Single-pion production in neutrino-matter collisions

M. Siddikov

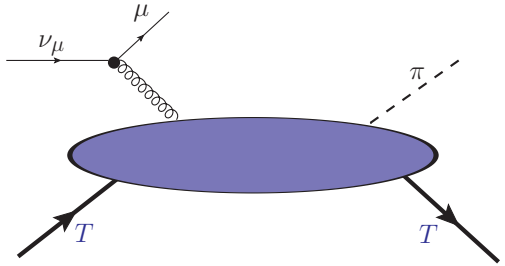
(In collaboration with B. Kopeliovich, I. Schmidt)

UNIVERSIDAD TECNICA
FEDERICO SANTA MARIA

Bonn, Germany, March 29, 2012

(PRD 84 (2011), 033012, PRC 84 (2011), 024608, arXiv: 1201.4053, to appear in PRD)

Kinematics



- Diffractive pion production,
  - CC: $\nu\, T \to l\, \pi^+\, T$
  - NC: $\nu\, T \to \nu\, \pi^0\, T$

$$E_\nu = \frac{p\cdot k_\nu}{m_N},\ \nu = \frac{p\cdot q_W}{M},\ y = \frac{p\cdot q_W}{p\cdot k}$$

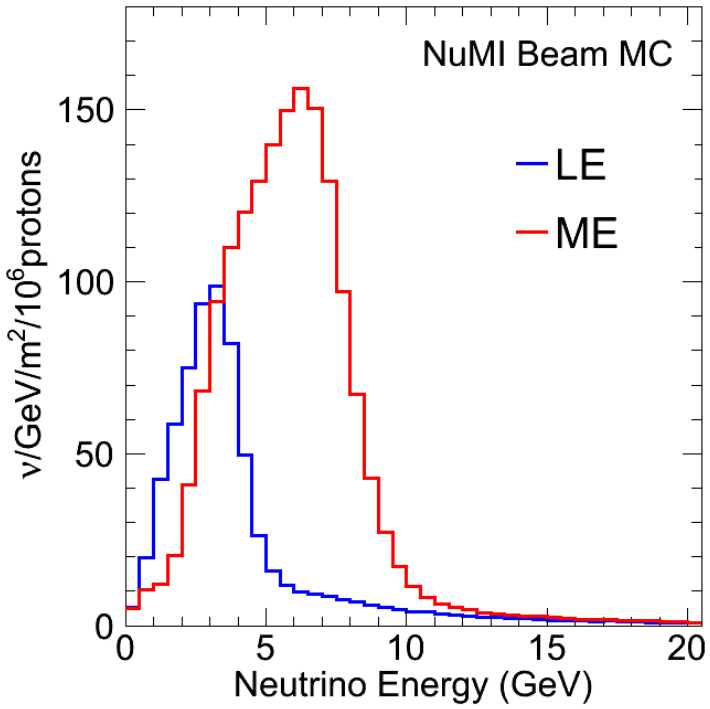
$$Q^2 = -q_W^2 = 4E_\nu(E_\nu - \nu)\sin^2\frac{\theta}{2} + \mathcal{O}\left(m_l^2\right)$$

$$t = (p' - p)^2 = \Delta^2 = t_{min} - \Delta_\perp^2$$

- ★ High statistics, *differential* x-sections are measured
- ★ Different targets (H_2O, He, C, CH, Fe, Pb)

(Tue. talks of J. Morfin, J. Mousseau)

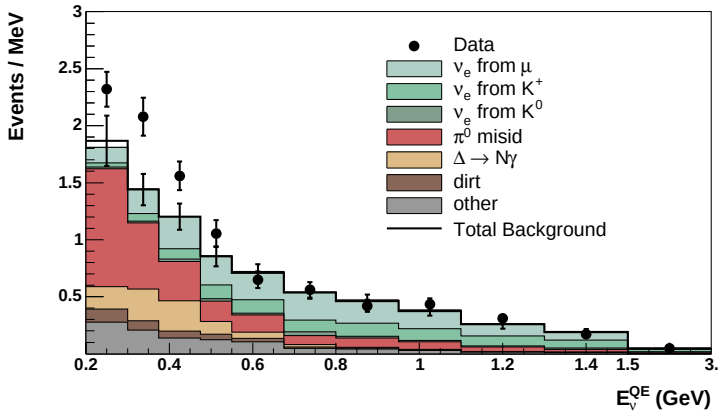
- (Minerva@Fermilab, 2011)

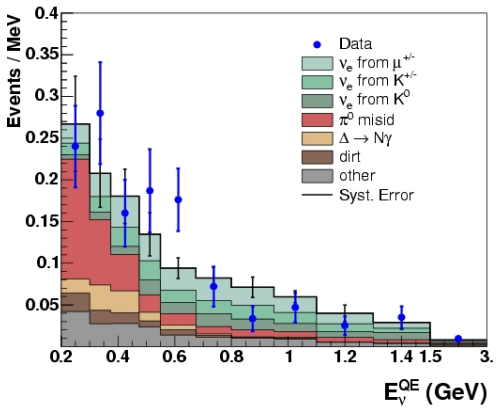


Why pion production ?

- Contribution of axial current dominates
- Background in $\nu_\mu \to \nu_e$ (misidentification of π_0, $\pi_0 \to 2\gamma$)

  (MiniBooNE [PLB 664, 41 (2008)], SciBooNE [PRD 81, 111102 (2010)])





(MiniBooNE [PRL 102, 101802 (2009); arXiv:1201.1519])

- Theory: $\sigma_{CC} \approx 2\sigma_{NC}$, experiment:
  - No evidence for CC ($\nu_\mu +^{12} C \to \mu^- + \pi^+ +^{12} C$, $E_\nu \sim 1$ GeV) (K2K [PRL 95, 252301 (2005)], SciBooNE [PRD 78, 112004 (2008)])
  - Signature for NC ($\nu_\mu +^{12} C \to \nu_\mu + \pi_0 +^{12} C$, $E_\nu \sim 1$ GeV) (MiniBooNE [PLB 664, 41 (2008)], SciBooNE [PRD 81, 111102 (2010)])

PCAC-based models: Adler relation

$$\left.\frac{d\sigma_{\nu T\to lF}}{d\nu dQ^2}\right|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T\to F}$$

(PR 135 (1964), 963)

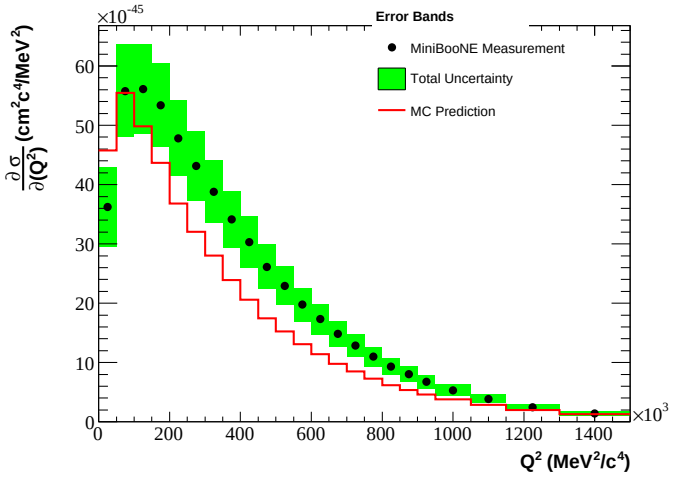
- In real measurements $q^2 \neq 0$, so AR requires extrapolation. AR $\neq$ Pion dominance:

$$T_\mu(\ldots) \sim \frac{q_\mu}{q^2 - m_\pi^2} + T_\mu^{non-pion}(\ldots),$$

but lepton currents are conserved, so

$$q_\mu L_{\mu\nu} = \mathscr{O}(m_l)$$

$\Rightarrow$AR survives due to heavier hadrons and χ-sym.



(MiniBooNE, PRD 83(2010), 052007)

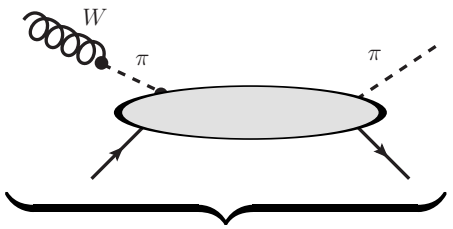
- $\sigma \sim (1 + Q^2/m_A^2)^{-2}$, $m_A \sim 1$ GeV

- Contributions from transverse part and from the vector part ($\mathscr{O}(q^2)$ for small q^2)

PCAC vs. black disk regime (high energy limit)

PCAC-based models are inconsistent with BDR:

$$\underbrace{\left.\frac{d\sigma_{\nu T\to l\pi T}}{d\nu dQ^2}\right|_{Q^2=0}}_{\text{diffractive production, } W\to\pi} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu-\nu}{E_\nu \nu} \underbrace{\sigma_{\pi T\to\pi T}}_{\text{elastic scattering}}$$



$$\sim \frac{q_\mu}{q^2 - m_\pi}$$

(diagrams with pions are suppressed by lepton mass, $\mathcal{O}(m_l)$)

PCAC vs. black disk regime (high energy limit)

