Diffractive Vector Meson Cross Sections from BK evolution with Impact Parameter

Jeffrey Berger

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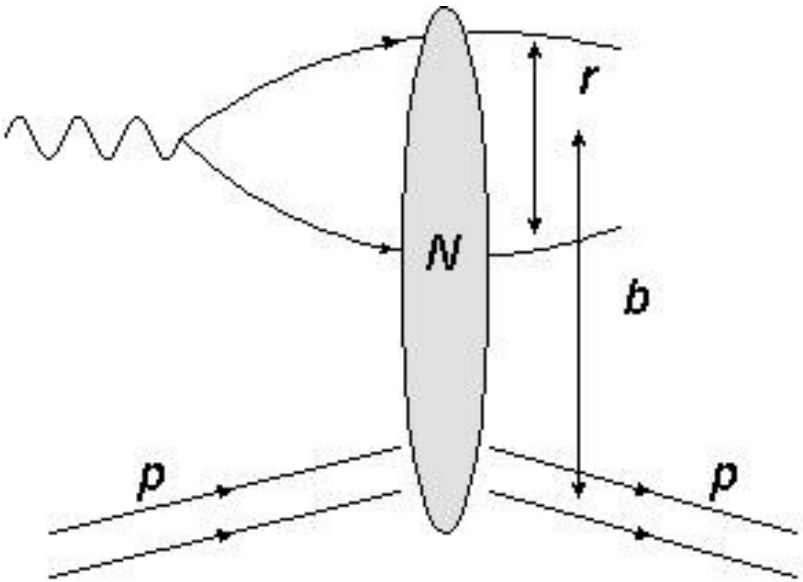
- BK with impact parameter
  - Dipole Model
  - Confinement effects
- Vector Meson Production
  - Phenomenological corrections
  - Differential and Integrated cross sections
- F_2 Structure function

Dipole Model



Photon splits into a color dipole of size r which interacts at impact parameter b with the target (nucleon)

Color dipole interacts with partons of the target through gluon exchange

[A.H.Mueller, Nucl. Phys B415 373 (1994)]

N(r,b,Y) is the scattering amplitude of the dipole interaction

- In the dipole model of small x scattering the evolution of the amplitude can be represented as a dipole cascade.

The BK equation

$$\frac{\partial N_{01}}{\partial Y} = \int d^2 x_2 K \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right]$$

- Enforces unitarity in the amplitude $N_{ij} = N(x_{ij}, b_{ij}, \vartheta_{ij}, Y)$
- Parent dipole $\boldsymbol{x_{01}} = \boldsymbol{x_0} - \boldsymbol{x_1}$ splits into two dipoles of $\boldsymbol{x_{02}}$ and $\boldsymbol{x_{12}}$
- Splitting is determined by the kernel $K = K(x_{01}, x_{02}, x_{12})$
- Impact parameter $\boldsymbol{b_{ij}} = \frac{1}{2}(\boldsymbol{x_i} + \boldsymbol{x_j})$ only dependence is in the amplitude
- Angle ϑ_{ij} is the angle between $\boldsymbol{x_{ij}}$ and $\boldsymbol{b_{ij}}$
- We take the full dependence of the amplitude on impact parameter into account

Modelling Confinement

- Large dipole sizes must be regulated
  - Saturation regime does not provide self-regulation as it does without impact parameter

$$K=\bar{\alpha}\left(x_{01}^2\right)\left[\frac{1}{x_{02}^2}\left(\frac{\alpha\left(x_{02}^2\right)}{\alpha\left(x_{12}^2\right)}-1\right)+\frac{1}{x_{12}^2}\left(\frac{\alpha\left(x_{12}^2\right)}{\alpha\left(x_{02}^2\right)}-1\right)+\frac{x_{01}^2}{x_{12}^2\,x_{02}^2}\right]\times\Theta\left(\frac{1}{m^2}-x_{02}^2\right)\Theta\left(\frac{1}{m^2}-x_{12}^2\right)$$

[I. Balitsky, Phys. Rev. D75, 014001 (2007)] [J Berger and A.M.Stasto Phys.Rev.D84 (2011)]

$$m=0.37\,GeV$$

- Large dipole sizes must be cut off at every step of the evolution
  - If only the initial condition is cut off then large dipoles will be generated during the evolution

Initial Conditions

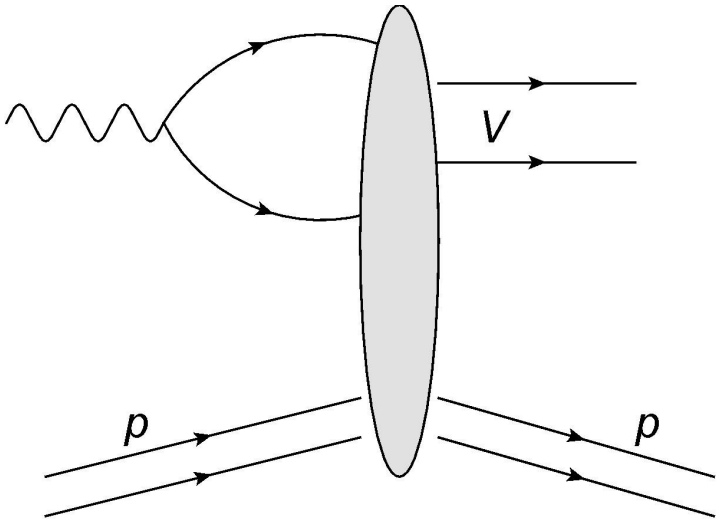
- We choose a Glauber-Mueller form for the initial condition

$$N(r,b,Y_0)=1-\exp\left(\frac{-\pi}{2N_c}r^2 x\, g(x,\eta^2)T(b)\right) \qquad T(b)=\frac{1}{8\pi}\exp\left(\frac{-b^2}{2B_G}\right)$$

$$B_G=\frac{1}{2\mathrm{m}^2}$$

- Initial rapidity taken to be $Y_0=\ln(1/x_0)$ with $x_0=10^{-2}$
- Large dipole sizes are cut off when $r\geqslant\frac{2}{m}$
- This choice gives a fixed cutoff that does not shift as the cutoff in the evolution is not on the parent dipole but on the daughter dipoles

Vector Meson Production

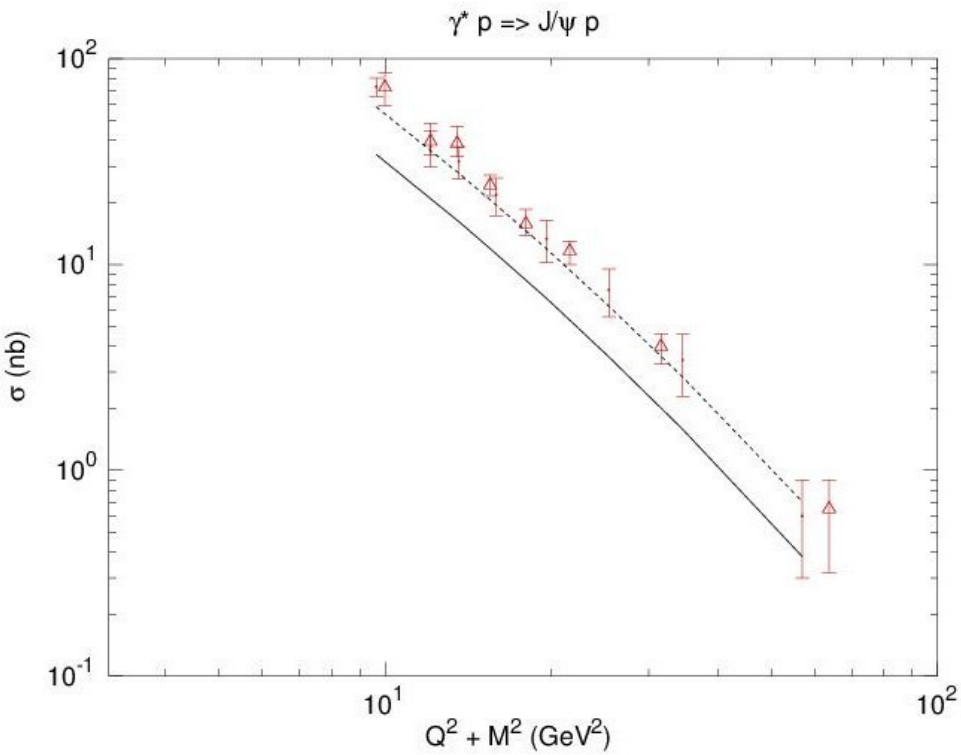


- We utilize the NNPZ prescription for the vector meson wavefunction [J.Nemchik,N.N.Nikolaev,E. Predazzi, and B. Zakharov Z.Phys.C75 (1997)]
- Parameters in WF constrained by normalization and lepton decay widths

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} |A(x, \Delta, Q^2)|^2$$

$$A(x, \Delta, Q^2) = \sum_{h,\bar{h}} \int d^2 r \int dz\, \Psi_{h,\bar{h}}(r, z, Q^2) N(x, r, \Delta) \Psi^V_{h,\bar{h}}(r, z)$$

