Single-pion production in neutrino-matter collisions

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Kinematics

- Diffractive pion production,
  - CC: $\nu T \rightarrow l\pi^+ T$
  - NC: $\nu T \rightarrow \nu \pi^0 T$

- High statistics, differential $x$-sections are measured
- Different targets (H$_2$O, He, C, CH, Fe, Pb)

(Minerva@Fermilab, 2011)

$$
E_\nu = \frac{p \cdot k_\nu}{m_N}, \quad \nu = \frac{p \cdot q_W}{M}, \quad y = \frac{p \cdot q_W}{p \cdot k} \\
Q^2 = -q^2_W = 4E_\nu(E_\nu - \nu)\sin^2 \frac{\theta}{2} + \mathcal{O}(m_l^2) \\
t = (p' - p)^2 = \Delta^2 = t_{\text{min}} - \Delta^2_{\perp}
$$

High statistics, *differential* $x$-sections are measured

Different targets (H$_2$O, He, C, CH, Fe, Pb)

(Tue. talks of J. Morfin, J. Mousseau)
Why pion production?

- Contribution of axial current dominates
- Background in $\nu_\mu \rightarrow \nu_e$ (misidentification of $\pi_0$, $\pi_0 \rightarrow 2\gamma$)

(MiniBooNE [PLB 664, 41 (2008)], SciBooNE [PRD 81, 111102 (2010)])

(MiniBooNE [PRL 102, 101802 (2009); arXiv:1201.1519])

- Theory: $\sigma_{CC} \approx 2\sigma_{NC}$, experiment:
  - No evidence for CC ($\nu_\mu + ^{12}C \rightarrow \mu^- + \pi^+ + ^{12}C$, $E_\nu \sim 1$ GeV) (K2K [PRL 95, 252301 (2005)], SciBooNE [PRD 78, 112004 (2008)])
  - Signature for NC ($\nu_\mu + ^{12}C \rightarrow \nu_\mu + \pi_0 + ^{12}C$, $E_\nu \sim 1$ GeV) (MiniBooNE [PLB 664, 41 (2008)], SciBooNE [PRD 81, 111102 (2010)])
PCAC-based models: Adler relation

\[
\frac{d\sigma_{\nu T\rightarrow lF}}{d\nu dQ^2} \bigg|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T\rightarrow F}
\]

(PR 135 (1964), 963)

- In real measurements \(q^2 \neq 0\), so AR requires extrapolation. AR \(\neq\) Pion dominance:

\[
T_\mu(...) \sim \frac{q_\mu}{q^2 - m_\pi^2} + T_\mu^{\text{non-pion}}(...),
\]

but lepton currents are conserved, so

\[
q_\mu L_{\mu \nu} = \mathcal{O}(m_l)
\]

\(\Rightarrow\) AR survives due to heavier hadrons and \(\chi\)-sym.

- Contributions from transverse part and from the vector part \(\mathcal{O}(q^2)\) for small \(q^2\)

\[
\sigma \sim (1 + Q^2/m_A^2)^{-2}, \quad m_A \sim 1 \text{ GeV}
\]
PCAC vs. black disk regime (high energy limit)

PCAC-based models are inconsistent with BDR:

\[
\left. \frac{d\sigma_{\nu T \rightarrow l\pi T}}{d\nu dQ^2} \right|_{Q^2=0} = \frac{G_F^2 f_\pi^2}{2\pi} \frac{E_{\nu \nu}}{E_{\nu \nu}} \sigma_{\pi T \rightarrow \pi T}
\]

diffractive production, $W \rightarrow \pi$

elastic scattering

\[
\sim \frac{q_\mu}{q^2 - m_\pi}
\]

(diagrams with pions are suppressed by lepton mass, $\mathcal{O}(m_l)$)
PCAC vs. black disk regime (high energy limit)

PCAC-based models are inconsistent with BDR:

\[
\left. \frac{d\sigma_{\nu T \rightarrow l\pi T}}{d\nu dQ^2} \right|_{Q^2=0} = \frac{G_F^2 f_{\pi}^2}{2\pi} \frac{E_{\nu} - m_{\pi}}{E_{\nu} \nu} \sigma_{\pi T \rightarrow \pi T}
\]

diffractive production, \( \mathcal{W} \rightarrow \pi \)

