal{N} = 4$ SYM contributions to QCD amplitudes given by the principle of maximal transcendentality
  
  [Kotikov, Lipatov, Onishchenko & Velizhanin'04]

- AdS/CFT computations point towards Regge behaviour at strong coupling [Brower, Polchinski, Strassler & Tan'07]

**We can expect learning important lessons for QCD from $\mathcal{N} = 4$ SYM** (in spite of broken SUSY and $\beta = 0$ in real world)

- A word of caution: Physics can be qualitatively different in some cases, e.g. at large $x$ [Hatta, Iancu & Mueller'07]
Mueller-Navelet Jets

- Two Forward Jets
- Need to Resum $[\alpha_s \ln (s/p^2)]^n$
- Dijet Cross Section in terms of BFKL Green’s Function

$$\frac{d\hat{\sigma}}{d^2q_1^2 d^2q_2^2} = \frac{\pi^2 \tilde{\alpha}_s^2}{2} \frac{f(q_1, q_2, Y)}{q_1^2 q_2^2},$$

$$f(q_1, q_2, Y) = \int \frac{d\omega}{2\pi i} e^{\omega Y} f(q_1, q_2, \omega)$$
NLO BFKL Equation
\[
\omega f(q_1^2, q_2^2, \omega) = \delta^2(q_1^2 - q_2^2) + \int d^2\kappa \, K_{\text{NLL}}(q_1, \kappa) f(\kappa, q_2, \omega)
\]

(LO) Eigenfunctions
\[
\langle q|n, \nu \rangle = \frac{1}{\pi^{1/2}} (q^2)^{i\nu - \frac{1}{2}} e^{in\vartheta}
\]

\(n = \text{Conformal Spin: labels } \text{SL}(2, \mathbb{C}) \text{ reps.}\)

[\text{Lipatov'86}]

NLO 'Eigenvalues'
\[
\langle n, \nu | \mathcal{K} | \nu', n' \rangle
\]

\[
= \bar{\alpha}_s, \text{MS} \left[ \chi_0 \left( |n'|, \frac{1}{2} + i\nu' \right) + \alpha_s, \text{MS} \chi_1 \left( |n'|, \frac{1}{2} + i\nu' \right) \right]
\]

\[
- \frac{\bar{\alpha}_s, \text{MS}}{8N_c} \beta_0 \frac{\chi_0 \left( |n'|, \frac{1}{2} + i\nu' \right) \left\{ -i \frac{\partial}{\partial \nu'} + i \frac{\partial}{\partial \nu} - 2 \ln \mu^2 \right\}}{8N_c} \delta_{n,n'} \delta(\nu - \nu')
\]

\[
\bar{\alpha}_s \chi_0(n=0, \nu); \quad \bar{\alpha}_s \equiv \frac{\alpha_s N_c}{\pi}
\]

\[
\tilde{\alpha}_s \chi_0 + \bar{\alpha}_s \chi_1(n=0, \nu)
\]
$N = 4$ SYM & QCD

NLO BFKL Dijet Production

Angular Coefficients and BLM Procedure

Conclusions

NLL BFKL Kernel in QCD and $N = 4$ SYM

$$\omega(n, \gamma) = \bar{\alpha}_s \chi_0(|n|, \gamma) + \bar{\alpha}_s^2 \chi_1(|n|, \gamma)$$

Eigenvalues of the Scale Invariant Sector of the BFKL Kernel
Good Properties of Higher Conformal Spins

★ For $n = 0$

- Large and Negative NLO Corrections
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Though in $\mathcal{N} = 4$ SYM...

- Corrections Not Large
- Non-Analytic Terms $\delta^0_n$ and $\delta^2_n$ Not Present
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Though in $\mathcal{N} = 4$ SYM...
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★ **While for $n \geq 1$**
- Asymptotically ($\gamma \sim \frac{1}{2}$) Hardly Sensitive to Radiative Corrections
- Suggests to Look for Observables Insensitive to $n = 0$
Good Properties of Higher Conformal Spins

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Angular Coefficients and Ratios

