Diffractive Vector Meson Cross Sections from BK evolution with Impact Parameter

Jeffrey Berger
Contents

- BK with impact parameter
  - Dipole Model
  - Confinement effects
- Vector Meson Production
  - Phenomenological corrections
  - Differential and Integrated cross sections
- $F_2$ Structure function
In the dipole model of small $x$ scattering, the evolution of the amplitude can be represented as a dipole cascade.

Photon splits into a color dipole of size $r$ which interacts at impact parameter $b$ with the target (nucleon)

Color dipole interacts with partons of the target through gluon exchange

$N(r,b,Y)$ is the scattering amplitude of the dipole interaction


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The BK equation

\[ \frac{\partial N_{01}}{\partial Y} = \int d^2 x_2 \ K \left[ N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right] \]

- Enforces unitarity in the amplitude
  \[ N_{ij} = N(x_{ij}, b_{ij}, \vartheta_{ij}, Y) \]
- Parent dipole \( x_{01} = x_0 - x_1 \) splits into two dipoles of \( x_{02} \) and \( x_{12} \)
- Splitting is determined by the kernel \( K = K(x_{01}, x_{02}, x_{12}) \)
- Impact parameter \( b_{ij} = \frac{1}{2} (x_i + x_j) \) only dependence is in the amplitude
- Angle \( \vartheta_{ij} \) is the angle between \( x_{ij} \) and \( b_{ij} \)
- We take the full dependence of the amplitude on impact parameter into account
Large dipole sizes must be regulated
  - Saturation regime does not provide self-regulation as it does without impact parameter

\[
K = \bar{\alpha}(x_{01}^2) \left[ \frac{1}{x_{02}^2} \left( \frac{\alpha(x_{02}^2)}{\alpha(x_{12}^2)} - 1 \right) + \frac{1}{x_{12}^2} \left( \frac{\alpha(x_{12}^2)}{\alpha(x_{02}^2)} - 1 \right) + \frac{x_{01}^2}{x_{12}^2 x_{02}^2} \right] \times \Theta\left(\frac{1}{m^2} - x_{02}^2\right) \Theta\left(\frac{1}{m^2} - x_{12}^2\right)
\]

[I. Balitsky, Phys. Rev. D75, 014001 (2007)]

\[m = 0.37 \text{GeV}\]

Large dipole sizes must be cut off at every step of the evolution
  - If only the initial condition is cut off then large dipoles will be generated during the evolution
Initial Conditions

- We choose a Glauber-Mueller form for the initial condition

\[ N(r, b, Y_0) = 1 - \exp \left( \frac{-\pi}{2N_c} r^2 x g(x, \eta^2) T(b) \right) \quad T(b) = \frac{1}{8\pi} \exp \left( \frac{-b^2}{2B_G} \right) \]

- Initial rapidity taken to be \( Y_0 = \ln(1/x_0) \) with \( x_0 = 10^{-2} \)

- Large dipole sizes are cut off when \( r \geq \frac{2}{m} \)

- This choice gives a fixed cutoff that does not shift as the cutoff in the evolution is not on the parent dipole but on the daughter dipoles

\[ B_G = \frac{1}{2m^2} \]
We utilize the NNPZ prescription for the vector meson wavefunction.

Parameters in WF constrained by normalization and lepton decay widths.

\[
\frac{d \sigma}{dt} = \frac{1}{16 \pi} |A(x, \Delta, Q^2)|^2
\]

\[
A(x, \Delta, Q^2) = \sum_{h, \bar{h}} \int d^2 r \int dz \Psi_{h, \bar{h}}(r, z, Q^2) N(x, r, \Delta) \Psi^V_{h, \bar{h}}(r, z)
\]
Skewed gluon distribution

**Important in the initial condition of the evolution**

\[
(xg(x, \eta^2))_{sk} = xg(x, \eta^2) \frac{2^{2\lambda_{sk}+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda_{sk} + 5/2)}{\Gamma(\lambda_{sk} + 4)}
\]

\[
\lambda_{sk} \equiv \frac{\partial \ln(xg(x, \eta^2))}{\partial \ln(1/x)}
\]


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Photon Wavefunction Corrections

- Additional contribution from photon wavefunction at low $Q^2$ required
- Universal for all mesons and processes

$$\left| \Psi_\gamma \right|^2 \rightarrow \left| \Psi_\gamma \right|^2 \left( \frac{1 + Be^{-\omega^2(x_{01} - R)^2}}{1 + Be^{-\omega^2 R^2}} \right)$$

\[ \sigma_V \left( Q^2 + M_V^2 \right) \]