\( \sim A^{1/3} \) \( \sim R_A \)

elastic scattering

\( \sim A^{2/3} \) \( \sim R_A^2 \)
The amplitude has a form

$$\mathcal{A}^{aT \rightarrow \pi T} = \bar{\Psi}_\pi (\beta', r') \otimes \mathcal{A}_T^d (\beta', r'; \beta, r) \otimes \Psi_a (\beta, r),$$

- $\mathcal{A}_T^d (\beta', r'; \beta, r)$ universal object, depends only on the target $T$, known from $\gamma p$ and $\gamma A$ processes. (Tuesday talks by Carlo Ewerz, Leszek Motyka)
- $\bar{\Psi}_\pi, \Psi_a$ are the distribution amplitudes of the initial and final states.
  - Need to take care of chiral symmetry.
  - For study of AR and its breaking, should be valid up to $Q^2 = 0$. 
On higher Fock states

Consider only $\bar{q}q$, contribution of $\bar{q}qg$ is suppressed for $\nu \leq 10^3$ GeV.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure}
\caption{Gluon shadowing in color dipole (PRD 62, 054022 (2000)) and in phenomenological D. de Florian, R. Sassot parametrization (PRD 69, 074028 (2004)). Shaded area: gluon uncertainty band from EPS’09 parametrization (JHEP 0904:065 (2009))}
\end{figure}
Distribution amplitudes of pion

Pion distribution amplitudes (P. Ball et al, 2006)

\[
\langle 0 \left| \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(x) \right| \pi(q) \rangle = \text{if}_\pi \int_0^1 du \ e^{i(u p \cdot y + \bar{u} p \cdot x)} \times \\
\times \left( p_\mu \phi_{2;\pi}(u) + \frac{1}{2} \frac{z_\mu}{(p \cdot z)} \psi_{4;\pi}(u) \right),
\]

\[
\langle 0 \left| \bar{\psi}(y) \gamma_5 \psi(x) \right| \pi(q) \rangle = -\text{if}_\pi \frac{m_\pi^2}{m_u + m_d} \int_0^1 du \ e^{i(u p \cdot y + \bar{u} p \cdot x)} \phi_{3;\pi}(u),
\]

\[
\langle 0 \left| \bar{\psi}(y) \sigma_{\mu \nu} \gamma_5 \psi(x) \right| \pi(q) \rangle = -\frac{i}{3} f_\pi \frac{m_\pi^2}{m_u + m_d} \int_0^1 du \ e^{i(u p \cdot y + \bar{u} p \cdot x)} \times \\
\times \frac{1}{p \cdot z} \left( p_\mu z_\nu - p_\nu z_\mu \right) \phi_{3;\pi}^{(\sigma)}(u),
\]
Distribution amplitude of pion

- Best known is leading twist DA $\phi_{2;\pi}(x)$

$\phi_{\pi}(u)$

$\phi_{\pi}(u)$

$\phi_{\pi}(u)$

close to $\phi_{as}(x) = 6x(1-x)$ (V. Yu. Petrov et. al., 1998)

- We take into account all the DAs in order not to kill the chiral
Distribution amplitudes of axial meson
Axial DAs (K.-C. Yang 2007)

\[
\langle 0 | \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(x) | A(q) \rangle = \text{i} f A m_A \int_0^1 du \, e^{i(q \cdot y + \bar{u} \cdot p \cdot x)} \times \\
\times \left( p \mu \frac{e^{(\lambda)} \cdot z}{p \cdot z} \Phi_{\parallel}(u) + e^{(\lambda=\perp)}_\mu g^{(a)}_{\perp}(u) - \frac{1}{2} z_\mu \frac{e^{(\lambda)} \cdot z}{(p \cdot z)^2} m_A^2 g_3(u) \right),
\]

\[
\langle 0 | \bar{\psi}(y) \gamma_\mu \psi(x) | A(q) \rangle = -\text{i} f A m_A \varepsilon_{\mu \nu \rho \sigma} e^{(\lambda)}_\nu p_\rho z_\sigma \int_0^1 du \, e^{i(q \cdot y + \bar{u} \cdot p \cdot x)} \frac{g^{(v)}_{\perp}(u)}{4}
\]