Skewed gluon distribution



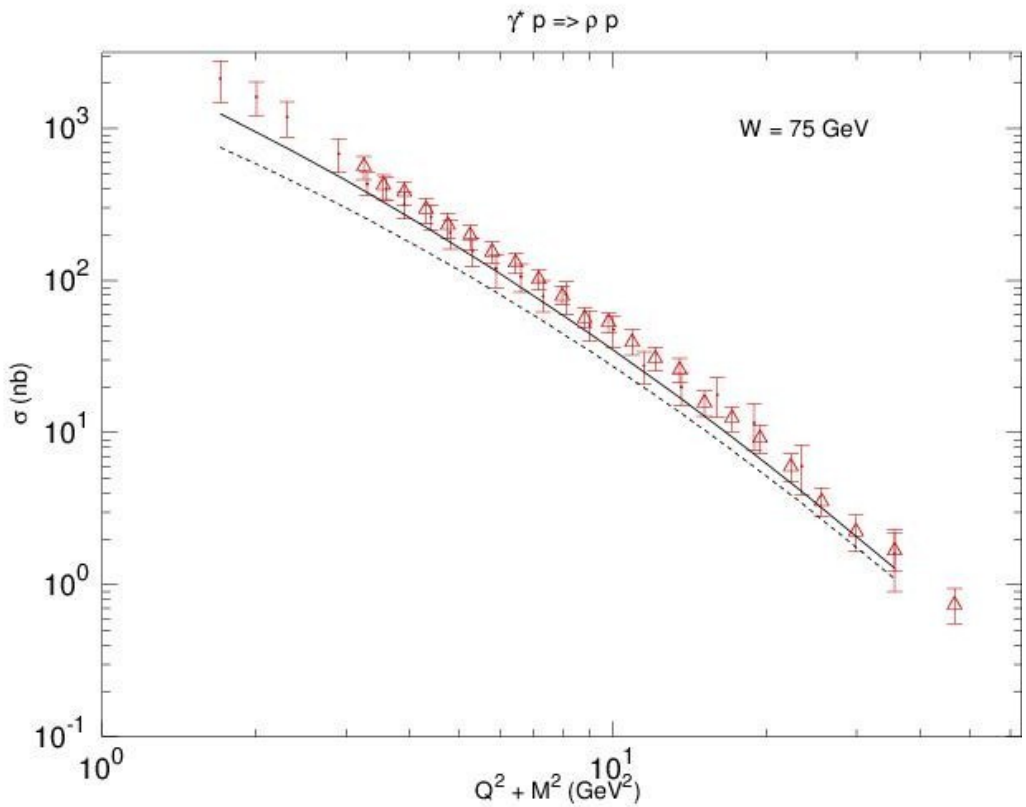
- Important in the initial condition of the evolution

$$\left(xg\left(x,\eta^2\right)\right)_{sk} = xg\left(x,\eta^2\right)\frac{2^{2\lambda_{sk}+3}}{\sqrt{\pi}}\frac{\Gamma\left(\lambda_{sk}+5/2\right)}{\Gamma\left(\lambda_{sk}+4\right)}$$

$$\lambda_{sk} \equiv \frac{\partial \ln\left(xg\left(x,\eta^2\right)\right)}{\partial \ln\left(1/x\right)}$$

[A.D.Martin,M.Ryskin, and T.Teubner Phys.Rev.D62 (2000)]

Photon Wavefunction Corrections

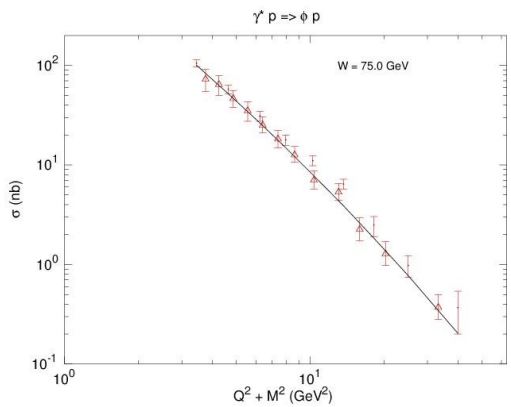


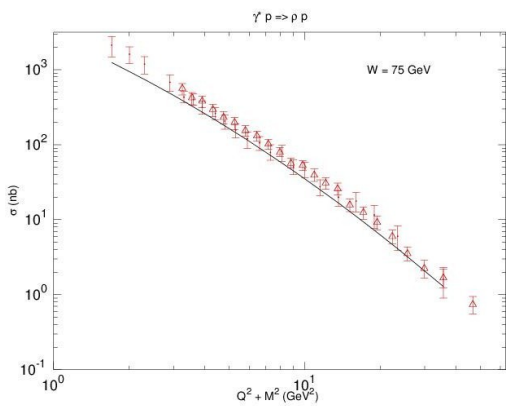
- Additional contribution from photon wavefunction at low Q^2 required
- Universal for all mesons and processes

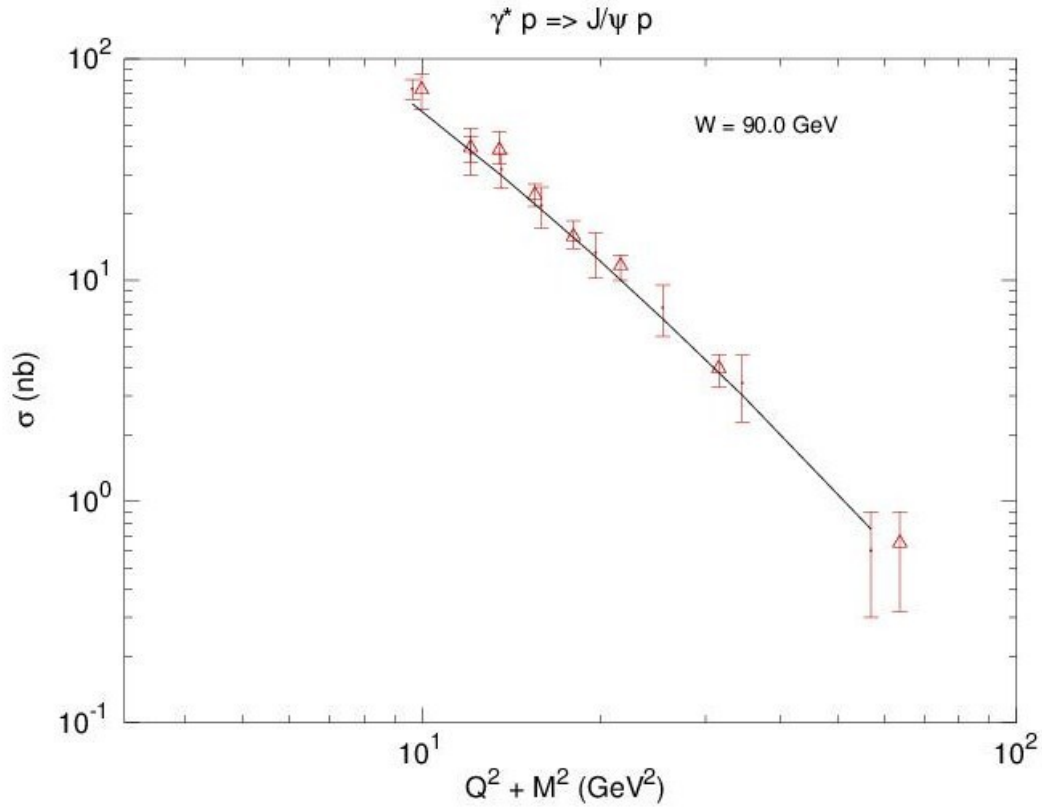
$$|\Psi_\gamma|^2 \rightarrow |\Psi_\gamma|^2 \left(\frac{1 + B e^{-\omega^2 (x_{01} - R)^2}}{1 + B\, e^{-\omega^2 R^2}} \right)$$

[J.R.Forshaw,R.Sandapen,and G.Shaw Phys.Rev.D69 (2004)]

$\sigma_V\left(Q^2+M_V^2\right)$



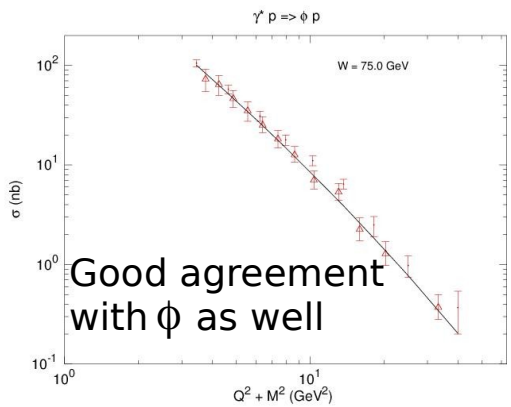




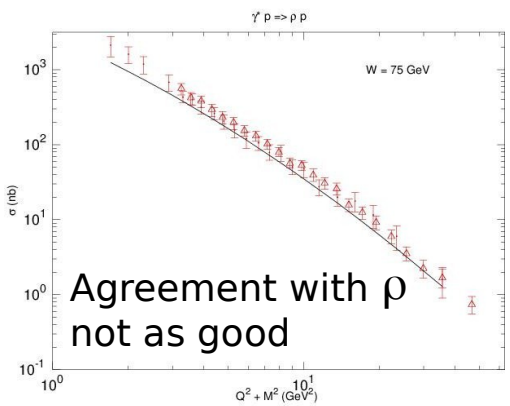
[H1 Collaboration Eur.Phys.J C13 (2000)]
[H1 Collaboration Eur.Phys.J C46 (2006)]
[H1 Collaboration JHEP 1005 (2010)]

[ZEUS Collaboration Nucl.Phys.B695 (2004)]
[ZEUS Collaboration Nucl.Phys.B718 (2005)]
[ZEUS Collaboration PMC Phys A1 (2007)]

