The statistical model for parton distributions

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Outline

- Basic procedure to construct the statistical polarized parton distributions
- Essential features from unpolarized and polarized Deep Inelastic Scattering data
- Predictions tested against new data : DIS, Semi-inclusive DIS and several hadronic processes
- Extension to transverse momentum dependence (TMD): Melosh-Wigner effects
- Conclusions

Collaboration with Claude Bourrely and Jacques Soffer

- A Statistical Approach for Polarized Parton Distributions
  Euro. Phys. J. C23, 487 (2002)
- Recent Tests for the Statistical Parton Distributions
  Mod. Phys. Letters A18, 771 (2003)
- The Statistical Parton Distributions: status and prospects
  Euro. Phys. J. C41,327 (2005)
- The extension to the transverse momemtum of the statistical parton distributions
  Mod. Phys. Letters A21, 143 (2006)
- Strangeness asymmetry of the nucleon in the statistical parton model
  Phys. Lett. B648, 39 (2007)
- How is transversity related to helicity for quarks and antiquarks in a proton?
  Mod. Phys. Letters A24, 1889 (2009)
- Semiinclusive DIS cross sections and spin asymmetries in the quantum statistical parton distributions approach
  Phys. Rev. D83, 074008 (2011)

Our motivation and goal

- Will propose a statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some QCD features

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- Will propose a statistical approach of the nucleon viewed as a gas of massless partons in equilibrium at a given temperature in a finite size volume.
- Will incorporate some QCD features
- Will parametrize our PDF in terms of a very few number of physical parameters, at variance with standard polynomial type parametrizations
- Will be able to construct **simultaneously** unpolarized and polarized PDF: a unique case on the market!
- Will be able to describe physical observables both in DIS and hadronic collisions

Basic procedure

Use a simple description of the PDF, at input scale Q_0^2, proportional to $[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$, *plus* sign for quarks and antiquarks, corresponds to a Fermi-Dirac distribution and *minus* sign for gluons, corresponds to a Bose-Einstein distribution. X_{0p} is a constant which plays the role of the *thermodynamical potential* of the parton *p* and $\bar{x}$ is the *universal temperature*, which is the same for all partons.

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From the chiral structure of QCD, we have two important properties, allowing to relate quark and antiquark distributions and to restrict the gluon distribution:

- Potential of a quark q^h of helicity *h* is opposite to the potential of the corresponding antiquark $\bar{q}^{-h}$ of helicity -*h*, $X_{0q}^h = -X_{0\bar{q}}^{-h}$.

- Potential of the gluon G is zero, $X_{0G} = 0$.

The polarized PDF at $Q_0^2 = 4\mathrm{GeV}^2$

For light quarks $q = u, d$ of helicity $h = \pm$, we take

$$xq^{(h)}(x, Q_0^2) = \frac{AX_{0q}^{h} x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A} x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} ,$$

consequently for antiquarks of helicity $h = \mp$

$$x\bar{q}^{(-h)}(x, Q_0^2) = \frac{\bar{A}(X_{0q}^h)^{-1} x^{2b}}{\exp[(x + X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A} x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} .$$

Note: $q = q^+ + q^-$ and $\Delta q = q^+ - q^-$ (idem for $\bar{q}$).

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For strange quarks and antiquarks, s and $\bar{s}$, given our poor knowledge on both unpolarized and polarized distributions, we first took in 2002

$xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) = \frac{1}{4}[x\bar{u}(x, Q_0^2) + x\bar{d}(x, Q_0^2)]$

and

$x\Delta s(x, Q_0^2) = x\Delta\bar{s}(x, Q_0^2) = \frac{1}{3}[x\Delta\bar{d}(x, Q_0^2) - x\Delta\bar{u}(x, Q_0^2)]$.

However given the strange quark asymmetry, this was improved in Phys. Lett. B648, 39 (2007).

For gluons we use a Bose-Einstein expression given by $xG(x, Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x})-1}$, with a vanishing potential and the same temperature $\bar{x}$. We also need to specify the polarized gluon distribution and we take the particular choice $x\Delta G(x, Q_0^2) = 0$.

Essential features from the DIS data

From well established features of u and d extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$ dominates over $d(x)$, so we should have $X_{0u}^{+} + X_{0u}^{-} > X_{0d}^{+} + X_{0d}^{-}$
- $\Delta u(x) > 0$, therefore $X_{0u}^{+} > X_{0u}^{-}$
- $\Delta d(x) < 0$, therefore $X_{0d}^{-} > X_{0d}^{+}$.

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- $u(x)$ dominates over $d(x)$, so we should have $X^+_{0u} + X^-_{0u} > X^+_{0d} + X^-_{0d}$
- $\Delta u(x) > 0$, therefore $X^+_{0u} > X^-_{0u}$
- $\Delta d(x) < 0$, therefore $X^-_{0d} > X^+_{0d}$.

So we expect X^+_{0u} to be the largest potential and X^+_{0d} the smallest one. In fact, from our fit we have obtained the following ordering (see below)

$$X^+_{0u} > X^-_{0d} \sim X^-_{0u} > X^+_{0d} \ .$$

This ordering has important consequences for $\bar{u}$ and $\bar{d}$, namely

- $\bar{d}(x) > \bar{u}(x)$, flavor symmetry breaking expected from Pauli exclusion principle. This was already confirmed by the violation of the Gottfried sum rule (NMC).
- $\Delta\bar{u}(x) > 0$ and $\Delta\bar{d}(x) < 0$, a PREDICTION in agreement with polarized DIS (see below) and will be more precisely checked at RHIC-BNL from $W^{\pm}$ production.

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- Note that since $u^{-}(x) \sim d^{-}(x)$, it follows that $\bar{u}^{+}(x) \sim \bar{d}^{+}(x)$, so we have

$$\Delta\bar{u}(x) - \Delta\bar{d}(x) \sim \bar{d}(x) - \bar{u}(x) \ ,$$

i.e. the flavor symmetry breaking is almost the same for unpolarized and polarized distributions ($\Delta\bar{u}$ and $\Delta\bar{d}$ contribute to about 10% to the Bjorken sum rule).

Nine free parameters

By performing a NLO QCD evolution of these PDF, we were able to obtain a good description of a large set of very precise data on $F_2^p(x,Q^2), F_2^n(x,Q^2), xF_3^{\nu N}(x,Q^2)$ and $g_1^{p,d,n}(x,Q^2)$, in correspondance with nine free parameters with some physical significance:

* the four potentials $X_{0u}^+,\ X_{0u}^-,\ X_{0d}^-,\ X_{0d}^+$,

* the universal temperature $\bar{x}$,

* and $b,\ \tilde{b},\ b_G,\ \tilde{A}$.

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* the universal temperature $\bar{x}$,

* and $b,\ \tilde{b},\ b_G,\ \tilde{A}$.

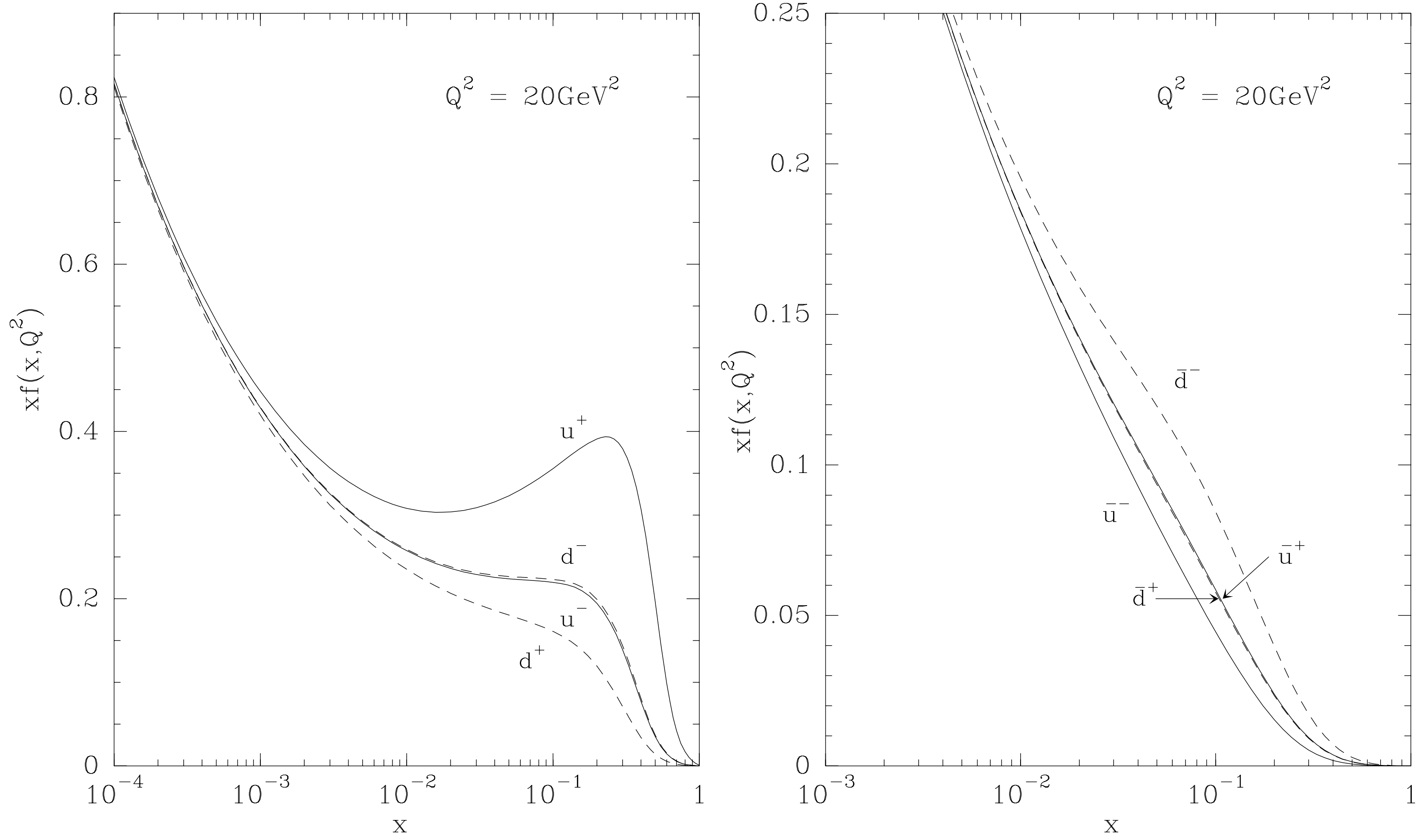
We also have three additional parameters, $A,\ \bar{A},\ A_G$, which are fixed by 3 normalization conditions .

