New determination of the nonperturbative form factor in QCD transverse-momentum resummation for vector boson production

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In the b_* model for the Collins-Soper-Sterman (CSS) resummation, the resummed form factor is accompanied by the nonperturbative gaussian form factor, which is known to exhibit strong dependence on the the vector boson mass. The nonperturbative form factor of similar nature arises in another approach for the CSS resummation, the "minimal prescription (MP)" based on analytic continuation to treat the impact parameter transform. We perform a global fit of the nonperturbative form factor in the MP resummation at the next-to-leading logarithmic accuracy, with the Z boson production data at the Tevatron and the low energy Drell-Yan data, and find weak dependence on the vector boson mass.

We consider the hadroproduction of vector bosons, $h_1 + h_2 \to V(Q, y, \cdots) + X$, where the vector bosons $V = \gamma^*, Z, W$ have momentum Q^μ and rapidity y. The differential cross section with the center of mass energy $\sqrt{S}$ of the two colliding hadrons $h_{1,2}$ is given as ($x_{1,2} = Qe^{\pm y}/\sqrt{S}$),

$$d\sigma \propto \sum\nolimits_q e_q^2 \left[q_{h_1}(x_1, Q^2)\bar{q}_{h_2}(x_2, Q^2) + \bar{q}_{h_1}(x_1, Q^2) q_{h_2}(x_2, Q^2) \right] + \cdots, \tag{1}$$

with the product of the (anti-)quark distributions for $h_{1,2}$ and the ellipses standing for the perturbative corrections, the contributions associated with the gluon distributions, etc. This is a benchmark process at the LHC; the comparison with experimental data gives constraints for the PDFs; this is also important for the new physics search. Thus, precise theoretical predictions are desirable. Now the perturbative QCD corrections are known up to NNLO not only for the total cross sections and the rapidity distributions, but also for fully differential cross sections [1].

We note that the vector bosons $V = \gamma^*, Z, W$ are mostly produced at small transverse momentum Q_T of typically a few GeV: the vector bosons with the large Q_T are obtained by the recoil from the hard emission and can be treated by the fixed-order perturbation theory. On the other hand, the large cross section at the small Q_T is obtained by the recoil from the emission of the soft gluons, whose contributions are accompanied by the logarithms $\alpha_s \ln^2 Q^2/Q_T^2$, $\alpha_s \ln Q^2/Q_T^2$, which become very large and diverge for small Q_T and have to be resummed to all orders in α_s to obtain meaningful results. The contributions due to the multiple gluon emission, where the total sum of the gluon's transverse momenta equals Q_T, are conveniently treated in the impact parameter b space conjugate to the transverse-momentum space with

$\delta^{(2)}(Q_T - k_{1T} - k_{2T} - \cdots - k_{nT}) = \int d^2 b e^{ib\cdot Q_T} \prod_n e^{-ib\cdot k_T}$. According to the Collins-Soper-Sterman (CSS) resummation formalism [2], the resummed contributions to all orders can be reorganized in terms of the quark and gluon PDFs, the perturbatively calculable coefficient functions, the hard vertex to produce the vector boson V, and the Sudakov factor due to the contributions of soft gluon radiation, which is given as exponentiation of the corresponding all-orders perturbation series. The resummation replaces the RHS of (1) by

$$\int d^2 b e^{i\mathbf{b}\cdot \mathbf{Q}_T} e^{S(b,Q)} \sum_q e_q^2 \left[q_{h_1}\left(x_1, \frac{b_0^2}{b^2}\right) \bar{q}_{h_2}\left(x_2, \frac{b_0^2}{b^2}\right) + \bar{q}_{h_1}\left(x_1, \frac{b_0^2}{b^2}\right) q_{h_2}\left(x_2, \frac{b_0^2}{b^2}\right)\right] + \cdots , \tag{2}$$

as the b-space Fourier transform back to the Q_T space. Here, $b_0 = 2e^{-\gamma_E}$ with γ_E being the Euler constant and the optimal scale for the PDFs is given by the order of $1/b$. We do not show explicitly the coefficient functions and the hard production vertex, with the corresponding higher-order perturbative corrections being contained in the ellipses, while we show the Sudakov factor $e^{S(b,Q)}$, which is universal with ($X(\alpha_s) = \sum_{n=1}^{\infty} (\alpha_s/2\pi)^n X^{(n)}$ with $X = A, B$)

$$S(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left\{ \left(\ln \frac{Q^2}{\mu^2} \right) A\left(\alpha_s(\mu^2)\right) + B\left(\alpha_s(\mu^2)\right) \right\},$$

where $A^{(1)} = 2C_F$ is the leading logarithmic (LL) contribution, and $A^{(2)} = 2C_F[(67/18 - \pi^2/6)C_G - 5N_f/9]$ and $B^{(1)} = -3C_F$ are the next-to-leading (NLL) level contributions, with $C_F = (N_c^2 - 1)/(2N_c)$, $C_G = N_c$, and N_f being the number of QCD massless flavors. In this work we employ the resummation at the NLL accuracy. The importance of the NLL accuracy is demonstrated in, e.g., Fig. 1 in the first paper in [8].