[H1 Collaboration JHEP 1005 (2010)]

[ZEUS Collaboration PMC Phys A1 (2007)]
\[ \sigma_V (Q^2 + M_V^2) \]

Good agreement with $\phi$ as well

Agreement with $\rho$ not as good

Good agreement with $J/\Psi$

[H1 Collaboration JHEP 1005 (2010)]

[ZEUS Collaboration PMC Phys A1 (2007)]
$\sigma(W)$
R ratio

- Ratio is sensitive to the form of the vector meson wavefunctions
- Is not strongly dependent on energy

\[
R = \frac{\sigma_L}{\sigma_T}
\]
$d\sigma/dt$ for $J/\Psi$
\[ B_D \left( Q^2 + M^2_V \right) \]

- Slope \( B_D \) is dimensionful
- Contains information on the distribution of the interaction region in transverse space

\[ \frac{d \sigma}{dt} \propto e^{-B_D |t|} \]
\[ B_D(W) \]

- Slope is a feature that is naturally inherent in our model and is sensitive to our cutoff mass \( b \propto \frac{1}{m^2} \)

- Intercept is sensitive to \( B_G \)

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\[ F_2(Q^2, x) = Q^2 4\pi^2 \alpha_{em} \int d^2 r \int_0^1 dz \times (|\Psi_T(r, z, Q^2)|^2 + |\Psi_L(r, z, Q^2)|^2) \times \sigma_{dipole}(r, x) \]

- Proton structure functions
- Photon WF correction included
- Overly strong growth
Conclusions

- Features of the HERA data on diffractive vector meson production are reproduced in both the differential and integrated cross sections.
- Phenomenological corrections for skewedness and non-perturbative effects in the photon wavefunction were found necessary.
- Good agreement is found in the data of $J/\Psi$ and $\phi$.
  - $\rho$ is consistently low, possibly requires an additional non-perturbative contribution.
- Diffusion in impact parameter space naturally produces slopes of $B_D(W)$. 

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Thank You

Special Thanks to: My Advisor Anna Stasto as well as Henry Kowalski for discussions and use of his code
Backup Slides
Features of BK with impact parameter

- Leading order kernel used
- Coupling fixed at \( \frac{N_c \alpha_s}{\pi} = 0.1 \)

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Large contributions at $x = 2b$

This behavior can be extracted from the representation in terms conformal eigenfunctions.

Nontrivial angular dependence.

Peak of the amplitude occurs when $x \parallel 2b$ and

- This behavior can be extracted from the representation in terms conformal eigenfunctions.

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Towards higher order

- Kinematical cut owing to a modification in the energy denominator
- The modified kernel slows the evolution by approximately 30%
- The modified kernel has almost no affect when the impact parameter dependence is neglected due to the saturation of all large dipole sizes.

This kernel reduces to the LO kernel at large rapidities or when $x_{01} > > x_{02}$.

$$K = \frac{dz}{z} \frac{N_c g_s}{2\pi^2} \frac{z}{x_{01}^2} \left[ K_1^2 \left( \frac{x_{02}}{x_{01}} \sqrt{z} \right) + K_1^2 \left( \frac{x_{12}}{x_{01}} \sqrt{z} \right) - \frac{2x_{02} \cdot x_{12}}{x_{02} \cdot x_{12}} K_1 \left( \frac{x_{02}}{x_{01}} \sqrt{z} \right) K_1 \left( \frac{x_{12}}{x_{01}} \sqrt{z} \right) \right]$$


LO (solid) vs Modified (dashed)
Saturation Scale

Saturation is when the parton density becomes large and recombination effects become important. Defined here as the amplitude becomes large and the nonlinear term becomes important.