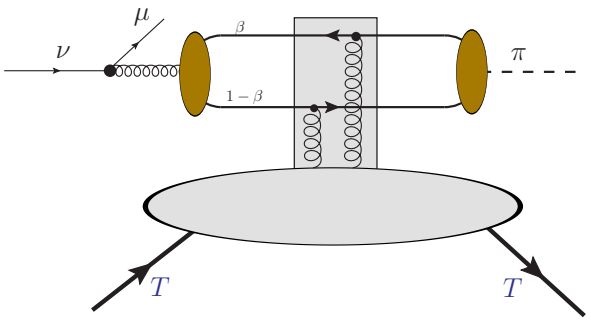
PCAC-based models are inconsistent with BDR:

$$\underbrace{\left.\frac{d\sigma_{\nu T\to l\pi T}}{d\nu dQ^2}\right|_{Q^2=0}}_{\text{diffractive production, } W\to\pi} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu-\nu}{E_\nu \nu} \underbrace{\sigma_{\pi T\to\pi T}}_{\text{elastic scattering}}$$

$\sim R_A$

$\sim R_A^2$

Color dipole and neutrino-proton interactions



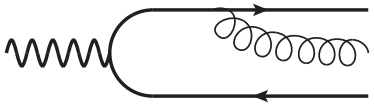
The amplitude has a form

$$\mathscr{A}^{aT\to\pi T} = \bar{\Psi}_\pi\left(\beta', r'\right) \otimes \mathscr{A}_T^d\left(\beta', r'; \beta, r\right) \otimes \Psi_a(\beta, r),$$

- $\mathscr{A}_T^d\left(\beta', r'; \beta, r\right)$ universal object, depends only on the target T, known from γp and γA processes. (Tuesday talks by Carlo Ewerz, Leszek Motyka)
- $\bar{\Psi}_\pi, \Psi_a$ are the distribution amplitudes of the initial and final states.
  - Need to take care of chiral symmetry.
  - For study of AR and its breaking, should be valid up to $Q^2 = 0$.

On higher Fock states

Consider only $\bar{q}q$, contribution of $\bar{q}qg$ is suppressed for $\nu \leq 10^3$ GeV.



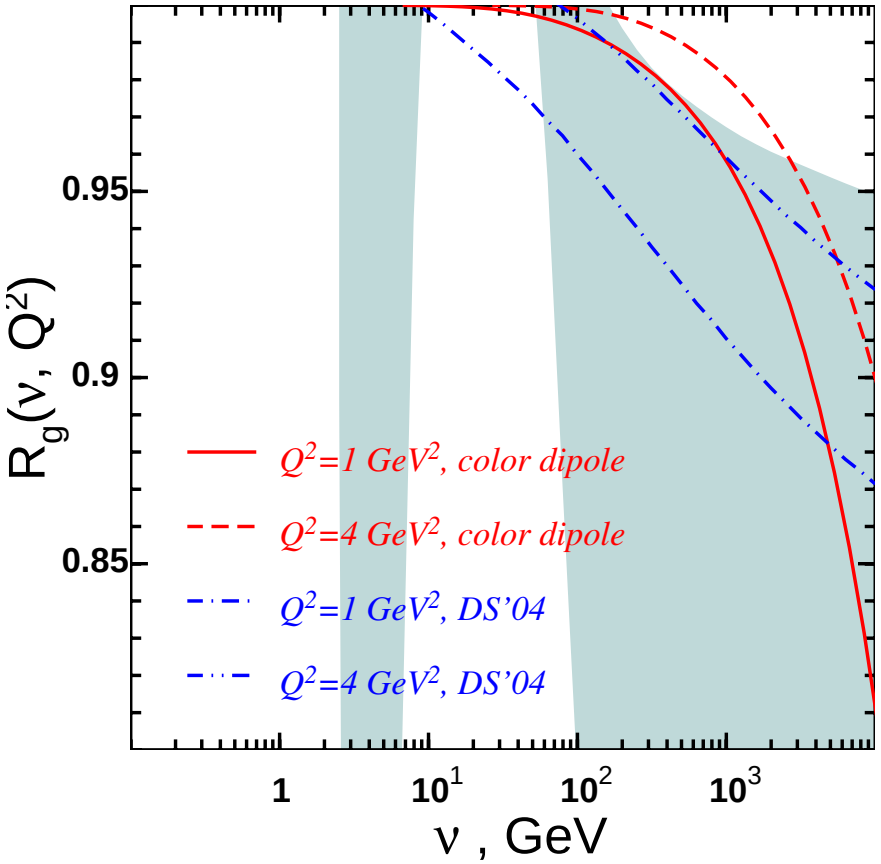


Figure: Gluon shadowing in color dipole (PRD 62, 054022 (2000)) and in phenomenological D. de Florian, R. Sassot parametrization (PRD 69, 074028 (2004)). Shaded area: gluon uncertainty band from EPS'09 parametrization (JHEP 0904:065 (2009))

Distribution amplitudes of pion

Pion distribution amplitudes (P. Ball *et al*, 2006)

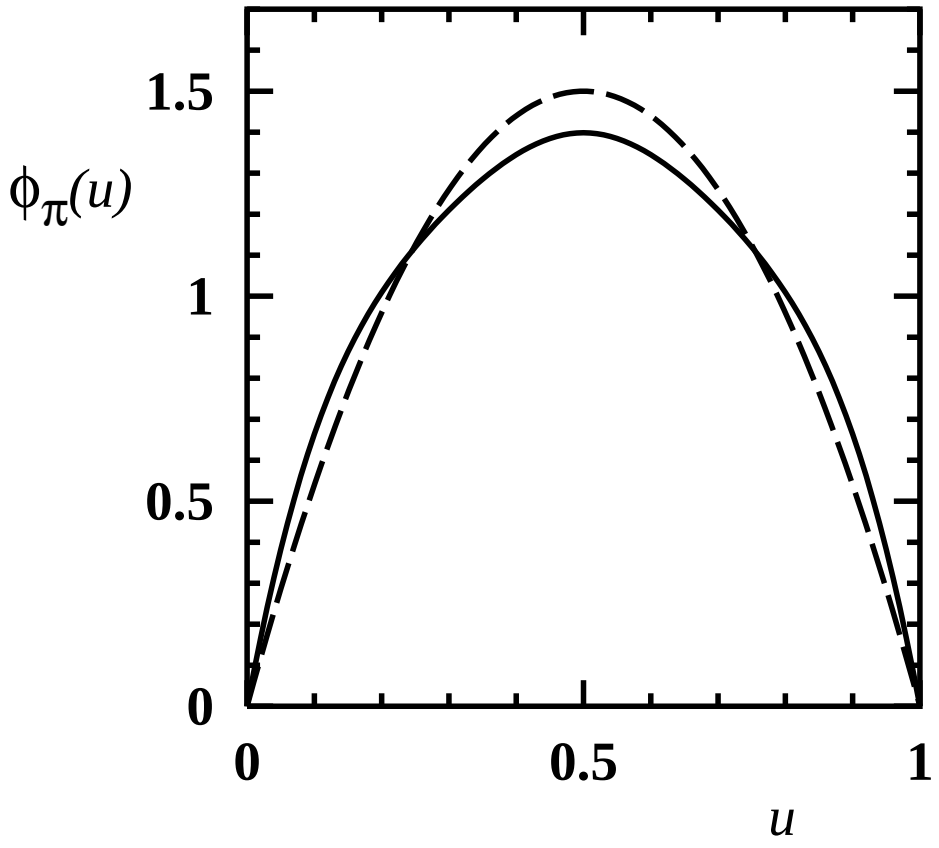
$$\left\langle 0 \left| \bar{\psi}(y)\,\gamma_\mu\gamma_5\,\psi(x) \right| \pi(q) \right\rangle = if_\pi \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \times \times \left(p_\mu \phi_{2;\pi}(u) + \frac{1}{2}\frac{z_\mu}{(p\cdot z)}\psi_{4;\pi}(u) \right),$$

$$\langle 0 | \bar{\psi}(y)\,\gamma_5\,\psi(x) | \pi(q) \rangle = -if_\pi \frac{m_\pi^2}{m_u+m_d} \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \phi_{3;\pi}^{(p)}(u).$$

$$\left\langle 0 \left| \bar{\psi}(y)\,\sigma_{\mu\nu}\gamma_5\,\psi(x) \right| \pi(q) \right\rangle = -\frac{i}{3} f_\pi \frac{m_\pi^2}{m_u+m_d} \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \times \times \frac{1}{p\cdot z}\left(p_\mu z_\nu - p_\nu z_\mu\right) \phi_{3;\pi}^{(\sigma)}(u),$$

Distribution amplitude of pion

- Best known is leading twist DA $\phi_{2;\pi}(x)$



close to $\phi_{as}(x) = 6x(1-x)$ (V. Yu. Petrov *et. al.*, 1998)

- We take into account all the DAs in order not to kill the chiral

Distribution amplitudes of axial meson

Axial DAs (K.-C. Yang 2007)

$$\langle 0 | \bar{\psi}(y)\gamma_\mu\gamma_5\psi(x) | A(q)\rangle = if_A m_A \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \times \times \left(p_\mu \frac{e^{(\lambda)}\cdot z}{p\cdot z}\Phi_{||}(u) + e_\mu^{(\lambda=\perp)} g_\perp^{(a)}(u) - \frac{1}{2} z_\mu \frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_A^2 g_3(u) \right),$$

$$\langle 0 | \bar{\psi}(y)\gamma_\mu\psi(x) | A(q)\rangle = -if_A m_A \varepsilon_{\mu\nu\rho\sigma} e_\nu^{(\lambda)} p_\rho z_\sigma \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \frac{g_\perp^{(v)}(u)}{4}$$

$$\langle 0 | \bar{\psi}(y)\sigma_{\mu\nu}\gamma_5\psi(x) | A(q)\rangle = f_A^\perp \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \left(e_{[\mu}^{(\lambda=\perp)} p_{\nu]} \Phi_\perp(u) + \frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_A^2 p_{[\mu} z_{\nu]} h_{||}^{(t)}(u) + \frac{1}{2} e_{[\mu}^{(\lambda)} z_{\nu]} \frac{m_A^2}{p\cdot z} h_3(u) \right),$$