Dijet Partonic Differential Cross Section

\( \phi = \vartheta_1 - \vartheta_2 - \pi, \ \vartheta_i \) Angles of the Tagged Jets

\[
\frac{d\hat{\sigma}(\bar{\alpha}_s, Y, p_{1,2}^2)}{d\phi} = \frac{\pi^2 \bar{\alpha}_s^2}{4 \sqrt{p_{1}^2 p_{2}^2}} \sum_{n=-\infty}^{\infty} e^{i n \phi} C_n(Y)
\]

\[
C_{n}^{\text{QCD}} (Y) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} e^{\bar{\alpha}_s(p^2)Y \left( \chi_0(|n|,\nu) + \bar{\alpha}_s(p^2) \left( \chi_1(|n|,\nu) - \frac{\beta_0}{8N_c} \frac{X_0(|n|,\nu)}{\left(\frac{1}{4} + \nu^2\right)} \right) \right)} \left(\frac{1}{4} + \nu^2\right)
\]

\[
C_{n}^{\text{SUSY}} (Y) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} e^{aY \left[ \chi_0(|n|,\nu) + a \chi_1^{\text{SUSY}}(|n|,\nu) \right]} \left(\frac{1}{4} + \nu^2\right) \quad (\beta_0 = 0)
\]
Total Cross Section Only Involves $n = 0$ as It Averages Over $\phi$

$$\hat{\sigma}(\alpha_s, Y, p_{1,2}^2) = \frac{\pi^3 \alpha_s^2}{2 \sqrt{p_{1}^2 p_{2}^2}} C_0(Y)$$

The Averages $\langle \cos(n\phi) \rangle$, $n \in \mathbb{Z}$ Project Out the Contribution of Higher $n$ Angular Components

$$\langle \cos(m\phi) \rangle = \frac{C_m(Y)}{C_0(Y)}$$
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In Order to Remove the Uncertainty Due to the Bad Convergence of the $n = 0$ Component

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Excellent Perturbative Convergence
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Excellent Perturbative Convergence
\[ \frac{c_2}{c_1} = \frac{\langle \cos 2\phi \rangle}{\langle \cos \phi \rangle} \]
The Rôle of the Renormalization Prescription

The Rôle of the Renormalization Prescription: the BLM Idea

- Renormalization Ambiguities Remain for Any Fixed-Order Computation
- Choice of Renormalization Scheme & Scale Can Render Small Perturbative Coefficients:
  First Orders Meaningful & Comparable with Physical Predictions
- Finite Renormalization $\leftrightarrow$ Redefinition of the Coupling

$$\alpha_s \rightarrow \alpha_s \left[ 1 + T \frac{\alpha_s}{\pi} \right]$$

[Brodsky, Lepage & Mackenzie’83]
The Rôle of the Renormalization Prescription

- To NLO, Redefinition of the Coupling = Rescaling of the Scale
  \[ \mu \rightarrow \bar{\mu} = \mu \exp \left( -\frac{T}{2\beta_0} \right) \]
  (Inverse Rescaling of Corresponding Landau Pole)
- BLM Prescription: Set the Scale by Redefining the Coupling to Absorb the Corrections Coming from Charge Renormalization.
- Perturbative Coefficients with the BLM Prescription Identical to Those of the Conformal Theory with \( \beta_0 = 0 \)
Application to the Pomeron Intercept

[Brody, Fadin, Kim, Lipatov & Pivovarov '99]

Original Computation of NLO BFKL Eigenvalues Gives

\[ \omega_{\text{MS}}(q^2,n,\nu) = N_c \chi_0(n,\nu) \frac{\alpha_{\text{MS}}(q^2)}{\pi} \left[ 1 + r_{\text{MS}}(n,\nu) \frac{\alpha_{\text{MS}}(q^2)}{\pi} \right] ; \quad r_{\text{MS}} = r_{\beta_{\text{MS}}} + r_{\text{conf}} \]

\[ r_{\beta_{\text{MS}}}(n,\nu) = -\frac{\beta_0}{4} \left[ \frac{\chi_0(n,\nu)}{2} - \frac{5}{3} \right] , \]

\[ r_{\text{conf}} = \frac{N_c}{4 \chi_0(n,\nu)} \left[ \frac{\pi^2}{3} - 4 \chi_0(n,\nu) - 6\zeta(3) - \left( \psi'' \left( \frac{n+1}{2} + i\nu \right) \right) + \left. \psi'' \left( \frac{n+1}{2} - i\nu \right) \right] \]

\[ - 2 \Phi \left( n, \frac{1}{2} + i\nu \right) - 2 \Phi \left( n, \frac{1}{2} - i\nu \right) + \frac{\pi^2}{2\nu} \text{sech}(\pi\nu) \text{tanh}(\pi\nu) \]

\[ \times \left\{ \left[ 3 + \left( 1 + \frac{N_f}{N_c^3} \right) \left( \frac{3}{4} - \frac{1}{16(1+\nu^2)} \right) \right] \delta_n - \left( 1 + \frac{N_f}{N_c^3} \right) \left( \frac{1}{8} - \frac{3}{32(1+\nu^2)} \right) \delta_n^2 \right\} . \]

\[ \overline{\text{MS}} + \text{Arbitrary Scale} \rightarrow \text{MOM Scheme} + \text{Optimal BLM Scale} \]
Application to the Pomeron Intercept