Numbers are consistent with analytical estimates:

\[
\langle N(r = \sqrt{Q_s(b, Y)}, b, \theta , Y) \rangle = 0.5
\]

Saturation scale was found to have the same impact parameter dependence at large \( b \) which leads us to a factorized form:

\[
Q_s^2(b, Y) = Q_0^2 e^{\alpha_s \lambda_s Y} S(b) \quad S(b) \sim \frac{1}{b^4}
\]

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<td>( \lambda_s )</td>
<td>4.4</td>
<td>3.6 ( \alpha_s ) = 0.1 (2.5 ( \alpha_s ) = 0.2)</td>
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Several different prescriptions for running coupling

Balitsky

\[ K = \frac{dz}{z} \frac{N_c \alpha_s(x_{01}^2)}{2\pi^2} \left[ \frac{x_{01}^2}{x_{02} x_{12}^2} + \frac{1}{x_{02}^2} \left( \frac{\alpha_s(x_{02}^2)}{\alpha_s(x_{12}^2)} - 1 \right) + \frac{1}{x_{12}^2} \left( \frac{\alpha_s(x_{12}^2)}{\alpha_s(x_{02}^2)} - 1 \right) \right] \]

[I. Balitsky, Phys. Rev. D75, 014001 (2007)]

Kovchegov-Weigert

\[ K = \frac{dz}{z} \frac{N_c}{2\pi^2} \left[ \frac{1}{x_{20}^2} \alpha_s \left( 4e^{-\frac{5\gamma}{27}} \right) + \frac{1}{x_{12}^2} \alpha_s \left( 4e^{-\frac{5\gamma}{27}} \right) \right] - 2 \frac{x_{12} \cdot x_{20}}{x_{20} x_{12}^2} \]

\[ \alpha_s \left( \frac{4e^{-\frac{5\gamma}{27}}}{x_{20}^2} \right) \alpha_s \left( \frac{4e^{-\frac{5\gamma}{27}}}{x_{20}^2} \right) \]

\[ \alpha_s \left( \frac{4e^{-\frac{5\gamma}{27}}}{x_{12}^2} \right) \alpha_s \left( \frac{4e^{-\frac{5\gamma}{27}}}{x_{12}^2} \right) \]


Parent Dipole

\[ K = \frac{dz}{z} \frac{N_c \alpha_s(x_{01}^2)}{2\pi^2} \left[ \frac{x_{01}^2}{x_{02} x_{12}^2} \right] \]

Minimum Dipole

\[ K = \frac{dz}{z} \frac{N_c \alpha_s(\min(x_{01}^2, x_{12}^2, x_{02}^2))}{2\pi^2} \left[ \frac{x_{01}^2}{x_{02} x_{12}^2} \right] \]

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Results with running coupling

- IR regularization of the kernel is important due to large dipole evolution
- Balitsky’s running coupling is well slower than the minimum dipole prescription

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Diffusion in impact parameter

- Increasing energy causes the dense region of the dipole cascade to expand in impact parameter space.
- Size of the dense or ‘black’ region characterized by a radius of this black disk.
- Fast increase in is partially due to the lack of scale in the solution currently.

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\[
\langle N(r, B_s = b, \theta, Y) \rangle = 0.5
\]

Growth of the black disk corresponds to growth of the cross section

\[
B_s^2(r, Y) = B_{s0}^2 e^{\bar{\alpha}_s \lambda_{sB} Y} F(r) \quad \sigma \approx e^{2\lambda_{sB} Y}
\]

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<td>(\lambda_{sB})</td>
<td>2.6</td>
<td>2.2 (\bar{\alpha}_s = 0.1)</td>
</tr>
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</table>
Adding mass parameter

- Full cut with theta function
  \[ K = \frac{dz}{z}\frac{N_c a_s}{2\pi^2} x_{01}^2 \theta \left( \sqrt{m^2 - x_{02}^2} \right) \theta \left( \sqrt{m^2 - x_{12}^2} \right) \]

- Splitting the theta function
  \[ K = \frac{dz}{z}\frac{N_c a_s}{2\pi^2} \left[ \frac{1}{x_{02}^2} \theta \left( \sqrt{m^2 - x_{02}^2} \right) + \frac{1}{x_{12}^2} \theta \left( \sqrt{m^2 - x_{12}^2} \right) - 2 \frac{x_{02} \cdot x_{12}}{x_{02} x_{12}^2} \theta \left( \sqrt{m^2 - x_{12}^2} \right) \theta \left( \sqrt{m^2 - x_{02}^2} \right) \right] \]

- Bessel function cut
  \[ K = \frac{dz}{z}\frac{N_c a_s m^2}{2\pi^2} \left[ K_1^2\left( m x_{02} \right) + K_1^2\left( m x_{12} \right) - 2K_1\left( m x_{02} \right)K_1\left( m x_{12} \right) \frac{x_{02} \cdot x_{12}}{x_{02} x_{12}} \right] \]