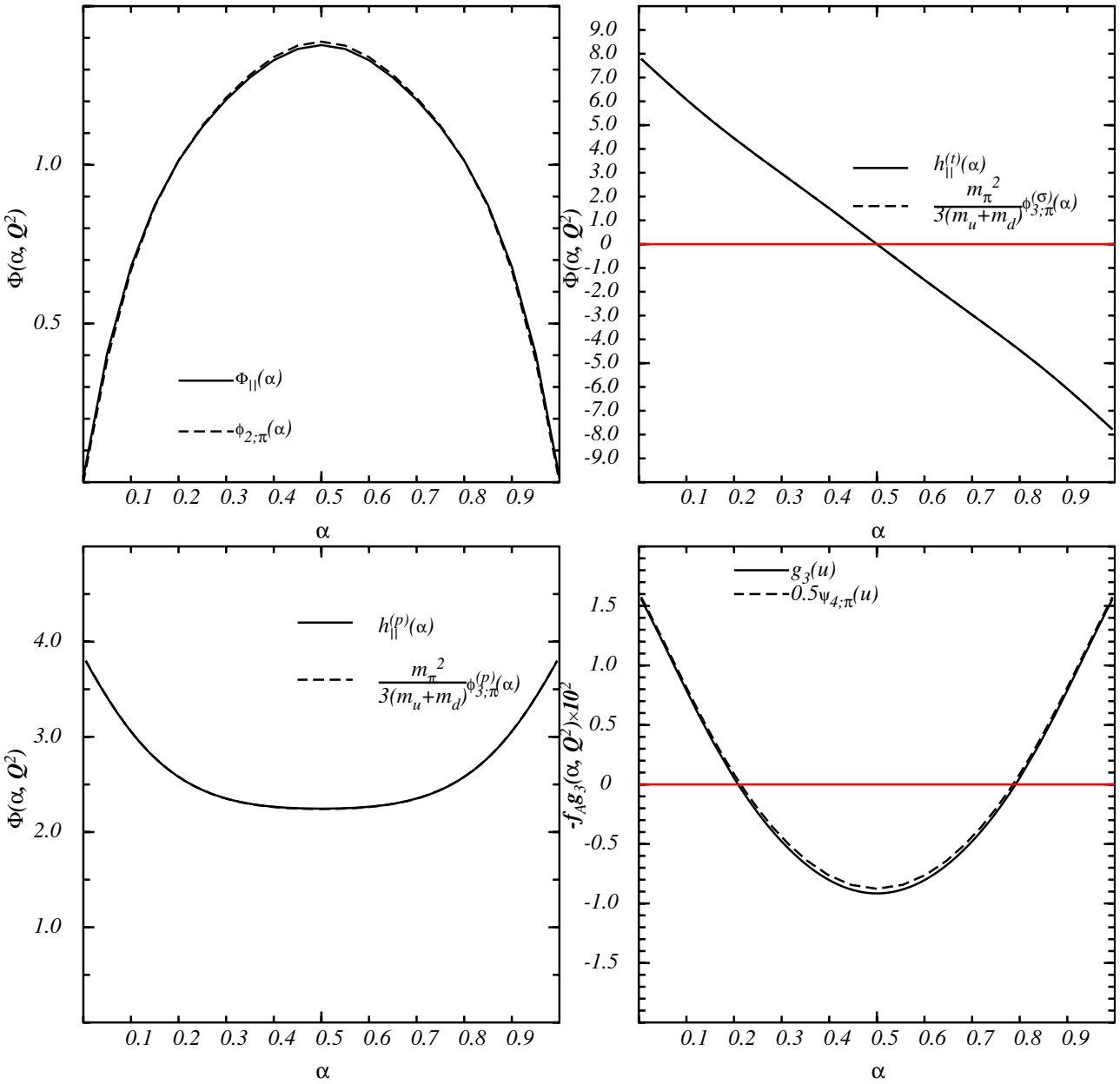
$$\langle 0 | \bar{\psi}(y)\gamma_5\psi(x) | A(q)\rangle = f_A^\perp m_A^2 e^{(\lambda)}\cdot z \int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)} \frac{h_{||}^{(p)}(u)}{2}.$$

PCAC relations for DAs

PCAC relates 4 DAs of the axial current and pion DAs:

$$
\begin{aligned}
\Phi_{||}\left(\alpha,\, q^2 = m_\pi^2\right) &= \phi_{2;\pi}(\alpha) \\
g_3\left(\alpha,\, q^2 = m_\pi^2\right) &= \frac{1}{2}\psi_{4;\pi}(\alpha) \\
h_{||}^{(t)}\left(\alpha,\, q^2 = m_\pi^2\right) &= -\frac{1}{3}\frac{m_\pi^2}{m_u + m_d}\phi_{3;\pi}^{(\sigma)}(\alpha) \\
h_{||}^{(p)}\left(\alpha,\, q^2 = m_\pi^2\right) &= \frac{2m_\pi^2}{m_u + m_d}\phi_{3;\pi}^{(p)}(\alpha)
\end{aligned}
$$

PCAC relations for DAs



Chiral symmetry & color dipole

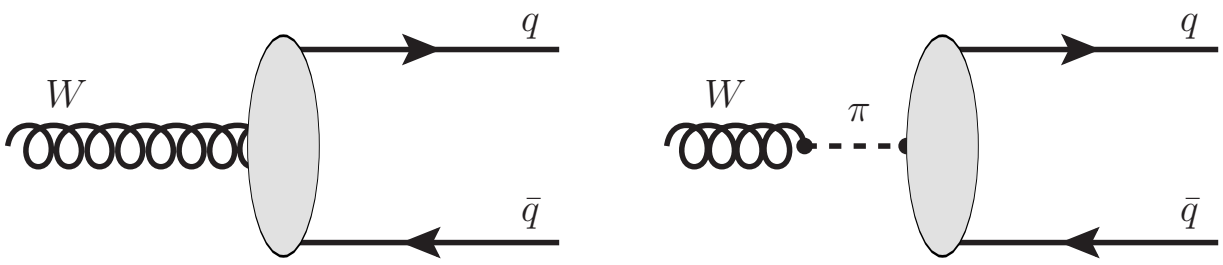


Figure: Relation between DAs of pion ($\pi\bar{q}q$), axial meson ($W\bar{q}q$) and $W\pi$ coupling guarantees that the amplitude has transverse structure required by chiral symmetry

$$T_\mu^{(a\to\pi)} = \left(\frac{q_\mu q_\nu}{q^2 - m_\pi^2} - g_{\mu\nu}\right) P_\nu T_{\pi\pi}(p,q) + \mathscr{O}\left(q^2\right),$$

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

At high energies $\frac{d\sigma}{dt d\ln\nu dQ^2} \sim s_{Wp}^{2\alpha} \Rightarrow \sigma_{\nu p \to l\pi p} \sim E_\nu^{2\alpha}$

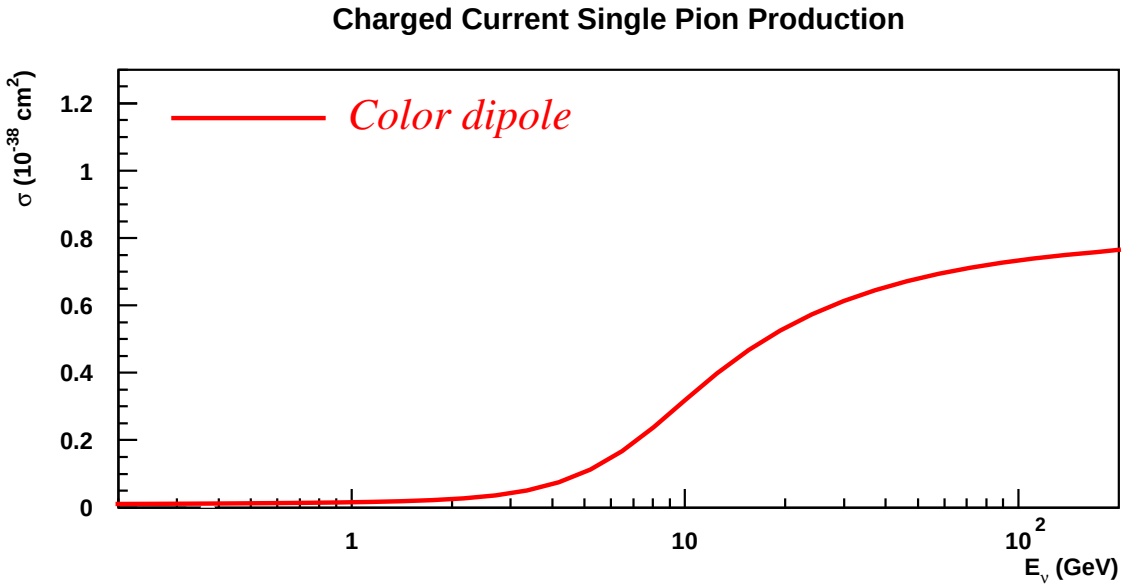


Figure: Total cross-section as a function of the neutrino energy E_ν.