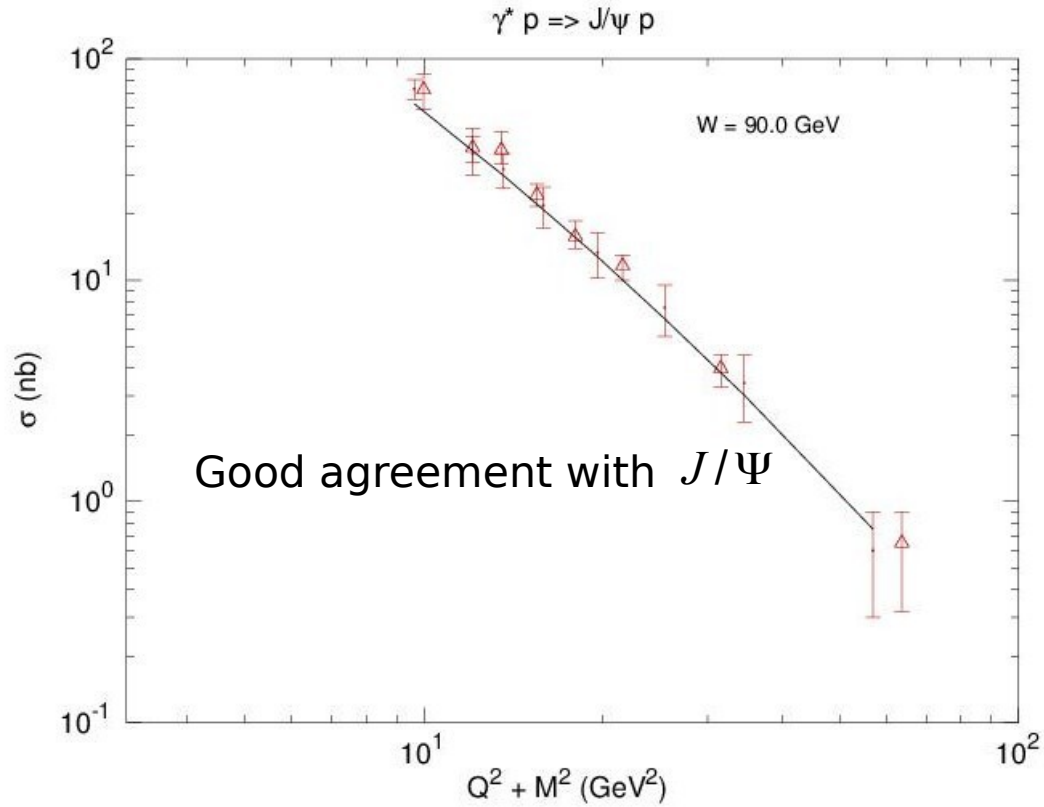
$\sigma_V(Q^2+M_V^2)$



Good agreement with ϕ as well



Agreement with ρ not as good

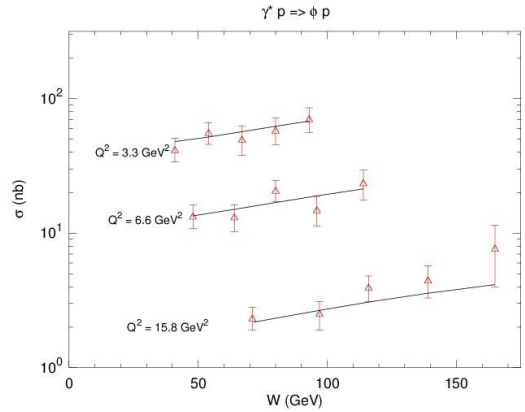


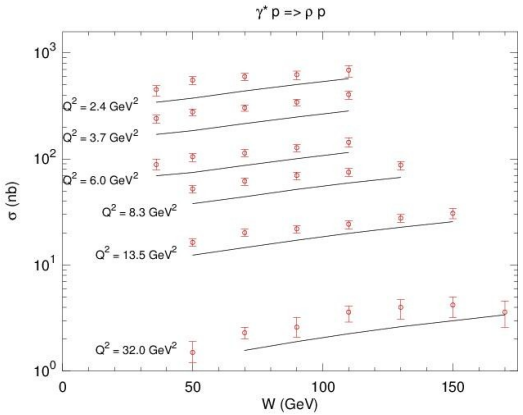
Good agreement with J/Ψ

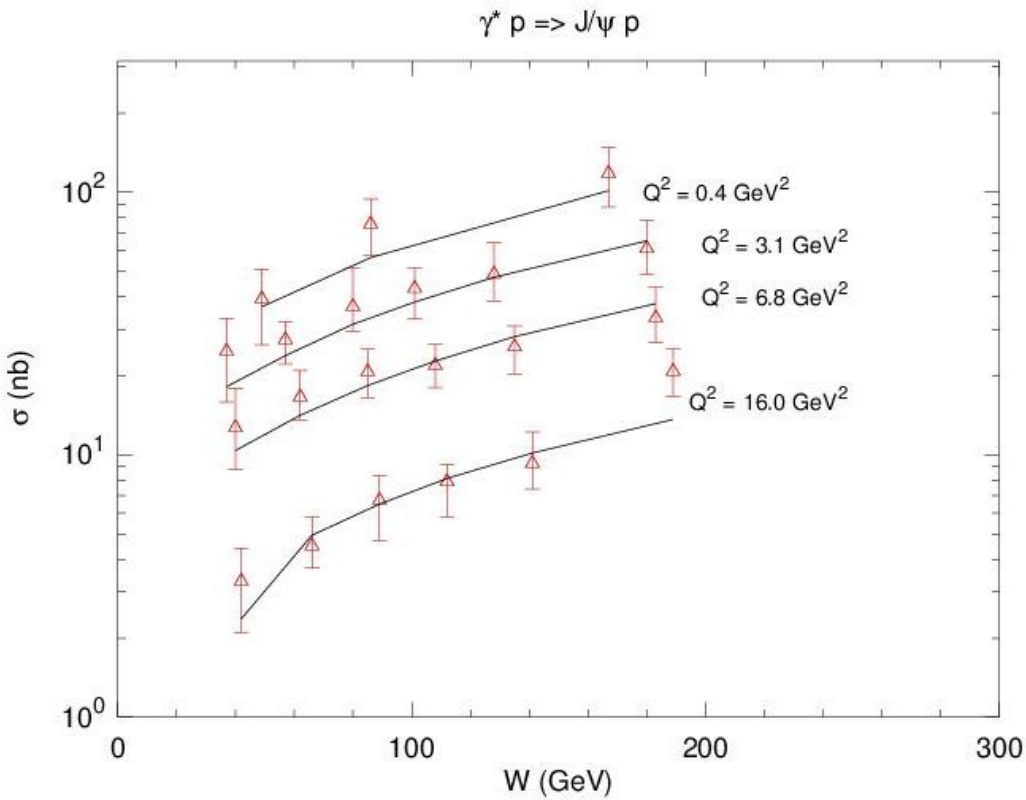
[H1 Collaboration Eur.Phys.J C13 (2000)]
[H1 Collaboration Eur.Phys.J C46 (2006)]
[H1 Collaboration JHEP 1005 (2010)]

[ZEUS Collaboration Nucl.Phys.B695 (2004)]
[ZEUS Collaboration Nucl.Phys.B718 (2005)]
[ZEUS Collaboration PMC Phys A1 (2007)]

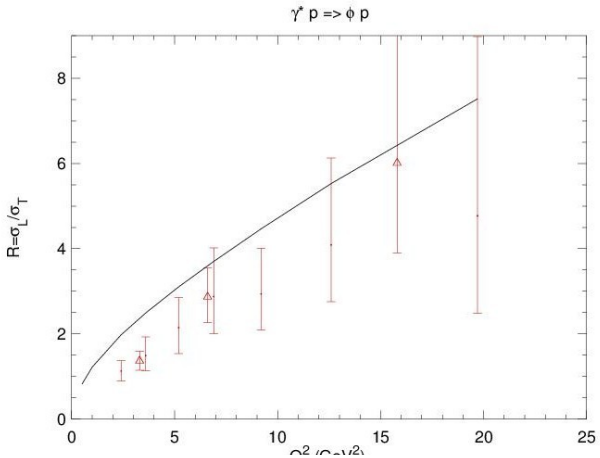
$\sigma(W)$


γ* p => φ p
σ (nb)
W (GeV)
Q² = 3.3 GeV²
Q² = 6.6 GeV²
Q² = 15.8 GeV²

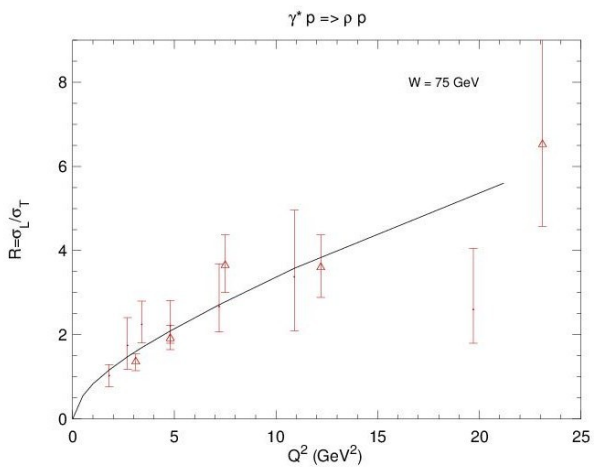

γ* p => ρ p
σ (nb)
W (GeV)
Q² = 2.4 GeV²
Q² = 3.7 GeV²
Q² = 6.0 GeV²
Q² = 8.3 GeV²
Q² = 13.5 GeV²
Q² = 32.0 GeV²

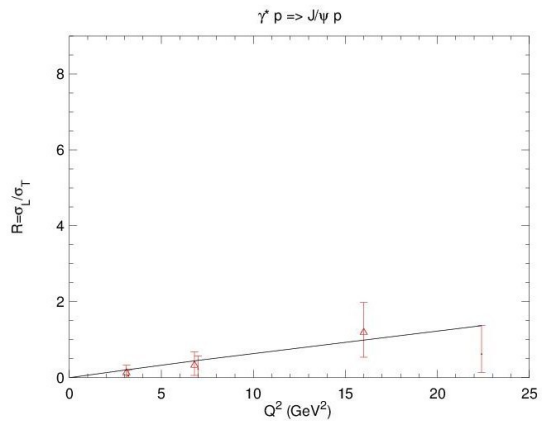

γ* p => J/ψ p
σ (nb)
W (GeV)
Q² = 0.4 GeV²
Q² = 3.1 GeV²
Q² = 6.8 GeV²
Q² = 16.0 GeV²