\[
\langle 0 | \bar{\psi}(y) \sigma_{\mu \nu} \gamma_5 \psi(x) | A(q) \rangle = f_{\perp} \int_0^1 du \, e^{i(q \cdot y + \bar{u} \cdot p \cdot x)} \left( e^{(\lambda=\perp)}_{[\mu} p_{\nu]} \Phi_{\perp}(u) \right. \right.
\left. + \frac{e^{(\lambda)} \cdot z}{(p \cdot z)^2} m_A^2 p[\mu z_\nu] h^{(t)}_{\parallel}(u) + \frac{1}{2} e^{(\lambda)}_{[\mu} z_{\nu]} \frac{m_A^2}{p \cdot z} h_3(u) \right),
\]

\[
\langle 0 | \bar{\psi}(y) \gamma_5 \psi(x) | A(q) \rangle = f_{\perp} m_A^2 e^{(\lambda)} \cdot z \int_0^1 du \, e^{i(q \cdot y + \bar{u} \cdot p \cdot x)} \frac{h^{(p)}_{\parallel}(u)}{2}.
\]
PCAC relations for DAs

PCAC relates 4 DAs of the axial current and pion DAs:

\[
\begin{align*}
\Phi_{||} (\alpha, q^2 = m_{\pi}^2) &= \phi_{2,\pi}(\alpha) \\
g_3 (\alpha, q^2 = m_{\pi}^2) &= \frac{1}{2} \psi_{4,\pi}(\alpha) \\
h_{||}^{(t)} (\alpha, q^2 = m_{\pi}^2) &= -\frac{1}{3} \frac{m_{\pi}^2}{m_u + m_d} \phi_{3,\pi}^{(\sigma)}(\alpha) \\
h_{||}^{(p)} (\alpha, q^2 = m_{\pi}^2) &= \frac{2m_{\pi}^2}{m_u + m_d} \phi_{3,\pi}^{(p)}(\alpha)
\end{align*}
\]
PCAC relations for DAs

\[ \varphi_2; \pi(\alpha) \quad \Phi(\alpha) \]

\[ m_{\pi}^2 \quad 3(m_u + m_d) \varphi_3; \pi(\sigma)(\alpha) \]

\[ h_\parallel^{(p)}(\alpha) \quad 0.5 \psi_4; \pi(u) \]

\[ g_3(u) \quad -0.5 \psi_4; \pi(u) \]

\[ M. Siddikov (UTFSM) Single-pion neutrino-production \]
Chiral symmetry & color dipole

**Figure:** Relation between DAs of pion ($\pi\bar{q}q$), axial meson ($W\bar{q}q$) and $W\pi$ coupling guarantees that the amplitude has transverse structure required by chiral symmetry

$$T^{(a\rightarrow \pi)}_\mu = \left(\frac{q_\mu q_\nu}{q^2 - m_\pi^2} - g_{\mu\nu}\right) P_\nu T_{\pi\pi}(p, q) + O(q^2),$$
Result for the $\nu p \rightarrow \mu^- \pi^+ p$ cross-section

At high energies

$$\frac{d\sigma}{dtd\ln\nu dQ^2} \sim s_{Wp}^{2\alpha} \Rightarrow \sigma_{\nu p \rightarrow l\pi p} \sim E_{\nu}^{2\alpha}$$

**Figure:** Total cross-section as a function of the neutrino energy $E_{\nu}$.

Reasonable value for energies $E_{\nu} > 10$ GeV, vanishes for $E_{\nu} < 10$ GeV
Result for the $\nu p \rightarrow \mu^- \pi^+ p$ cross-section

\[ \sigma \sim s^2 \alpha W p \Rightarrow \sigma_{\nu p \rightarrow l \pi p} \sim E^2 \alpha \nu \]

Figure: Total cross-section as a function of the neutrino energy $E_\nu$.