Reasonable value for energies $E_\nu > 10$ GeV, vanishes for $E_\nu < 10$ GeV

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

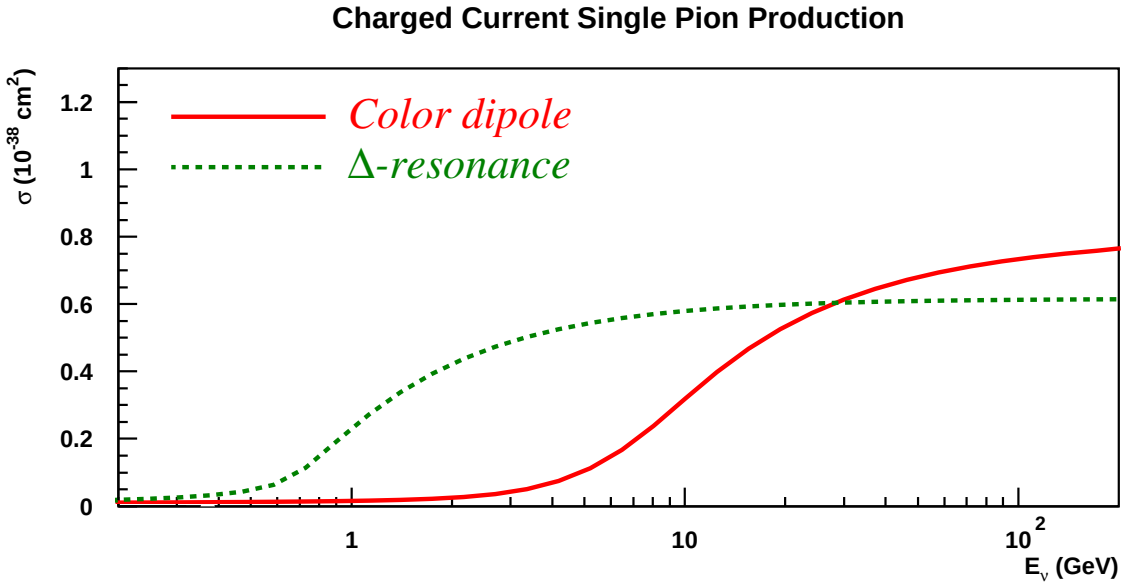


Figure: Total cross-section as a function of the neutrino energy E_ν.

Low-energy region is dominated by $\Delta(1232)$

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

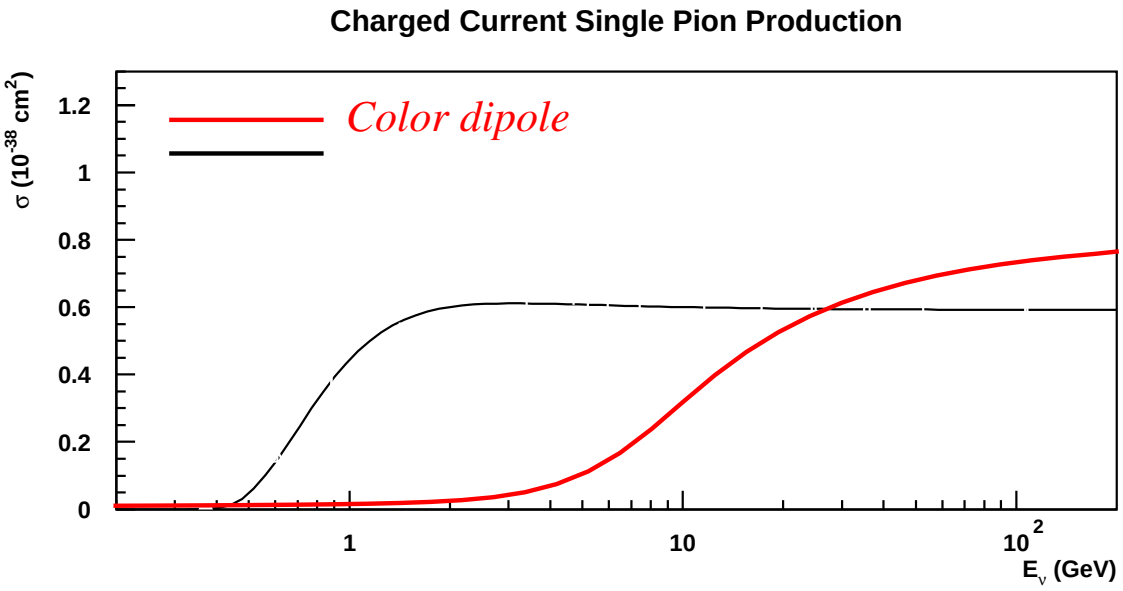


Figure: Total cross-section as a function of the neutrino energy E_ν.

Resonance-based models include just a few resonances, the total cross-section is constant above all thresholds

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

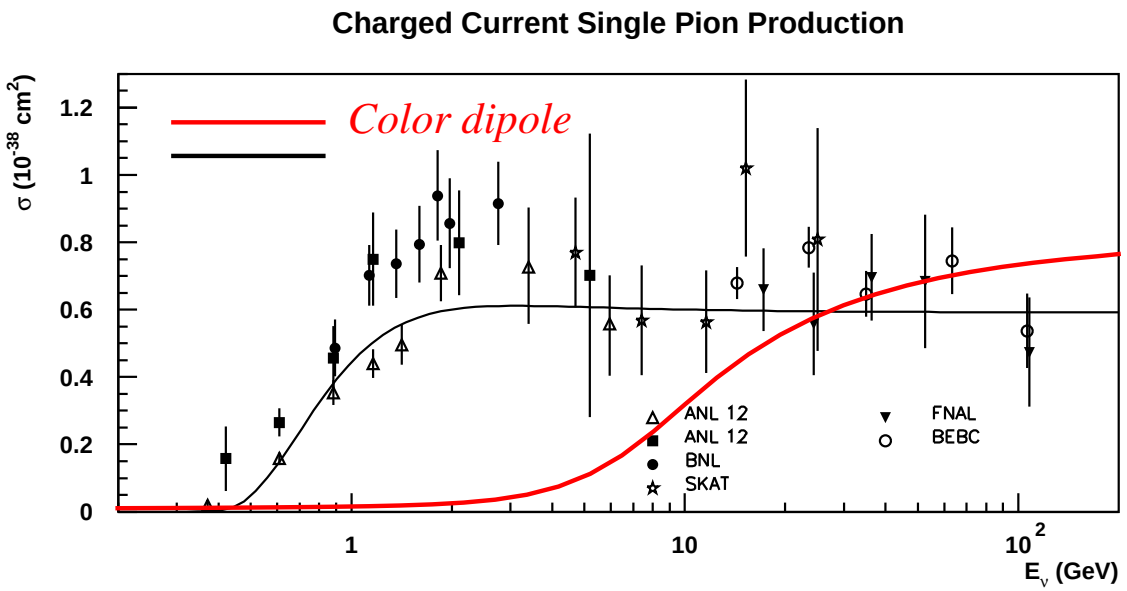


Figure: Total cross-section as a function of the neutrino energy E_ν. *Compilation of experimental data from* *(Minerva proposal, 2004)*

Total cross-section does not distinguish diffractive and resonance contributions. Differential cross-section would give much more information

WA21 experiment @ CERN $\nu p \to \mu^- \pi^+ p$

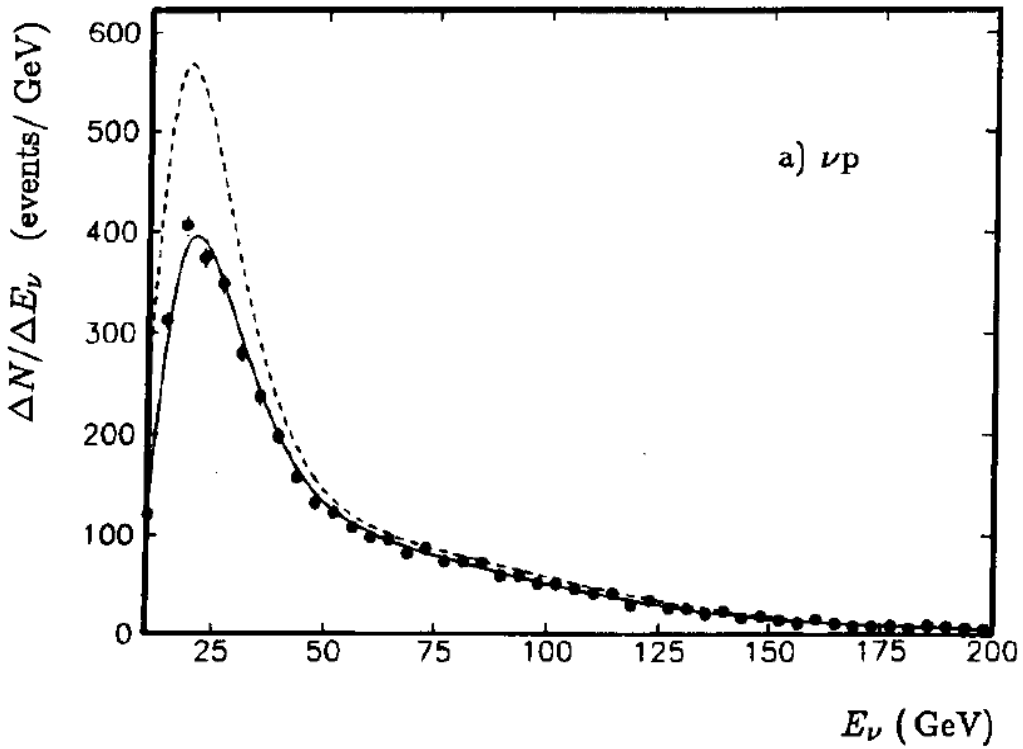


Figure: Neutrino spectrum at BEBC (CERN) (Allen et.al., 1985)

