Medium-induced soft gluon radiation in DIS

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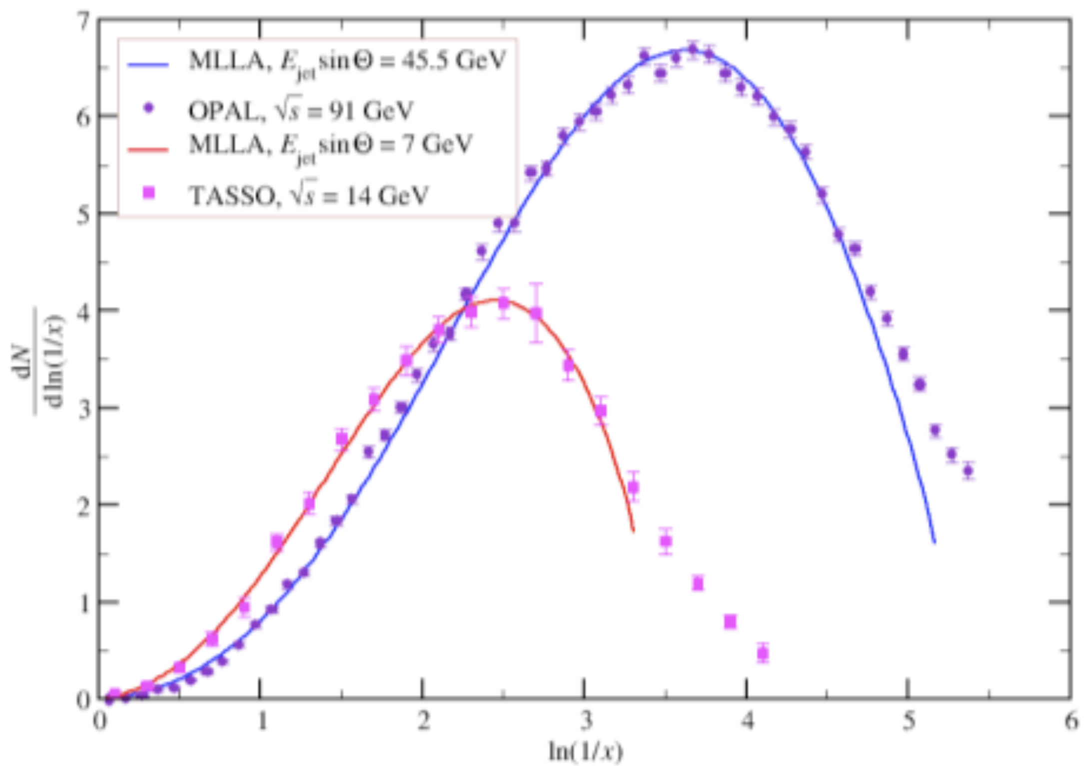
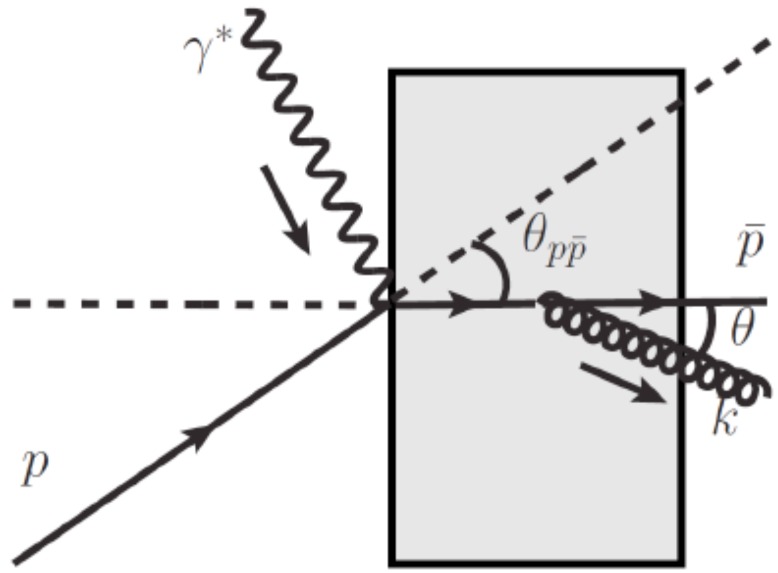
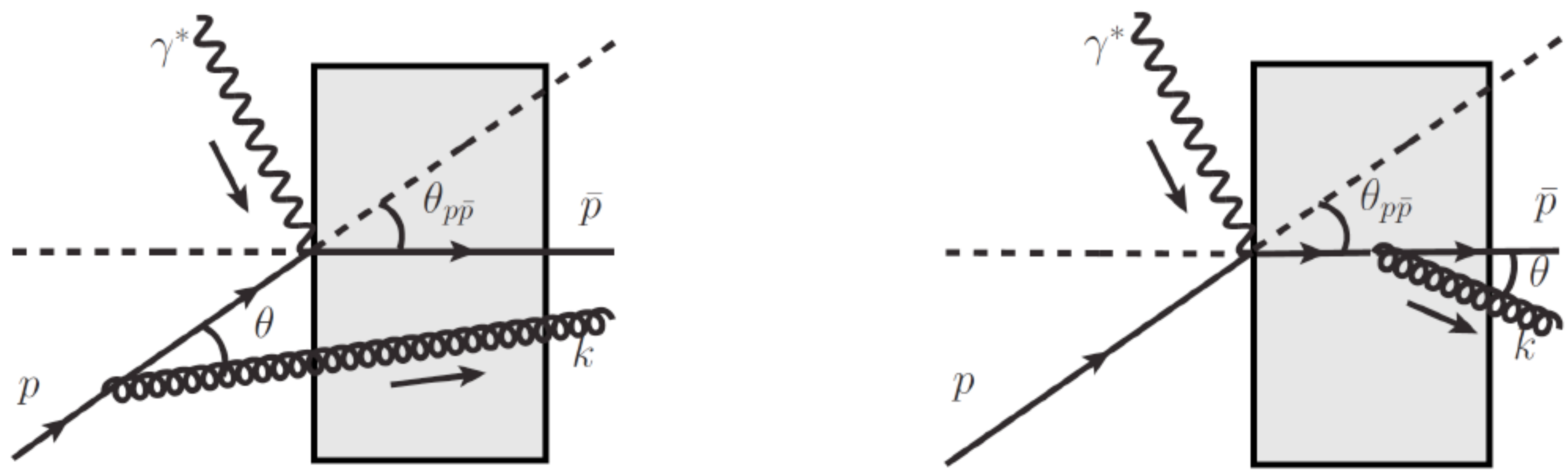
Motivation





