Diffractive pQCD mechanisms of exclusive production of $b\bar{b}$ dijets and $W^+W^-$ pairs in proton-proton collisions

Antoni Szczurek

Institute of Nuclear Physics (PAN), Kraków, Poland and
Rzeszów University, Rzeszów, Poland

Deep-inelastic scattering 2012
Bonn, March 26 - 30, 2012
The $pp \rightarrow ppW^+W^-$ reaction was studied recently assuming two-photon fusion (Royon et al.)

**triple** $\gamma WW$ and **quartic** $\gamma\gamma W^+W^-$ couplings in the Standard Model and beyond is interesting and fundamental problem

**Exclusive reaction:** $pp \rightarrow pXp$

($X = H, Z, \eta', \eta_c, \eta_b, \chi_c, \chi_b, jj, c\bar{c}, b\bar{b}$).

At high energy - **one of many open channels (!)**

$\Rightarrow$ **rapidity gaps.**
The QCD mechanism for exclusive Higgs production

3-body process
KMR: on-shell matrix element
Pasechnik-Szczurek-Teryaev: off-shell matrix element
Maciula, Pasechnik, Szczurek,
The QCD mechanism for exclusive $q\bar{q}$ production

$q\bar{q} = b\bar{b}$: background to exclusive Higgs production 4-body process with exact matrix element (without $J_z = 0$ selection rule) with exact kinematics in the full phase space
$M_{bb}$ spectrum, cuts

$p\ p \rightarrow p\ p\ b\ b\ \sqrt{s} = 14$ TeV

$|\eta| \leq 1.0$

$E_{\text{D}}\ D$ with CTEQ6 NLO

$E_{\text{D}}\ D \rightarrow \gamma\ \gamma\ \text{CEP}$

$Higgs$

$M_{bb}$ (GeV)

$|\eta| \leq 2.5$, $|\eta_{b} - \eta_{\bar{b}}| \leq 2.0$

$|\eta| \leq 2.5$

$p_{T} > 40$ GeV

$E_{\text{D}}\ D$ with CTEQ6 NLO

$E_{\text{D}}\ D \rightarrow \gamma\ \gamma\ \text{CEP}$

$Higgs$

$M_{bb}$ (GeV)

$p_{T} > 40$ GeV

$|p_{T} | > 0.2$ GeV

$E_{\text{D}}\ D$ with CTEQ6 NLO

$E_{\text{D}}\ D \rightarrow \gamma\ \gamma\ \text{CEP}$ completely removed

$Higgs$

$M_{bb}$ (GeV)
The QCD mechanism for exclusive $W^+W^-$ production

Similar in spirit to exclusive production of $H, q\bar{q}, gg$. Lebiedowicz, Pasechnik, Szczurek, arXiv:1203.1832
Kinematics

Sudakov decomposition:

\[ q_1 = x_1 p_1 + q_1 \perp, \quad q_2 = x_2 p_2 + q_2 \perp, \quad 0 < x_1, 2 < 1, \]
\[ q_0 = x' p_1 - x' p_2 + q_0 \perp \simeq q_0 \perp, \quad x' \ll x_1, 2, \]

(1)

where \( x_1, 2, x' \) are the longitudinal momentum fractions for active and screening gluons

\[ q_\perp^2 \simeq -|q|^2. \]

In the forward scattering limit:

\[ t_{1,2} = (p_{1,2} - p'_{1,2})^2 \simeq p'_{1,2}^2 \to 0, \]
\[ q_0 \perp \simeq -q_1 \perp \simeq q_2 \perp. \]

(2)
It is convenient to introduce the Sudakov expansions for $W^\pm$ boson momenta

\begin{align}
  k_+ &= x_1^+ p_1 + x_2^+ p_2 + k_+\perp, \\
  k_- &= x_1^- p_1 + x_2^- p_2 + k_-\perp
\end{align}

leading to

\begin{align}
  x_{1,2} &= x_{1,2}^+ + x_{1,2}^-, \\
  x_{1,2}^+ &= \frac{m_{+\perp}}{\sqrt{s}} e^{+y_+}, \\
  x_{1,2}^- &= \frac{m_{-\perp}}{\sqrt{s}} e^{-y_-},
\end{align}

\begin{align}
  m_{\perp\perp}^2 &= m_W^2 + |k_{\perp\perp}|^2, \\
  \text{In the forward limit:} &amp; \\
  k_{+\perp} &= -k_{-\perp}.
\end{align}

\begin{align}
  \sigma_{l+\nu l^-\nu} &\approx \sigma_{WW} \times BR(W^+ \to l^+\nu) \times BR(W^- \to l^-\nu),
\end{align}
Diffractive amplitude for $pp \to ppW^+W^-$