R ratio



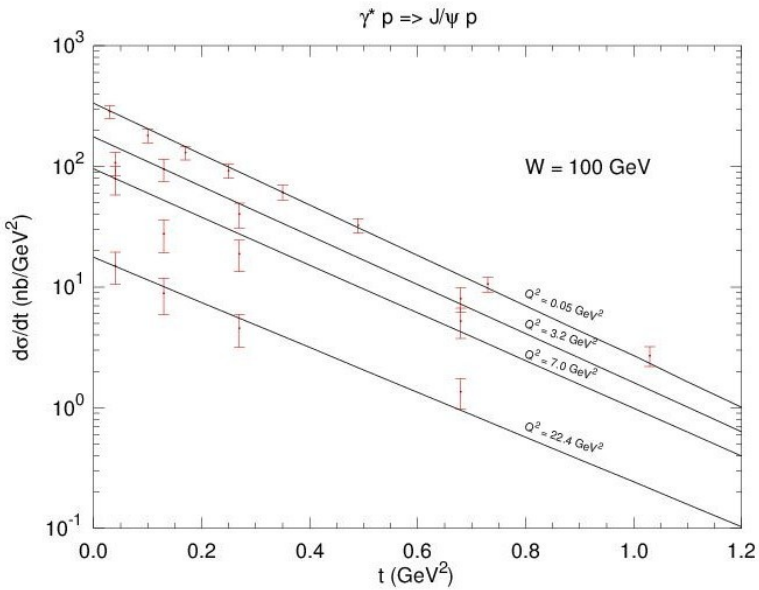
- Ratio is sensitive to the form of the vector meson wavefunctions
- Is not strongly dependent on energy

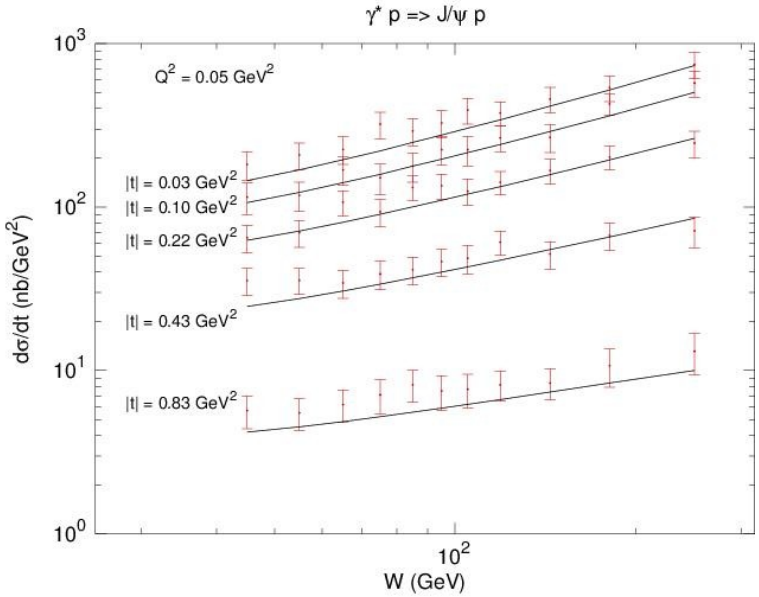




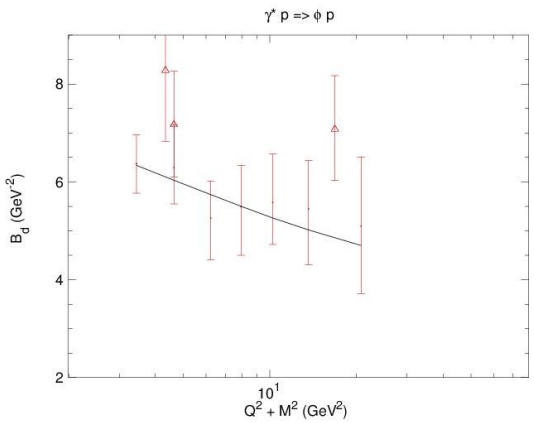
$$R = \frac{\sigma_L}{\sigma_T}$$

$d\sigma/dt$ for J/Ψ

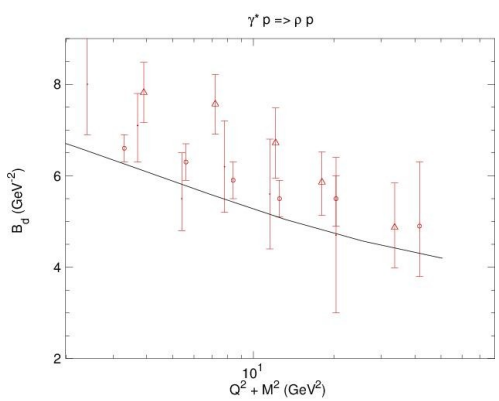


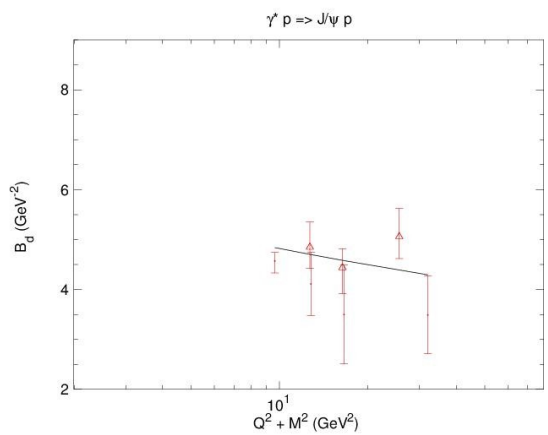


$B_D(Q^2+M_V^2)$



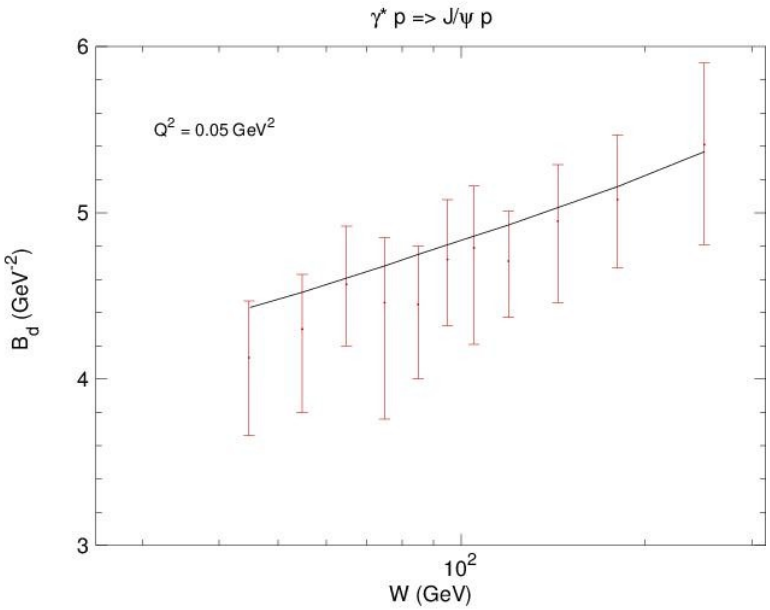
- Slope B_D is dimensionful
- Contains information on the distribution of the interaction region in transverse space