WA21 experiment @ CERN $\nu p \to \mu^- \pi^+ p$

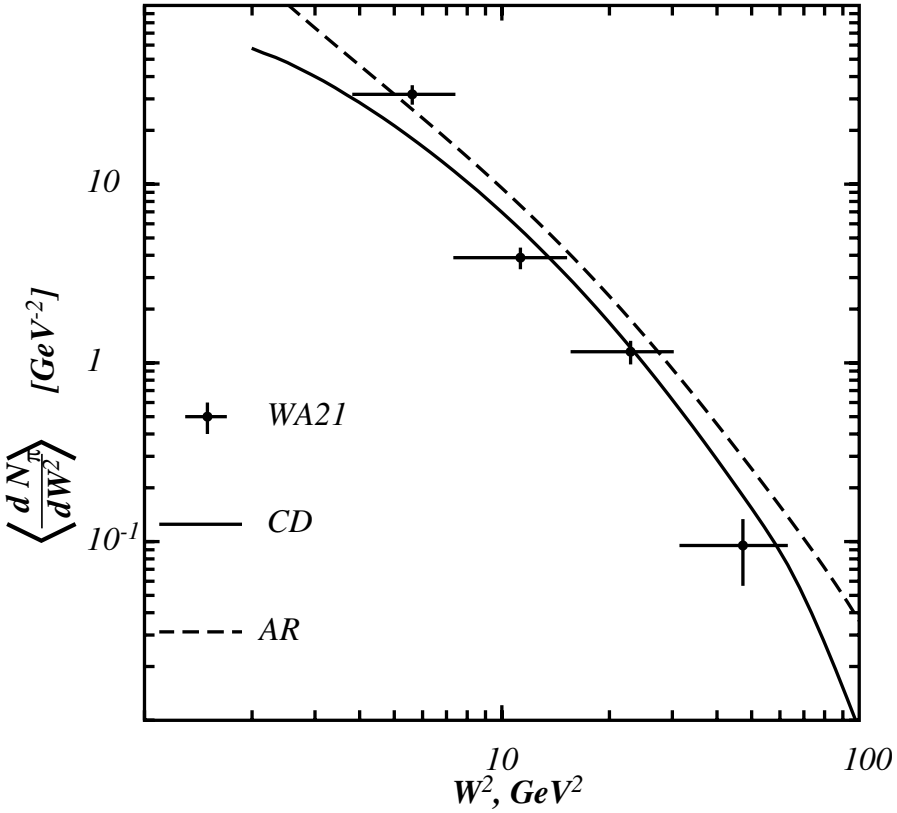
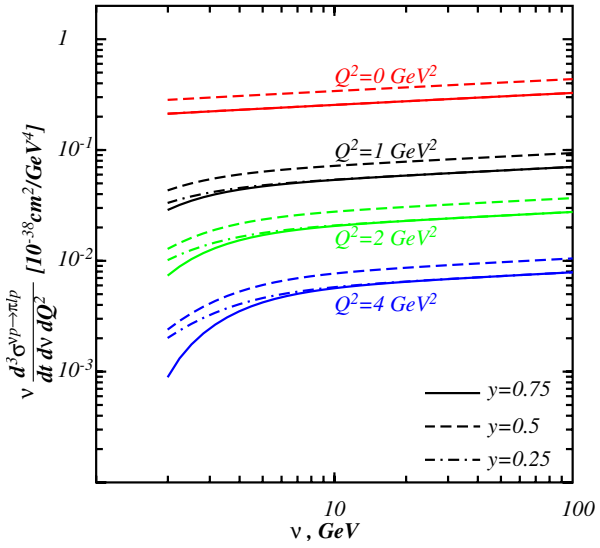


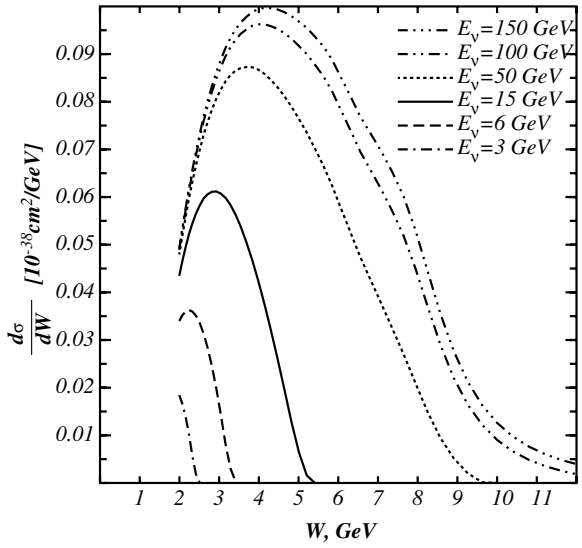
Figure: Color dipole vs. BEBC (CERN) (Allen et.al., 1985)

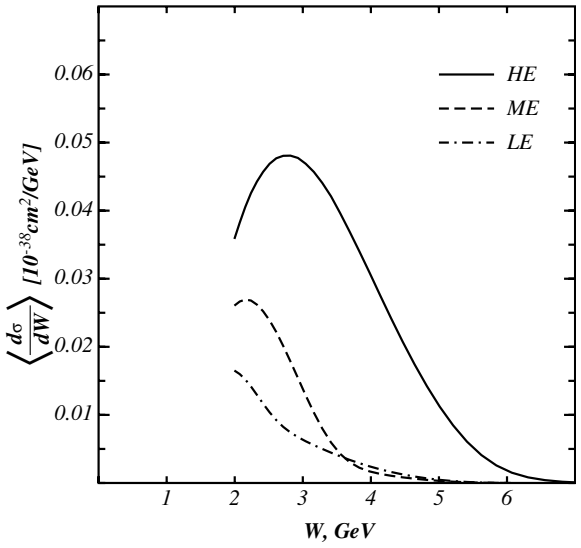
Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

Predictions for Minerva kinematics



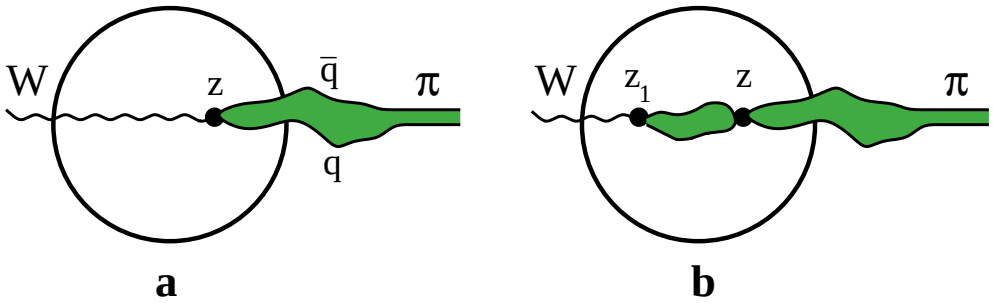
$$\frac{d^3\sigma}{dt\,d\ln\nu\,dQ^2} \sim \frac{\sigma^{tot}_{\pi p}(s)^2}{\left(1+Q^2/m_A^2\right)^2} e^{-B\Delta_\perp^2}$$





Coherent neutrino-nuclear scattering

- Use Gribov-Glauber approach



- Two scales: coherence length of the pion and effective axial meson

$$l_c^\pi = \frac{2\nu}{m_\pi^2 + Q^2}, \quad l_c^a = \frac{2\nu}{m_a^2 + Q^2}.$$

- For large Q^2, $l_c^\pi \approx l_c^a$, so this case is similar to photon-nuclear processes, we have only two regimes: $l_c \gg R_A$ and $l_c \ll R_A$.
- For small $m_\pi^2 \lesssim Q^2 \ll m_a^2$, $\quad l_c^a \ll l_c^\pi$, appears a third regime, when $l_c^a \ll R_A \ll l_c^\pi$.

$\nu A \to l\pi^+ A$ cross-section in a toy model

(Phys. Rev. C84 (2011), 024608)

Cross-check in a simple *toy model*:

- Assume there are only pion and a_1 mesons
  - Numerically, vector dominance does not work in axial channel,