$$u - \bar{u} = 2,\ \ d - \bar{d} = 1$$

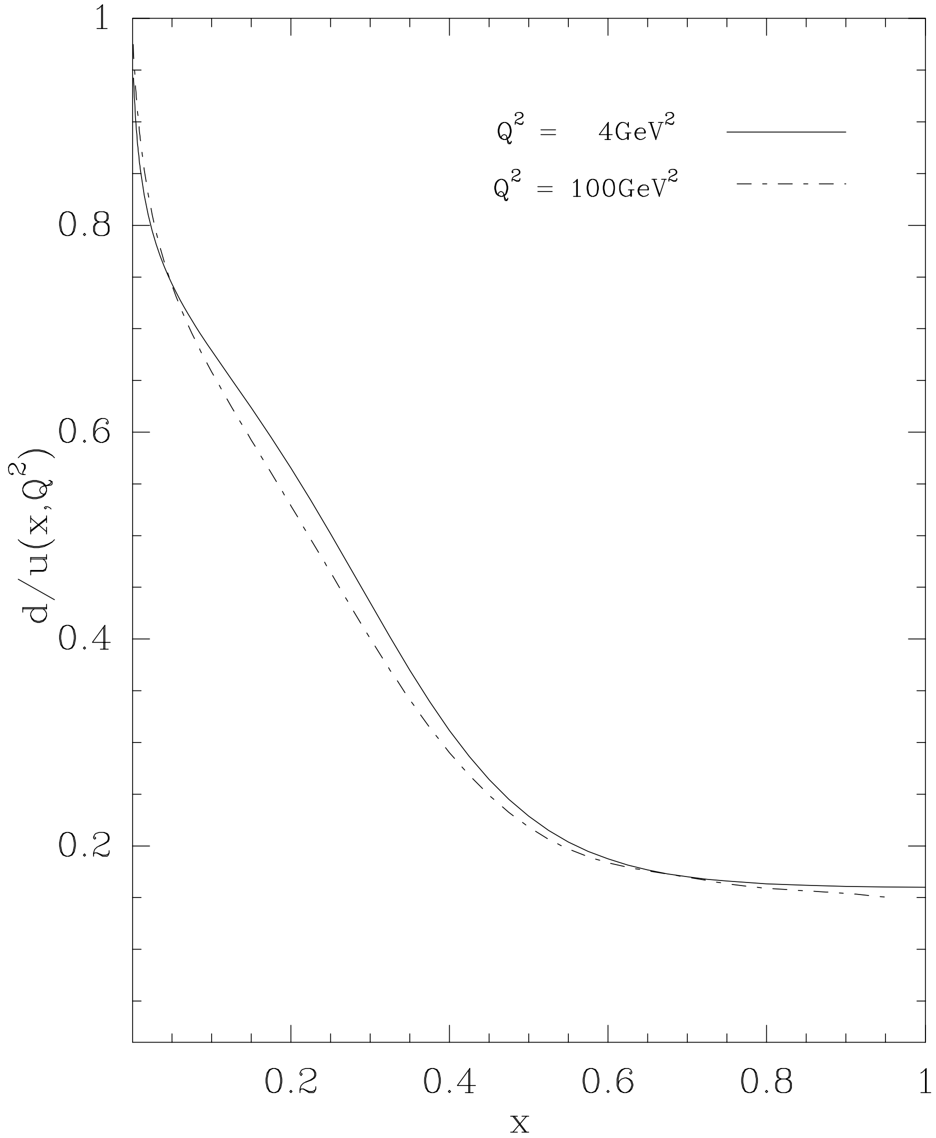
and the momentum sum rule.

Polarized light quarks distributions versus x

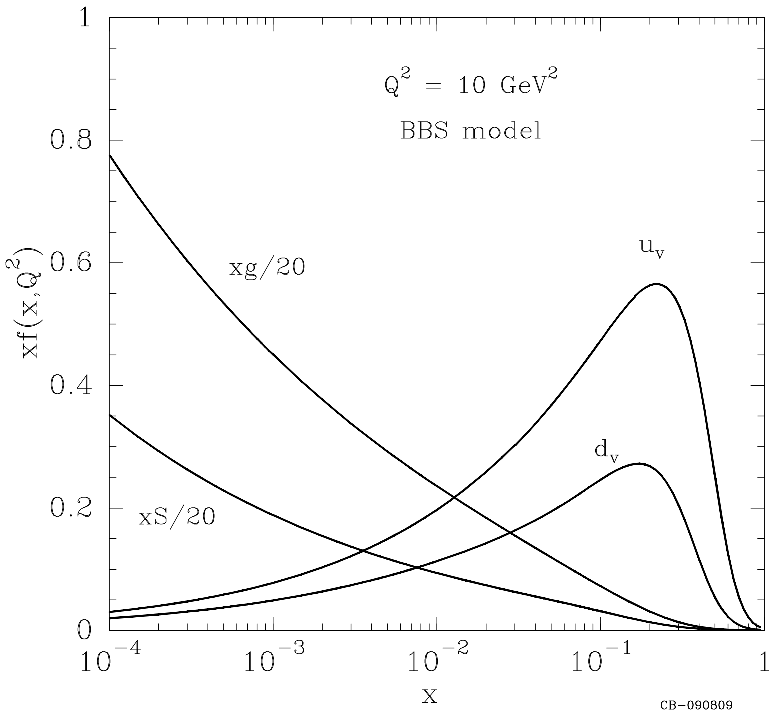
As we could anticipated u^+, is the largest one, is maximum near $x = 0.3$ and $u^- \simeq d^-$.
Therefore for the antiquarks $\bar{d}^-$ is the largest one and $\bar{u}^+ \simeq \bar{d}^+$
Moreover we find $\Delta\Sigma(Q_0^2) = 0.28$