- In vacuum, the color coherence effects are indicated by TASSO and OPAL experiments, i.e. depletion of particle energy spectrum in the soft part.
- Medium-induced soft gluon radiation off a single quark in the final state: BDMPS-Z-W spectrum

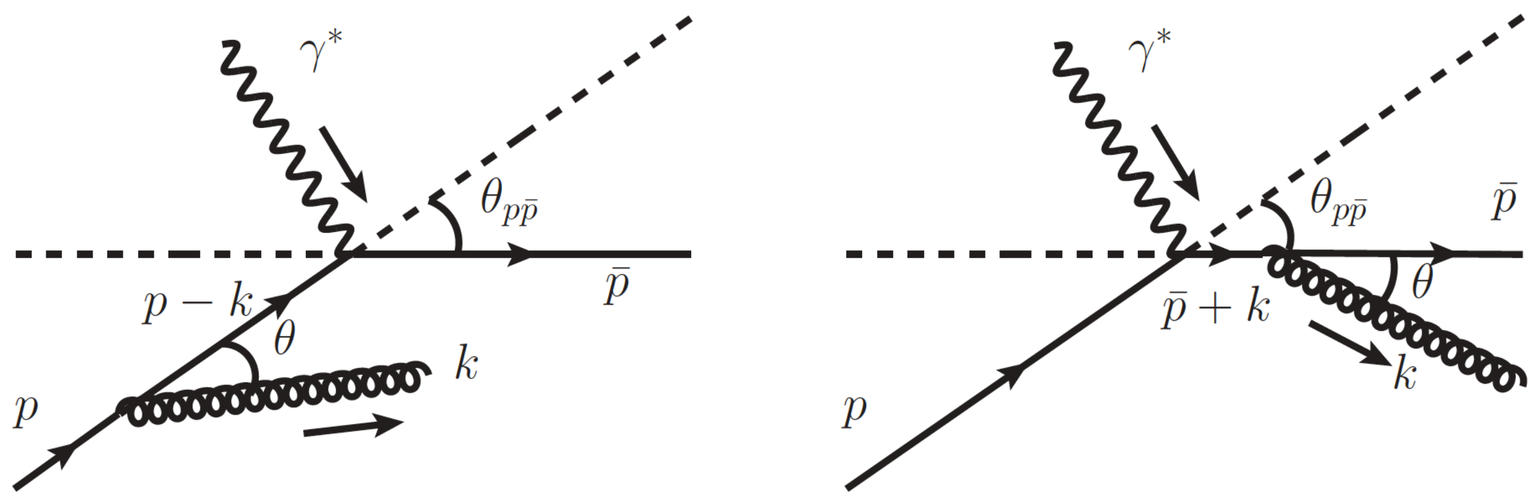
Motivation



- To check color coherence between initial and final state quarks in computing particle production in DIS on nucleus—a different setup complementary to the antenna in s-channel
- The process we study happens in eA collisions: LHeC and EIC.
- When $\theta_{p\bar{p}} = 0$, our calculation matches the one for gluon production in the totally coherent limit in the CGC framework:

Yuri V. Kovchegov and A. H. Mueller, Nucl. Phys. B529 (1998) 451.

Semiclassical method

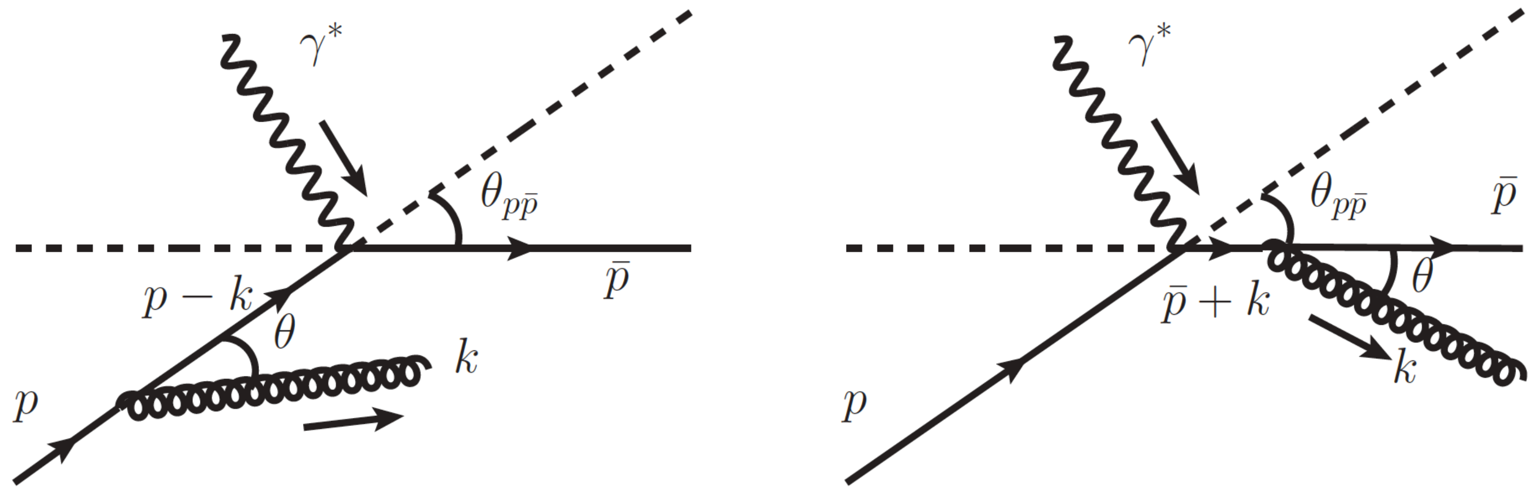


- Eikonal approximation: $p^+ \sim \bar{p}^+ \gg k^+ \gg |\mathbf{k}|$
- The light-cone gauge: $n = (0, 1, \mathbf{0}),\ n \cdot A^a = A_a^+ = 0$
- The incoming and outgoing quarks with $p^+ \sim \bar{p}^+ \gg k^+$ are described as color currents, which act as color sources for soft gluon field.
- The emitted soft gluon is described as a classical gauge field.
- The classical gauge field obeys the classical Yang-Mills equations:

$$(D_\mu F^{\mu\nu})_a = J_a^\nu$$

Current conservation: $(D \cdot J)_a = 0$

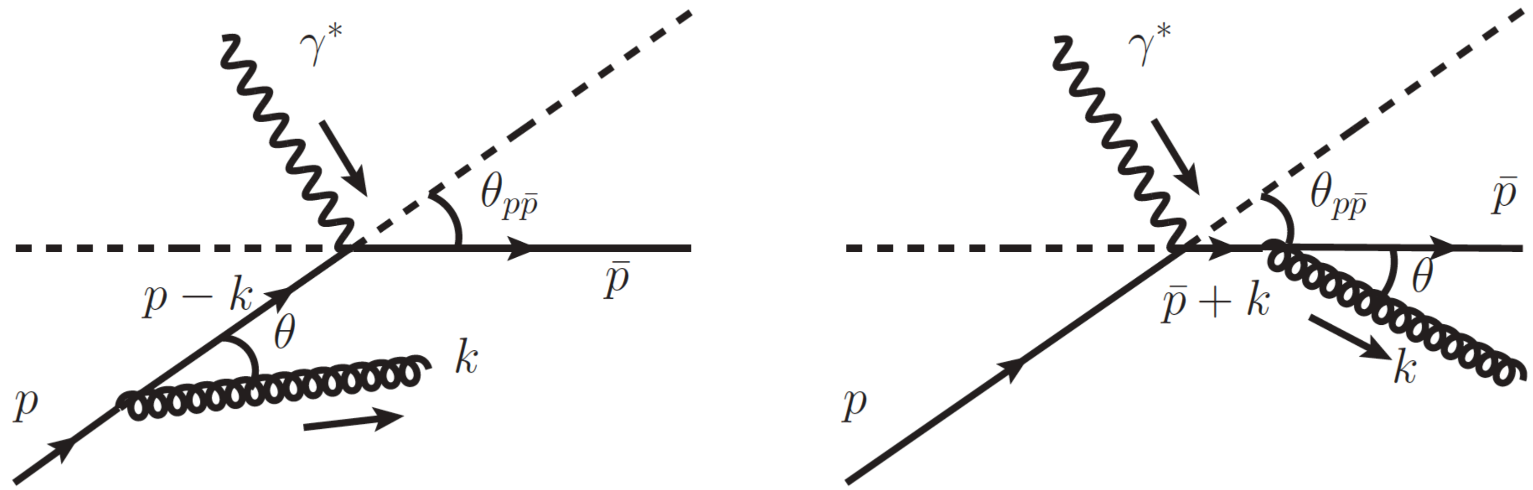
Antenna radiation in t-channel in vacuum



- Infinite momentum frame + small scattering angle: $\theta_{p\bar{p}} \ll 1$
- ɣ* is absorbed by an incoming quark with transverse size: $|\Delta\mathbf{x}| \sim \dfrac{1}{|\bar{\mathbf{p}}-\mathbf{p}|}$
- The cross section in vacuum:

$$\omega \frac{dN^{\mathrm{vac}}}{d^3\vec{k}} = \frac{\alpha_s C_F}{(2\pi)^2\,\omega^2}\,\left(\mathcal{R}_{in} + \mathcal{R}_{out} - 2\mathcal{J}\right)$$

Antenna radiation in t-channel in vacuum



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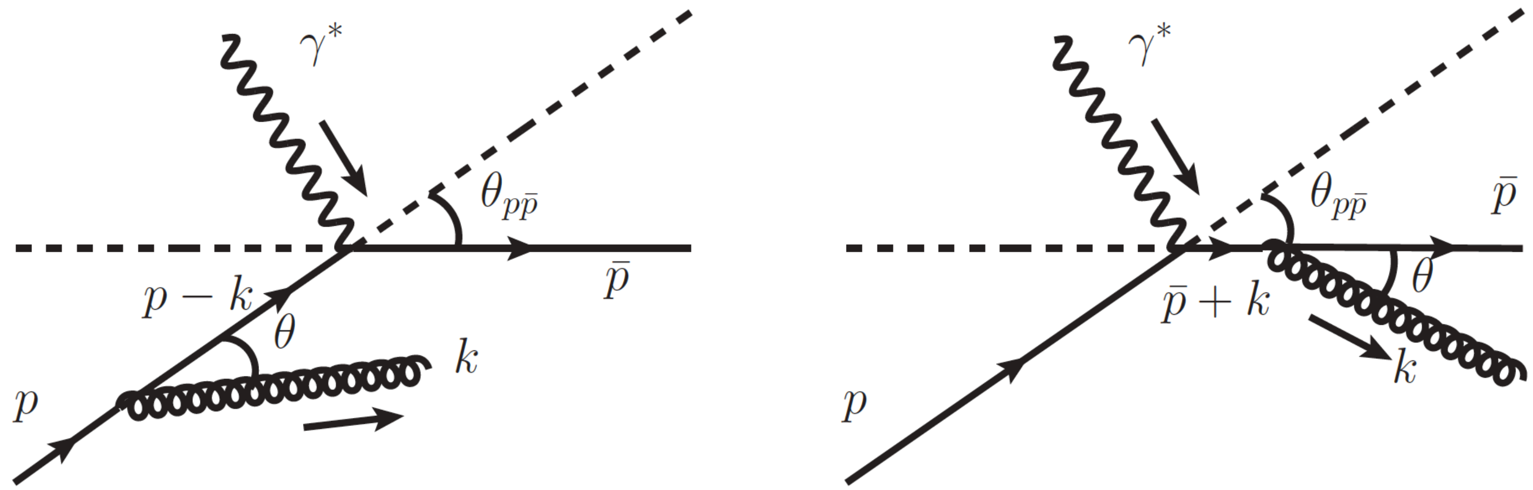
Independent radiation off incoming quark: $\mathcal{R}_{in} = 2\omega^2 \dfrac{1}{x(p\cdot k)}$