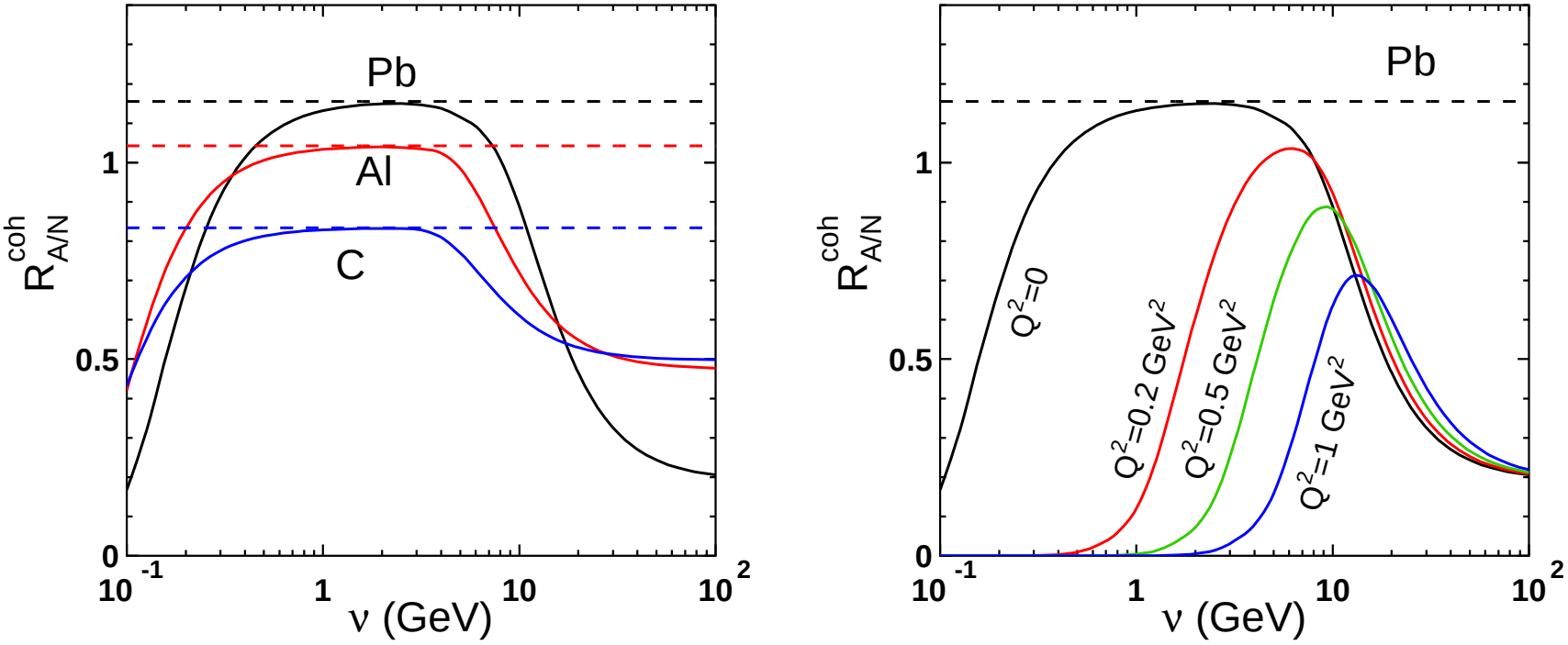
$$\sigma_{a_1(1236)p} \approx \sigma_{\pi p}/30$$

(Piketty, Stodolsky, NPB 15(1970), 571); the axial meson should be understood as an effective $\rho\pi$ state (Deck, PRL 13 (1964), 164)

- Assume Adler relation is valid for nucleon at $Q^2 = 0$
- Use Gribov-Glauber approach for nuclei.

Result for the $\nu A \to l\pi^+ A$ cross-section (toy model)

$$R^{coh}_{A/N} = \frac{1}{A}\frac{d\sigma_A/d\nu dQ^2}{d\sigma_N/d\nu dQ^2}$$



Adler relation works in the region $\nu \leq 10$ GeV; for high energies results broken due to absorptive corrections

Result for the $\nu A \to l\pi^+ A$ differential cross-section (color dipole)

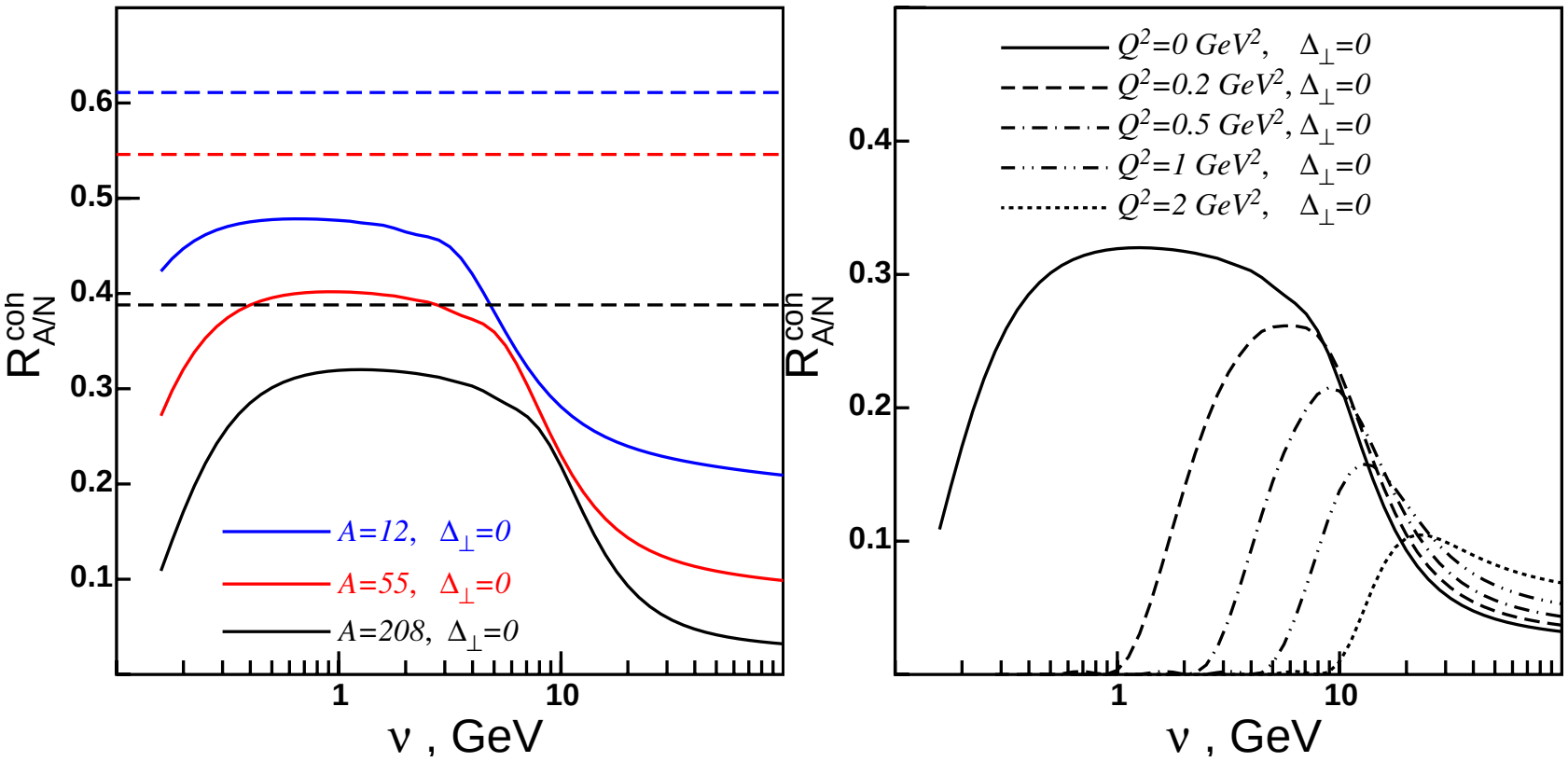
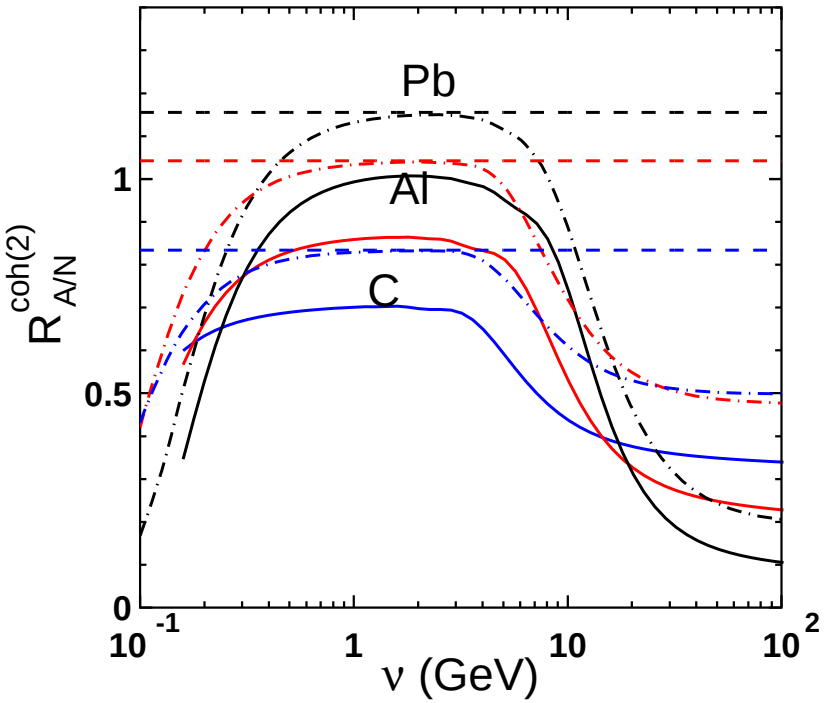


Figure: Ratio of cross-sections on the nucleus and proton.

Results similar to toy model, but *Adler relation on nuclei is always broken*.