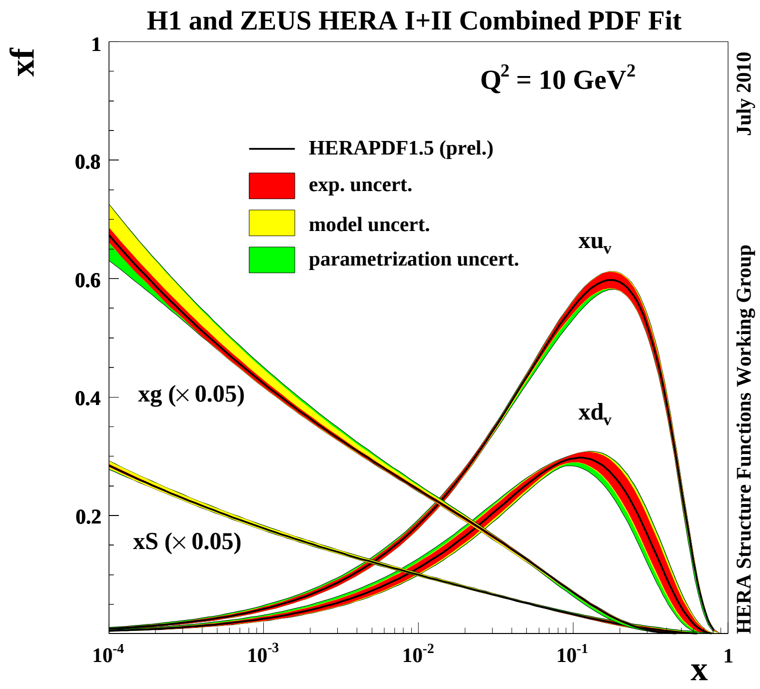


The d/u ratio versus x



A global view of the unpolarized parton distributions



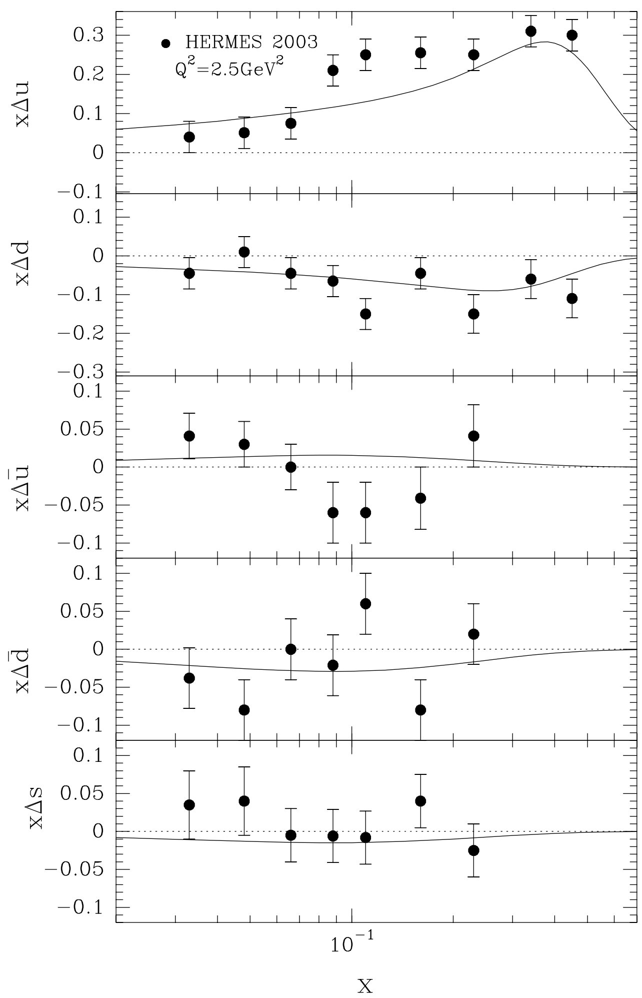


Predictions tested against some data 2002 - 2005

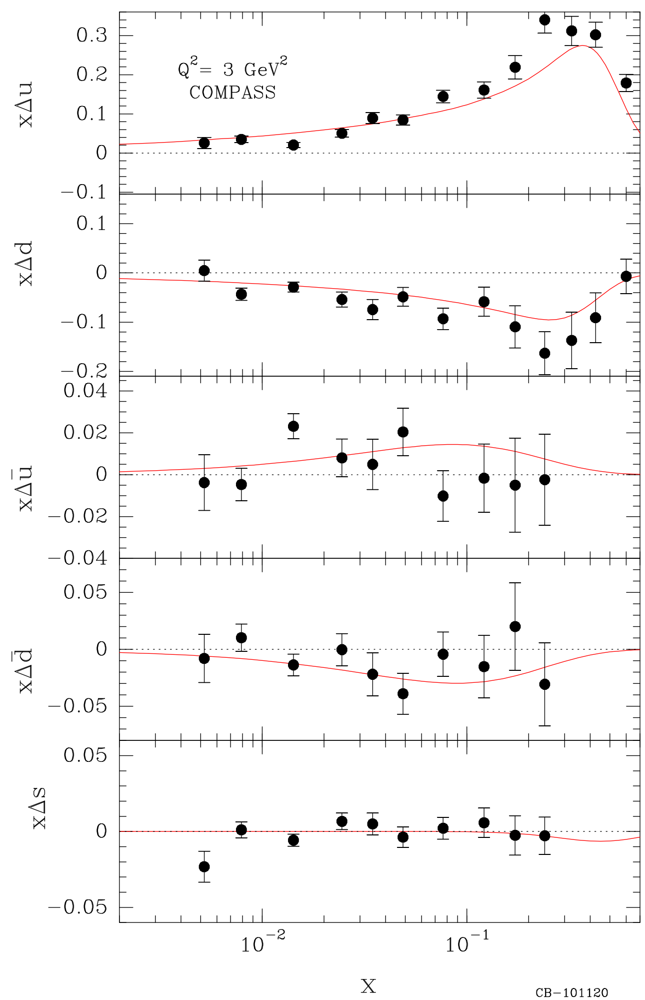
- Deep Inelastic Scattering
- Hadronic Collisions

Helicity distributions versus x at DESY, JLab and COMPASS

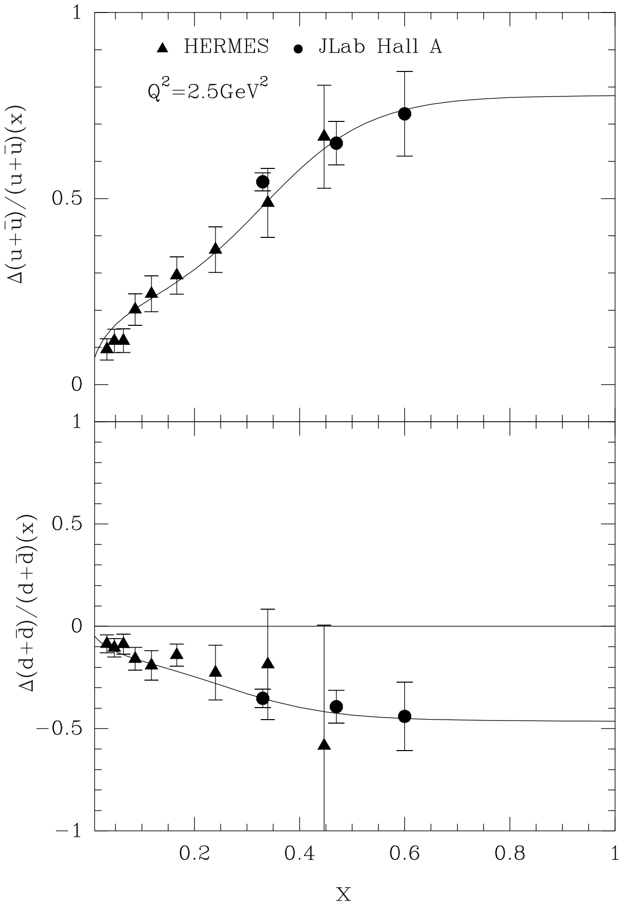
2003



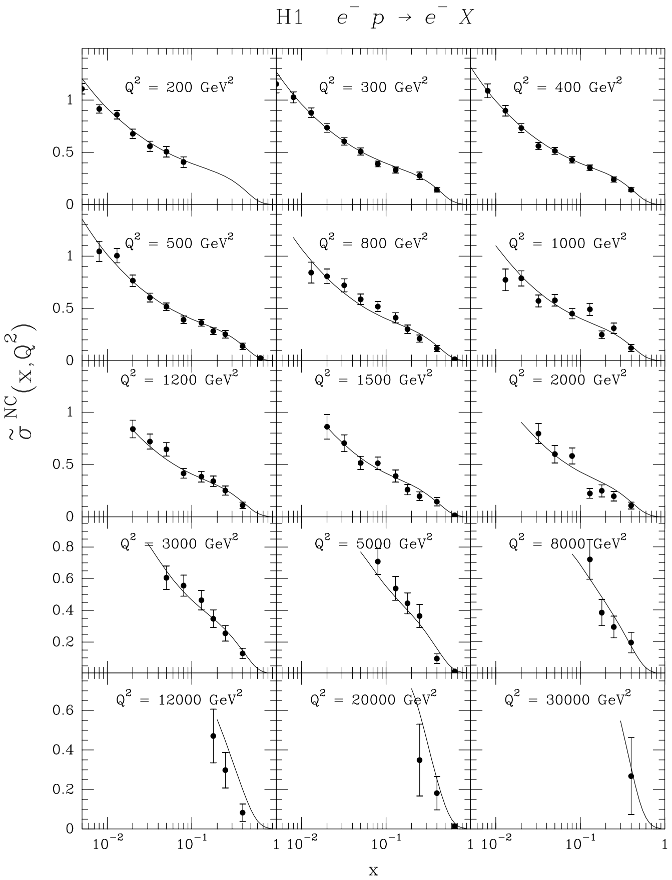
2010

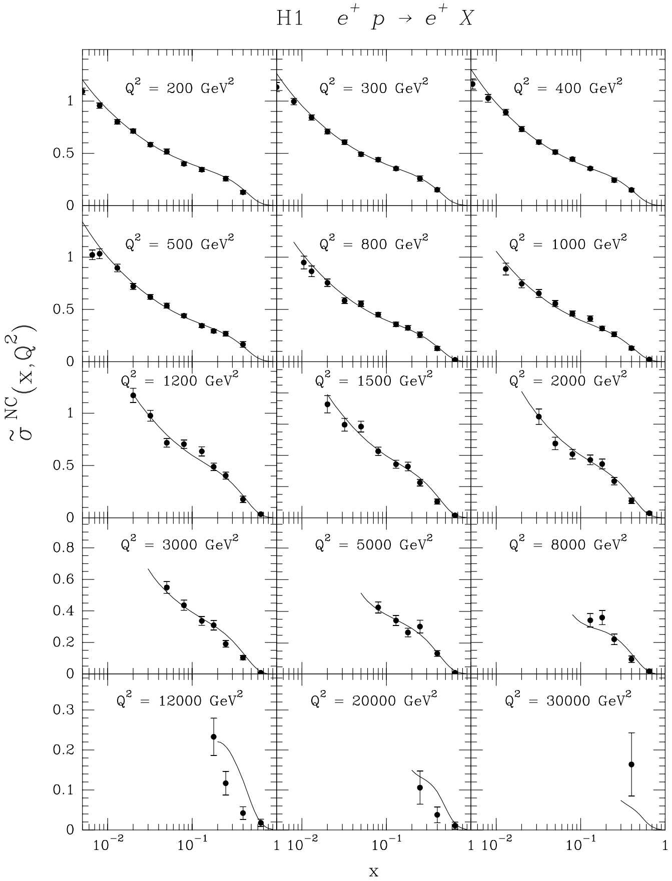


Helicity distributions versus x at DESY and JLab (2004)