- Running coupling with theta function
  \[ K = \frac{dz}{z}\frac{N_c a_s}{2\pi^2} \left[ \frac{x_{01}^2}{x_{02} x_{12}^2} + \frac{1}{x_{02}^2} \left( a_s(x_{12}^2) - 1 \right) + \frac{1}{x_{12}^2} \left( a_s(x_{02}^2) - 1 \right) \right] \theta \left( \sqrt{m^2 - x_{12}^2} \right) \theta \left( \sqrt{m^2 - x_{02}^2} \right) \]

- Modified kernel with theta function
  \[ K = \frac{dz}{z}\frac{N_c a_s}{2\pi^2} \frac{z}{x_{01}^2} \left[ K_1^2\left( \frac{x_{02} \sqrt{z}}{x_{01}} \right) + K_1^2\left( \frac{x_{12} \sqrt{z}}{x_{01}} \right) - 2\frac{x_{02} \cdot x_{12}}{x_{02} x_{12}} K_1\left( \frac{x_{02} \sqrt{z}}{x_{01}} \right) K_1\left( \frac{x_{12} \sqrt{z}}{x_{01}} \right) \right] \theta \left( \sqrt{m^2 - x_{12}^2} \right) \theta \left( \sqrt{m^2 - x_{02}^2} \right) \]
Impact parameter tails

- Power-like tails are generated during the evolution
- Initial impact parameter dependence $N = 1 - e^{-x^2} e^{-b^2}$ is quickly forgotten
- There is a clear ‘ankle’ where dependence of the amplitude on impact parameter become power-like

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Larger dipole sizes have slightly different saturation scale exponents

More thinking to be done on this result...

Equation has two solutions now! Same Parameterization

\[ Q_{sL}^2(b, Y) = Q_{0L}^2 e^{-\bar{a}_s \lambda_{sL}} S_L(b) \]

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<td>5.8</td>
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<td>( \bar{a}_s = 0.1 )</td>
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<td>(5.2 ( \bar{a}_s = 0.2 ))</td>
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</table>
Saturation Scale – B dependence

\[
\langle N(r = \frac{1}{Q_s(b,Y)}, b, \theta, Y) \rangle = 0.5
\]

Saturation scale was found to have the same impact parameter dependence at large \( b \) which leads us to a factorized form

\[
Q_s^2(b, Y) = Q_0^2 e^{\bar{\alpha}_s \lambda_s} Y S(b)
\]

- Large impact parameters yield similar slopes with similar dependence

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Angular Dependence

- Angular dependence only comes in when $x = 2b$
- Enhancements when $\cos(\theta) = +1, -1$

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Unusual slowness of the coupling

- Naive analysis leads us to believe the equivalence of the minimum dipole size coupling and Balitsky’s.
- Numerical analysis reveals this not to be true.

When one daughter dipole is small there are regions where one prescription dominates when \( \cos(\theta) = +1 \) [left] the minimum dipole size method dominates while when \( \cos(\theta) = -1 \) [right] the Balitsky prescription for running coupling dominates, however these regions are not equal in BK.

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Surprising behaviors of Balitsky’s kernel

Increasing the $\mu$ decreases the coupling but in the case of the Balitsky kernel this increases the amplitude.

Increasing the $\mu$ decreases the coupling but in the case of the Balitsky kernel this increases the amplitude.

\[ \alpha_s(x^2) = \frac{1}{b \ln \left( \frac{1}{\Lambda^2} \left( \frac{1}{x^2} + \mu^2 \right) \right)} \]

Using a $\mu$ factor to regularize the coupling or a sharp cutoff was found to change the amplitude by much more than expected (a factor of 2 or more in some cases), indicating a great sensitivity to the specific form the coupling takes.
Impact Parameter is so important!

- Impact parameter corresponds to momentum transfer, neglecting impact parameter is equivalent to setting momentum transfer $\text{ALT} \ 0$
- With BFKL this is self consistent
  - Only linear terms (two pomeron vertex)
- This assumption with BK causes problems
  - Nonlinear term (three pomeron vertex)
  - Momentum transfer cannot stay zero without altering the interaction
Conformal Symmetry?

- LO Kernel is conformally invariant
- Expect evolution in small dipole and large dipole directions to be the same
- Additional angular dependence? Numerics say no dice

- Need higher order corrections?

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