Insights into the nucleon spin from lattice QCD

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DIS

Bonn, March 28, 2012

Outline

- Introduction: what insights can the lattice give?
- Lattice calculations.
- Results for Δq, focus on Δs.
- Summary and outlook.

Based on arXiv:1112.3354 (QCDSF Collaboration).

Spin of the nucleon

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_\psi + J_g$$

Composed of the contribution of the quark spin,

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s + \cdots ,$$

the quark orbital and gluon total angular momentum, L_ψ, and J_g.

Notation: Δq contains both, the spin of the quarks q and of the antiquarks $\bar{q}$.

Naïve non-relativistic SU(6) quark model: $\Delta\Sigma = 1$, $L_\psi = J_g = \Delta s = 0$

What can be calculated on the lattice?

The individual quark spin contributions ($q = u, d, s$)

$$\frac{\Delta q}{2} \quad s_\mu \quad = \quad \frac{1}{m_N} \langle N, s | \bar{q} \gamma_\mu \gamma_5 \frac{1}{2} q | N, s \rangle$$

So that one can construct the axial charges:

$$\begin{aligned} a_3 &= -s_\mu \frac{1}{m_N} \langle N, s | \bar{\psi} \gamma_\mu \gamma_5 \lambda_3 \psi | N, s \rangle = \Delta u - \Delta d = g_A \\ a_8 &= -s_\mu \frac{\sqrt{3}}{m_N} \langle N, s | \bar{\psi} \gamma_\mu \gamma_5 \lambda_8 \psi | N, s \rangle = \Delta u + \Delta d - 2\Delta s \\ a_0(Q^2) &= -s_\mu \frac{1}{m_N} \langle N, s | \bar{\psi} \gamma_\mu \gamma_5 \mathbb{1} \psi | N, s \rangle = \Delta u + \Delta d + \Delta s = \Delta\Sigma(Q^2) \end{aligned}$$

where $\psi = (u, d, s)$, λ_j are Gell-Mann flavour matrices.

Total quark angular momentum, $J_q = \frac{1}{2}\Delta q + L_q$, can be extracted from generalized form factors at $q^2 = 0$,

$$J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

where $A_{20}^q(q^2)$ and $B_{20}^q(q^2)$ extracted from matrix elements of the form $\langle N, s', p+q|\bar{q}\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2\}} q|N, s, p\rangle$.

Can infer L_q and J_g.

Also, $\langle x\rangle_\Delta$ and $\langle x\rangle_\delta$. For review of recent results see, e.g. C. Alexandrou arXiv:1111.5960.

Focus on Δq and Δs in particular.

Extraction of Δu, Δd, Δs from experiment

DIS expt. measures spin structure function of proton and neutron, $g_1^{p,n}(x,Q^2)$.

First moment related to a_i's via OPE (leading twist),

$$\Gamma_1^{p,n}(Q^2) = \int_0^1 \mathrm{d}x\, g_1^{p,n}(x,Q^2) \quad = \quad \frac{1}{36}\left[(a_8 \pm 3a_3)C_{NS} + 4a_0 C_S\right]$$

Use models to extrapolate g_1 from expt. x_{min} to $x = 0$.

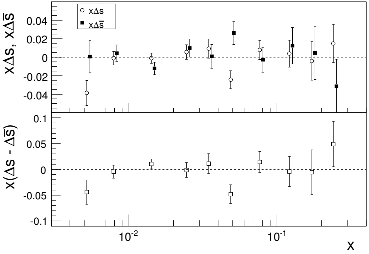
$C_{S/NS} = C_{S/NS}(\alpha_s(Q^2))$. a_3 known from neutron β decay.

Hyperon β decays, assuming $SU(3)$ flavour symmetry, used to obtain a_8.

Combinations of a_i give Δq's.
e.g. $\Delta s(5GeV^2) = \frac{1}{3}(a_0 - a_8) = -0.085(13)(8)(9)$ HERMES (2006).

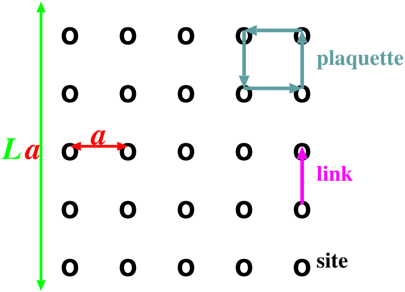
SIDIS offers direct measurement of $\Delta q(x)$s using pion and kaon beams.

