A holographic light-cone wavefunction for the ρ meson

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Aim

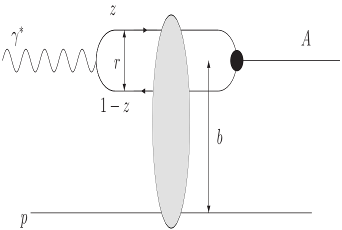
Compute rates for diffractive ρ production using an AdS/QCD holographic meson wavefunction with no free parameters and compare with current HERA data

Paper to appear on arXiv tomorrow : March 29 2012

Previous work on diffractive ρ production :
J. R. Forshaw and RS, JHEP 1110 :093, 2011
J. R. Forshaw and RS, JHEP 1011 :037,2010
J. R. Forshaw, RS and G. Shaw, PRD 69 :094013,2004.

Review article on the AdS/QCD correspondence and light-cone holography :
G. F. de Teramond and S. J. Brodsky (2012), 1203.4025.

Diffractive ρ production in the dipole model



- $A = \rho$
- r : transverse dipole size
- z : fraction of photon's light-cone momentum carried by quark

At high energy ($s \gg t, Q^2, M_\rho^2$), amplitude factorises

$$\Im\mathrm{m}\mathcal{A}_\lambda(s,t;Q^2) = \sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\mathrm{d}z \Psi^{\gamma^*,\lambda}_{h,\bar{h}} \Psi^{\rho,\lambda *}_{h,\bar{h}} e^{-iz\mathbf{r}.\mathbf{\Delta}} \mathcal{N}(x,\mathbf{r},\mathbf{\Delta})$$

Universal dipole cross-section

$$\hat{\sigma}(x,\mathbf{r}) = \mathcal{N}(x,\mathbf{r},\mathbf{0})/s$$

$\hat{\sigma}$ is well constrained by very precise F_2 HERA data

Dipole models

Color Glass Condensate (CGC)

Soyez (2007)

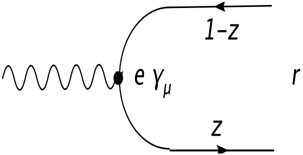
$$\mathcal{N}(rQ_s, x, 0) = \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left[\gamma_s + \frac{\ln(2/rQ_s)}{\kappa\lambda \ln(1/x)}\right]} \quad \text{for} \quad rQ_s \le 2$$

$$= \{1 - \exp[-a \ln^2(brQ_s)]\} \quad \text{for} \quad rQ_s > 2$$

Saturation scale $Q_s = (x_0/x)^{\lambda/2}$

- CGC[0.74] : anomalous dimension $\gamma_s = 0.74$ (fitted)
- Other dipole models which fit F_2 give similar results

Light cone wavefunctions



Photon

γ^{μ}

QED

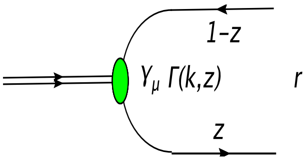
Spinor × Scalar

$$\Psi^{\gamma\{\lambda\}}_{h,\bar{h}}(\mathbf{k}, z; Q^2) \propto S^{\gamma,\lambda}_{h,\bar{h}}(\mathbf{k}, z) \times \phi_{\gamma}(\mathbf{k}, z; Q^2)$$

$$S^{\gamma,\lambda}_{h,\bar{h}}(\mathbf{k}, z) = \frac{\bar{u}_h(\mathbf{k})}{\sqrt{z}} \gamma^{\mu} \cdot \varepsilon^{\lambda}_{\mu} \frac{v_{\bar{h}}(-\mathbf{k})}{\sqrt{1-z}}$$

▷ Sensitive to phenomenological quark mass m_f as $Q^2 \to 0$
▷ Here $m_f = 0.14$ GeV as fixed in the fits to extract dipole cross-section from F_2

Light cone wavefunctions



Meson

$\gamma^{\mu}\Gamma(\mathbf{k}, z)$

QCD

Spinor × Scalar

$$\Psi^{\rho\{\lambda\}}_{h,\bar{h}}(\mathbf{k}, z) \propto S^{\rho,\lambda}_{h,\bar{h}}(\mathbf{k}, z) \times \phi(\mathbf{k}, z)$$

$$S^{\rho,\lambda}_{h,\bar{h}}(\mathbf{k}, z) = \frac{\bar{u}_h(\mathbf{k})}{\sqrt{z}} \gamma^{\mu} \cdot e^{\lambda}_{\mu} \frac{v_{\bar{h}}(-\mathbf{k})}{\sqrt{1-z}}$$