Antenna radiation in t-channel in vacuum



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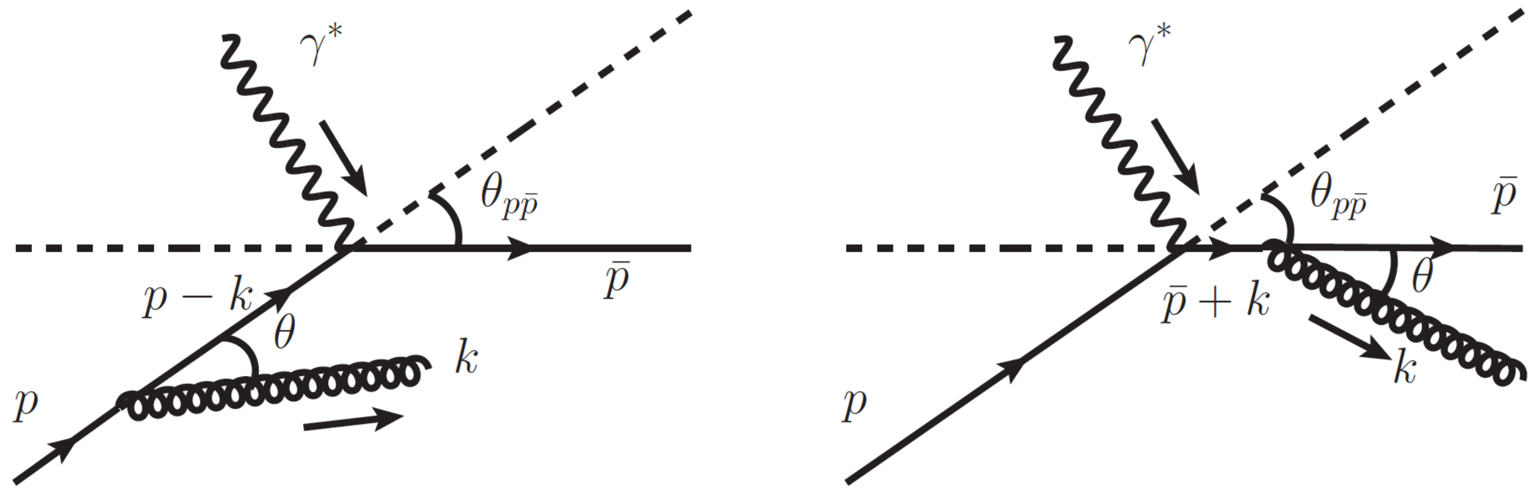
Independent radiation off outgoing quark: $\mathcal{R}_{out} = 2\omega^2 \dfrac{1}{\bar{x}(\bar{p}\cdot k)}$

Antenna radiation in t-channel in vacuum



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- The cross section in vacuum:

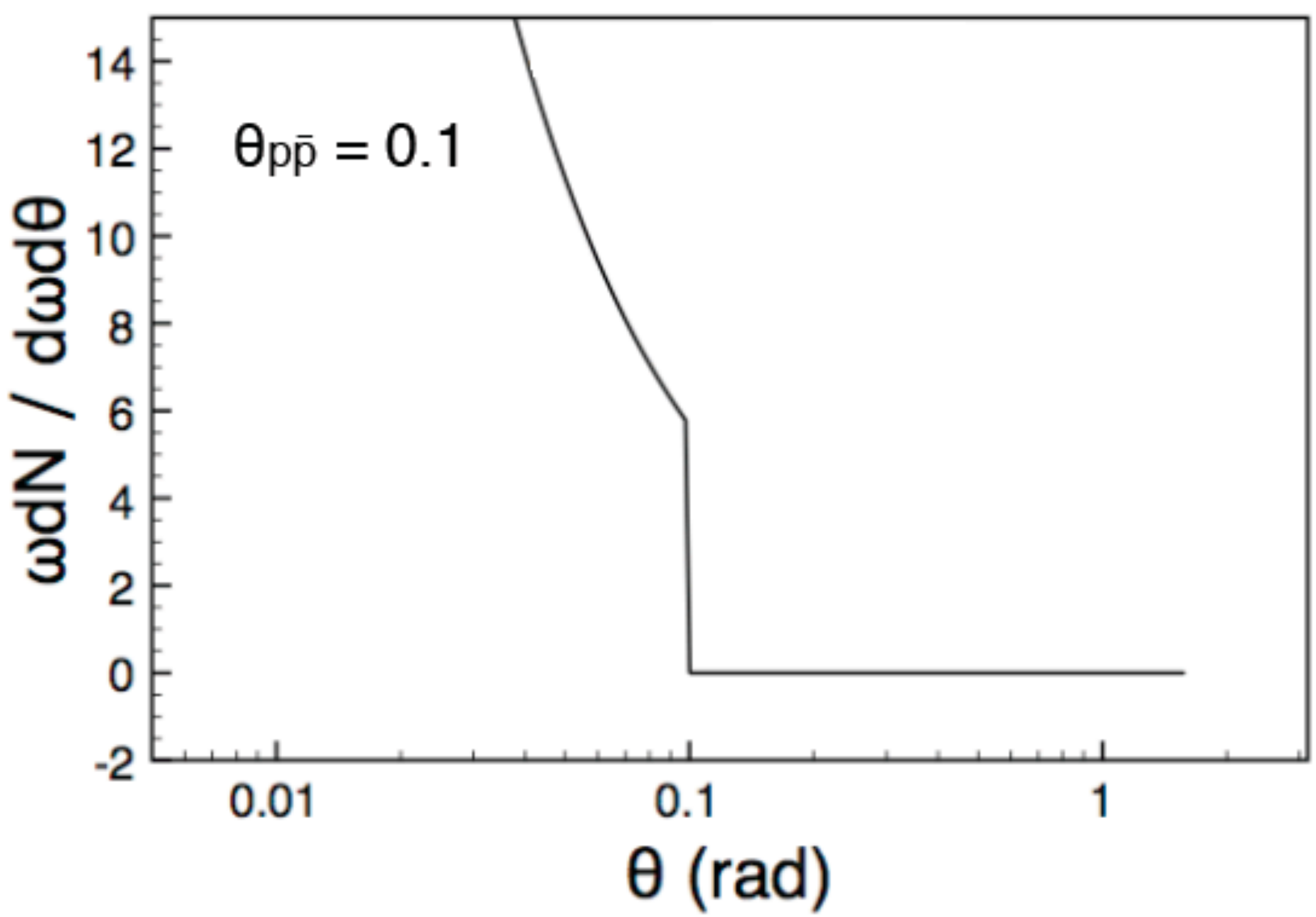
$$\omega \frac{dN^{\text{vac}}}{d^3\vec{k}} = \frac{\alpha_s C_F}{(2\pi)^2\, \omega^2} \left(\mathcal{R}_{in} + \mathcal{R}_{out} - 2\mathcal{J}\right)$$