Limitation is knowledge of fragmentation functions.
COMPASS (2010)



	Naive Extrap.	DSSV
$\Delta s + \Delta\bar{s}$	$-0.02 \pm 0.02 \pm 0.02$	$-0.10 \pm 0.02 \pm 0.02$

Lattice QCD



continuum limit: $a \to 0$, La fixed

infinite volume: $La \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\, O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": average over a *representative* ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

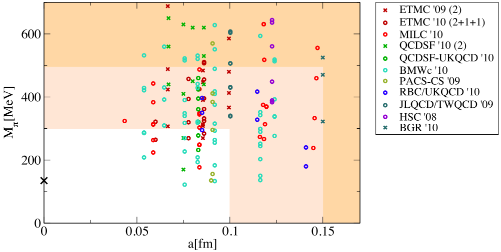
Systematic uncertainties

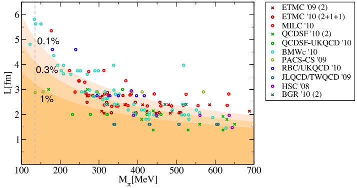
- **Sea quarks**: present simulations with $N_f = 2$ (u/d), $N_f = 2+1$ $(u/d+s)$, $N_f = 2+1+1$ $(u/d+s+c)$.
- **Finite lattice spacing**: need results for several a (> 3) in order to perform $a \to 0$.
- **Finite volume**: need to perform simulations on several volumes, want $Lm_\pi \geq 3.5 - 4$.
- **Non-physical quark masses**: typically $m_\pi > 200$ MeV, requiring chiral extrapolation of results.
- **Renormalisation constants**: needed to relate lattice matrix elements to continuum results in the $\overline{\mathrm{MS}}$ scheme.

Small a, large V, $m_\pi \to 138$ MeV, simulation costs increase very rapidly.

However, balance between statistical and systematic error, for some quantities (stat.)>(sys.).

Landscape of current lattice simulations





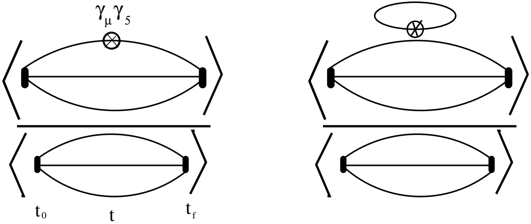
Figures taken from C Hölbling, Lattice 2010

Results will be presented with $a \approx 0.072$ fm, $La \approx 2.9$ fm, $m_\pi \approx 285$ MeV.

Spin Content

$$\langle N, s|(\bar{q}\gamma_\mu\gamma_5 q)^{lat}|N, s\rangle \quad = \quad 2m_N s_\mu \frac{\Delta q^{lat}}{2}$$

On the lattice extracted using ratio of (3pt fn)/(2pt fn)



(a) connected, $R^{con}(t_0, t, t_f)$ (b) disconnected, $R^{dis}(t_0, t, t_f)$

For Δs^{lat} $\exists$ only the disconnected contribution.

Extract matrix element from ratios (at zero momentum):

For $t_f \gg t \gg t_0$:

$$R^{con}(t_0, t, t_f) + R^{dis}(t_0, t, t_f) \to 2\frac{\langle N, s|(\bar{q}\gamma_j\gamma_5 q)^{lat}|N, s\rangle}{2m_N} = \Delta q^{lat}$$

Disconnected quark line diagrams computationally expensive: involves calculating quark propagators from $(\vec{x}, t) \to (\vec{y}, t')$, "all-to-all".

In the past:

- Contribution ignored or quote differences. e.g. $g_A = \Delta u - \Delta d$, J_{u-d}, $\langle x\rangle_{\Delta u - \Delta d}$.

Methods now developed to calculate these disconnected terms: e.g. Δs!

Simulation details

Quark action: Non-perturbatively improved (clover) action. $O(a^2)$ leading order discretisation effects.

Configurations: QCDSF, $N_f = 2$ (u/d) sea quarks, $a \sim 0.072$ fm, $m_\pi \sim 285$ MeV.

