Photon impact factor for BFKL pomeron at next-to-leading order

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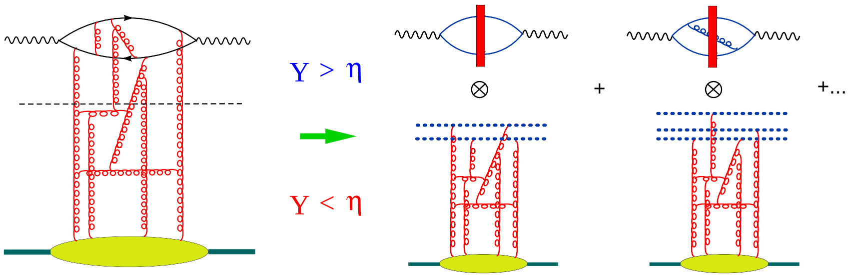
Outline

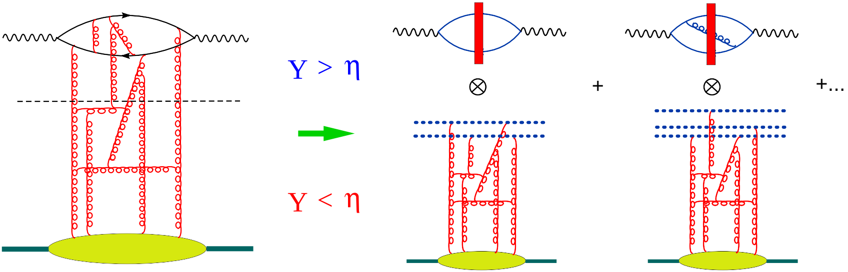
- High-energy scattering and Wilson lines formalism.
- Factorization in rapidity.
- NLO Photon Impact Factor in momentum representation.
- Linearized forward NLO BK equation in momentum space.
- Conclusions.



η - rapidity factorization scale

Rapidity Y > η - coefficient function ("impact factor")
Rapidity Y < η - matrix elements of (light-like) Wilson lines with rapidity divergence cut by η

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$
$$+\int d^2z_1 d^2z_2 d^2z_3\ I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$



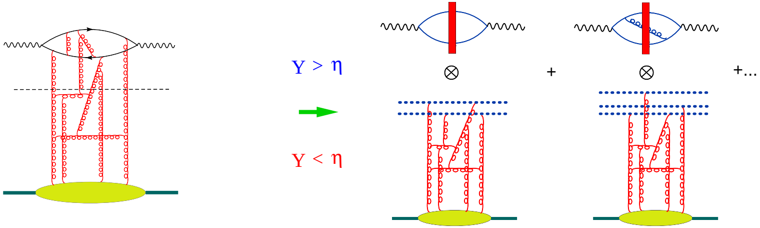
η - rapidity factorization scale

Evolution equation for color dipoles

$$\frac{d}{d\eta}\mathrm{tr}\{U^\eta_x U^{\dagger\eta}_y\} = \frac{\alpha_s}{2\pi^2}\int d^2z \frac{(x-y)^2}{(x-z)^2(y-z)^2}[\mathrm{tr}\{U^\eta_x U^{\dagger\eta}_y\}\mathrm{tr}\{U^\eta_x U^{\dagger\eta}_y\} - N_c\mathrm{tr}\{U^\eta_x U^{\dagger\eta}_y\}] + \alpha_s K_{\mathrm{NLO}}\mathrm{tr}\{U^\eta_x U^{\dagger\eta}_y\} + O(\alpha_s^2)$$

(Linear part of K_{NLO} = $K_{\mathrm{NLO\ BFKL}}$)

Expansion of $F_2(x)$ in color dipoles in the next-to-leading order



$$F_2(x_B) \simeq \int d^2z_1 d^2z_2 \; I^{LO}(z_1,z_2)\langle \mathrm{tr}\{U^{\eta}_{z_1}U^{\dagger\,\eta}_{z_2}\}\rangle \qquad \eta = \ln\frac{1}{x_B}$$