$$M_{\lambda_+\lambda_-}(s, t_1, t_2) \simeq is\frac{\pi^2}{2} \int d^2q_0 V_{\lambda_+\lambda_-}(q_1, q_2, k_+, k_-)$$

$$\frac{f_g(q_0, q_1; t_1)f_g(q_0, q_2; t_2)}{q_0^2 q_1^2 q_2^2}$$

where $\lambda_+, \lambda_- = \pm 1, 0$ are the polarisation states of the produced $W^\pm$ bosons. $f_g(r_1, r_2; t)$ is the off-diagonal unintegrated gluon distribution function (UGDF), which depends on the longitudinal and transverse components of both gluon momenta $r_1$ and $r_2$ emitted from the proton lines.
Diffractive amplitude for $pp \rightarrow ppW^+W^-$

The gauge-invariant $gg \rightarrow W^+_{\lambda_+} W^-_{\lambda_-}$ hard subprocess amplitude $V_{\lambda_+\lambda_-}(q_1, q_2, k_+, k_-)$ is given by

$$V_{\lambda_+\lambda_-} = n_+^\mu n_-^\nu V_{\lambda_+\lambda_-}^{\mu\nu} = \frac{4}{s} \frac{q_1^{\nu} q_2^\mu}{x_1 x_2} V_{\lambda_+\lambda_-,\mu\nu},$$

$$q_1^{\nu} V_{\lambda_+\lambda_-,\mu\nu} = q_2^\mu V_{\lambda_+\lambda_-,\mu\nu} = 0,$$  \hspace{1cm} (7)

where $n_{\mu}^{\pm} = p_{1,2}^{\mu}/E_{p,\text{cms}}, E_{p,\text{cms}} = \sqrt{s}/2$.

$$V_{\lambda_+\lambda_-}^{\mu\nu}(q_1, q_2, k_+, k_-) = \epsilon^*_{\rho}(k_+, \lambda_+) \epsilon^*_{\sigma}(k_-, \lambda_-) V_{\rho\sigma}^{\mu\nu},$$ \hspace{1cm} (8)

$V_{\rho\sigma}^{\mu\nu}$ calculated with automatic programs.
The amplitude for $pp \rightarrow ppQ\bar{Q}$

$\epsilon^*_\mu(k_+, \lambda_+)$ and $\epsilon^*_\nu(k_-, \lambda_-)$ can be defined easily in the proton-proton center-of-mass frame with $z$-axis along the proton beam as

$$\epsilon(k, 0) = \frac{E_W}{m_W} \left( \frac{k}{E_W}, \cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta \right),$$

$$\epsilon(k, \pm 1) = \frac{1}{\sqrt{2}} (0, i \sin \phi \mp \cos \theta \cos \phi, -i \cos \phi \mp \cos \theta \sin \phi, \pm \sin \theta).$$

such that $\epsilon^\mu(\lambda)\epsilon^*_\mu(\lambda) = -1$ and

$\epsilon^*_\mu(k_+, \lambda_+)k^\mu_+ = \epsilon^*_\nu(k_-, \lambda_-)k^\nu_- = 0$.

In the forward limit $\phi_- = \phi_+ + \pi$. 

The amplitude for $pp \rightarrow ppQ\bar{Q}$

The diffractive amplitude is averaged over the color indices and over the two transverse polarizations of the incoming gluons. The relevant color factor which includes summing over colors of quarks in the loop (triangle or box) and averaging over gluon colors according to the definition of unintegrated gluon distribution function is

$$\frac{1}{N_c^2 - 1} \times \sum_{ij} t^a_{ij} t^a_{ji}.$$  \hspace{1cm} (9)

The matrix element $V_{\lambda_+,\lambda_-}$ contains twice the strong coupling constant $g_s^2 = 4\pi\alpha_s$. We take the running coupling constant $\alpha_s(M_{WW}^2)$.
Mechanisms of exclusive diffractive production

Figure: Representative mechanisms of the hard subprocess \( gg \rightarrow W^\pm W^{\mp} \).
Intermediate Higgs contribution

\[ V_{gg \rightarrow h^0 \rightarrow W^+ W^-}(q_1, q_2, k_+, k_-) = \delta^{(4)}(q_1 + q_2 - k_+ - k_-) \times \]

\[ V_{gg \rightarrow h^0}(q_1, q_2, p_{h^0}) \frac{i}{M_{WW}^2 - m_{h^0}^2 + i M_{WW} \Gamma_{h}^{\text{tot}}} V_{h^0 \rightarrow W^+ W^-}(k_+, k_-, \lambda_+, \lambda_-), \]

\[ V_{gg \rightarrow h^0} \simeq \frac{i \delta^{ab}}{v} \frac{\alpha_s(\mu_F^2)}{\pi} (q_{1 \perp} \cdot q_{2 \perp}) \frac{2}{3} \left( 1 + \frac{7}{120} \frac{M_{WW}^2}{m_{\text{top}}^2} \right), \quad v = \left( G_F \sqrt{2} \right)^{-1/2}. \]