Volumes: $La \sim 2.3$ fm $(Lm_\pi = 3.4)$ and 2.9 fm $(Lm_\pi = 4.2)$.

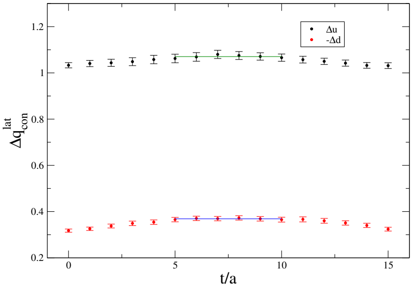
Statistics: approx. 2000×8 for disconnected quark line diagram, 2000×2 for connected.

Quark mass: input mass parameter in quark action such that,

- disconnected loop, $m_\pi \sim 285$ $(u/d$ sea), 449 and 720 (s) MeV,
- valence quark, $m_\pi \sim 285$, 449 and 720 MeV.

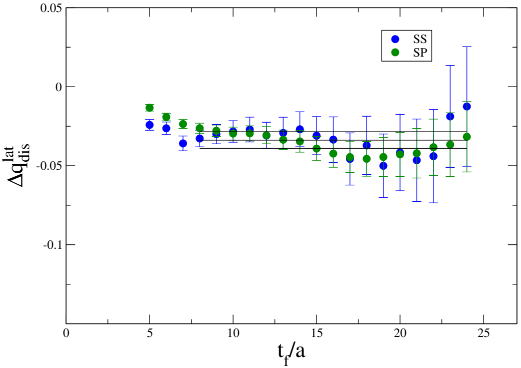
$R^{con}(t_0 = 0, t, t_f = 15a)$

- Create all states (g.s. + excited) with same QNs, ground state dominates for $t_f \gg t \gg t_0$.
- Reduce excited state contribution using "smeared" source ($t_0 = 0$) and sink ($t_f = 15a$) nucleon operators.
- No renormalisation.



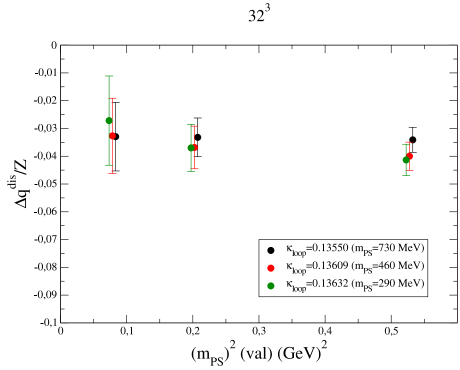
$R^{dis}(t_0 = 0, t = 4a, t_f)$

- $t_f \gg t$, t fixed.
- No renormalisation.
- Δs in sss nucleon.



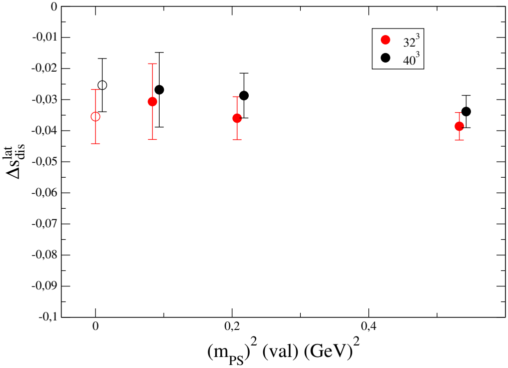
Dependence on loop and valence quark masses

- All loop and valence quark mass combinations shown.
- No statistically significant dependence seen, suggests our results may also approx. apply to physical light-quark masses.



Volume dependence of Δs^{lat}

- No statistically significant finite size effects.