Interference between two quarks: $\mathcal{J} = \omega^2 \frac{\boldsymbol{\kappa} \cdot \bar{\boldsymbol{\kappa}}}{x\bar{x}(p \cdot k)(\bar{p} \cdot k)}$

Notation:

$\boldsymbol{\kappa} = \boldsymbol{k} - x\,\boldsymbol{p}$

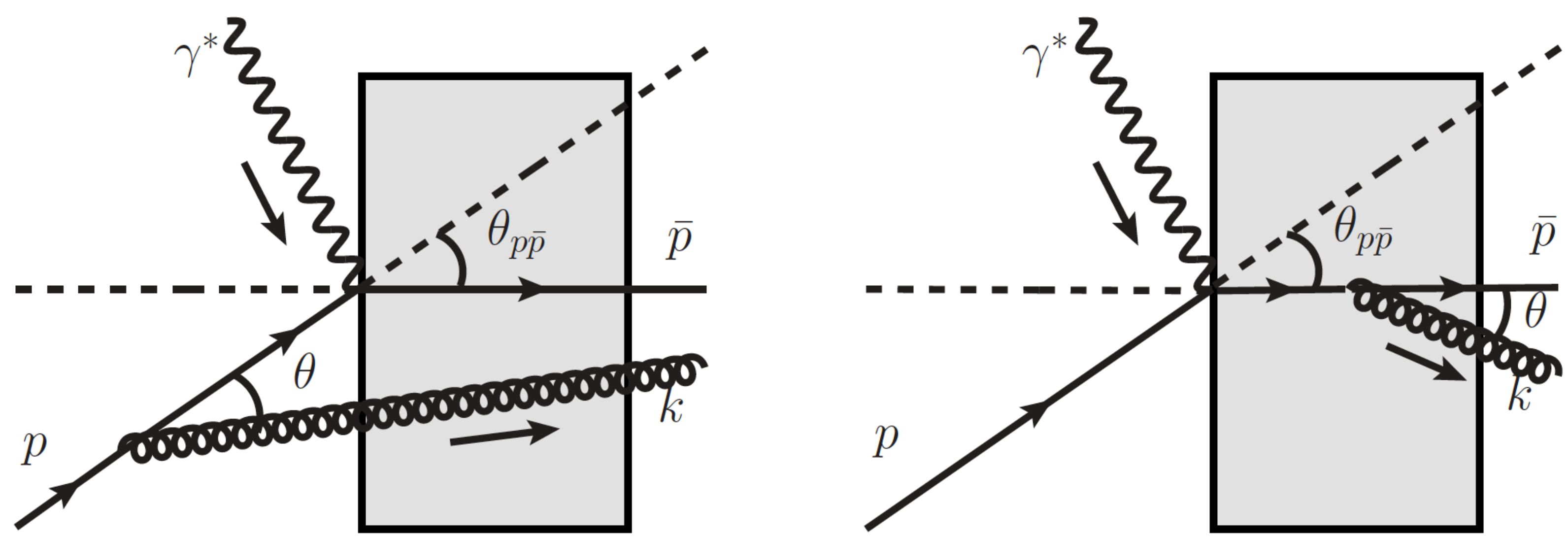
Angular constraint in t-channel in vacuum



- The soft gluon spectrum off outgoing quark in vacuum in which the azimuthal angle is integrated with respect to the direction of outgoing quark (analogously for the incoming quark):

$$dN_{out}^{\rm vac} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{\sin\theta\, d\theta}{1-\cos\theta}\, \Theta(\cos\theta - \cos\theta_{p\bar{p}})$$

Medium-induced antenna radiation in t-channel



- Infinite momentum frame + small scattering angle: $\theta_{p\bar{p}} \ll 1$
- Eikonal approximation + the light-cone gauge
- ɣ* is absorbed by an incoming quark with transverse size: $|\Delta \mathbf{x}| \sim \frac{1}{|\bar{\mathbf{p}} - \mathbf{p}|}$
- Description for the medium is model-independent.
- Dilute medium scenario: at 1st order in opacity expansion, i.e. $\frac{L}{\lambda}$
- Hadronization is considered to occur outside the medium.

Medium-induced antenna spectrum in t-channel

$$\omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{\alpha_s C_F\, \hat{q}}{\pi} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{V}^2(\boldsymbol{q}) \int_0^{+\infty} dx^+$$

$$\left[\frac{\boldsymbol{\nu}^2}{x^2\,(p\cdot v)^2} - \frac{\boldsymbol{\kappa}^2}{x^2\,(p\cdot k)^2} \right.$$

$$+ \frac{2}{\bar{x}^2} \left(\frac{\bar{\boldsymbol{\nu}}^2}{(\bar{p}\cdot v)^2} - \frac{\bar{\boldsymbol{\kappa}}\cdot\bar{\boldsymbol{\nu}}}{(\bar{p}\cdot v)(\bar{p}\cdot k)} \right) (1-\cos[\Omega_{\bar{p}} x^+])$$

$$+ \frac{2}{x\,\bar{x}} \left(\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot k)} - \frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot v)} \right.$$

$$\left.\left. + \left(\frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot v)} - \frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\nu}}}{(\bar{p}\cdot v)\,(p\cdot v)} \right) (1-\cos[\Omega_{\bar{p}} x^+]) \right) \right]$$

Notation: $\Omega_{\bar{p}} = \frac{(\mathbf{k}-\mathbf{q})^2}{2\,k^+} \approx 1/t_{form}$