The second tree-level \( H^0 \rightarrow W^+ W^- \) “decay” amplitude reads:

\[ V_{H^0 \rightarrow W^+ W^-} \simeq i m_W \frac{e}{\sin \theta_W} \epsilon^*(k_+, \lambda_+) \cdot \epsilon^*(k_-, \lambda_-), \quad (11) \]
Gluon $k_{\perp}$-dependent densities in the forward limit

\[ f_g(q_0, q_{1,2}; t_{1,2}) = R_g f_g(x_{1,2}, q^2, \mu_F^2) \exp(bt_{1,2}/2) = \]

\[ R_g \frac{\partial}{\partial \ln q^2} \left[ xg(x_{1,2}, q^2) \sqrt{T_g(q^2, \mu_F^2)} \right] \exp(bt_{1,2}/2), \]

where the diffractive slope $b = 4$ GeV$^{-2}$

$T_g$ is the Sudakov form factor

\[ T_g(q^2, \mu_F^2) = \exp\left(- \int_{q^2}^{\mu_F^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^{1-\Delta} \left[ zP_{gg}(z) + \sum_q P_{qg}(z) \right] dz \right) \]

where

\[ \Delta = \frac{|k|}{|k| + M_{WW}}. \quad (12) \]

We take $\mu_F^2 = M_{WW}^2$.

$T_g(q^2, \mu_F^2)$ for extremely large scales $\mu_F^2$ is needed

the integration is performed in $\log_{10}(k^2/k_0^2)$. 
Phase space in the forward limit

\[ \sigma = \int \frac{1}{2s} |M|^2 (2\pi)^4 \delta^4 (p_1 + p_2 - p'_1 - p'_2 - p_+ - p_-) \]

\[ \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \frac{d^3 p'_2}{(2\pi)^3 2E'_2} \frac{d^3 p_+}{(2\pi)^3 2E_+} \frac{d^3 p_-}{(2\pi)^3 2E_-}, \]

where \( |M|^2 = \sum_{\lambda_+ \pm \lambda_-} M(\lambda_+, \lambda_-) M^*(\lambda_+, \lambda_-). \)

The evaluation of the corresponding hard subprocess amplitude \( V_{\lambda_+ \lambda_-} \) and convolution with the gluon uPDFs in the diffractive amplitude and full phase space integration is extremely time consuming.

**Simplification:**

\[ d\sigma \equiv \frac{1}{2s} |M|^2 d^4PS \]

\[ = \frac{1}{2s} |M|^2 \frac{1}{2^4 (2\pi)^8} \frac{1}{E'_1 E'_2} \frac{1}{4} dt_1 dt_2 d\phi_1 d\phi_2 \frac{p_{m\perp}}{4} \mathcal{J}^{-1} dy_+ dy_- dp_{m\perp} d\phi_m, \]

where \( p_{m\perp} = |p_{+\perp} - p_{-\perp}| \), \( \phi_m \) is the corresponding azimuthal angle.
Phase space in the forward limit

For the sake of simplicity, assuming exponential slope of $t$-dependence of the KMR UGDFs and in the consequence approximately exponential dependence of the cross section on $t_1$ and $t_2$ ($\exp(bt_1)$ and/or $\exp(bt_2)$)

$$d\sigma \approx \frac{1}{2s} |\mathcal{M}|^2 \bigg|_{t_1, t_2=0} \frac{1}{2^4} \frac{1}{(2\pi)^8} \frac{1}{E_1'E_2'} \frac{1}{4} \frac{1}{b^2} (2\pi)^2 \frac{p_{m\perp}}{4} \mathcal{J}^{-1} dy_+ dy_- dp_{m\perp} d\phi_m.$$  

In this approximation – no correlations between outgoing protons on $\phi_m$ – the phase space integration can be further reduced to three dimensions.

The Jacobian $\mathcal{J}$ is given by

$$J = \left| \frac{p'_{1z}}{\sqrt{m_p^2 + p'^2_{1z}}} - \frac{p'_{2z}}{\sqrt{m_p^2 + p'^2_{2z}}} \right|.$$  

(13)
γγ → W⁺W⁻ process

pp → ppW⁺W⁻ calculated already by Royon et al.