Model for scalar part

Light-cone wavefunctions

Longitudinal polarisation

$$\Psi^{L}_{h,\bar{h}} = \frac{1}{2\sqrt{2}}\delta_{h,-\bar{h}} \left(1 + \frac{m_f^2 - \nabla^2}{M_\rho^2\, z(1-z)}\right) \phi(z,r)$$

Impose current conservation

$$\Psi^{L}_{h,\bar{h}} \to \Psi^{L}_{h,\bar{h}} = \frac{1}{\sqrt{2}}\delta_{h,-\bar{h}}\, \phi(z,r)$$

Transverse polarisation

$$\Psi^{T=\pm}_{h,\bar{h}} = \pm[ie^{\pm i\theta}(z\delta_{h\pm,\bar{h}\mp} - (1-z)\delta_{h\mp,\bar{h}\pm})\partial_r + m_f\delta_{h\pm,\bar{h}\pm}]\frac{\phi(z,r)}{2z(1-z)}$$

Light-cone wavefunction

Brodsky and de Téramond (2009)

$$\phi(z,\zeta,\varphi) = \frac{\Phi(\zeta)}{\sqrt{2\pi\zeta}} f(z) \mathrm{e}^{iL\varphi}$$

Impact variable : transverse separation at equal light-cone time

$$\zeta = \sqrt{z(1-z)}r$$

Light-cone Schroedinger equation for transverse modes

$$\left(-\frac{\mathrm{d}^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\Phi(\zeta) = M^2\Phi(\zeta)$$

$U(\zeta)$ is the confining potential at equal light-front time

AdS/QCD correspondance

Impact variable ζ maps onto the fifth dimension z_5 in AdS space

$$\zeta \Leftrightarrow z_5$$

Schroedinger equation for spin J string modes in AdS space

$$\left(-\frac{d^2}{dz_5^2} - \frac{1-4L^2}{4z_5^2} + U(z_5)\right)\Phi(z_5) = M^2\Phi(z_5)$$

Soft wall dilaton field (A. Karch et al. (2006))

$$U(z_5) = \kappa^4 z_5^2 + 2\kappa^2(L+S-1)$$

Correctly reproduces Regge-like mass spectrum

$$M^2 = 4\kappa^2(n+L+S/2)$$

For vector mesons, $\kappa = 0.55$ GeV (best fit value)

AdS/QCD holographic ρ wavefunction

ρ meson : $n = L = 0$, $S = 1$

$$M_\rho^2 = 2\kappa^2 \qquad \Phi(\zeta) = \kappa\sqrt{2\zeta}\exp\left(-\frac{\kappa^2\zeta^2}{2}\right)$$

Compare pion form factor in LCQCD and in AdS space

$$f(z) = \mathcal{N}\sqrt{z(1-z)}$$

Final form of the holographic wavefunction

$$\phi(z,\zeta) = \mathcal{N}\sqrt{z(1-z)}\exp\left(-\frac{m_f^2}{2\kappa^2 z(1-z)}\right)\exp\left(-\frac{\kappa^2\zeta^2}{2}\right)$$

Normalization condition (leading Fock state dominance)

$$\int \mathrm{d}^2\mathbf{r}\,\mathrm{d}z\,|\phi(z,\zeta)|^2 = 1$$

Normalization issues

Our normalization condition

$$\sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\,\mathrm{d}z\, |\Psi^L_{h,\bar{h}}(r,z)|^2 = 1$$

Method A uses

$$\Psi^L_{h,\bar{h}}(r,z) = \frac{1}{\sqrt{2}}\delta_{h,-\bar{h}}\ \phi(z,\zeta)$$

Method B uses

$$\Psi^L_{h,\bar{h}}(r,z) = \frac{1}{2\sqrt{2}}\delta_{h,-\bar{h}}\left(1 + \frac{m_f^2 - \nabla^2}{M_\rho^2\ z(1-z)}\right)\phi(z,\zeta)\ ,$$