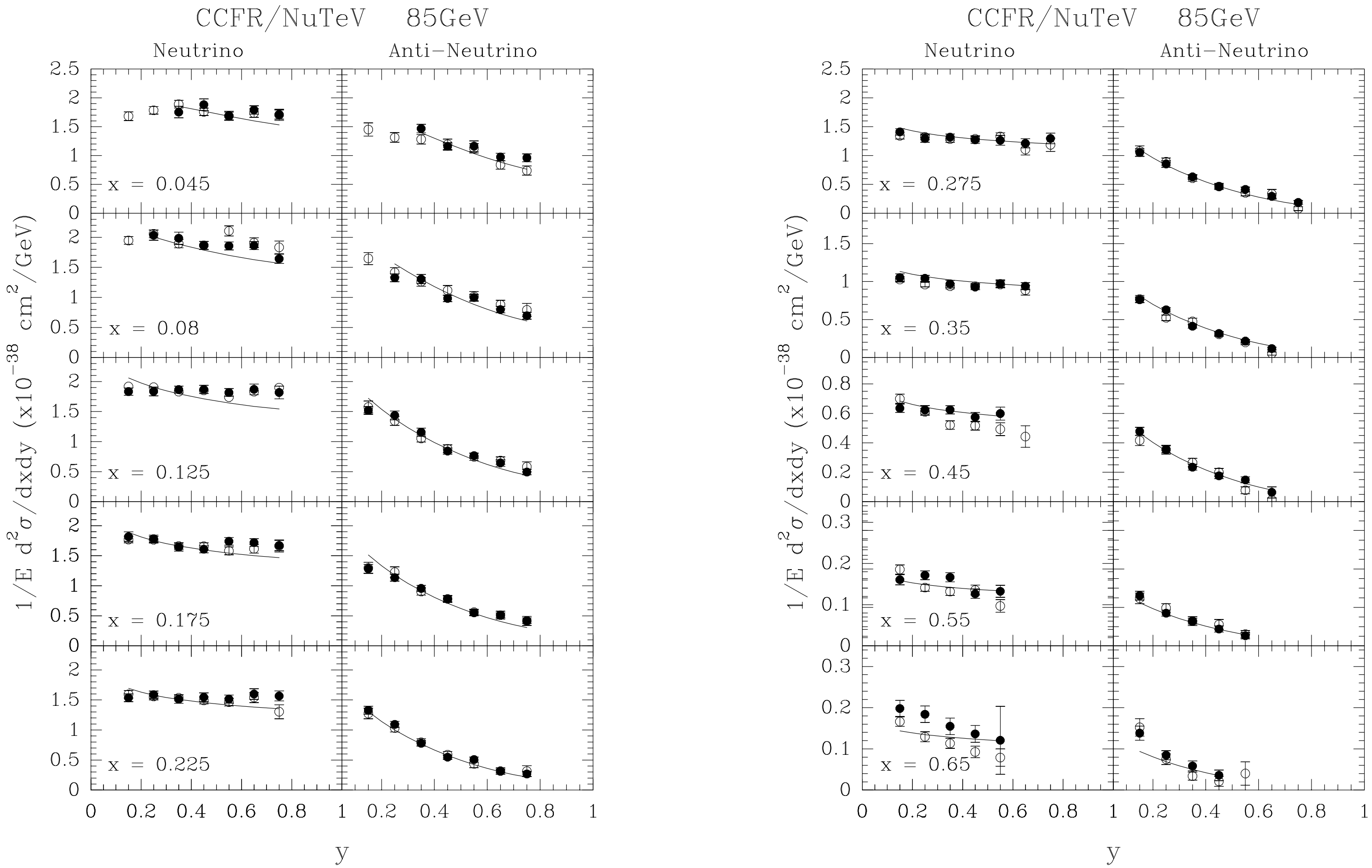


Neutral current in $e^{\pm}p$ collisions (H1) (2003)



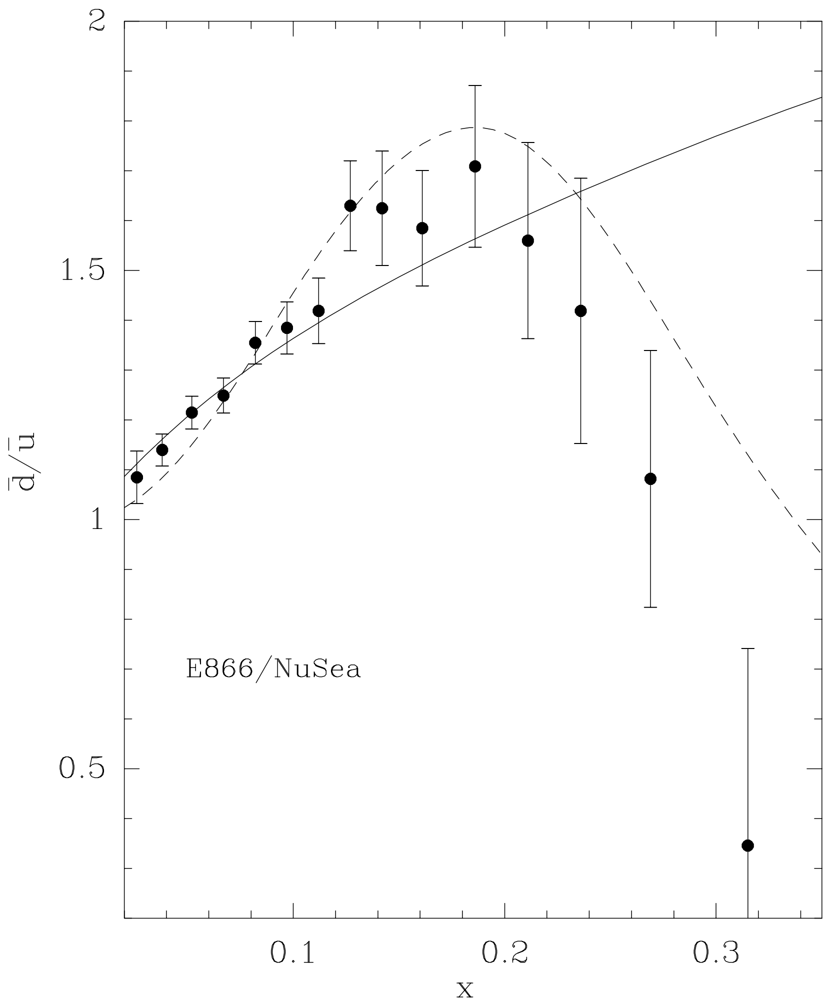


Charged current neutrino cross sections at FNAL (2004)