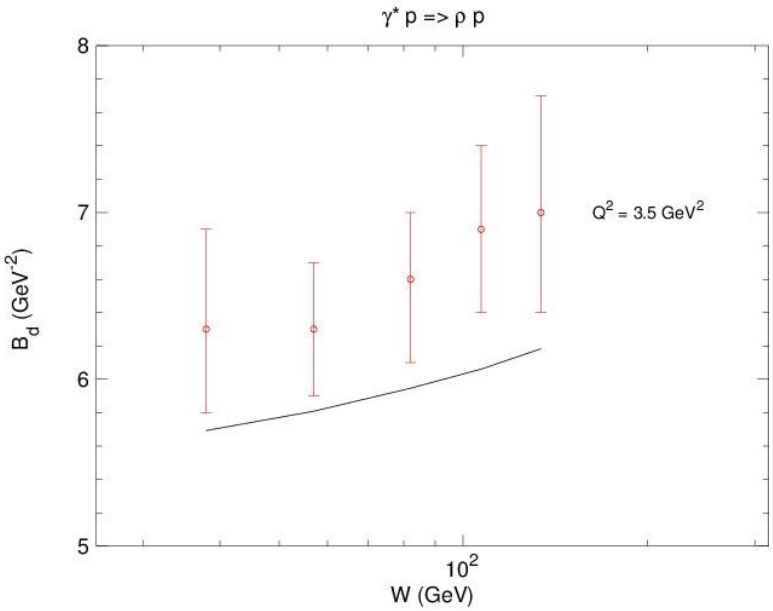




$$\frac{d\sigma}{dt} \propto e^{-B_D|t|}$$

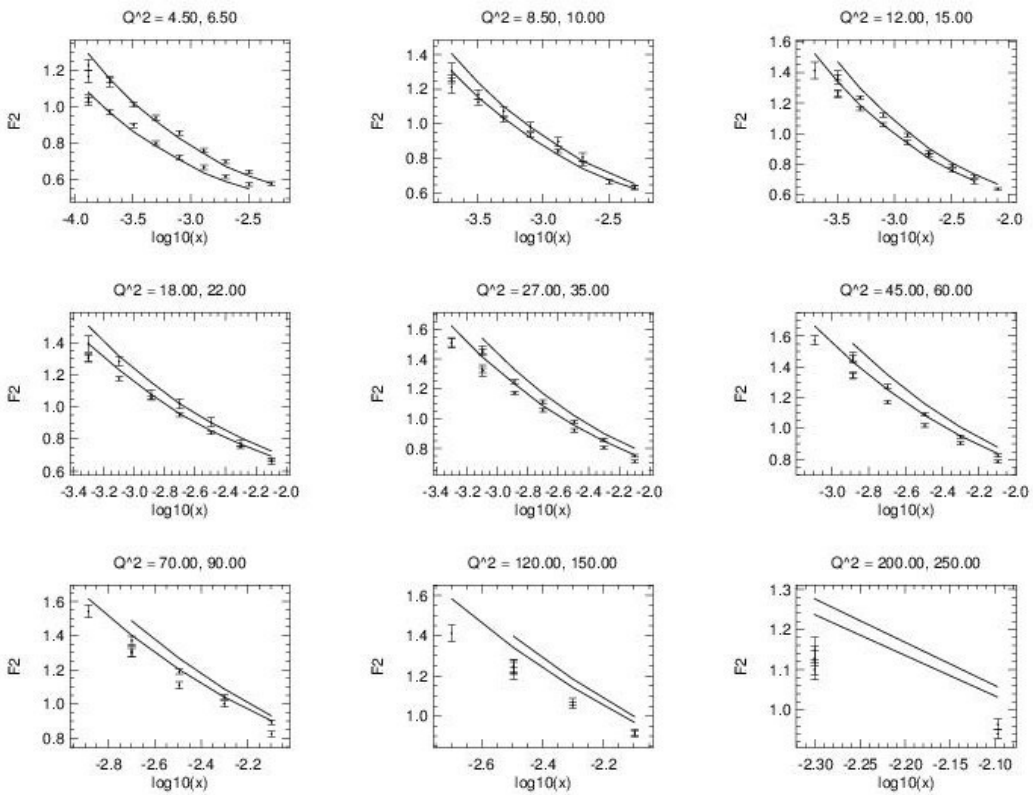
$B_D(W)$





- Slope is a feature that is naturally inherent in our model and is sensitive to our cutoff mass $b \propto \frac{1}{m^2}$
- Intercept is sensitive to B_G

F_2



- Proton structure functions
- Photon WF correction included
- Overly strong growth

$$F_2(Q^2, x) = Q^2 4\pi^2 \alpha_{em} \int d^2 r \int_0^1 dz$$

$$\times(|\Psi_T(r, z, Q^2)|^2 + |\Psi_L(r, z, Q^2)|^2)$$

$$\times \sigma_{dipole}(r, x)$$

Conclusions

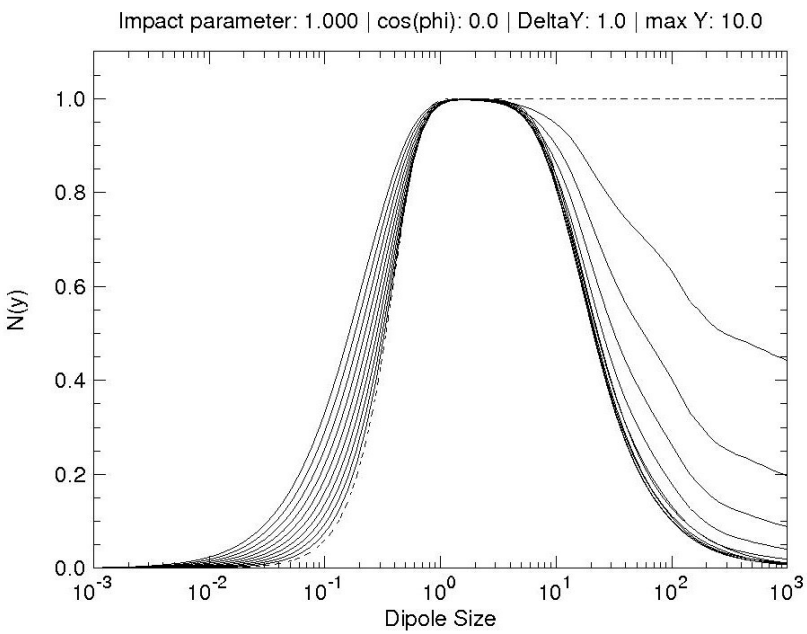
- Features of the HERA data on diffractive vector meson production are reproduced in both the differential and integrated cross sections
- Phenomenological corrections for skewedness and non-pertubrative effects in the photon wavefunction were found necessary
- Good agreement is found in the data of J/Ψ and ϕ
  - ρ is consistently low, possibly requires an additional non-perturbative contribution
- Diffusion in impact parameter space naturally produces slopes of $B_D(W)$

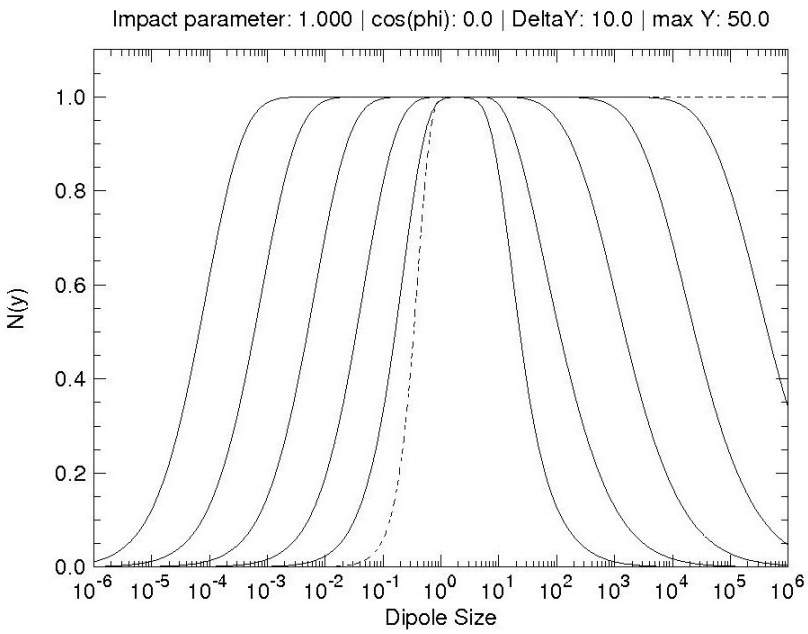
Thank You

Special Thanks to : My Advisor Anna Stasto as well as Henry Kowalski for discussions and use of his code

Backup Slides

Features of BK with impact parameter

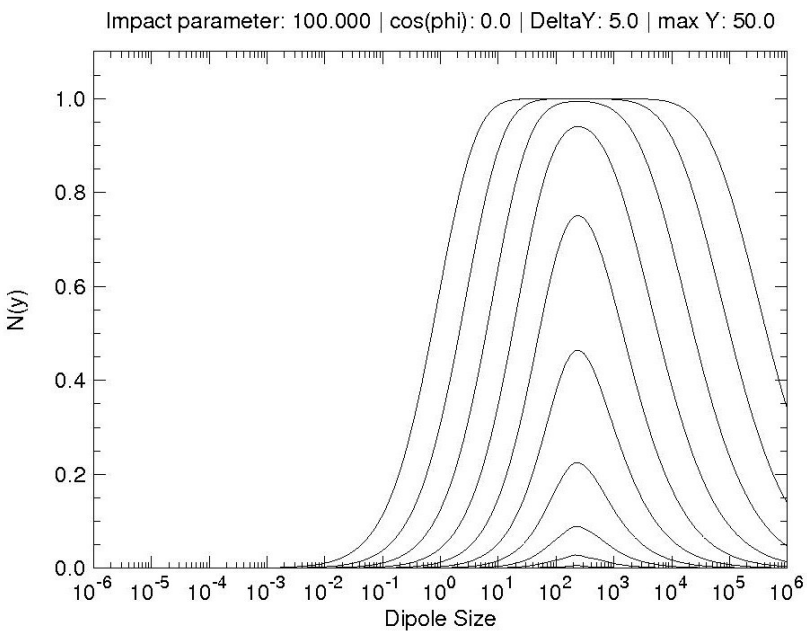


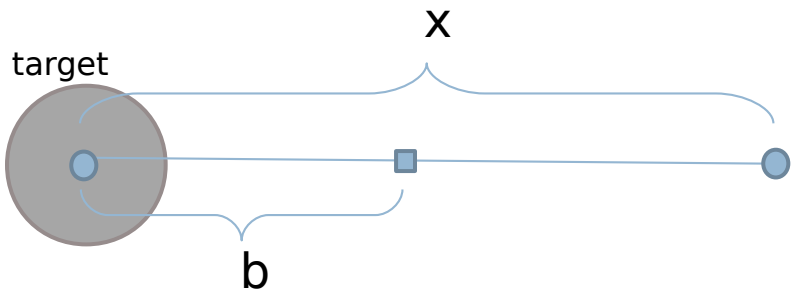


- Leading order kernel used
- Coupling fixed at $\frac{N_c \alpha_s}{\pi} = 0.1$

$$K = \frac{dz}{z} \frac{N_c \alpha_s}{2\pi^2} \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$

Large contributions at x = 2b





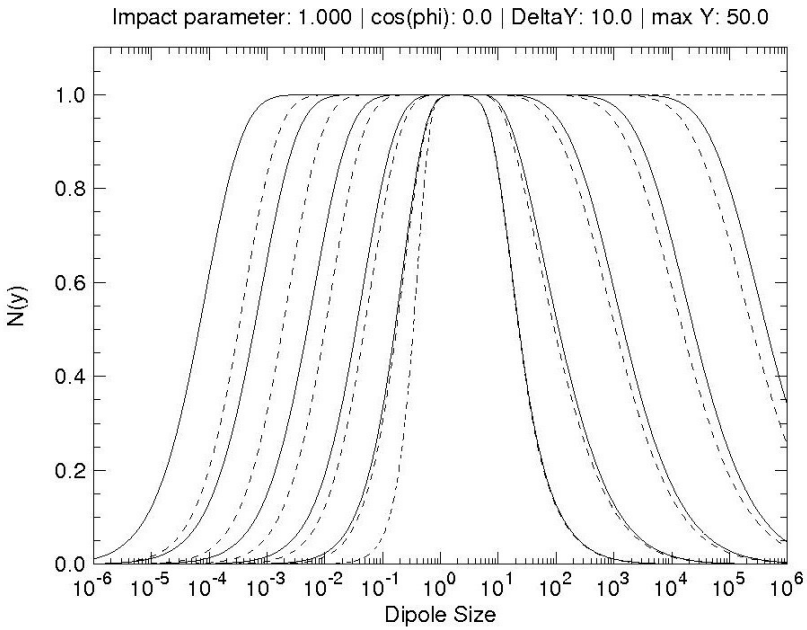
Nontrivial angular dependence.