Medium-induced antenna spectrum in t-channel

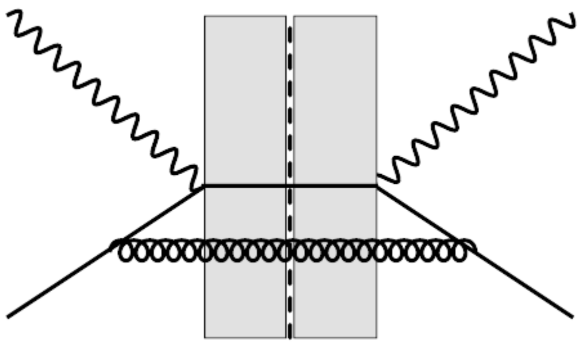
$$\omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{\alpha_s C_F\,\hat{q}}{\pi} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{V}^2(\boldsymbol{q}) \int_0^{+\infty} dx^+$$

$$\Bigg[\frac{\boldsymbol{\nu}^2}{x^2\,(p\cdot v)^2} - \frac{\boldsymbol{\kappa}^2}{x^2\,(p\cdot k)^2}$$

$$+\frac{2}{\bar{x}^2}\left(\frac{\bar{\boldsymbol{\nu}}^2}{(\bar{p}\cdot v)^2} - \frac{\bar{\boldsymbol{\kappa}}\cdot\bar{\boldsymbol{\nu}}}{(\bar{p}\cdot v)(\bar{p}\cdot k)}\right)(1-\cos[\Omega_{\bar{p}}x^+])$$

$$+\frac{2}{x\,\bar{x}}\left(\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot k)} - \frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot v)}\right.$$

$$\left.+\left(\frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot v)} - \frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\nu}}}{(\bar{p}\cdot v)\,(p\cdot v)}\right)(1-\cos[\Omega_{\bar{p}}x^+])\right)\Bigg]$$



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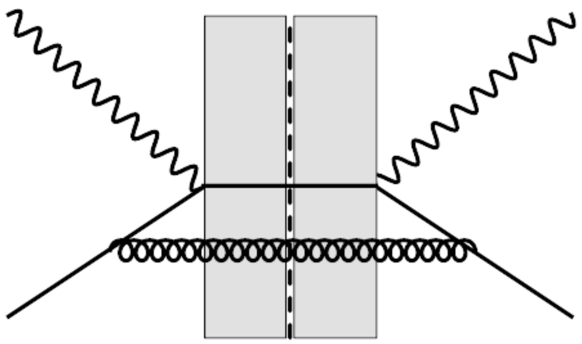
Medium-induced antenna spectrum in t-channel



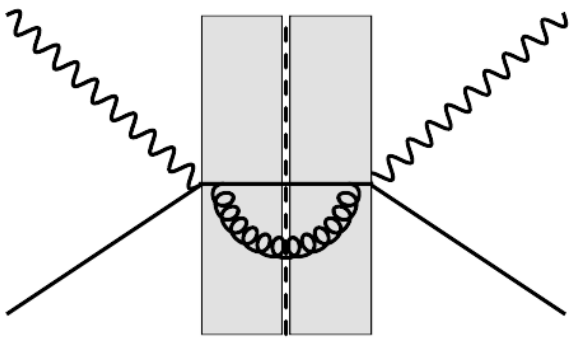
$$\omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{\alpha_s C_F\, \hat{q}}{\pi} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{V}^2(\boldsymbol{q}) \int_0^{+\infty} dx^+$$

$$\left[\frac{\boldsymbol{\nu}^2}{x^2\,(p\cdot v)^2} - \frac{\boldsymbol{\kappa}^2}{x^2\,(p\cdot k)^2} \right.$$

$$+ \frac{2}{\bar{x}^2} \left(\frac{\bar{\boldsymbol{\nu}}^2}{(\bar{p}\cdot v)^2} - \frac{\bar{\boldsymbol{\kappa}}\cdot\bar{\boldsymbol{\nu}}}{(\bar{p}\cdot v)(\bar{p}\cdot k)} \right) \left(1 - \cos[\Omega_{\bar{p}} x^+]\right)$$

$$+ \frac{2}{x\,\bar{x}} \left(\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot k)} - \frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot v)} \right.$$

$$\left.\left. + \left(\frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot v)} - \frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\nu}}}{(\bar{p}\cdot v)\,(p\cdot v)} \right) \left(1 - \cos[\Omega_{\bar{p}} x^+]\right) \right) \right]$$



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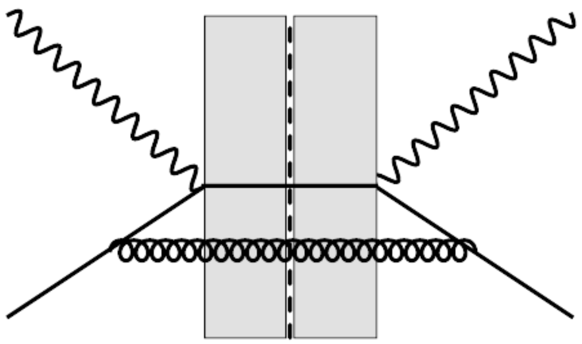
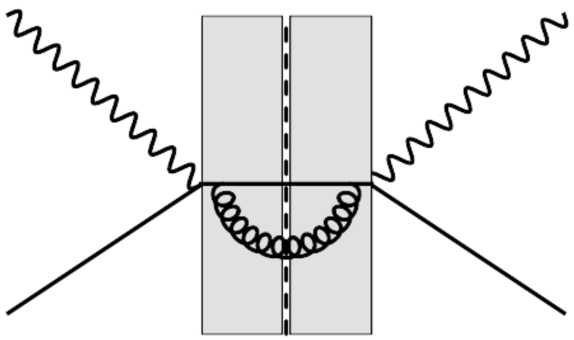
Medium-induced antenna spectrum in t-channel

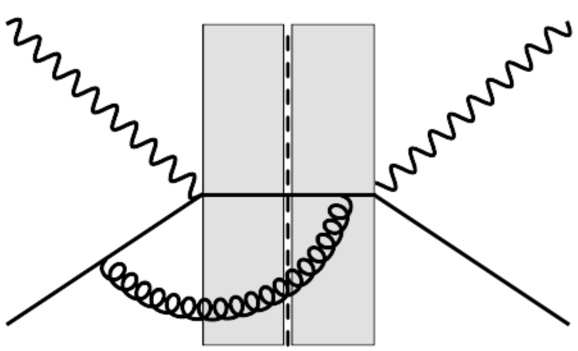


Ini.