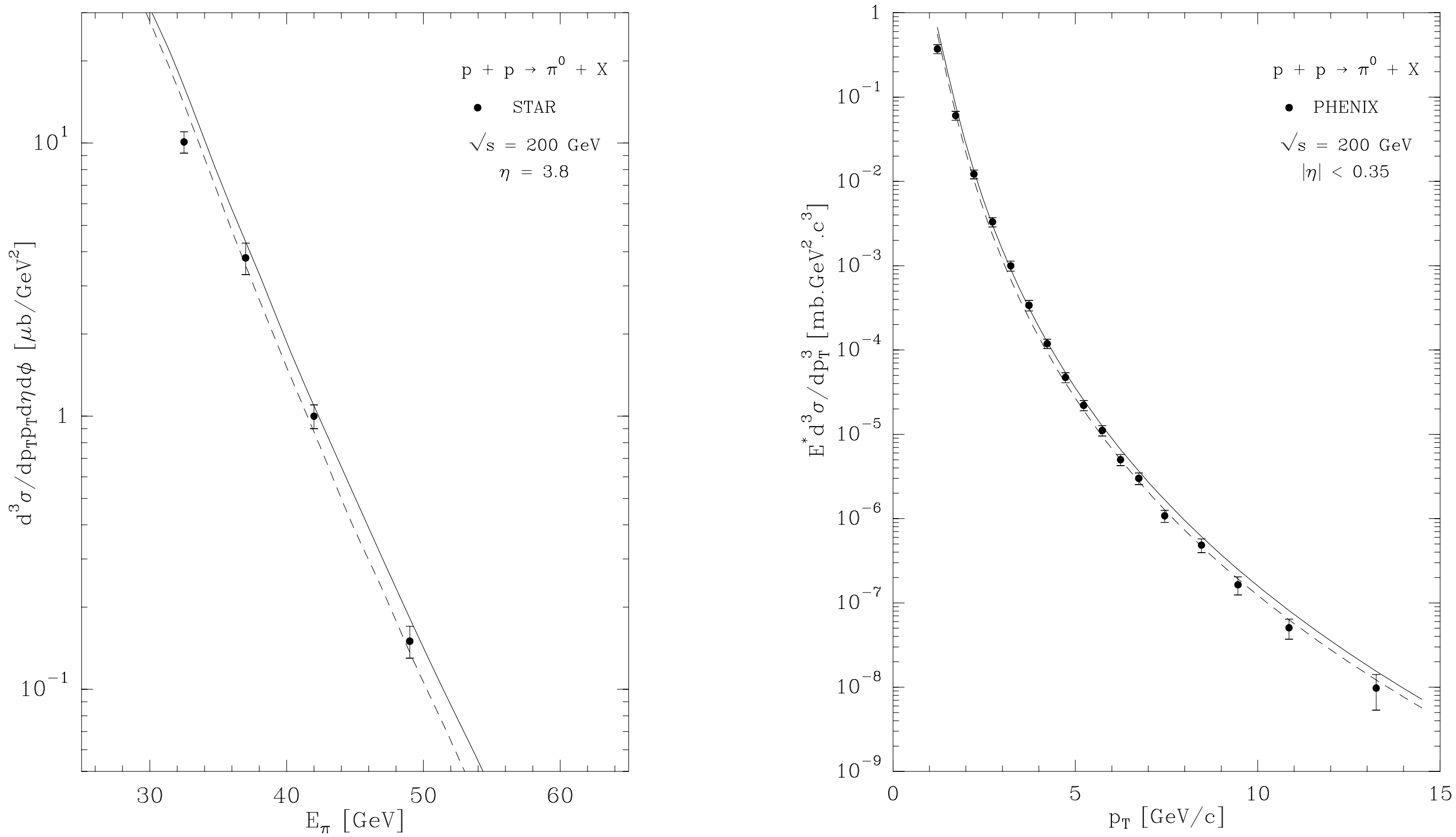


The important issue of $\bar{d}/\bar{u}$ at large x

$Q^2 = 54\text{GeV}^2$

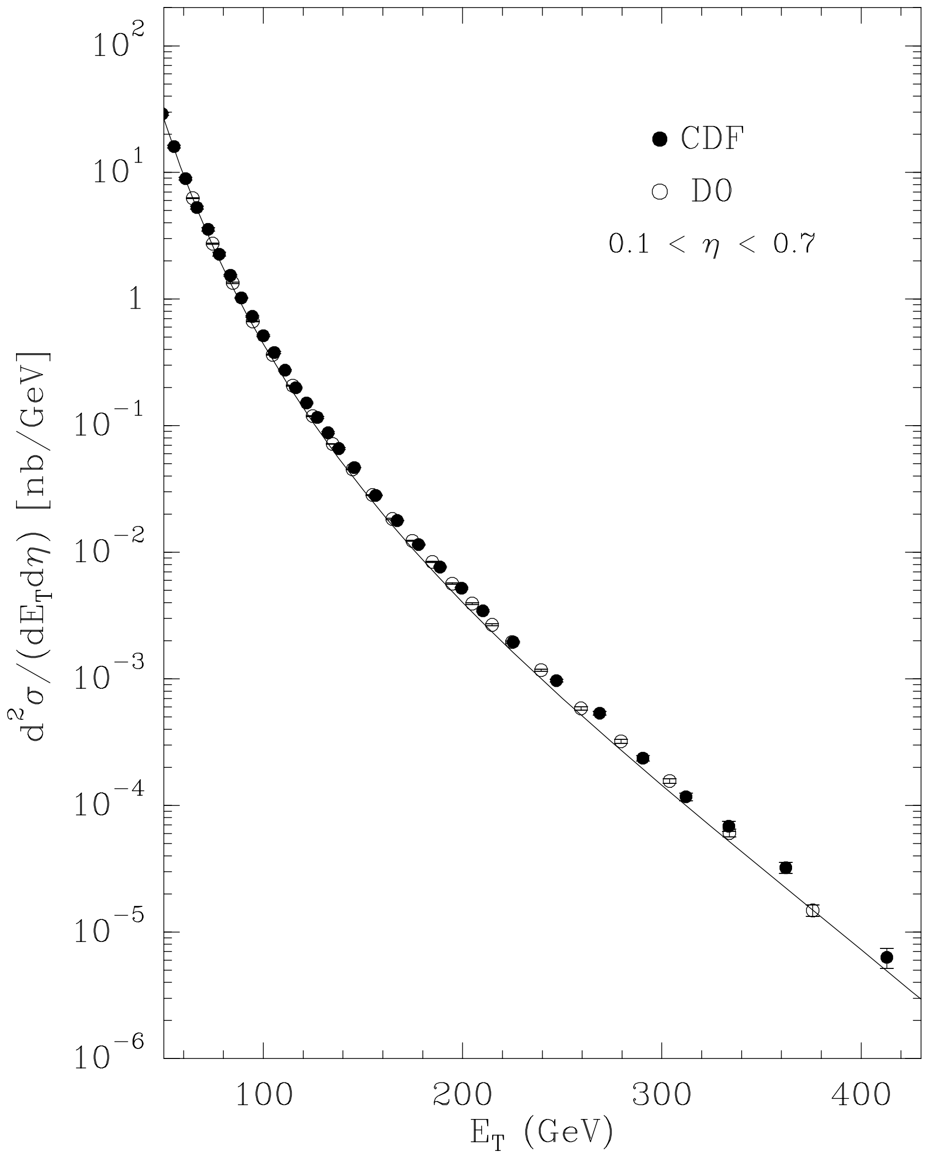


Inclusive π^0 production in pp collisions at RHIC (2003)



Mid-rapidity and central region

Single-jet production in $\bar{p}p$ collisions at FNAL



Predictions tested against some very recent data

- Unpolarized Deep Inelastic Scattering

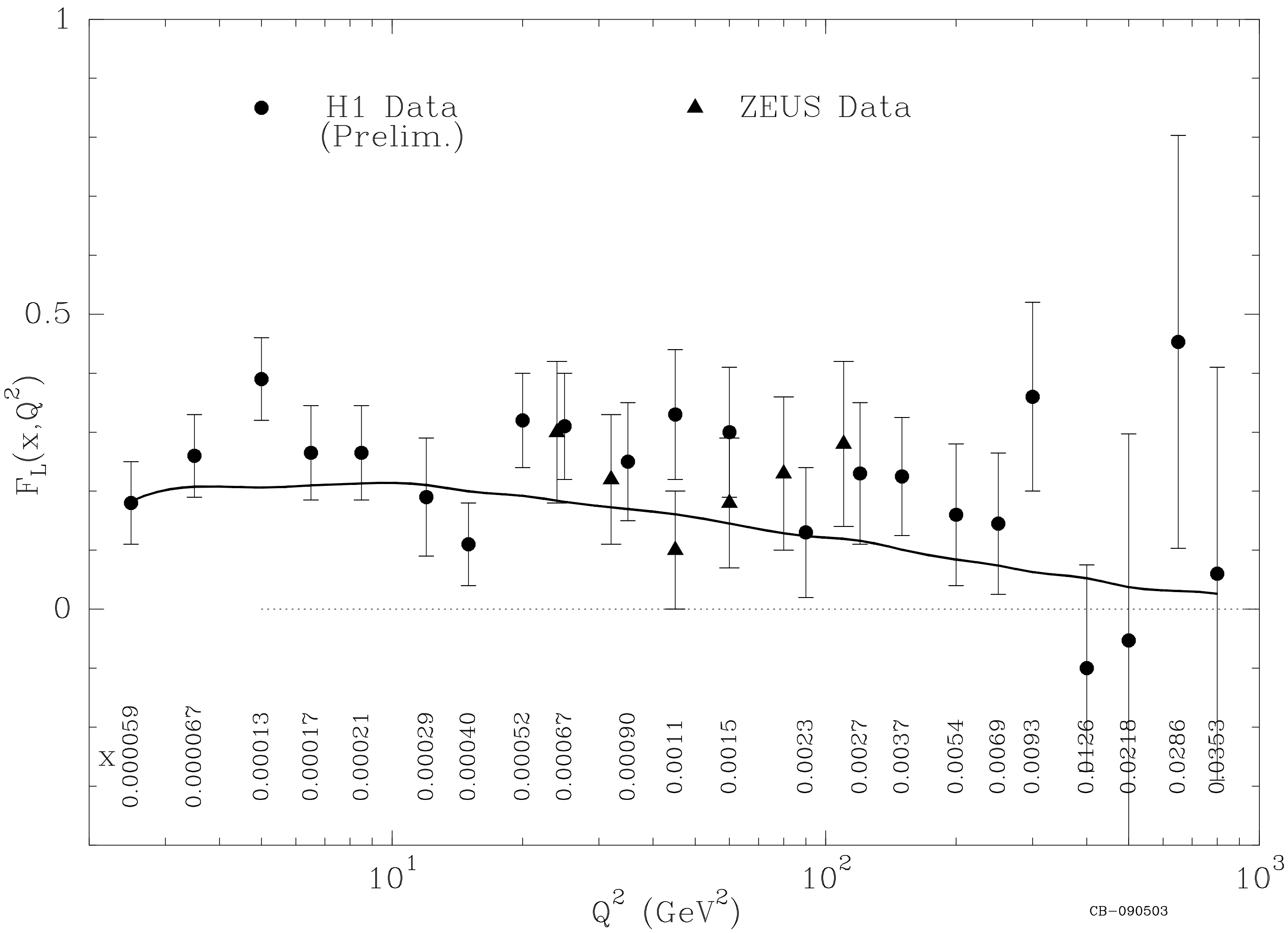
  * The structure function F_L is a direct sensitivity to the gluon: $F_L = 0$ in quark-parton model, but $F_L \neq 0$ in NLO pQCD