$\nu A \to l\pi^+ A$ differential cross-section: CD vs. 2-channel



At low energies, AR works for toy model but not for color dipole. Reason: axial current contains a mixture of states with different masses; for light dipoles coherence length is large. At higher energies, there are absorptive corrections

High energy limit

Adler relation is broken due to absorptive corrections. Assume the limit $R_A \ll l_a \ll l_\pi$.

$$\underbrace{\left.\frac{d\sigma_{\nu T \to l \pi T}}{d\nu\, dQ^2}\right|_{Q^2=0}}_{\text{diffractive production, } W \to \pi} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \underbrace{\sigma_{\pi T \to \pi T}}_{\text{elastic scattering}}$$

$\sim R_A$

$\sim R_A^2$

Result for the $\nu A \to l\pi^+ A'$ incoherent differential cross-section

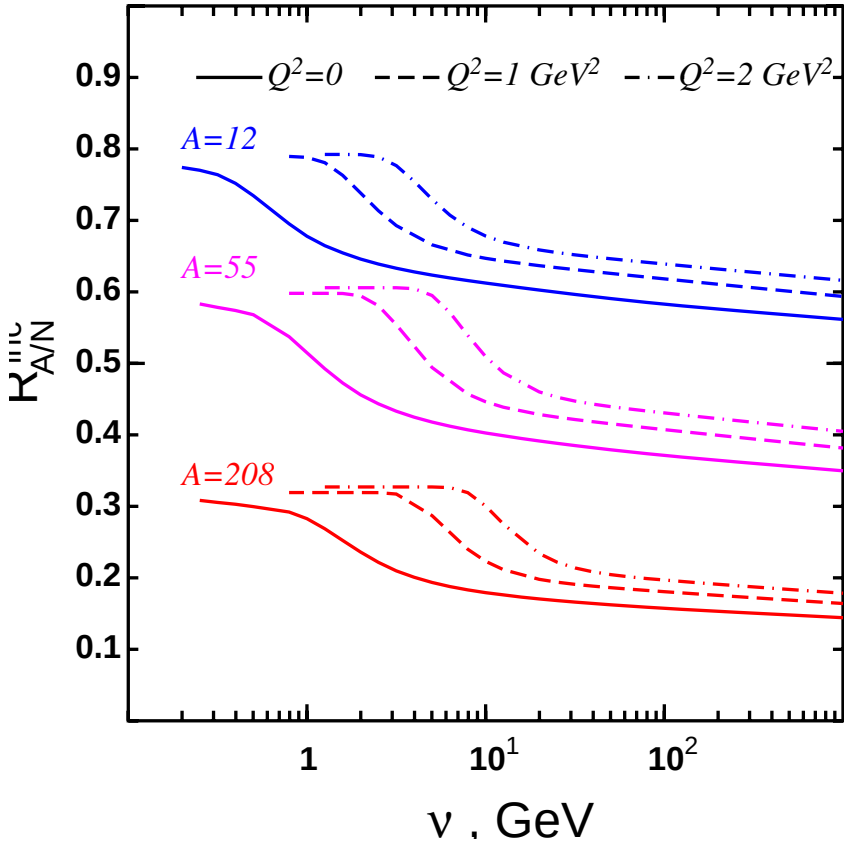


Figure: Incoherent cross-section, is controlled by l_c^a.

Conclusion

- The AR is broken on the nuclei
- We evaluated cross-sections for the processes $\nu p \to \mu \pi p$, $\nu A \to \mu \pi A$, $\nu A \to \mu \pi A'$.
  - Our approach does not use AR (though is consistent with it at small energies and $Q^2 = 0$).
  - At high energies, the cross-section is suppressed due to shadowing correcions
- Further direction for study–strange and charmed mesons (in progress)

- Thank You for your attention !

Models

All the models used for description of the coherent $\nu \to \pi$-production fall into three categories:

- PCAC-based models
- Low-energy microscopic models
- High-energy microscopic models

Rein-Sehgal model

(Ann. Phys. 133 (1981) 79, NPB 223 (1983), 29)

- For the nucleons

$$\left.\frac{d\sigma_{\nu p\to l\pi p}}{d\nu dQ^2}\right|_{Q^2\neq 0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi p\to F} \frac{1}{\left(1+Q^2/m_A^2\right)^2}$$

- For the nuclei:

$$\left.\frac{d\sigma_{\nu A\to l\pi A}}{d\nu dQ^2}\right|_{Q^2\neq 0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T\to F} \frac{1}{\left(1+Q^2/m_A^2\right)^2} |F_A(t)|^2 F_{abs}(A)$$

  - $F_A(t)$-nuclear formfactor
  - $F_{abs}(A)$-absorptive factor, depends only on geometry (A) but not on energy

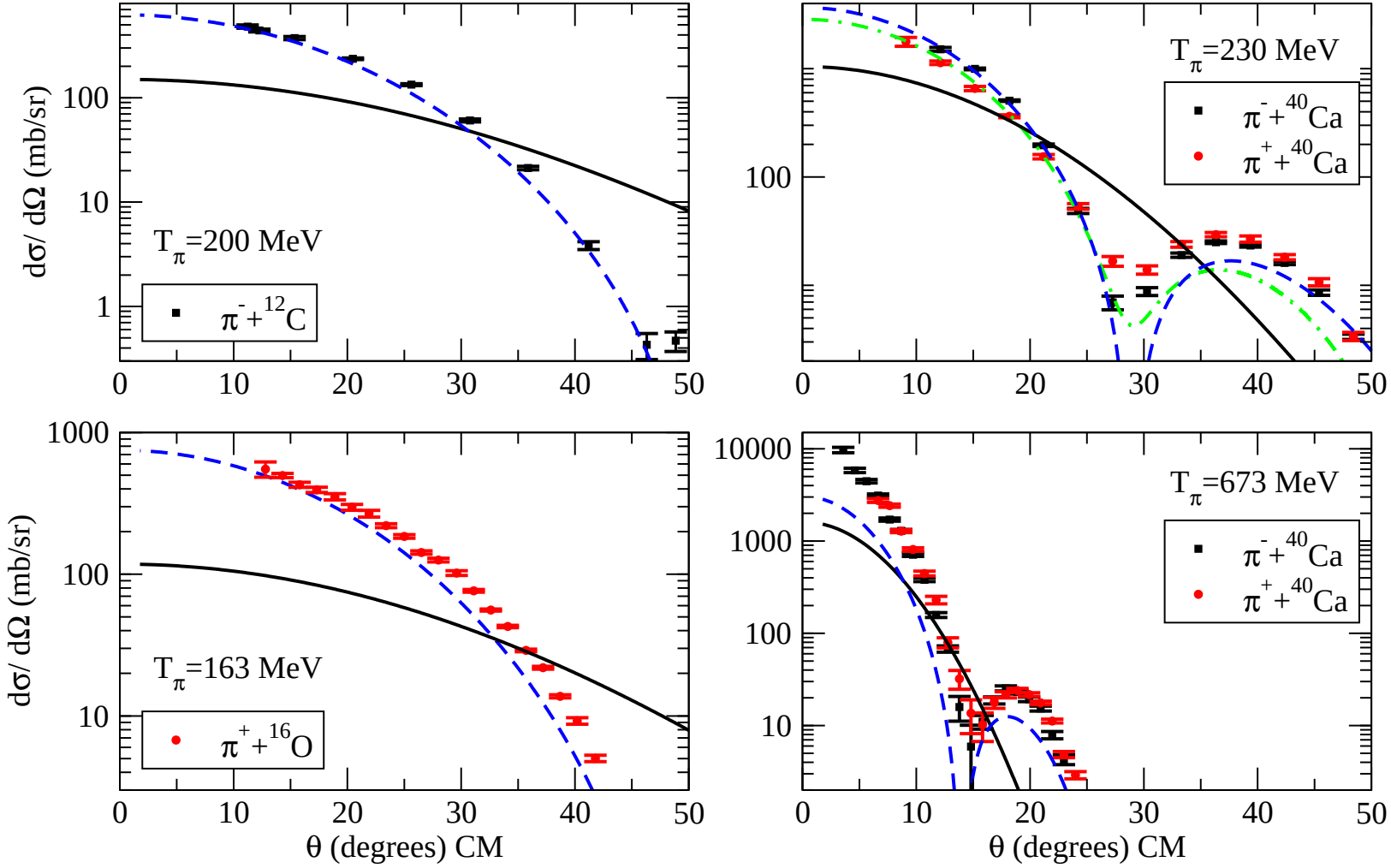
$$F_{abs}(A) \sim \exp\left(-const\, A^{1/3}\right)$$

- Main advantage–simplicity, easy to implement in MC generators

Rein-Sehgal model

(Ann. Phys. 133 (1981) 79, NPB 223 (1983), 29)

Fails to describe the angular dependence (E. Hernandez, J. Nieves, M. Vicente-Vacas, PRD 80 (2009), 013003)



Other PCAC-based models

- Model of Hernandez, Nieves-Vicente, Vacas

  (PRD 80 (2009), 013003)

  Modification of the absorption factor

  $F_A(t)F_{abs}(A) \to \tilde{F}_A(t) = \int d^3r\, e^{i\Delta\cdot r}\rho_A(r)\exp(-\frac{\sigma_{inel}}{2}T_A(b))$
- Kartavtsev-Paschos, Paschos-Shalla models

  (PRD 74 (2006), 054007, PRD 80 (2009), 033005) (π-dominance + PCAC)
- Models of resonant production where PCAC is used for some transition formfactors

  (Will be discussed later)

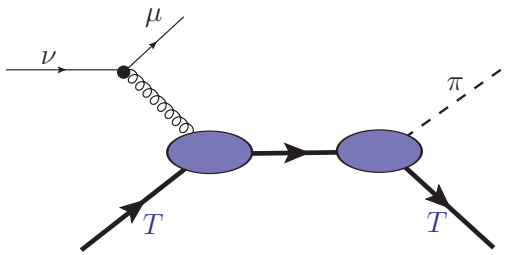
- Low-energy microscopic models

Low-energy microscopic models

Sufficient experimental data

(K2K [PRL 95 (2005), 252301], SciBooNE [PRD 78 (2008), 112004, PRD 81 (2010), 111102], MiniBooNE [PLB 664, 41 (2008)], ...)