Low-energy region is dominated by $\Delta(1232)$
Result for the $\nu p \rightarrow \mu^- \pi^+ p$ cross-section

**Figure**: Total cross-section as a function of the neutrino energy $E_\nu$.

Resonance-based models include just a few resonances, the total cross-section is constant above all thresholds.
Result for the $\nu p \rightarrow \mu^- \pi^+ p$ cross-section

**Figure:** Total cross-section as a function of the neutrino energy $E_\nu$. *Compilation of experimental data from (Minerva proposal, 2004)*

Total cross-section does not distinguish diffractive and resonance contributions. Differential cross-section would give much more information.
WA21 experiment @ CERN $\nu p \rightarrow \mu^- \pi^+ p$

Figure: Neutrino spectrum at BEBC (CERN) (Allen et.al., 1985)
WA21 experiment @ CERN $\nu p \rightarrow \mu^- \pi^+ p$

Figure: Color dipole vs. BEBC (CERN) (Allen et.al., 1985)
Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

Predictions for Minerva kinematics

\[ \frac{d^3\sigma}{dt d\ln v dQ^2} \sim \frac{\sigma_{\pi p}^\text{tot}(s)^2}{(1+Q^2/m_A^2)^2} e^{-B\Delta^2} \]
Coherent neutrino-nuclear scattering

- Use Gribov-Glauber approach

\[ \begin{align*}
q_z & \quad q_z \\
\pi & \quad \pi \\
\end{align*} \]

- Two scales: coherence length of the pion and effective axial meson

\[ \begin{align*}
\ell_\pi^a &= \frac{2v}{m_\pi^2 + Q^2}, & \ell_\pi^a &= \frac{2v}{m_a^2 + Q^2}.
\end{align*} \]

- For large \( Q^2 \), \( \ell_\pi^a \approx \ell_\pi^a \), so this case is similar to photon-nuclear processes, we have only two regimes: \( \ell_c \gg R_A \) and \( \ell_c \ll R_A \).

- For small \( m_\pi^2 \lesssim Q^2 \ll m_a^2 \), \( \ell_\pi^a \ll \ell_\pi^a \), appears a third regime, when \( \ell_\pi^a \ll R_A \ll \ell_\pi^a \).
$\nu A \rightarrow l\pi^+ A$ cross-section in a toy model


Cross-check in a simple toy model:

- Assume there are only pion and $a_1$ mesons
  - Numerically, vector dominance does not work in axial channel,

\[ \sigma_{a_1(1236)p} \approx \sigma_{\pi p}/30 \]

(Piketty, Stodolsky, NPB 15(1970), 571); the axial meson should be understood as an effective $\rho\pi$ state (Deck, PRL 13 (1964), 164)

- Assume Adler relation is valid for nucleon at $Q^2 = 0$
- Use Gribov-Glauber approach for nuclei.
Result for the $\nu A \rightarrow l\pi^+ A$ cross-section (toy model)

$$R^\text{coh}_{A/N} = \frac{1}{A} \frac{d\sigma_A/d\nu dQ^2}{d\sigma_N/d\nu dQ^2}$$

Adler relation works in the region $\nu \leq 10$ GeV; for high energies results broken due to absorptive corrections
Result for the $\nu A \rightarrow l\pi^+ A$ differential cross-section (color dipole)

![Graph]

**Figure:** Ratio of cross-sections on the nucleus and proton.

Results similar to toy model, but *Adler relation on nuclei is always broken.*
At low energies, AR works for toy model but not for color dipole. Reason: axial current contains a mixture of states with different masses; for light dipoles coherence length is large. At higher energies, there are absorptive corrections
High energy limit

Adler relation is broken due to absorptive corrections. Assume the limit $R_A \ll l_a \ll l_\pi$.

$$\left. \frac{d\sigma_{\nu T \rightarrow \pi T}}{d\nu dQ^2} \right|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_{\nu} - E_{\nu'}}{E_{\nu} E_{\nu'}} \sigma_{\pi T \rightarrow \pi T}$$

diffractive production, $W \rightarrow \pi$

elastic scattering

$\sim A^{1/3}$

$\sim R_A$

$\sim A^{2/3}$

$\sim R_A^2$
Result for the $\nu A \rightarrow l\pi^+ A'$ incoherent differential cross-section

![Graph showing incoherent cross-section](image)