- Polarized Deep Inelastic Scattering

  * Polarized valence light quarks from Semi-inclusive DIS on Deuterium

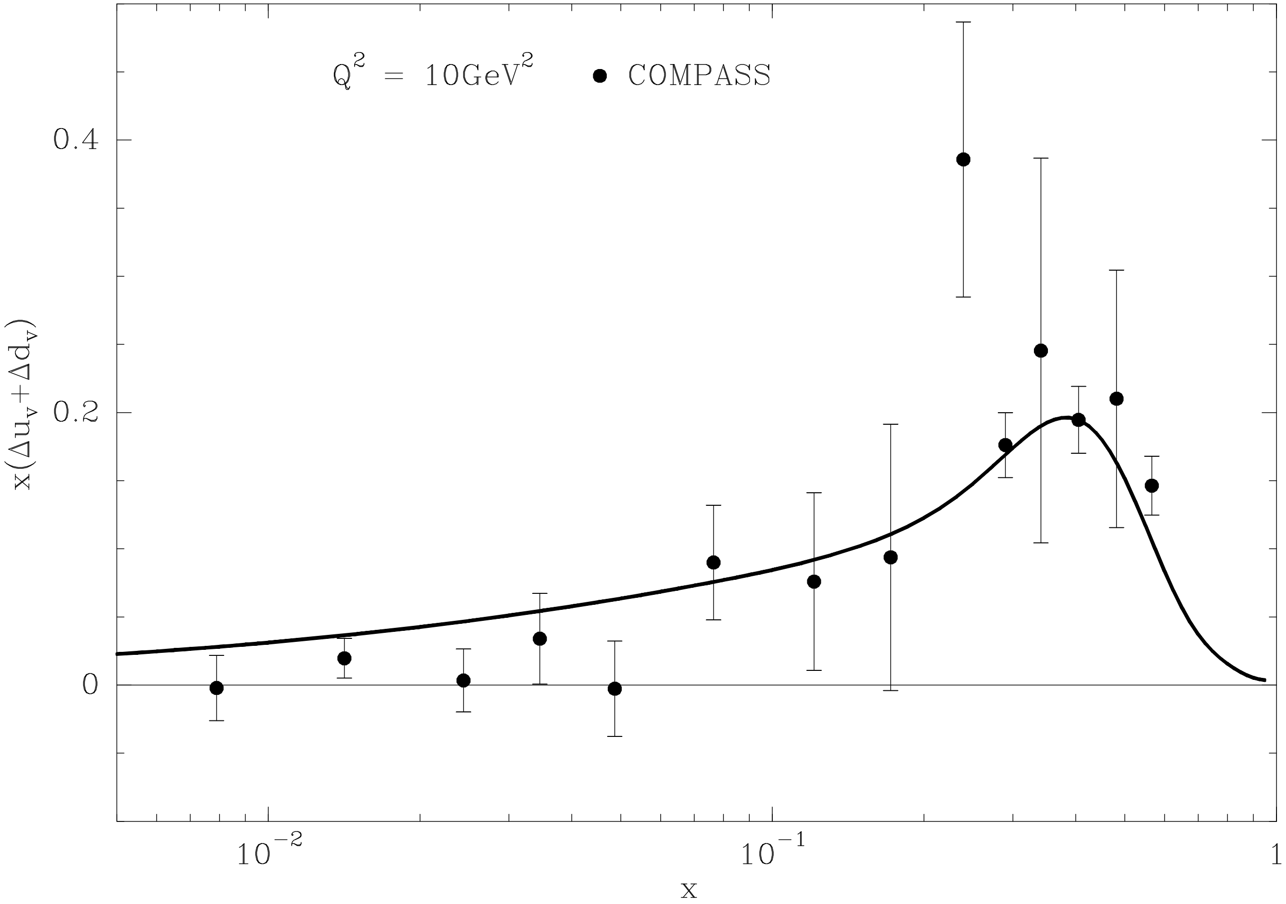
  * Non-symmetric polarized sea quarks

The longitudinal structure function F_L

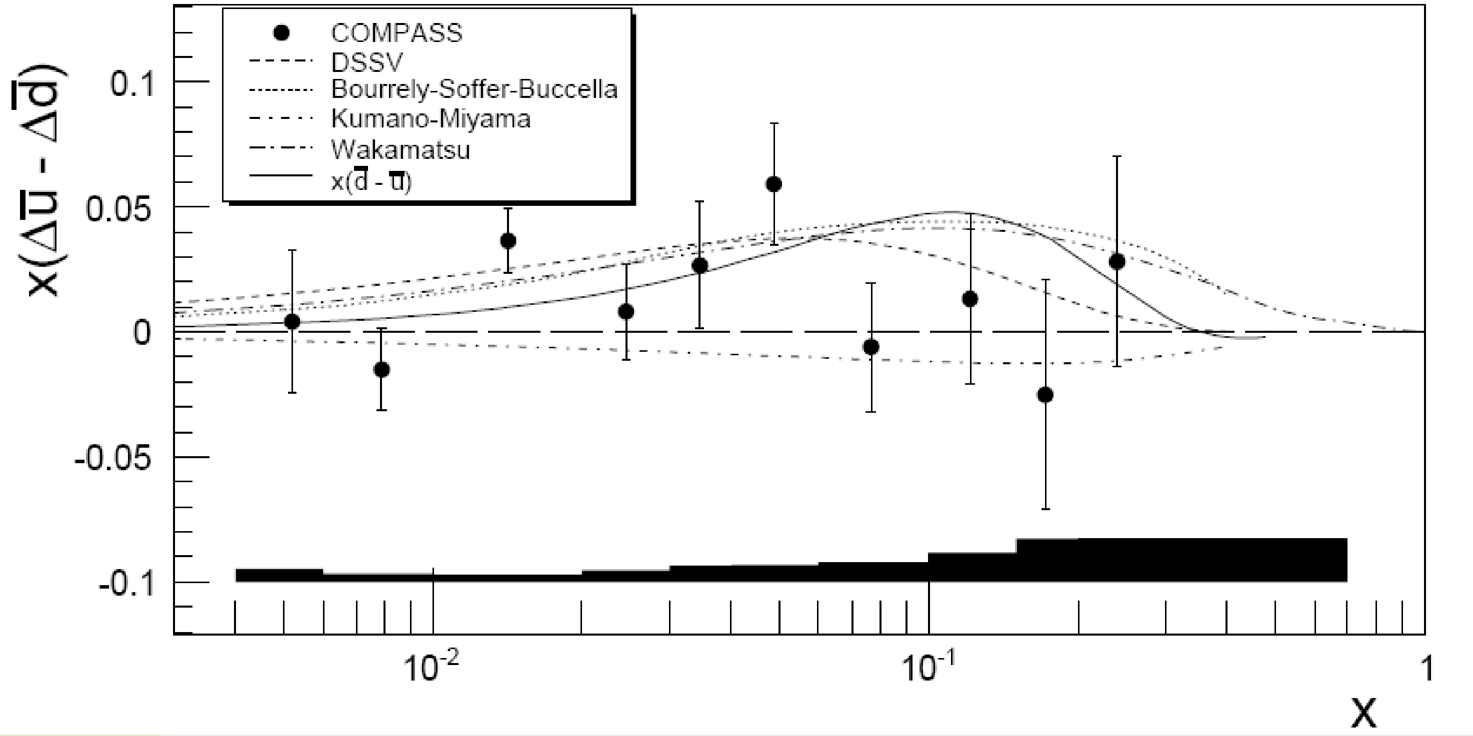


The valence quark helicity distributions versus x

From semi-inclusive DIS $\mu p \to \mu h^{\pm} X$ can determine the valence quark helicity distributions
Combined with g_1^d it leads to $\Delta\bar{u} + \Delta\bar{d} = 0.0 \pm 0.04 \pm 0.03$, *i.e.* non-symmetric polarized sea



The difference $x\Delta\bar{u}(x) - x\Delta\bar{d}(x)$ versus x



Comparison with new COMPASS data

Transverse momentum dependence (TMD) of the PDF

How to introduce the TMD of the PDF ?

- Currently one assumes factorization and a simple Gaussian behavior for the PDF

$$q(x,k_T) = q(x)\frac{1}{\pi\mu_0^2}\exp[-k_T^2/\mu_0^2]\ ,$$

and also for the fragmentation function

$$D(z,q_T) = D(z)\frac{1}{\pi\mu_D^2}\exp[-q_T^2/\mu_D^2]\ .$$

A simplifiing assumption which has no theoretical justification

- In our statistical distributions for quarks and antiquarks we assume NO factorization

(TMD) in the statistical approach

The parton distributions $p_i(x, p_T^2)$,of momentum p_T, must obey the momentum sum rule

$$\sum_i \int_0^1 dx \int x p_i(x, p_T^2) dp_T^2 = 1 ,$$

and also the transverse energy sum rule

$$\sum_i \int_0^1 dx \int p_i(x, p_T^2) \frac{p_T^2}{x} dp_T^2 = M^2 .$$

From the general method of statistical thermodynamics we are led to put $p_i(x, p_T^2)$ in correspondance with the following expression

$$\exp(\frac{-x}{\bar{x}} + \frac{-p_T^2}{x\mu^2}) ,$$

where μ^2 is a parameter which will be determined by the second sum rule, as we will see.

So we have now the main ingredients for the extension to the TMD of the statistical PDF, with clearly NO factorization

(TMD) in the statistical approach

The quantum statistics distributions for quarks and antiquarks read in this case

$$xq^h(x,k_T^2) = \frac{F(x)}{\exp(x - X_{0q}^h)/\bar{x} + 1} \frac{1}{\exp(k_T^2/x\mu^2 - Y_{0q}^h) + 1} ,$$

$$x\bar{q}^h(x,k_T^2) = \frac{\bar{F}(x)}{\exp(x + X_{0q}^{-h})/\bar{x} + 1} \frac{1}{\exp(k_T^2/x\mu^2 + Y_{0q}^{-h}) + 1}, \qquad (1)$$

where

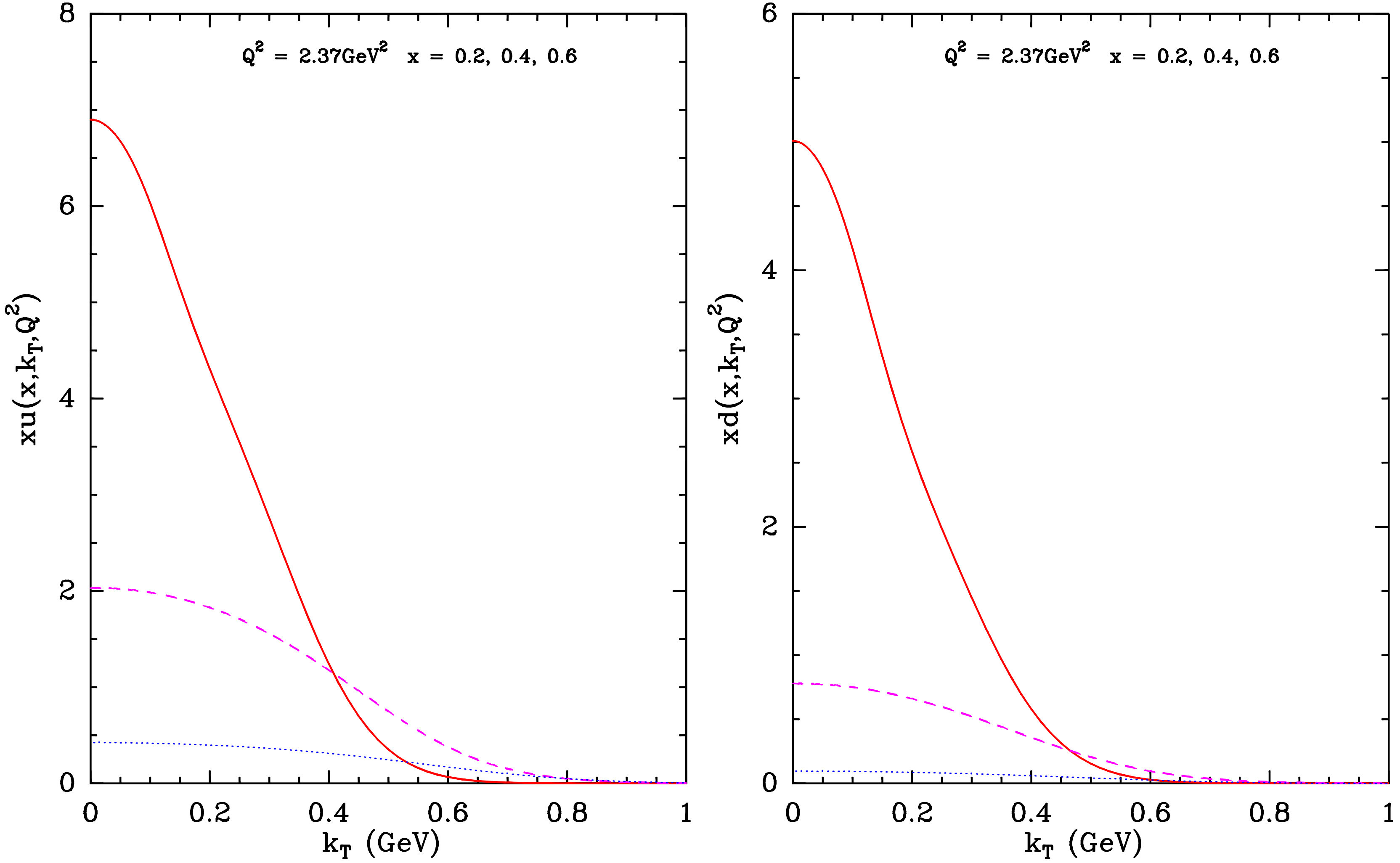
$$F(x) = \frac{Ax^{b-1}X_{0q}^h}{\ln(1+\exp Y_{0q}^h)\mu^2} = \frac{Ax^{b-1}}{k\mu^2} ,$$

because Y_{0q}^h are the thermodynamical potentials chosen such that $\ln(1+\exp Y_{0q}^h) = kX_{0q}^h$, in order to recover the factors X_{0q}^h, introduced earlier.
Similarly for $\bar{q}$ we have $\bar{F}(x) = \bar{A}x^{2b-1}/k\mu^2$. This determination of the 4 potentials Y_{0q}^h can be achieved with the choice $k = 3.05$ and they have the following values