$$\omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{\alpha_s C_F\, \hat{q}}{\pi} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{V}^2(\boldsymbol{q}) \int_0^{+\infty} dx^+ \left[\frac{\boldsymbol{\nu}^2}{x^2\,(p\cdot v)^2} - \frac{\boldsymbol{\kappa}^2}{x^2\,(p\cdot k)^2} + \frac{2}{\bar{x}^2}\left(\frac{\bar{\boldsymbol{\nu}}^2}{(\bar{p}\cdot v)^2} - \frac{\bar{\boldsymbol{\kappa}}\cdot\bar{\boldsymbol{\nu}}}{(\bar{p}\cdot v)(\bar{p}\cdot k)} \right)(1-\cos[\Omega_{\bar{p}} x^+]) + \frac{2}{x\,\bar{x}} \left(\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot k)} - \frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot v)} + \left(\frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\kappa}}}{(\bar{p}\cdot k)\,(p\cdot v)} - \frac{\boldsymbol{\nu}\cdot\bar{\boldsymbol{\nu}}}{(\bar{p}\cdot v)\,(p\cdot v)} \right)(1-\cos[\Omega_{\bar{p}} x^+]) \right) \right]$$



Med.



Int.

Notation: $\Omega_{\bar{p}} = \dfrac{(\mathbf{k}-\mathbf{q})^2}{2\,k^+} \approx 1/t_{form}$

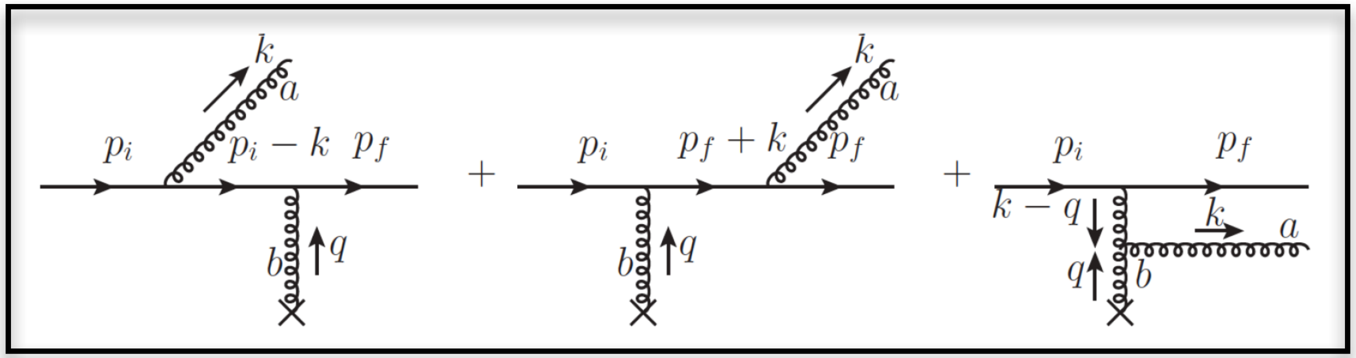
Medium-induced antenna spectrum in t-channel

- When $\theta_{p\bar{p}} = 0$, the medium-induced antenna spectrum in t-channel is:

$$\omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{4\,\alpha_s C_F\,\hat{q}\,L^+}{\pi} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \mathcal{V}^2(\mathbf{q})\,\boldsymbol{L}^2$$

- The transverse component of Lipatov vertex in the light-cone gauge:

$$\boldsymbol{L} = \frac{\boldsymbol{\kappa} - \boldsymbol{q}}{(\boldsymbol{\kappa} - \boldsymbol{q})^2} - \frac{\boldsymbol{\kappa}}{\boldsymbol{\kappa}^2}$$



1. Medium-induced radiation off an on-shell quark
2. The Lipatov vertex is gauge invariant.

- Multiple soft scattering approach:

Yuri V. Kovchegov and A. H. Mueller, Nucl. Phys. B529 (1998) 451.

Soft gluon emission limit

- The total spectrum in the soft gluon emission limit:

$$\omega \frac{dN^{\text{vac}}}{d^3\vec{k}} + \omega \frac{dN^{\text{med}}}{d^3\vec{k}}$$
$$= \frac{4\,\alpha_s\,C_F}{(2\,\pi)^2}\left[(1-\Delta)\left(\frac{1}{\boldsymbol{\kappa}^2} - \frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\,\bar{\boldsymbol{\kappa}}^2}\right) + \frac{1}{\bar{\boldsymbol{\kappa}}^2} - (1-\Delta)\,\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\,\bar{\boldsymbol{\kappa}}^2}\right]$$

Medium parameter at 1st order in opacity: $\Delta = \frac{\hat{q}\,L^+}{m_D^2}$

Notation: $\hat{q} = \alpha_s C_A n_0 m_D^2$

Soft gluon emission limit

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$$\omega \frac{dN^{\rm vac}}{d^3\vec{k}} + \omega \frac{dN^{\rm med}}{d^3\vec{k}}$$
$$= \frac{4\,\alpha_s\,C_F}{(2\,\pi)^2}\left[(1-\Delta)\left(\frac{1}{\boldsymbol{\kappa}^2} - \frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\,\bar{\boldsymbol{\kappa}}^2}\right) + \frac{1}{\bar{\boldsymbol{\kappa}}^2} - (1-\Delta)\,\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\,\bar{\boldsymbol{\kappa}}^2}\right]$$

Angular constraint for the reduced number of soft gluon emission off incoming quark if one performs the azimuthal angle integration