Peak of the amplitude occurs when $\boldsymbol{x} \parallel 2\boldsymbol{b}$ and $x = 2b$

- This behavior can be extracted from the representation in terms conformal eigenfunctions

Towards higher order

LO (solid) vs Modified (dashed)



$$K = \frac{dz}{z}\frac{N_c \alpha_s}{2\pi^2}\frac{z}{x_{01}^2}\left[K_1^2\left(\frac{x_{02}}{x_{01}}\sqrt{z}\right) + K_1^2\left(\frac{x_{12}}{x_{01}}\sqrt{z}\right) - \frac{2x_{02}\cdot x_{12}}{x_{02}x_{12}}K_1\left(\frac{x_{02}}{x_{01}}\sqrt{z}\right)K_1\left(\frac{x_{12}}{x_{01}}\sqrt{z}\right)\right]$$

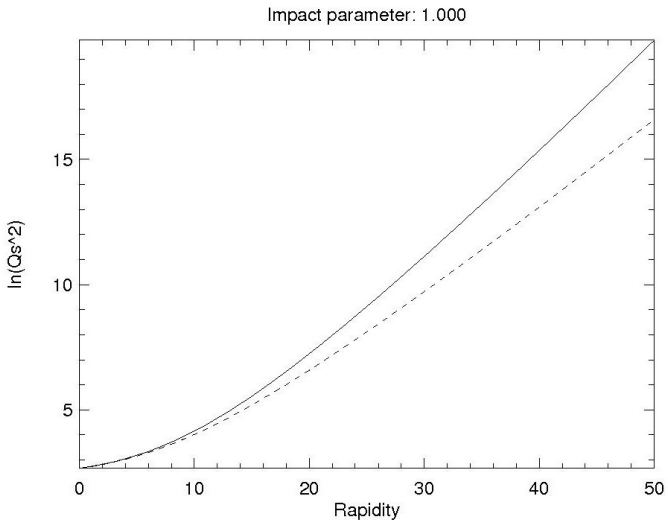
[L. Motyka and A. M. Stasto, Phys. Rev. D79, 085016 (2009)]

This kernel reduces to the LO kernel at large rapidities or when $x_{01} \gg x_{02}, x_{12}$

- Kinematical cut owing to a modification in the energy denominator
- The modified kernel slows the evolution by approximately 30%
- The modified kernel has almost no affect when the impact parameter dependence is neglected due to the saturation of all large dipole sizes.

Saturation Scale

LO (solid) vs Modified (dashed)



$$\left\langle N\left(r = 1/Q_s(b,Y), b, \theta, Y\right)\right\rangle = 0.5$$

Saturation scale was found to have the same impact parameter dependence at large b which leads us to a factorized form

$$Q_s^2(b,Y) = Q_0^2 e^{\bar{\alpha}_s \lambda_s Y} S(b) \qquad S(b) \sim \frac{1}{b^4}$$

	LO	Modified
λ_s	4.4	3.6 $\bar{\alpha}_s = 0.1$ (2.5 $\bar{\alpha}_s = 0.2$)

- Saturation is when the parton density becomes large and recombination effects become important
  - Defined here as the amplitude becomes large and the nonlinear term becomes important.
- Numbers are consistent with analytical estimates

[S. Munier and R. B. Peschanski, Phys. Rev. D69, 034008 (2004)]
[A. H. Mueller and D. N. Triantafyllopoulos, Nucl. Phys. B640, 331]

Running coupling

- Several different prescriptions for running coupling
- Balitsky

$$K = \frac{dz}{z}\frac{N_c \alpha_s(x_{01}^2)}{2\pi^2}\left[\frac{x_{01}^2}{x_{02}^2 x_{12}^2} + \frac{1}{x_{02}^2}\left(\frac{\alpha_s(x_{02}^2)}{\alpha_s(x_{12}^2)} - 1\right) + \frac{1}{x_{12}^2}\left(\frac{\alpha_s(x_{12}^2)}{\alpha_s(x_{02}^2)} - 1\right)\right]$$

[I. Balitsky, Phys. Rev. D75, 014001 (2007)]

- Kovchegov -Weigert

$$K = \frac{dz}{z}\frac{N_c}{2\pi^2}\left[\frac{1}{x_{20}^2}\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{x_{20}^2}\right) + \frac{1}{x_{12}^2}\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{x_{12}^2}\right) - 2\frac{\mathbf{x_{12}}\cdot\mathbf{x_{20}}}{x_{20}^2 x_{12}^2}\frac{\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{x_{20}^2}\right)\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{x_{20}^2}\right)}{\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{R^2(x_0,x_1;x_2)}\right)}\right]$$

[Y. V. Kovchegov and H. Weigert, Nucl. Phys. A784, 188 (2007]

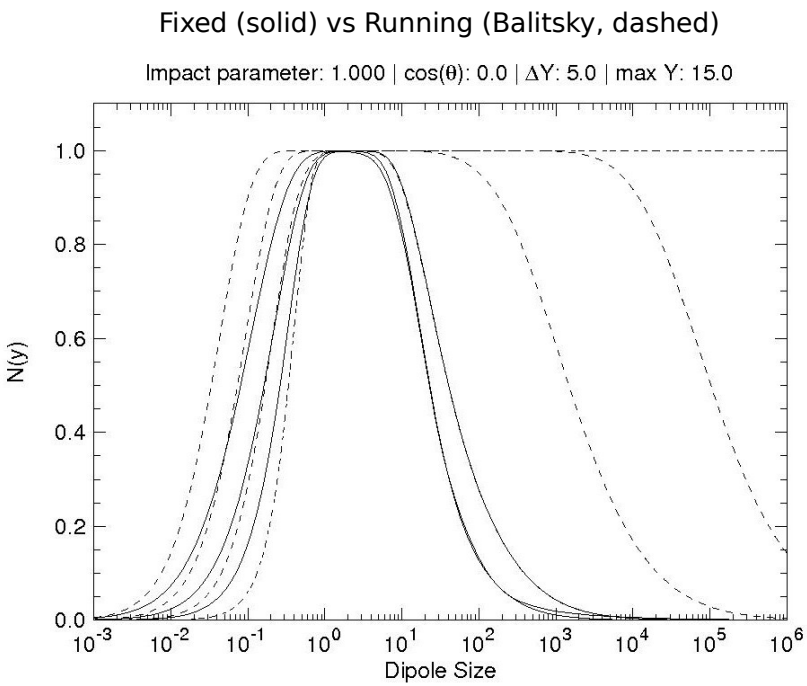
- Parent Dipole

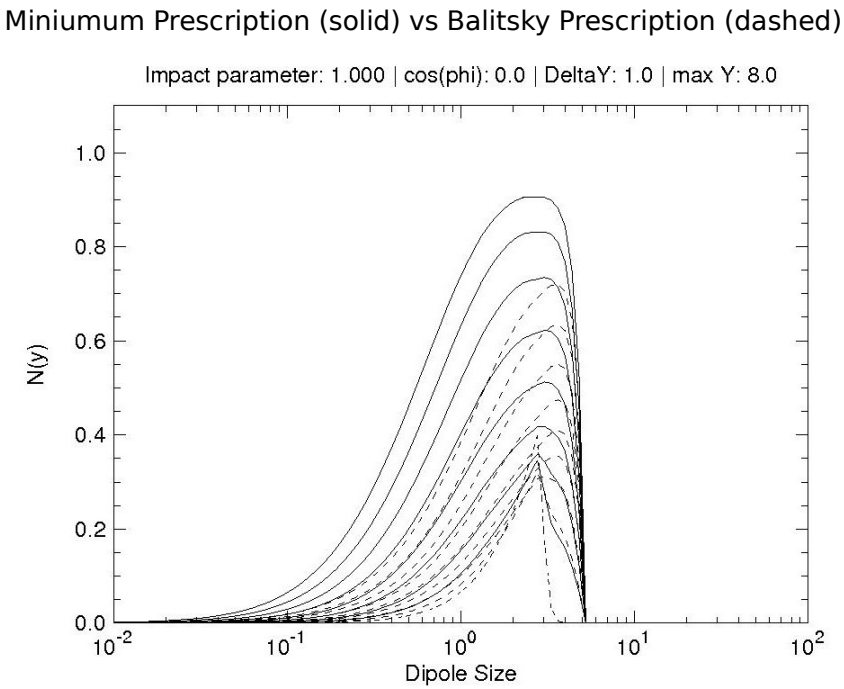
$$K = \frac{dz}{z}\frac{N_c \alpha_s\left(x_{01}^2\right)}{2\pi^2}\frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$

- Minimum Dipole

$$K = \frac{dz}{z}\frac{N_c \alpha_s\left(\min\left(x_{01}^2, x_{12}^2, x_{02}^2\right)\right)}{2\pi^2}\frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$

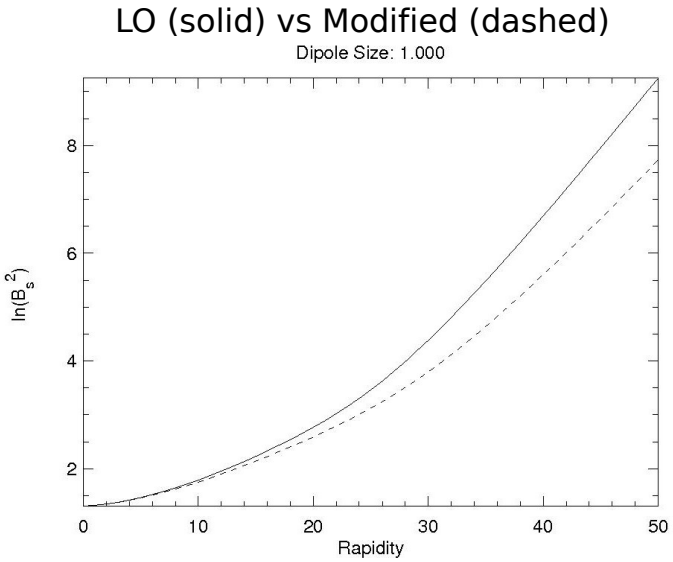
Results with running coupling





- IR regularization of the kernel is important due to large dipole evolution
- Balitsky's running coupling is well slower than the minimum dipole prescription