The Sudakov factor associated with all-orders resummation should be eventually accompanied by the nonperturbative form factor, which is usually taken as a gaussian form and would be considered as originating from the intrinsic k_T of partons inside hadron. This implies the following replacement in (2), with Q_0 denoting a certain fixed momentum,

$$e^{S(b,Q)} \to e^{S(b,Q)} e^{-g_{NP} b^2}, \qquad g_{NP} = g_1 + g_2 \ln \frac{Q}{2Q_0}, \tag{3}$$

where the linear dependence of g_{NP} on $\ln Q$ is obeyed by the renormalization group. We have, at least, two nonperturbative parameters g_1, g_2 associated with the resummed form factor.

The participation of the nonperturbative form factor is also signaled by the infrared Landau pole arising in the integrand of (2) at $b \simeq (1/Q)e^{1/[2\beta_0 \alpha_s(Q^2)]}$ from the all-orders resummation embodied by the Sudakov factor, where β_0 is the first coefficient of the QCD β function. A conventional approach to avoid the Landau pole is to introduce the cut-off $b_{\rm max}$ in the b integration: making the replacement $b \to b_* = b/\sqrt{1 + b^2/b_{\rm max}^2}$ with $b_{\rm max} \simeq 0.5$ GeV^{-1} in the Sudakov factor and the PDFs in (2), the b integration is effectively frozen before reaching the Landau pole. Based on this, the resummed cross sections are fitted to the experimental data and the results of this global fit give $g_1 \simeq -0.08$ GeV^2, $g_2 \simeq 0.67$ GeV^2 [3] and $g_1 \simeq 0.016$ GeV^2, $g_2 \simeq 0.54$ GeV^2 [4] for $Q_0 = 1.6$ GeV, exhibiting the strong $\ln Q$ dependence of g_{NP}.

Another approach to circumvent the Landau pole is based on the deformation of the b-integration contour in the complex b space [5, 6, 7, 8, 9]. Its advantages are that it leaves unchanged the perturbative expansion to any (and arbitrarily-high) fixed order in α_s, and that it does not require any infrared cut-off, like $b_{\rm max}$, in the b-space integration, so this approach is often called the "minimal prescription (MP)". However, now we need the PDFs at the complex

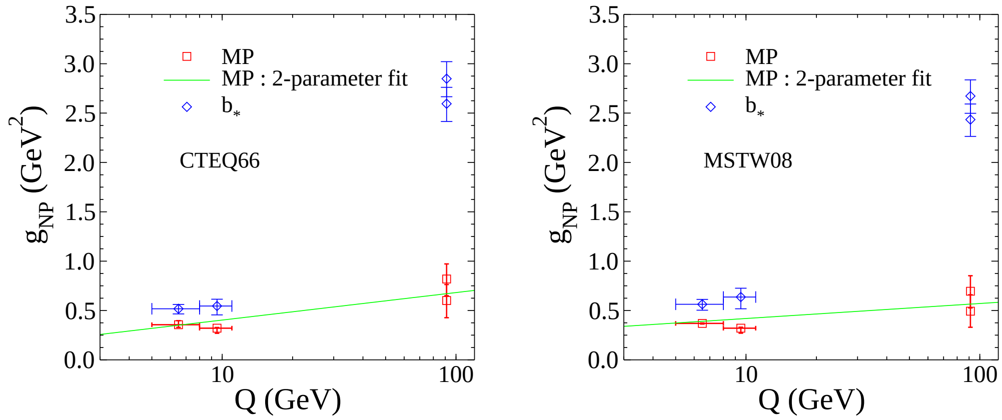

Figure 1: Fit of g_{NP} to R209, and CDF and D0 Z data, using CTEQ6.6M (left panel) and MSTW2008 (right panel) sets for the input PDFs to calculate the NLL resummed cross sections.

scale b_0/b (see (2)) and the numerical calculations become complicated. The values of g_1, g_2 based on the global fit have been unknown in the MP. Performing the matching of, e.g., the Q_T-integrated cross section, $\int dQ_T d\sigma/dQ_T$, and the average Q_T, $\int dQ_T Q_T d\sigma/dQ_T$, between the above two schemes, the b_* model and the MP, the results indicate that the values of g_1, g_2 in the MP are largely different from those in the b_* model [10].