**Figure:** Incoherent cross-section, is controlled by $l^a_c$. 
Conclusion

- The AR is broken on the nuclei
- We evaluated cross-sections for the processes $\nu p \rightarrow \mu \pi p$, $\nu A \rightarrow \mu \pi A$, $\nu A \rightarrow \mu \pi A'$.  
  - Our approach does not use AR (though is consistent with it at small energies and $Q^2 = 0$).  
  - At high energies, the cross-section is suppressed due to shadowing corrections
- Further direction for study—strange and charmed mesons (in progress)
Thank You for your attention!
All the models used for description of the coherent $\nu \rightarrow \pi$-production fall into three categories:

- PCAC-based models
- Low-energy microscopic models
- High-energy microscopic models
Rein-Sehgal model


For the nucleons

\[
\frac{d\sigma_{\nu p \rightarrow l\pi p}}{d\nu dQ^2} \bigg|_{Q^2 \neq 0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi p \rightarrow F} \frac{1}{\left(1 + Q^2/m_A^2\right)^2}
\]

For the nuclei:

\[
\frac{d\sigma_{\nu A \rightarrow l\pi A}}{d\nu dQ^2} \bigg|_{Q^2 \neq 0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T \rightarrow F} \frac{1}{\left(1 + Q^2/m_A^2\right)^2} |F_A(t)|^2 F_{abs}(A)
\]

- $F_A(t)$-nuclear formfactor
- $F_{abs}(A)$-absorptive factor, depends only on geometry ($A$) but not on energy

\[
F_{abs}(A) \sim \exp\left(-\text{const} A^{1/3}\right)
\]

- Main advantage–simplicity, easy to implement in MC generators
Rein-Sehgal model


Fails to describe the angular dependence (E. Hernandez, J. Nieves, M. Vicente-Vacas, PRD 80 (2009), 013003)
Other PCAC-based models

- Model of Hernandez, Nieves-Vicente, Vacas
  (PRD 80 (2009), 013003)
  Modification of the absorption factor
  \[ F_A(t) F_{abs}(A) \rightarrow \tilde{F}_A(t) = \int d^3 r \, e^{i \Delta \cdot r} \rho_A(r) \exp\left( -\frac{\sigma_{inel}}{2} T_A(b) \right) \]

- Kartavtsev-Paschos, Paschos-Shalla models
  (PRD 74 (2006), 054007, PRD 80 (2009), 033005) \( (\pi\text{-dominance + PCAC}) \)

- Models of resonant production where PCAC is used for some transition formfactors
  (Will be discussed later)
Low-energy microscopic models
Low-energy microscopic models

Sufficient experimental data

(K2K [PRL 95 (2005), 252301], SciBooNE [PRD 78 (2008), 112004, PRD 81 (2010), 111102], MiniBooNE [PLB 664, 41 (2008)], ...)

In the small-$\nu$ dominant contribution comes from $s$-channel resonances

\[
\begin{align*}
\nu \rightarrow \mu + \pi^{-} \\
\text{crossed diagrams give nonresonant background}
\end{align*}
\]

- Number of resonances required increases rapidly with $s$
  - Just a few resonances which give largest contributions:
    Spin-3/2: $\Delta(1232), N(1520), \Delta(1600), \Delta(1620), ...$
    Spin-1/2: $N(1440), N(1535), N(1650), ...$

Low-energy microscopic models

Sufficient experimental data

(K2K [PRL 95 (2005), 252301], SciBooNE [PRD 78 (2008), 112004, PRD 81 (2010), 111102], MiniBooNE [PLB 664, 41 (2008)], ...)

In the small-$\nu$ dominant contribution comes from $s$-channel resonances

![Diagram](image)

crossed diagrams give nonresonant background

- Number of resonances required increases rapidly with $s$
- Couplings are NOT local, for $\Delta(1232)$ alone there are 8 transitional formfactors

\[
\langle \Delta^{++}|J^\nu|p\rangle = \sqrt{3} \bar{\psi}_A (p') d^{\lambda \nu} u(p)
\]

\[
d^{\lambda \nu} = g^{\lambda \nu} \left[ \frac{C_V}{m_N} \hat{q} + \frac{C_A}{m^2_N} (p' q) + \frac{C_V}{m^2_N} (p q) + C_6 \right] \gamma_5 - q^\lambda \left[ \frac{C_V}{m_N} \gamma^\nu + \frac{C_A}{m^2_N} p'^\nu + \frac{C_V}{m^2_N} p^\nu \right] \gamma_5 
\]

\[
+ g^{\lambda \nu} \left[ \frac{C_A}{m_N} \hat{q} + \frac{C_A}{m^2_N} (p' q) \right] - q^\lambda \left[ \frac{C_A}{m_N} \gamma^\nu + \frac{C_A}{m^2_N} p'^\nu \right] + g^{\lambda \nu} C_5 + q^\lambda q^\nu \frac{C_A}{m^2_N}.
\]
Low-energy microscopic models