We Pass to MOM Scheme\[\alpha_{\text{MOM}} = \alpha_{\text{MS}} \left[1 + T_{\text{MOM}} \frac{\alpha_{\text{MS}}}{\pi}\right]\]

\[\omega^{\text{MOM}}(q^2_{\text{BLM}}, n, \nu) = \chi_0(n, \nu) \frac{\alpha_{\text{MS}}(q^2_{\text{BLM}})}{\pi} \left[1 + r_{\text{MOM}}(n, \nu) \frac{\alpha_{\text{MS}}(q^2_{\text{BLM}})}{\pi}\right];\]

\[r_{\text{MOM}}(n, \nu) = r_{\text{MS}}(n, \nu) + T_{\text{MOM}}\]

and Then Fix the Scale With BLM

\[q^2_{\text{BLM}}(n, \nu) = q^2 \exp \left[-\frac{4r_{\text{MOM}}^\beta(n, \nu)}{\beta_0}\right].\]

- More Sensible Result for Pomeron Intercept
- BLM Intercept Has Very Weak Dependence on the Energy Scale \(p^2\):
  1. In Agreement With Regge Theory
  2. Nearly Insensitive to Non-Perturbative Effects
  3. Approach to Conformal Behaviour
Angular Coefficients with BLM Setting and Physical Renormalization Prescription

Angular Coefficients in QCD+BLM & MSYM

[Angioni, Chachamis, JDM & Sabio Vera '11]

- Apply the Same Procedure for Our Angular Coefficients and Ratios
- Compare to Those Obtained in $\mathcal{N} = 4$ SYM.
- $\mathcal{N} = 4$ Coupling Going from $a = \bar{\alpha}_s(p^2/4)$ [MSYM$_-$] to $a = \bar{\alpha}_s(4p^2)$ [MSYM$_+$].

- Predictions for $\langle \cos m\phi \rangle$ Are 'Spoiled' By Bad Convergence of the $n = 0$ Contribution.
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Angular Coefficients with BLM Setting and Physical Renormalization Prescription

\[ \langle \cos \phi \rangle \]

- NLL (MOM, BLM)
- NLL $\overline{\text{MS}}$
- $\text{MSYM}_-$
- $\text{MSYM}_+$
- $\text{MSYM}$

NLO BFKL Dijet Production
Angular Coefficients and BLM Procedure
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High-Energy Dijets in $\mathcal{N} = 4$ SYM

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Angular Coefficients with BLM Setting and Physical Renormalization Prescription

\[ \langle \cos(2\phi) \rangle \]

- NLL (MOM, BLM)
- NLL MS
- MSYM-
- MSYM+
- MSYM

\( Y \)
Angular Coefficients with BLM Setting and Physical Renormalization Prescription
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\[ \frac{C_3}{C_2} \]

For different settings and prescriptions:
- **MSYM**
- **NLL (MOM, BLM)**
- **NLL MS**
- **SYM**
- **SYM+**

The graph shows the behavior of these coefficients as a function of the variable $Y$. The $N = 4$ SYM QCD framework is used for this analysis.
Angular Coefficients with BLM Setting and Physical Renormalization Prescription
Conclusions

- We computed observables relevant for dijet production in \( \mathcal{N} = 4 \) SYM.
- The ratios \( \frac{\langle \cos m\phi \rangle}{\langle \cos n\phi \rangle} \) provide most promising observables to test conformal structure of gauge theories in the high energy limit.

Open questions...

- May AdS/CFT help in collider phenomenology for well-chosen observables?
- SL(2, \( \mathbb{C} \)) and 4d conformal invariance related? (Not obvious)
- BLM as a natural prescription also beyond Multi-Regge regime?
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- The Ratios $\frac{\langle \cos m\phi \rangle}{\langle \cos n\phi \rangle}$ Provide Most Promising Observables to Test Conformal Structure of Gauge Theories in the High Energy Limit
- A BLM Choice of the Renormalization Scale Within a Physical MOM Scheme Renders Systematically Closer the QCD NLO Results for These Ratios to Those of Truly Conformal MSYM
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