We perform a global fit of the nonperturbative form factor in the MP. We calculate the Q_T-differential and y-integrated cross sections at the NLL accuracy in the MP using the method described in Appendix in [6]. The experimental data sets we use are the available rapidity-integrated cross section data: the low-energy Drell-Yan data (R209 measured at CERN in the different Q ranges, $5 < Q < 8$ GeV and $8 < Q < 11$ GeV) and the Tevatron Z-boson production data (CDF Run-0, Run-1 and D0 Run-1). The cross section calculated with the nonperturbative form factor $e^{-g_{NP}b^2}$ in (3) is compared with each of the above data sets, associated with different Q ranges, and we perform the 1-parameter fit of g_{NP} for each data sets, allowing us to adjust overall normalization factor of the calculated cross section [10]. Plotting the results as a function of the vector boson mass Q, we can extract the Q-dependence of g_{NP}. The symbol $\Box$ in Fig. 1 shows the fitted results of g_{NP} as a function of Q: the two symbols at low Q are obtained using the R209 data, while the upper and lower symbols at high Q are obtained using the CDF and D0 data, respectively [10]. We perform those fits using the two different sets of NLO PDFs, CTEQ6.6M and MSTW2008. For comparison, we also plot the results of the similar fits in the b_* model by the symbol $\Diamond$, which show the strong $\ln Q$ dependence corresponding to the above-mentioned values [3, 4] of g_1, g_2 in the b_* model. We see that the results in the MP have rather mild $\ln Q$ dependence, with some dependence on the PDFs.

We also use the 2-paramater form (3) of the nonperturbative form factor and perform the 2-parameter fit of g_1, g_2 using all the above-mentioned data sets, allowing us to adjust the overall normalization factor of the calculated cross section similarly as in the case of the 1-parameter fit [10]. We obtain good description of the Q_T distributions for Drell-Yan and Z-boson productions using the 2-paramater form (3), with $g_1 = 0.241^{+0.026}_{-0.028}$ GeV^2, $g_2 = 0.121^{+0.041}_{-0.038}$ GeV^2 for the CTEQ6.6M PDF and $g_1 = 0.330^{+0.024}_{-0.026}$ GeV^2, $g_2 = 0.066^{+0.039}_{-0.037}$ GeV^2 for the MSTW2008 PDF; here, the errors in the results of g_1, g_2 corresponds to the 1-σ deviation from the χ^2 minimum. Our best fit value of g_1, g_2 gives the solid line in Fig. 1, and the mild $\ln Q$ dependence in the MP reflects that the value of g_2 in the MP is smaller than that in the b_* model by the factor 4 or more.

We note that the Fourier transform of the nonperturbative form factor in (3) gives the intrinsic transverse-momentum distribution $e^{-k_T^2/(4g_{NP})}$, which implies $\langle k_T^2 \rangle = 4g_{NP}$ for the average k_T^2. The mild Q dependence in the MP gives $\langle k_T^2 \rangle = 4g_{NP} \lesssim 2$ GeV^2 over wide range of Q. Because this represents the combined contributions from the two protons $h_{1,2}$, we obtain $\langle k_T^2 \rangle_{\text{1-proton}} \lesssim 1$ GeV^2. This suggests that the nonperturbative form factor in the MP can be naturally interpreted as arising from the intrinsic k_T of partons inside hadrons.

To summarize, we have discussed the NLL resummation in the vector boson production, which is crucial for reliable prediction of the transverse-momentum Q_T distribution. We have the Sudakov factor and the associated nonperturbative form factor which is parameterized by the two nonperturbative parameters g_1, g_2. We employed the MP based on analytic continuation procedure in the impact-parameter space, instead of using the conventional b_* model. In the MP, we performed a first systematic determination of g_1, g_2 by the global fit of the NLL-resummed cross section to experimental data. The results are obtained for the two popular sets of the PDFs, and exhibit the significantly weaker $\ln Q$ dependence of the nonperturbative form factor than that in the b_* model. We mention that the so-called "revised b_* model" using the cut-off $b_{\max}$ which is three times larger than the usual choice $b_{\max} \simeq 0.5$ GeV^{-1} gives the small value of g_2 [11] similar to the present result, so the investigation of the relation between the MP and the revised b_* model would be interesting. We also found that the nonperturbative form factor in the MP can be naturally interpreted as arising from the intrinsic k_T. For more detailed analysis, more data, in particular, the low energy Drell-Yan data, are desirable. g_1, g_2 in the MP determined by us are applicable to the production of the colorless final states, W, Higgs, diboson, etc.

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