Sufficient experimental data

\( \text{K2K [PRL 95 (2005), 252301], SciBooNE [PRD 78 (2008), 112004, PRD 81 (2010), 111102], MiniBooNE [PLB 664, 41 (2008)], ...} \)

In the small-\( \nu \) dominant contribution comes from \( s \)-channel resonances

\[ \begin{array}{c}
\nu \\
\mu \\
\pi \\
\end{array} \]

\[ \begin{array}{c}
\bullet \\
\bullet \\
\end{array} \]

\[ \begin{array}{c}
\text{Number of resonances required increases rapidly with } s \\
\text{Couplings are NOT local, for } \Delta(1232) \text{ alone there are } 8 \text{ transitional formfactors} \\
\quad \begin{array}{c}
\text{\quad Completely neglect the nonlocality (Amaro et. al, PRD 79(2009),013002)} \\
\text{Parameterize everything in dipole-like form (O. Lalakulich, et. al. [PRD 71 (2005), 074003; PRD 74(2006), 014009])} \\
\text{\quad} \begin{array}{c}
\text{\quad Too many formfactors, uncertainty in parameters}
\end{array}
\end{array}
\end{array} \]

crossed diagrams give nonresonant background

closeup of a diagram showing a neutrino, muon, and pion interacting through a pair of identical diagrams.
Low-energy microscopic models

Sufficient experimental data

(K2K [PRL 95 (2005), 252301], SciBooNE [PRD 78 (2008), 112004, PRD 81 (2010), 111102], MiniBooNE [PLB 664, 41 (2008)], ...)

In the small-\(\nu\) dominant contribution comes from \(s\)-channel resonances

\[
\begin{align*}
\nu & \rightarrow \pi^- \\
\pi^- & \rightarrow \mu^+ \\
\mu^+ & \rightarrow T \\
\text{crossed diagrams give nonresonant background}
\end{align*}
\]

- Number of resonances required increases rapidly with \(s\)
- Couplings are NOT local, for \(\Delta(1232)\) alone there are 8 transitional formfactors
- Open questions: non-resonant background, modification of resonances inside the nuclei, ...
High-energy microscopic model
High-energy coh\(\pi\) neutrino-production

Limited experimental data

- The only high-statistics experiment is Minerva@Fermilab

(Minerva@Fermilab, 2011) (Minerva proposal, 2004)
High-energy coh$\pi$ neutrino-production

Limited experimental data

- The only high-statistics experiment is Minerva@Fermilab

Older experiments have poor statistics and measure total cross-sections:

- BEBC ($E_\nu \lesssim 200$ GeV) (Allen et. al. [NPB 264 (1986),221]; Marage et.al. [ZPC 31 (1986),191, ZPC 43 (1989),523])
High-energy coh $\pi$ neutrino-production

Limited experimental data

- The only high-statistics experiment is Minerva@Fermilab

Older experiments have poor statistics and measure total cross-sections:

- BEBC ($E_\nu \lesssim 200$ GeV)
- FNAL ($E_\nu \lesssim 250$ GeV) (Aderholz et. al. [PRL 63(1989),2349]; Wilocq et.al. [PRD 47(1993),2661])
High-energy coh$\pi$ neutrino-production

Limited experimental data

- The only high-statistics experiment is Minerva@Fermilab

Older experiments have poor statistics and measure total cross-sections:

- BEBC ($E_{\nu} \lesssim 200$ GeV)
- FNAL ($E_{\nu} \lesssim 250$ GeV)
- CHARM & CHARM-II ($E_{\nu} \lesssim 300$ GeV) (Bergsma et. al. [PLB 157(1985),469], Vilain et. al. [PLB 313(1993),267])