Renormalisation

$N_F = 2+1$:

$$a_0 = \Delta\Sigma^{\overline{MS}}(\mu) = (\Delta u + \Delta d + \Delta s)^{\overline{MS}}(\mu) = Z_A^s(\mu, a)(\Delta u + \Delta d + \Delta s)^{\mathrm{lat}}(a)\,,$$
$$a_8 = (\Delta u + \Delta d - 2\Delta s)^{\overline{MS}} = Z_A^{ns}(a)(\Delta u + \Delta d - 2\Delta s)^{\mathrm{lat}}(a)\,,$$
$$a_3 = g_A = (\Delta u - \Delta d)^{\overline{MS}} = Z_A^{ns}(a)(\Delta u - \Delta d)^{\mathrm{lat}}(a)\,,$$

$$\begin{pmatrix} \Delta u(\mu) \\ \Delta d(\mu) \\ \Delta s(\mu) \end{pmatrix}^{\overline{MS}} = \begin{pmatrix} Z_A^{ns} + \frac{z(\mu)}{3} & \frac{z(\mu)}{3} & \frac{z(\mu)}{3} \\ \frac{z(\mu)}{3} & Z_A^{ns} + \frac{z(\mu)}{3} & \frac{z(\mu)}{3} \\ \frac{z(\mu)}{3} & \frac{z(\mu)}{3} & Z_A^{ns} + \frac{z(\mu)}{3} \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta d \\ \Delta s \end{pmatrix}^{\mathrm{lat}}\,,$$

where $z(\mu) = Z_A^s(\mu) - Z_A^{ns}$.

$N_F = 2$: ($\Delta u + \Delta d$ is now the singlet.)

$$\begin{pmatrix} \Delta u(\mu) \\ \Delta d(\mu) \\ \Delta s(\mu) \end{pmatrix}^{\overline{MS}} = \begin{pmatrix} Z_A^{ns} + \frac{z(\mu)}{2} & \frac{z(\mu)}{2} & 0 \\ \frac{z(\mu)}{2} & Z_A^{ns} + \frac{z(\mu)}{2} & 0 \\ \frac{z(\mu)}{2} & \frac{z(\mu)}{2} & Z_A^{ns} \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta d \\ \Delta s \end{pmatrix}^{\mathrm{lat}}\,.$$

$O(a)$ improvement:

$$Z_A^{ns} \mapsto Z_A^{ns}(1 + b_A am)\,,$$
$$z(\mu) \mapsto z(\mu)[1 + (b_A^s - b_A)am]\,,$$

where $b_A = b_A^s + O(\alpha_s^2)$. In total,

$$\Delta q^{\overline{MS}}(\mu) = Z_A^{ns}(1 + b_A am_q)\Delta q^{\mathrm{lat}} + \frac{z(\mu)}{2}(\Delta u + \Delta d)^{\mathrm{lat}}$$

- $Z_A^{ns} = 0.76485(64)(73)$ determined non-perturbatively.
- $1 + b_A am = 1.004 - 1.032$ to $O(\alpha)$ for quark mass range. Omit $b_A^s - b_A$.
- $z(\mu = \sqrt{7.4}\,\mathrm{GeV}) = 0.0055(27)$ to $O(\alpha^2)$, very slow running with μ.
- We allow for a 50 % error on all perturbative results.

$\Delta q^{\overline{\text{MS}}}(\mu = \sqrt{7.4} GeV)$

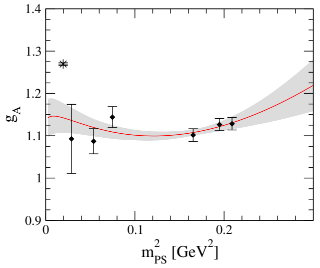
For $La = 2.9$ fm, first error statistical, second is from renormalisation.

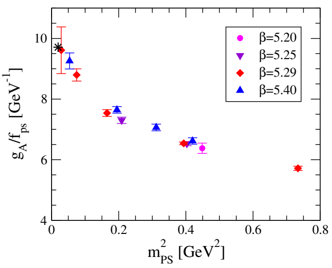
q	$\Delta q^{\text{lat}}_{\text{con}}$	$\Delta q^{\text{lat}}_{\text{dis}}$	$\Delta q^{\overline{\text{MS}}}(\mu)$	DSSV x_{min}	DSSV 0
u	1.071(15)	-0.049(17)	0.787(18)(2)	0.793(12)	0.814
d	-0.369(9)	-0.049(17)	-0.319(15)(1)	-0.416(11)	-0.458
s	0	-0.027(12)	**-0.020(10)(1)**	-0.012(24)	-0.114
a_3	1.439(17)	0	**1.105(13)(2)**	1.21	1.272
a_8	0.702(18)	-0.044(19)	**0.507(20)(1)**	0.401	0.583
Σ	0.702(18)	-0.124(44)	**0.448(37)(2)**	0.366(17)	0.242

Mixing effects are very small. DSSV : de Florian et al. (2009), $x_{min} = 0.001$.