Diffusion in impact parameter



$$\left\langle N\left(r, B_s = b, \theta, Y\right)\right\rangle = 0.5$$

Growth of the black disk corresponds to growth of the cross section

$$B_s^2(r,Y) = B_{s0}^2 e^{\bar{\alpha}_s \lambda_{sB} Y} F(r) \qquad \sigma \approx e^{2\lambda_{sB} Y}$$

	LO	Modified
λ_{sB}	2.6	2.2 $\bar{\alpha}_s = 0.1$ (2.0 $\bar{\alpha}_s = 0.2$)

- Increasing energy causes the dense region of the dipole cascade to expand in impact parameter space
- Size of the dense or ‘black’ region characterized by a radius of this black disk
- Fast increase in is partially due to the lack of scale in the solution currently

Adding mass parameter

- Full cut with theta function

$$K = \frac{dz}{z}\frac{N_c\alpha_s}{2\pi^2}\frac{x_{01}^2}{x_{02}^2x_{12}^2}\theta\left(1/m^2 - x_{02}^2\right)\theta\left(1/m^2 - x_{12}^2\right)$$

- Splitting the theta function

$$K = \frac{dz}{z}\frac{N_c\alpha_s}{2\pi^2}\left[\frac{1}{x_{02}^2}\theta\left(1/m^2 - x_{02}^2\right) + \frac{1}{x_{12}^2}\theta\left(1/m^2 - x_{12}^2\right) - 2\frac{x_{02}\cdot x_{12}}{x_{02}^2x_{12}^2}\theta\left(1/m^2 - x_{12}^2\right)\theta\left(1/m^2 - x_{02}^2\right)\right]$$

- Bessel function cut

$$K = \frac{dz}{z}\frac{N_c\alpha_s m^2}{2\pi^2}\left[K_1^2(mx_{02}) + K_1^2(mx_{12}) - 2K_1(mx_{02})K_1(mx_{12})\frac{x_{02}\cdot x_{12}}{x_{02}x_{12}}\right]$$

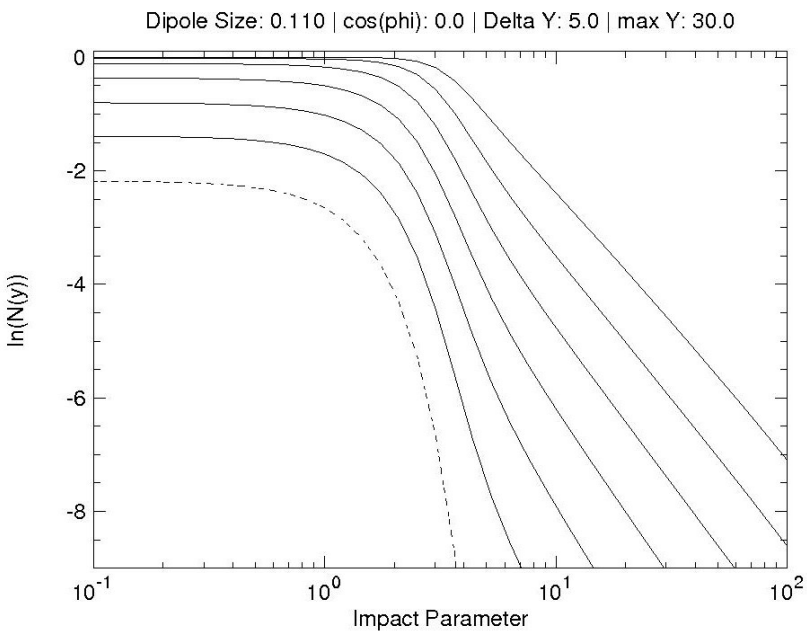
- Running coupling with theta function

$$K = \frac{dz}{z}\frac{N_c\alpha_s(x_{01}^2)}{2\pi^2}\left[\frac{x_{01}^2}{x_{02}^2x_{12}^2} + \frac{1}{x_{02}^2}\left(\frac{\alpha_s(x_{02}^2)}{\alpha_s(x_{12}^2)} - 1\right) + \frac{1}{x_{12}^2}\left(\frac{\alpha_s(x_{12}^2)}{\alpha_s(x_{02}^2)} - 1\right)\right]\theta\left(1/m^2 - x_{12}^2\right)\theta\left(1/m^2 - x_{02}^2\right)$$

- Modified kernel with theta function

$$K = \frac{dz}{z}\frac{N_c\alpha_s}{2\pi^2}\frac{z}{x_{01}^2}\left[K_1^2\left(\frac{x_{02}}{x_{01}}\sqrt{z}\right) + K_1^2\left(\frac{x_{12}}{x_{01}}\sqrt{z}\right) - \frac{2x_{02}\cdot x_{12}}{x_{02}x_{12}}K_1\left(\frac{x_{02}}{x_{01}}\sqrt{z}\right)K_1\left(\frac{x_{12}}{x_{01}}\sqrt{z}\right)\right]\theta\left(1/m^2 - x_{12}^2\right)\theta\left(1/m^2 - x_{02}^2\right)$$

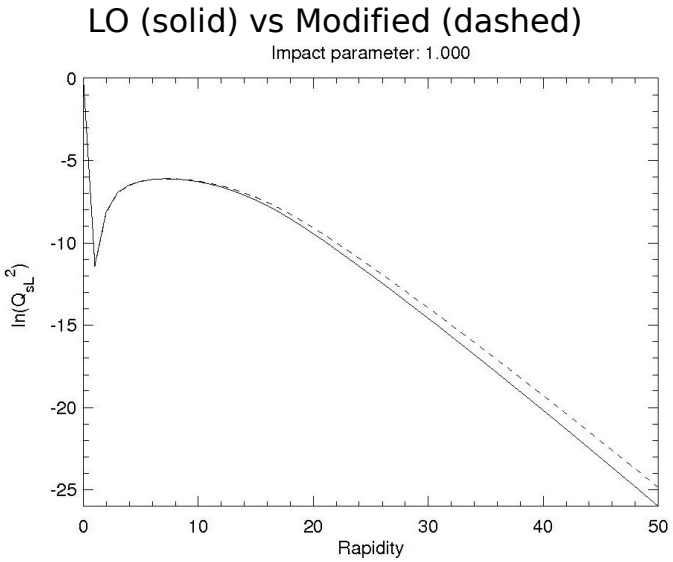
Impact parameter tails



- Power-like tails are generated during the evolution
- Initial impact parameter dependence $N = 1 - e^{-x^2 e^{-b^2}}$ is quickly forgotten

- There is a clear ‘ankle’ where dependence of the amplitude on impact parameter become power-like

A Second Saturation Scale



$$\left\langle N\left(r = 1/Q_{sL}(b,Y), b, \theta, Y\right)\right\rangle = 0.5$$

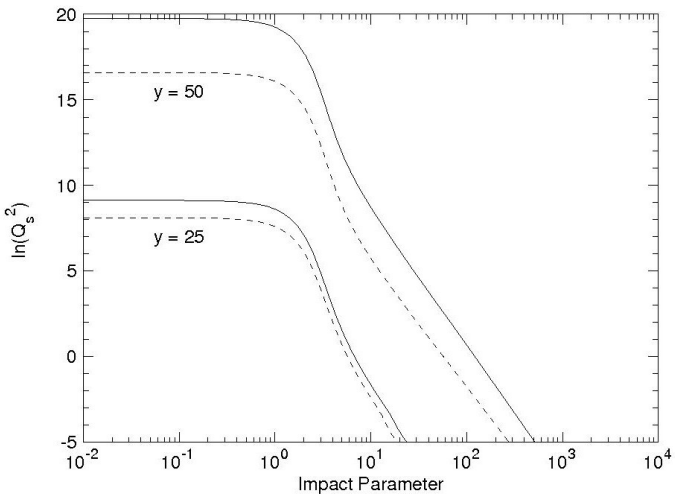
Equation has two solutions now! Same Parameterization

$$Q_{sL}^2(b,Y) = Q_{0L}^2 e^{-\bar{\alpha}_s \lambda_{sL} Y} S_L(b)$$

	LO	Modified
λ_{sL}	6.0	5.8 $\bar{\alpha}_s = 0.1$ (5.2 $\bar{\alpha}_s = 0.2$)

- Larger dipole sizes have slightly different saturation scale exponents
  - More thinking to be done on this result…