The Standard Model couplings:

\[ \mathcal{L}_{WWγ} = -ie(A_μW_ν^- \bar{\partial}^μ W^{+ν} + W_μ^- W_ν^+ \bar{\partial}^μ A^ν + W_μ^+ A_ν \bar{\partial}^μ W^{-ν}) \]

\[ \mathcal{L}_{WWγγ} = -e^2(W_μ^- W^{+μ} A_ν A^ν - W_μ^- A^μ W_ν^+ A^ν) \]  

where the asymmetric derivative has the form

\[ X \bar{\partial}^μ Y = X\partial^μ Y - Y\partial^μ X. \]

**Figure:** The Born diagrams for the γγ → W±W∓ subprocess.
\( \gamma \gamma \rightarrow W^+ W^- \) mechanism

\[
\frac{d\sigma}{d\Omega} = \frac{3\alpha^2 \beta}{2s} \left( 1 - \frac{2s(2s + 3m_W^2)}{3(m_W^2 - t)(m_W^2 - u)} + \frac{2s^2(s^2 + 3m_W^4)}{3(m_W^2 - t)^2(m_W^2 - u)^2} \right), \tag{15}
\]

where \( \beta = \sqrt{1 - 4m_W^2/s} \) is the velocity of the \( W \) bosons in their center-of-mass frame.

In the EPA approximation the total cross section for the \( pp \rightarrow pp(\gamma \gamma) \rightarrow W^+ W^- \) can be written as in parton model

\[
\sigma = \int dx_1 dx_2 \; f_1^{WW}(x_1) \; f_2^{WW}(x_2) \; \sigma_{\gamma \gamma \rightarrow W^+ W^-}(\hat{s}). \tag{16}
\]

We take Weizsäcker-Williams equivalent photon fluxes of protons from Drees and Zeppenfeld.

\[
x_1 = \frac{m_\perp}{\sqrt{s}} (\exp(y_+) + \exp(y_-)) ,
\]

\[
x_2 = \frac{m_\perp}{\sqrt{s}} (\exp(-y_+) + \exp(-y_-)) . \tag{17}
\]
$\gamma \gamma \rightarrow W^+ W^-$ mechanism

**Figure:** The lines were calculated within EPA approximation with Drees and Zeppenfeld photon fluxes. $\xi_{1,2} = \log_{10}(x_{1,2})$ where $x_{1,2}$ are photon longitudinal fractions with respect to parent protons.
Inclusive production of $W^+W^-$ pairs

\[
\frac{d\sigma}{dy_+ dy_- d^2 p_{W\perp}} = \frac{1}{16\pi^2 s^2} x_1 g(x_1, \mu_F) x_2 g(x_2, \mu_F) |M_{gg\rightarrow W^+W^-}(\lambda_1, \lambda_2, \lambda_+ , \lambda_-)|^2
\]

The distributions in rapidity of $W^+ (y_+)$, rapidity of $W^- (y_-)$ and transverse momentum of one of them $p_{W\perp}$ can be calculated. The distribution in invariant mass can be obtained by appropriate binning.

The matrix element obtained from automatic code calculation and checked against existing calculations.
Results

**Figure:** Rapidity distribution of $W$ bosons.

$M_H = 120 \text{ GeV}$ in triangles
**Figure:** Polarization components to rapidity distribution of $W$ bosons.
Figure: Distribution in transverse momentum of one of the \(W\) bosons.
Results

**Figure:** Distribution in $W^+W^-$ invariant mass. The result when Sudakov form factor is put to 1 is shown for comparison.
Results

\[ \text{Figure: Two-dimensional distribution in rapidity of } W^+ \text{ and } W^- \text{ bosons. Diffractive mechanism (left panel), two-photon mechanism (right panel).} \]
Conclusions

- We have calculated differential cross sections for \( pp \rightarrow ppW^+W^- \)
- Two mechanisms have been considered:
  (a) photon-photon fusion known from the literature,
  (b) a new diffractive mechanism
- Two diffractive subprocesses have been considered:
  (a) intermediate boxes,
  (b) intermediate triangles with intermediate virtual Higgs boson.
- The cross section for the diffractive mechanism is much smaller than that for the two-photon mechanism.
- We observe negative interference between box contribution and triangle contribution.
- Experimentally one could focus on the diffractive component but the cross section is small.
- The \( pp \rightarrow ppW^+W^- \) reaction is therefore very promising laboratory for testing triple and quartic boson coupling.