Systematic uncertainties

- Uncertainty from finite volume and renormalisation not significant.
- Non physical light quark mass ($m_{u/d} \sim 4m_{u/d}^{phys}$) likely to be dominant source: $a_3 = g_A = \Delta u - \Delta d$ underestimated.

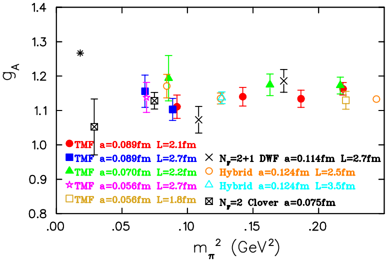




QCDSF: D. Pleiter et al. (2010)

Similar underestimates seen in results from other collaborations.

Review by C. Alexandrou (2011)



We underestimate g_A by 13%. We add 20% systematic certainty for finite a effects, $m_{u/d} \neq m_{u/d}^{phys}$ and neglecting strange sea quark.

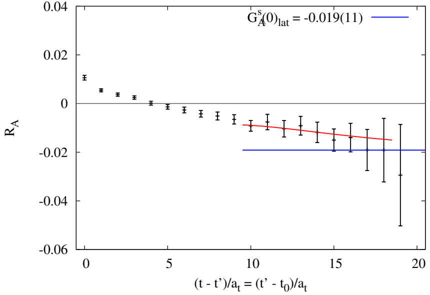
We find for $\overline{\mathrm{MS}}$ at $\mu = \sqrt{7.4}$ GeV:

$$\begin{aligned} \Delta\Sigma = \Delta u + \Delta d + \Delta s &= 0.45(4)(9) \\ \Delta s &= -0.020(10)(4) \end{aligned}$$

For the strange spin content the statistical error dominates.

Other modern lattice calculation of Δs

Babich et al. (2010), unrenormalised, for $N_f = 2$, $m_\pi = 400$ MeV, $a_t \sim 0.036$ fm, $a_s = 0.108$ fm.

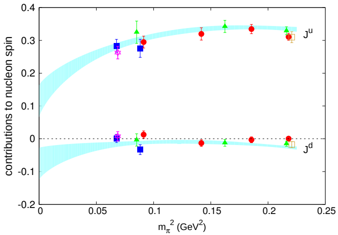


- Local operator: $\Delta s^{lat} = -0.024(15)$.
- Point split: $\Delta s^{lat} = -0.019(11)$

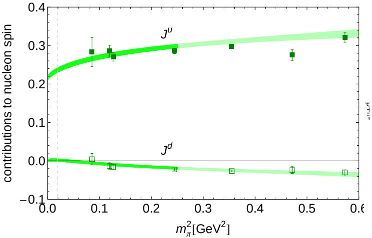
$J_u = \frac{1}{2}[A^u_{20}(0) + B^u_{20}(0)]$ and J_d

$$\langle N(p', s')|O^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A_{20}(q^2)\,\gamma^{\{\mu}(p+p')^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha(p+p')^{\nu\}}}{2m} + C_{20}(q^2)\,\frac{1}{m}q^{\{\mu}q^{\nu\}}\Big]u_N(p,s)$$

Assuming sea quark contribution small down to $m_\pi \sim 300$ MeV:
$J_u \sim 0.2$ and $J_d \sim 0$.



ETMC (2011).



LHPC (2010).

Summary and Outlook

- Lattice calculations have an important role to play in determining the spin content of the nucleon: Δq, $\Delta\Sigma$, J_q, ...
- Previously, the disconnected quark line diagrams were not calculated: differences quoted, g_A, J_{u-d}, ..., and no Δs, $\Delta\Sigma$ or a_8.
- New methods now make it possible to calculate these diagrams.
- We suggest to use Δs result to constrain polarized PDFs.

In the future:

- Investigate lattice spacing and light quark mass dependence of the results. Also move to $N_f = 2 + 1$ and improve statistical uncertainty.
- Calculate other quantities, J_q, $\langle x \rangle_\Delta$ and $\langle x \rangle_\delta$.