Saturation Scale – B dependence



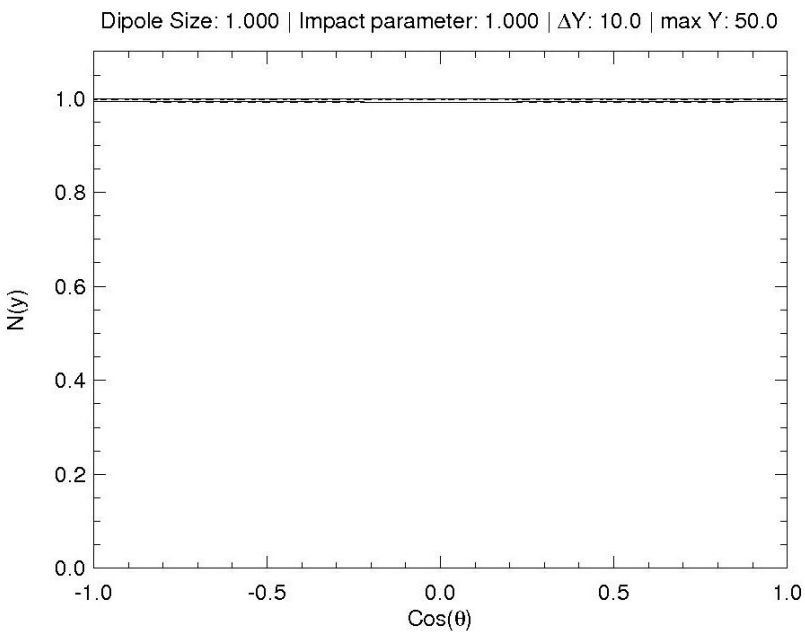
$$\left\langle N\left(r = 1/Q_s(b,Y), b, \theta, Y\right)\right\rangle = 0.5$$

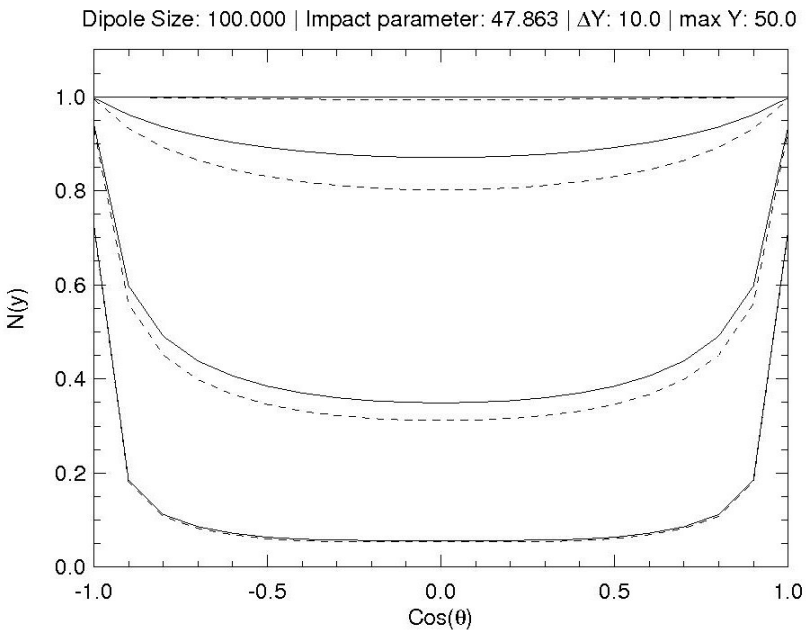
Saturation scale was found to have the same impact parameter dependence at large b which leads us to a factorized form

$$Q_s^2(b,Y) = Q_0^2 e^{\bar{\alpha}_s \lambda_s Y} S(b)$$

- Large impact parameters yield similar slopes with similar dependence

Angular Dependence

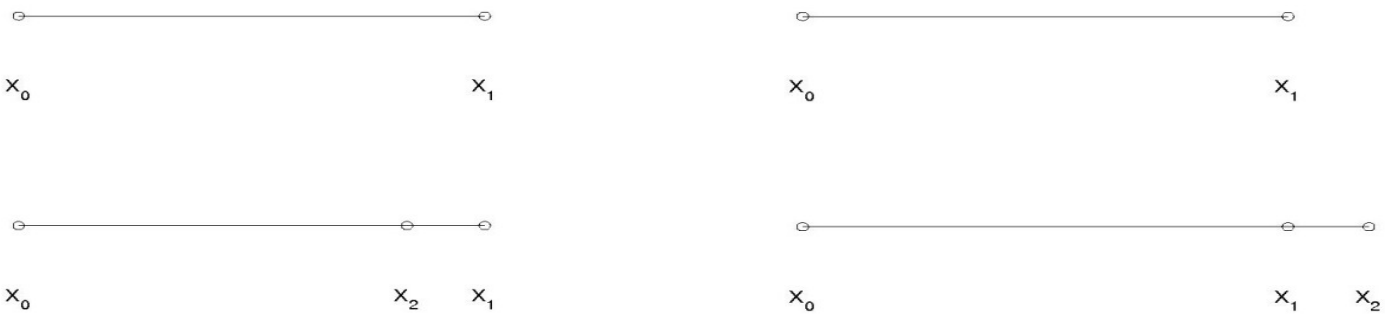




- Angular dependence only comes in when x = 2b
- Enhancements when $\cos(\theta) = +1, -1$

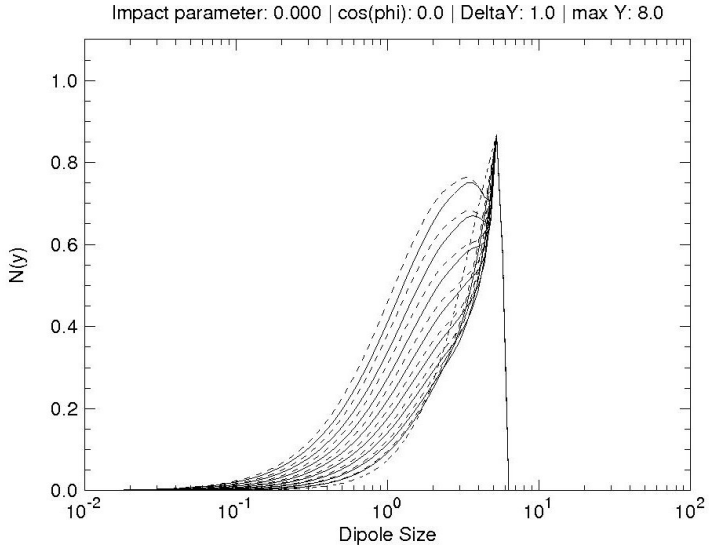
Unusual slowness of the coupling

- Naive analysis leads us to believe the equivalence of the minimum dipole size coupling and Balitsky's
- Numerical analysis reveals this not to be true



When one daughter dipole is small there are regions where one prescription dominates when $\cos(\theta) = +1$ [left] the minimum dipole size method dominates while when $\cos(\theta) = -1$ [right] the Balitsky prescription for running coupling dominates, however these regions are not equal in BK.

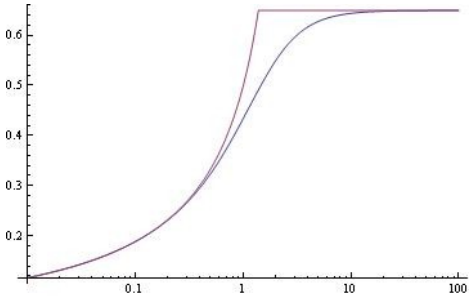
Surprising behaviors of Balitsky's kernel



$$\alpha_s(x^2) = \frac{1}{b\ln\left(\frac{1}{\Lambda^2}\left(\frac{1}{x^2}+\mu^2\right)\right)}$$

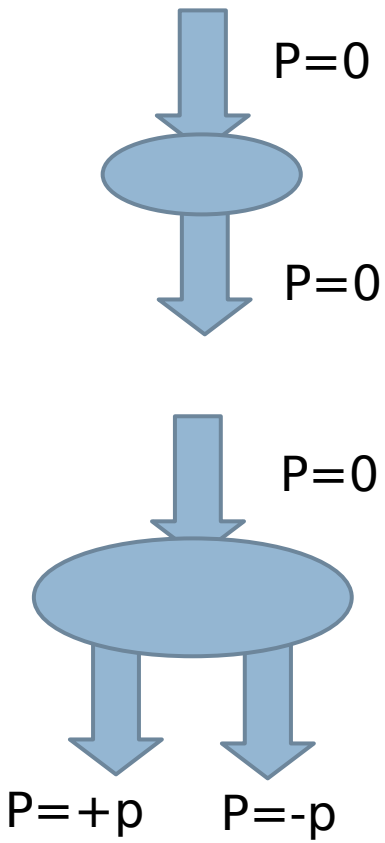
Using a μ factor to regularize the coupling or a sharp cutoff was found to change the amplitude by much more than expected (a factor of 2 or more in some cases), indicating a great sensitivity to the specific form the coupling takes.

Increasing the μ decreases the coupling but in the case of the Balitsky kernel this increases the amplitude



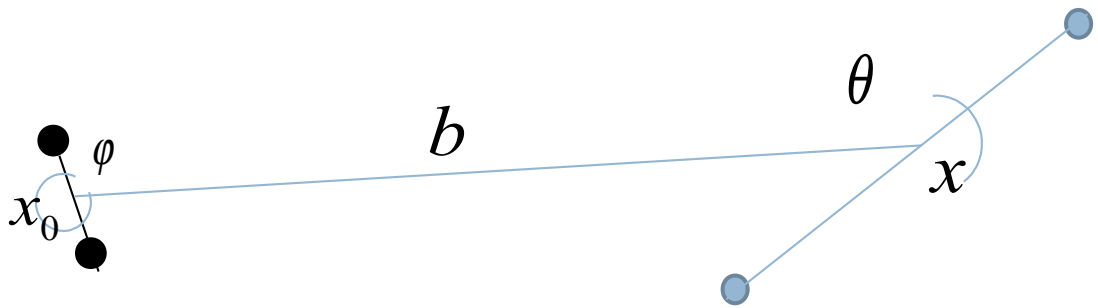
Impact Parameter is so important!

- Impact parameter corresponds to momentum transfer, neglecting impact parameter is equivalent to setting momentum transfer ALT 0
- With BFKL this is self consistent
  - Only linear terms (two pomeron vertex)
- This assumption with BK causes problems
  - Nonlinear term (three pomeron vertex)
  - Momentum transfer cannot stay zero without altering the interaction



Conformal Symmetry?

- LO Kernel is conformally invariant
- Expect evolution in small dipole and large dipole directions to be the same
- Additional angular dependence? Numerics say no dice



- Need higher order corrections?