Normalization issues

Method A implies

$$\int \mathrm{d}^2\mathbf{r}\;\mathrm{d}z\;|\phi(z,\zeta)|^2 = 1$$

Method B implies

$$\int \mathrm{d}^2\mathbf{r}\;\mathrm{d}z\;|\phi(z,\zeta)|^2 \neq 1$$

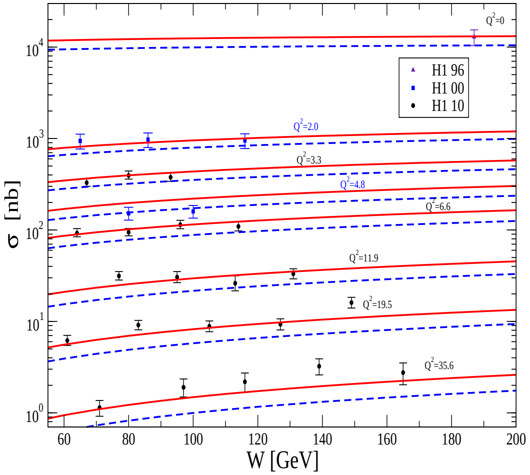
Small deviation from unity

Both methods imply

$$\sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\,\mathrm{d}z\,|\Psi^{T}_{h,\bar{h}}(r,z)|^2 \neq 1$$

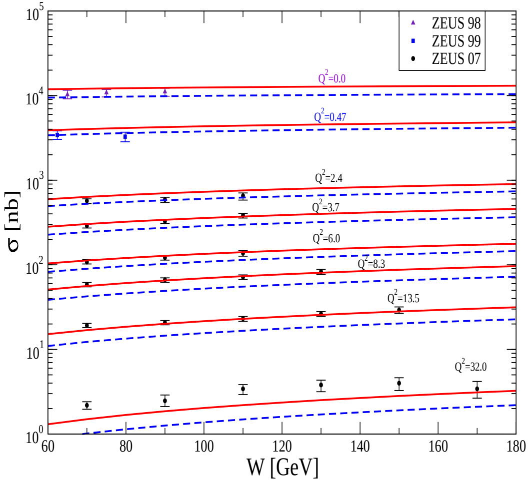
Small deviation from unity

Predictions with the AdS/QCD holographic wavefunction



- Blue : method A
- Red : method B
- No free parameters
- Expect to undershoot at high Q^2

Predictions with the AdS/QCD holographic wavefunction



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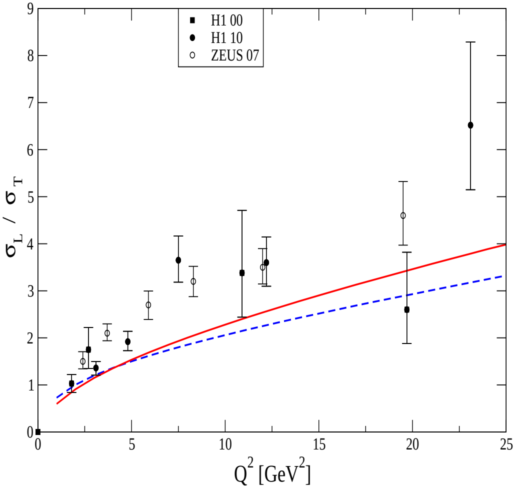

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Distribution Amplitude

DA is related to LCWF : J. R. Forshaw and RS (2010,11)

Method A

$$\varphi(z,\mu) = \left(\frac{N_c}{\pi}\right)^{1/2} \frac{1}{f_\rho} \int \mathrm{d}r \; \mu J_1(\mu r) \; \phi(z,\zeta)$$

Method B

$$\varphi(z,\mu) = \left(\frac{N_c}{\pi}\right)^{1/2} \frac{1}{2f_\rho} \int \mathrm{d}r \; \mu J_1(\mu r) \left(1 + \frac{m_f^2 - \nabla^2}{z(1-z)M_\rho^2}\right) \phi(z,\zeta)$$

Lowest moment

$$\langle \xi^2 \rangle_\mu = \int_0^1 \mathrm{d}z \xi^2 \varphi(z,\mu) \qquad \xi = 2z - 1$$

Moment of the twist-2 DA

Approach	Scale μ	$\langle \xi^2 \rangle_\mu$
Method A	~ 1 GeV	0.217
Method B	~ 1 GeV	0.228
Sum Rules	3 GeV	0.24 ± 0.02
Lattice	2 GeV	0.24 ± 0.04

Sum Rules : Ball, Braun and Lenz (2007)
Lattice : RBC Collaboration, P. A. Boyle et al. (2008)

Conclusions & Outlook

- Agreement with QCD Sum Rules and lattice predictions for the moment of the corresponding DA
- Parameter-free predictions agree reasonably well with HERA data
- Better fits possible by tuning quark mass and/or allowing the normalization to be polarization dependent
- Expect better agreement at high Q^2 if perturbative evolution is included in holographic wavefunction

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