Medium parameter at 1st order in opacity: $\Delta = \frac{\hat{q}\,L^+}{m_D^2}$

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Soft gluon emission limit

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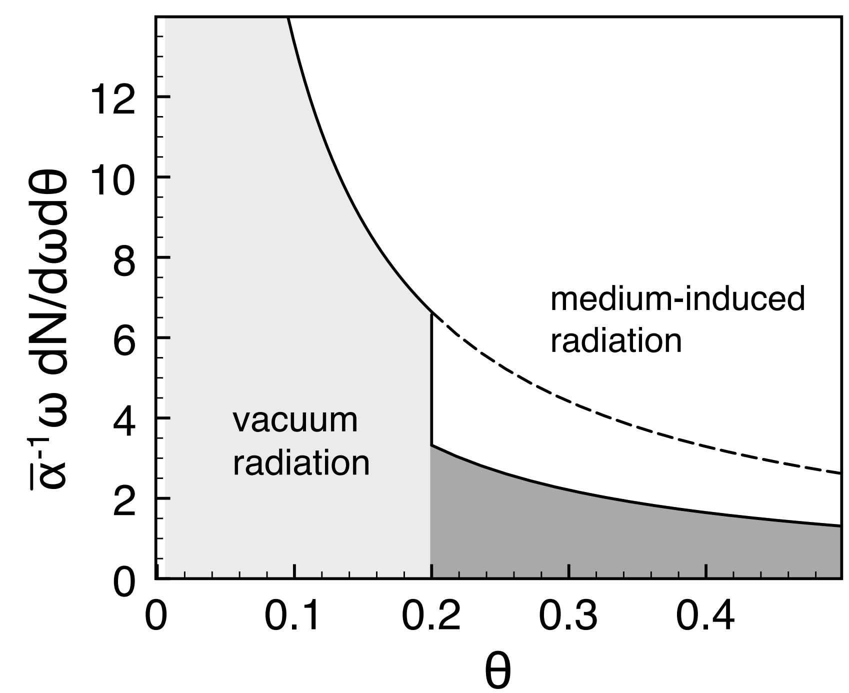
Soft gluon emission off outgoing quark: vacuum + medium-induced

Medium parameter at 1st order in opacity: $\Delta = \frac{\hat{q}\,L^+}{m_D^2}$

Notation: $\hat{q} = \alpha_s C_A n_0 m_D^2$

Notation: $\bar{\alpha} \equiv \alpha_s C_F/\pi$



Soft gluon emission limit

- The total spectrum in the soft gluon emission limit:

$$\omega\frac{dN^{\rm vac}}{d^3\vec{k}}+\omega\frac{dN^{\rm med}}{d^3\vec{k}}$$
$$=\frac{4\,\alpha_s\,C_F}{(2\,\pi)^2}\left[(1-\Delta)\left(\frac{1}{\boldsymbol{\kappa}^2}-\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\,\bar{\boldsymbol{\kappa}}^2}\right)+\frac{1}{\bar{\boldsymbol{\kappa}}^2}-(1-\Delta)\,\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\,\bar{\boldsymbol{\kappa}}^2}\right]$$

- When the medium is switched off: $\Delta\to 0$

  one naturally gets the vacuum contribution only: $\omega\frac{dN^{\rm vac}}{d^3\vec{k}}$

Soft gluon emission limit

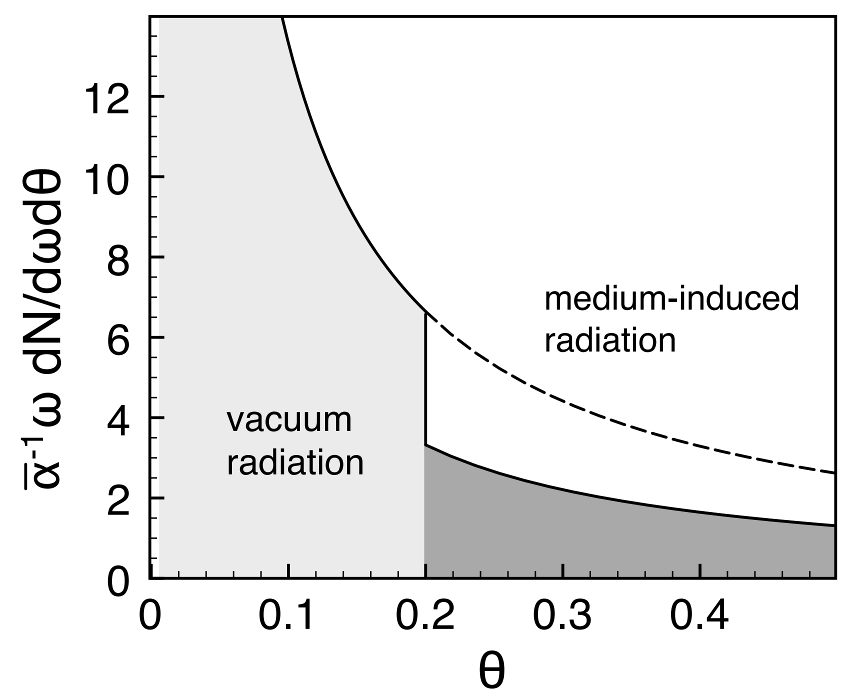
- The total spectrum in the soft gluon emission limit:

$$\omega \frac{dN^{\rm vac}}{d^3\vec{k}} + \omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{4\,\alpha_s\,C_F}{(2\,\pi)^2}\left[(1-\Delta)\left(\frac{1}{\boldsymbol{\kappa}^2} - \frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\,\bar{\boldsymbol{\kappa}}^2}\right) + \frac{1}{\bar{\boldsymbol{\kappa}}^2} - (1-\Delta)\,\frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2\,\bar{\boldsymbol{\kappa}}^2}\right]$$

- In the opaque medium limit: $\Delta \to 1$

1. The soft part of the medium-induced gluon energy spectrum is suppressed.
2. Complete decoherence for the outgoing quark:

$$\omega \frac{dN^{\rm vac}}{d^3\vec{k}} + \omega \frac{dN^{\rm med}}{d^3\vec{k}} = \frac{4\,\alpha_s\,C_F}{(2\,\pi)^2}\,\frac{1}{\bar{\boldsymbol{\kappa}}^2}$$



Notation: $\bar{\alpha} \equiv \alpha_s C_F/\pi$

Conclusions

- We setup the medium-induced antenna radiation in t-channel at 1st order in opacity expansion to study the coherence effects between two quark emitters.
- Angular constraint is embedded in the antenna spectrum in t-channel.
- The transition from coherence to decoherence is studied.
- The antenna in t-channel parallels the efforts done in the antenna in s-channel:

  Y. Mehtar-Tani, C. A. Salgado and K. Tywoniuk, Phys. Rev. Lett. 106 (2011) 122002.

  J. Casalderrey-Solana and E. Iancu, JHEP 1108 (2011) 015.

  N. Armesto, H. Ma, Y. Mehtar-Tani, C. A. Salgado and K. Tywoniuk, JHEP 1201 (2012) 109.

  Y. Mehtar-Tani, C. A. Salgado and K. Tywoniuk, Phys. Lett. B707 (2012) 156.

- Prospect: antenna spectrum in t-channel in multiple soft scattering approach.