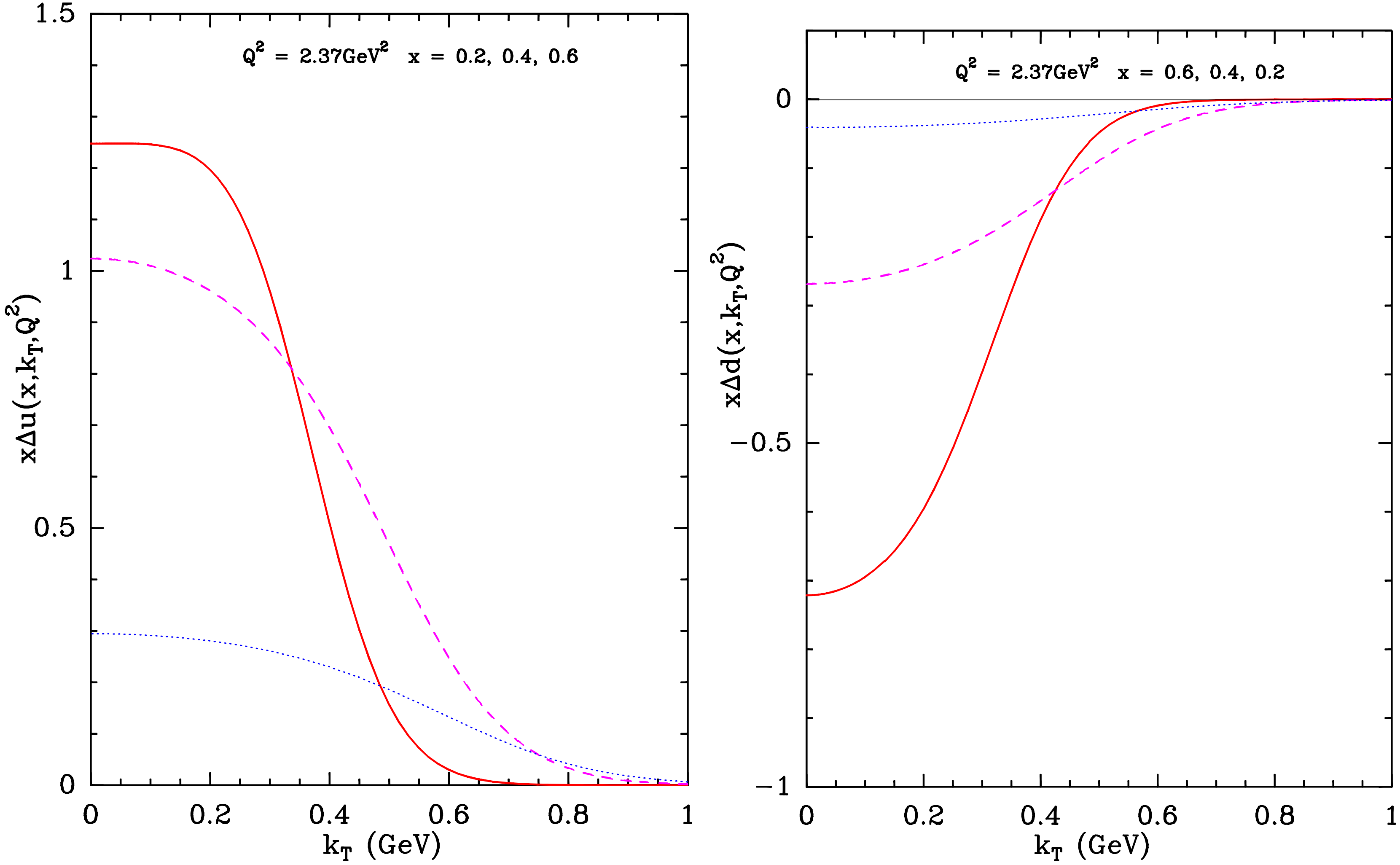
$$Y_{0u}^+ = 1.122,\ Y_{0u}^- = 0.388,\ Y_{0d}^- = 0.409,\ Y_{0d}^+ = 0.010 .$$

Finally μ^2 will be determined by the transverse energy sum rule and one finds $\mu^2 = 0.198\mathrm{GeV}^2$.

The statistical distributions u and d vs k_T



The statistical distributions Δu and Δd vs k_T



Melosh-Wigner effects

The basic effect in a situation where $k_T \neq 0$ is the Melosh-Wigner rotation which mixes the components $q^{\pm}$ in the following way

$$q^{+\prime} = \cos^2\theta \; q^+ + \sin^2\theta \; q^- \quad \text{and} \quad q^{-\prime} = \cos^2\theta \; q^- + \sin^2\theta \; q^+$$

where $2\theta = \text{Arctg}(\kappa k_T / xM)$ and κ is a dimensionless parameter.

Consequently $q = q^+ + q^-$ remains unchanged $q' = q$, whereas we have

$$\Delta q' = (\cos^2\theta - \sin^2\theta) \, \Delta q = \cos 2\theta \, \Delta q = \cos \text{Arctg}(\kappa k_T / xM) \, \Delta q$$

So we finally get

$$\Delta q' = \frac{1}{\sqrt{1 + (\kappa k_T / xM)^2}} \, \Delta q$$

This effect which clearly vanishes at $k_T = 0$, was taken into account with $\kappa = 1.35$.

Cross sections and asymmetries in SIDIS

$$\frac{d^5\sigma^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy\,dz\,d^2p_{hT}} = \frac{2\alpha^2}{xy^2s}\,\{\mathcal{H}_1 + \lambda\,S_L\mathcal{H}_2\}\,,$$

where the arrows indicate the direction of the lepton ($\rightarrow$) and target nucleon ($\Leftarrow$) polarizations, with respect to the lepton momentum; λ, and S_L are the magnitudes of the longitudinal beam polarization and the longitudinal target polarization, respectively. We have

$$\mathcal{H}_1(p_{hT}) = \sum_q e_q^2 \int d^2\mathbf{k_T} q(x,k_T)\,\pi y^2\,\frac{\hat{s}^2+\hat{u}^2}{Q^4}\,D_q^h(z,q_T),$$

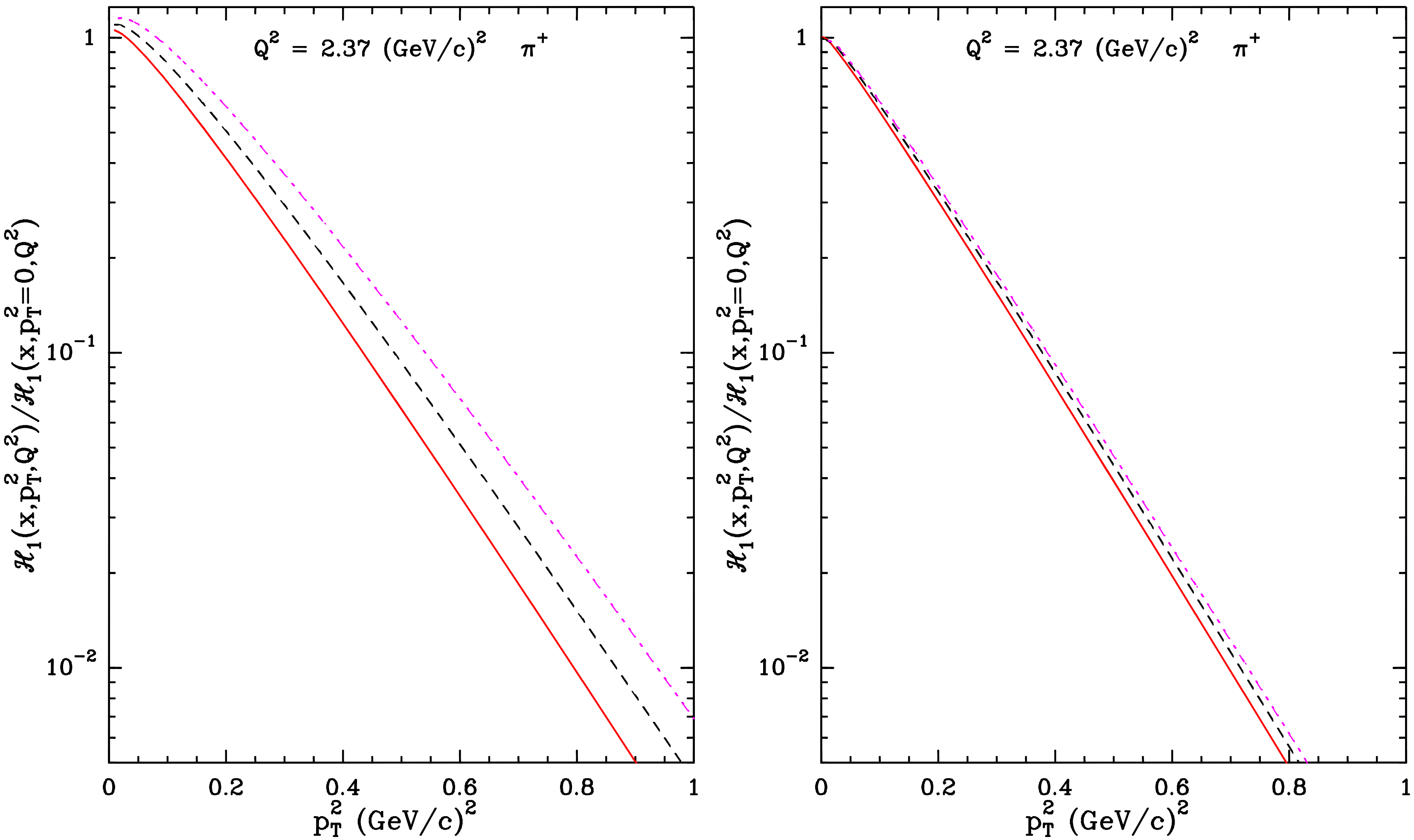
$$\mathcal{H}_2(p_{hT}) = \sum_q e_q^2 \int d^2\mathbf{k_T}\Delta q(x,k_T)\,\pi y^2\,\frac{\hat{s}^2-\hat{u}^2}{Q^4}\,D_q^h(z,q_T),$$