In the small-ν dominant contribution comes from s-channel resonances



crossed diagrams give nonresonant background

- Number of resonances required increases rapidly with s
  - Just a few resonances which give largest contributions:
    Spin-3/2: $\Delta(1232), N(1520), \Delta(1600), \Delta(1620), ...$
    Spin-1/2: $N(1440), N(1535), N(1650), ...$

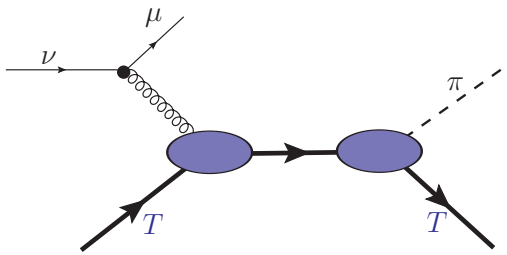
    (E. Paschos [PRD 80 (2009) 033005], O. Lalakulich, et. al. [PRD 71 (2005), 074003; PRD 74(2006), 014009], S. Nakamura, 2011;)

Low-energy microscopic models

Sufficient experimental data

(K2K [PRL 95 (2005), 252301], SciBooNE [PRD 78 (2008), 112004, PRD 81 (2010), 111102], MiniBooNE [PLB 664, 41 (2008)], ...)

In the small-ν dominant contribution comes from s-channel resonances



crossed diagrams give nonresonant background

- Number of resonances required increases rapidly with s
- Couplings are NOT local, for $\Delta(1232)$ alone there are ***8*** transitional formfactors

$$\langle \Delta^{++}|J^\nu|p\rangle = \sqrt{3}\bar{\psi}_\lambda(p')d^{\lambda\nu}u(p)$$

$$d^{\lambda\nu} = g^{\lambda\nu}\left[\frac{C_3^V}{m_N}\hat{q} + \frac{C_4^V}{m_N^2}(p'q) + \frac{C_5^V}{m_N^2}(pq) + C_6^V\right]\gamma_5 - q^\lambda\left[\frac{C_3^V}{m_N}\gamma^\nu + \frac{C_4^V}{m_N^2}p'^\nu + \frac{C_5^V}{m_N^2}p^\nu\right]\gamma_5$$

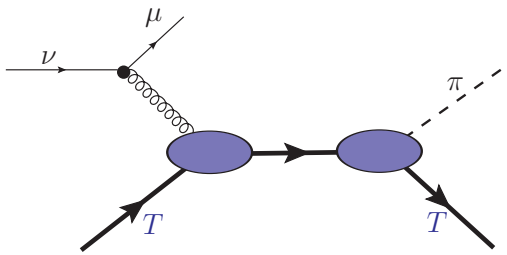
$$+g^{\lambda\nu}\left[\frac{C_3^A}{m_N}\hat{q} + \frac{C_4^A}{m_N^2}(p'q)\right] - q^\lambda\left[\frac{C_3^A}{m_N}\gamma^\nu + \frac{C_4^A}{m_N^2}p'^\nu\right] + g^{\lambda\nu}C_5^A + q^\lambda q^\nu\frac{C_6^A}{m_N^2}.$$

Low-energy microscopic models

Sufficient experimental data

(K2K [PRL 95 (2005), 252301], SciBooNE [PRD 78 (2008), 112004, PRD 81 (2010), 111102], MiniBooNE [PLB 664, 41 (2008)], ...)

In the small-ν dominant contribution comes from s-channel resonances



crossed diagrams give nonresonant background

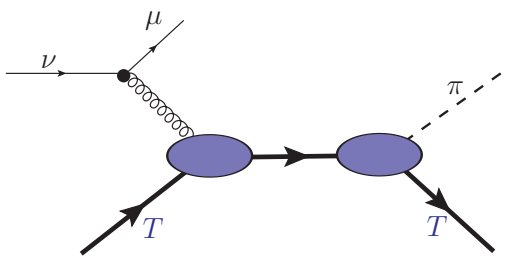
- Number of resonances required increases rapidly with s
- Couplings are NOT local, for $\Delta(1232)$ alone there are ***8*** transitional formfactors
  - Completely neglect the nonlocality (Amaro et. al, PRD 79(2009),013002)
  - Parameterize everything in dipole-like form (O. Lalakulich, et. al. [PRD 71 (2005), 074003; PRD 74(2006), 014009])
    - ★ Too many formfactors, uncertainty in parameters

Low-energy microscopic models

Sufficient experimental data

(K2K [PRL 95 (2005), 252301], SciBooNE [PRD 78 (2008), 112004, PRD 81 (2010), 111102], MiniBooNE [PLB 664, 41 (2008)], ...)

In the small-ν dominant contribution comes from s-channel resonances



crossed diagrams give nonresonant background

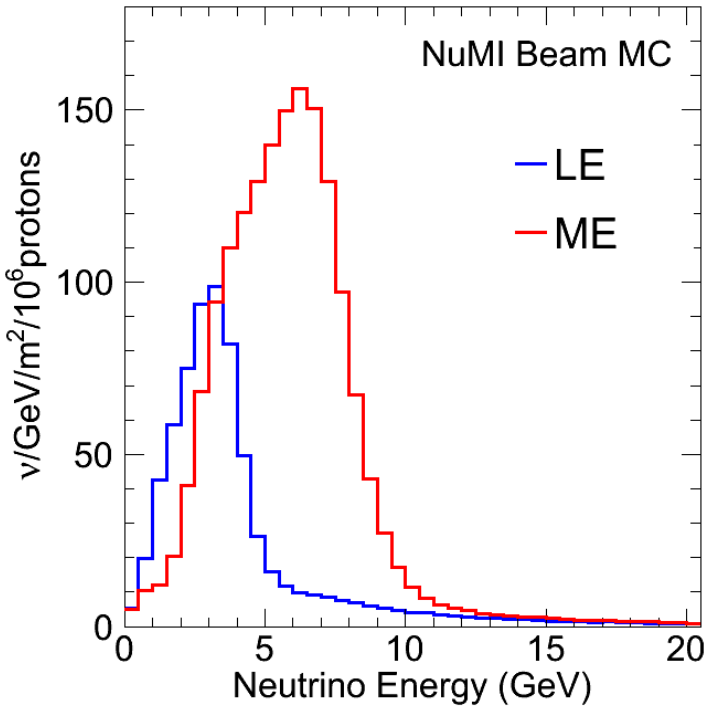
- Number of resonances required increases rapidly with s
- Couplings are NOT local, for $\Delta(1232)$ alone there are ***8*** transitional formfactors
- Open questions: non-resonant background, modification of resonances inside the nuclei, ...

- High-energy microscopic model

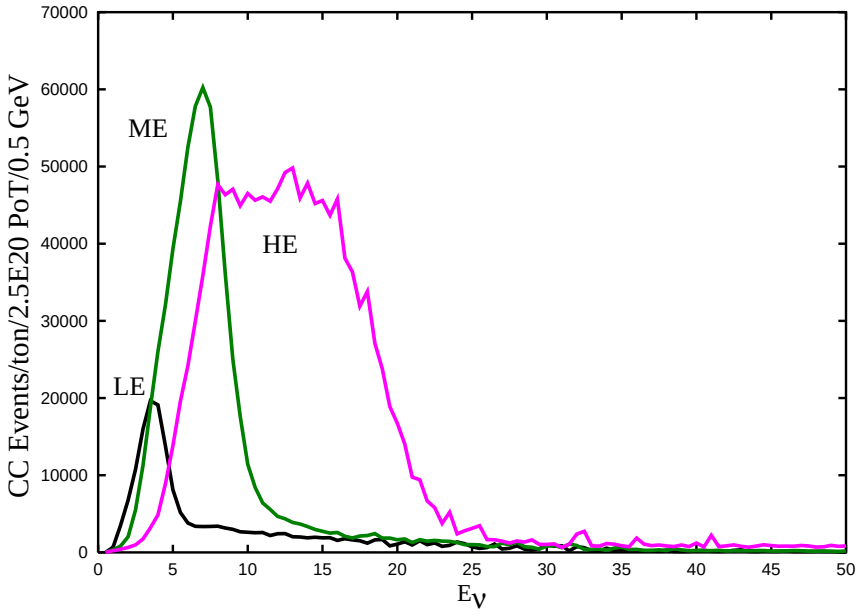
High-energy cohπ neutrino-production

Limited experimental data

- The only high-statistics experiment is Minerva@Fermilab



(Minerva@Fermilab, 2011)



(Minerva proposal, 2004)

High-energy cohπ neutrino-production

Limited experimental data

- The only high-statistics experiment is Minerva@Fermilab

Older experiments have poor statistics and measure total cross-sections:

- BEBC ($E_\nu \lesssim 200$ GeV) (Allen et. al. [NPB 264 (1986),221]; Marage et.al. [ZPC 31 (1986),191, ZPC 43 (1989),523])

High-energy cohπ neutrino-production

Limited experimental data

- The only high-statistics experiment is Minerva@Fermilab

Older experiments have poor statistics and measure total cross-sections:

- BEBC ($E_\nu \lesssim 200$ GeV)
- FNAL ($E_\nu \lesssim 250$ GeV) (Aderholz et. al. [PRL 63(1989),2349]; Wilocq et.al. [PRD 47(1993),2661])

High-energy cohπ neutrino-production

Limited experimental data

- The only high-statistics experiment is Minerva@Fermilab

Older experiments have poor statistics and measure total cross-sections:

- BEBC ($E_\nu \lesssim 200$ GeV)
- FNAL ($E_\nu \lesssim 250$ GeV)
- CHARM & CHARM-II ($E_\nu \lesssim 300$ GeV) (Bergsma et. al. [PLB 157(1985),469], Vilain et. al. [PLB 313(1993),267])