where $p_{hT} = q_T + zk_T$ is the intrinsic transverse momentum of the hadron h with respect to the fragmenting quark direction.

The first two contributions, give, respectively, the unpolarized cross section and the helicity asymmetry

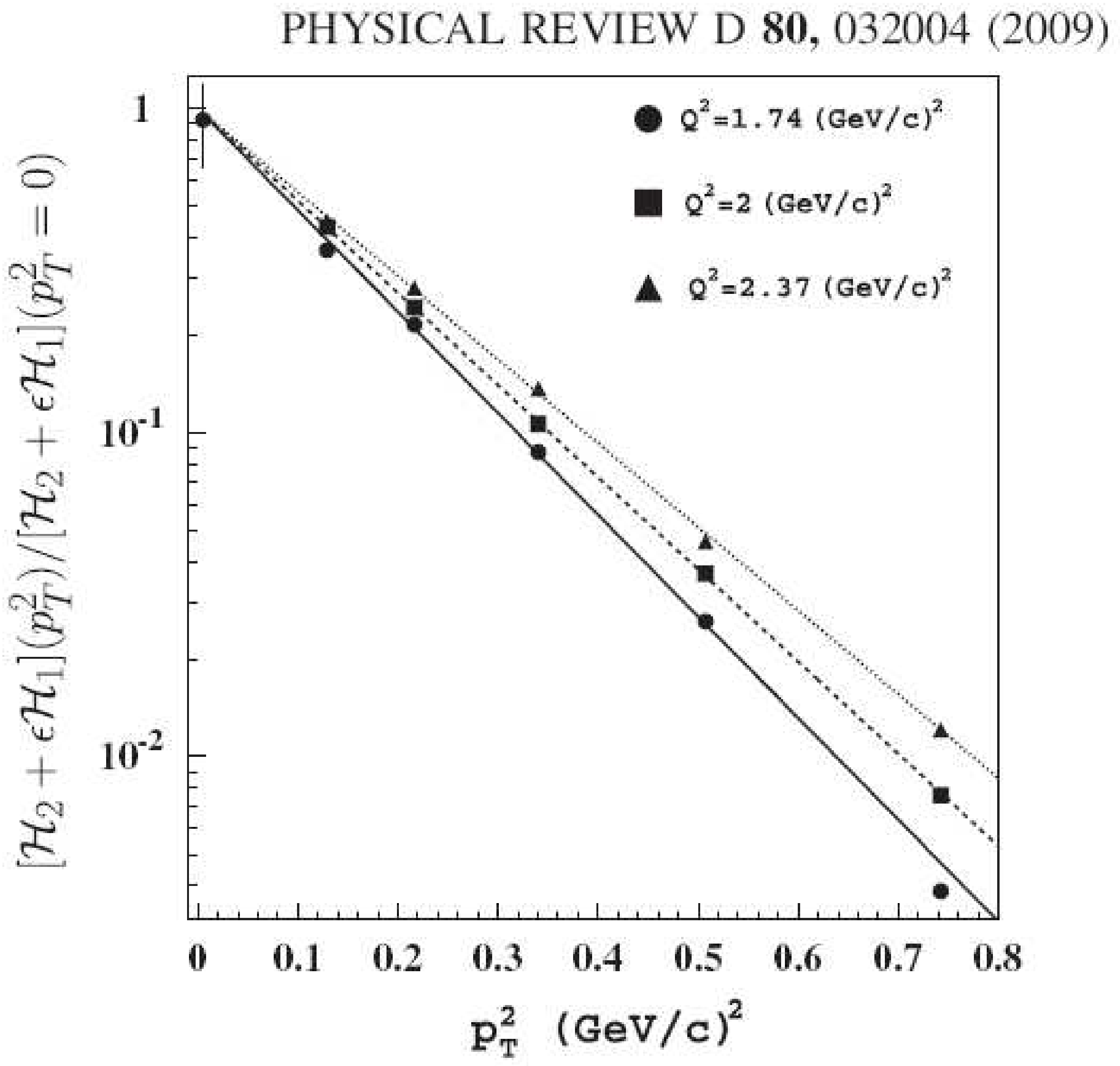
$$\frac{d^5\sigma}{dx\,dy\,dz\,d^2p_{hT}} = \frac{2\alpha^2}{x\,y^2s}\,\mathcal{H}_1 \qquad \frac{d^5\sigma^{++}}{dx\,dy\,dz\,d^2p_{hT}} - \frac{d^5\sigma^{+-}}{dx\,dy\,dz\,d^2p_{hT}} = \frac{4\alpha^2}{x\,y^2s}\,\mathcal{H}_2\,,$$

Predicted cross sections $e + p \rightarrow e + \pi^{\pm} + X$

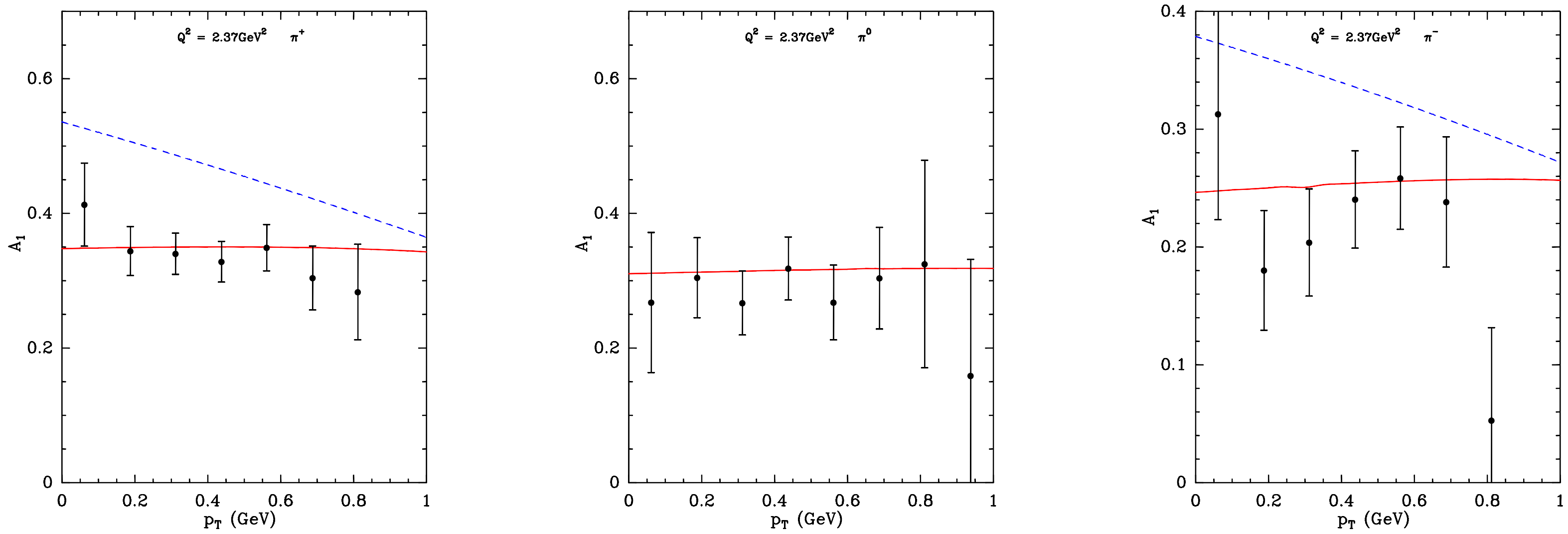


In the absence of statistical fragmentation functions, we have used $D(z, q_T) = D(z)\frac{1}{\pi\mu_D^2}\exp[-q_T^2/\mu_D^2]$ with $\mu_D^2 = 0.155\text{GeV}^2$

Cross section CLAS data



Predicted asymmetries $e + p \to e + \pi^{\pm,0} + X$



Data from CLAS Coll. arXiv:1003.4549 [hep-ex]

Conclusions

- A new set of PDF is constructed in the framework of a statistical approach of the nucleon.
- All unpolarized and polarized distributions depend upon nine free parameters, with some physical meaning.
- New tests against experimental (unpolarized and polarized) data on DIS, Semi-inclusive DIS and hadronic processes are very satisfactory.
- Good predictive power but some special features remain to be verified, specially in the high x region.
- Extension to TMD successful for SIDIS and must be tested, for example, in the p_T dependence of the DY cross section.

For practical use of our PDF see www.cpt.univ-mrs.fr/∼ bourrely/research/bbs-dir/bbs.html