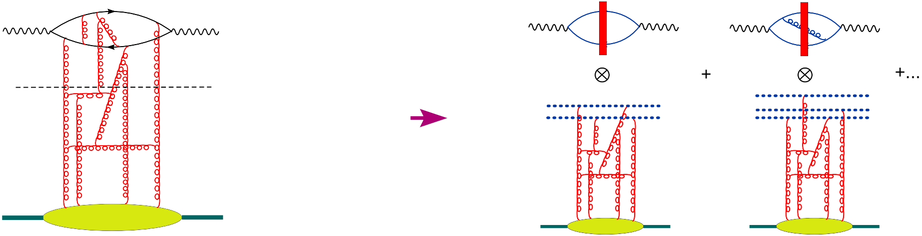
$$+\frac{\alpha_s}{\pi}\int d^2z_1 d^2z_2 d^2z_3 \; I^{NLO}(z_1,z_2,z_3)\langle \mathrm{tr}\{U^{\eta}_{z_1}U^{\dagger\eta}_{z_3}\}\mathrm{tr}\{U_{z_3}U^{\dagger\eta}_{z_2}\}\rangle$$

plan

- Calculate the NLO photon impact factor.
- Calculate the NLO evolution of color dipole.
- Convolute the solution with the initial conditions for the evolution and get the amplitude.

Propagation in the shock wave: Wilson line (Spectator frame)

Each path is weighted with the gauge factor $Pe^{ig\int dx_\mu A^\mu}$. Since the external field exists only within the infinitely thin wall, quarks and gluons do not have time to deviate in the transverse direction $\Rightarrow$ we can replace the gauge factor along the actual path with the one along the straight-line path.



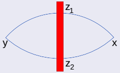
Plan for the calculation of the momentum space impact factor

- Apply the Operator product expansion up to NLO in α_s to get the the NLO Impact factor in coordinate representation.
- Obtain the Mellin rpresentation of the NLO Impact Factor for unpolarized (conformal spin $n = 0$) and for polarized (conformal spin $n = 2$) structure function.
- Perform the Fourier transform in momentum space and put the result in the k_{T}-factorization form.

Gauge invariance is checked at each step.

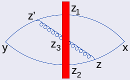
$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\, I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$
$$+ \int d^2z_1 d^2z_2 d^2z_3\, I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

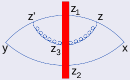
LO Impact Factor diagram: $I^{\rm LO}$



$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\, I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$
$$+ \int d^2z_1 d^2z_2 d^2z_3\, I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

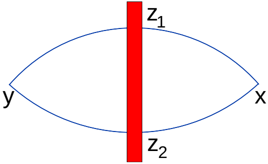
NLO Impact Factor diagrams: $I^{\rm NLO}$





LO Impact Factor

Conformal invariance: $(x^+, x_\perp)^2 = -x_\perp^2 \Rightarrow$ after the inversion $x_\perp \to x_\perp/x_\perp^2$ and $x^+ \to x^+/x_\perp^2$



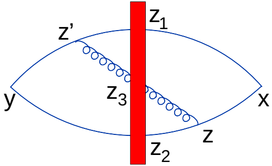
Conformal vectors:

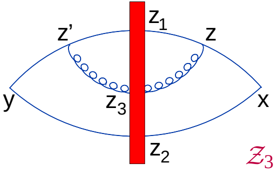
$$\kappa = \frac{\sqrt{s}}{2x_*}\left(\frac{p_1}{s} - x^2 p_2 + x_\perp\right) - \frac{\sqrt{s}}{2y_*}\left(\frac{p_1}{s} - y^2 p_2 + y_\perp\right)$$

$$\zeta_1 = \left(\frac{p_1}{s} + z_{1\perp}^2 p_2 + z_{1\perp}\right), \qquad \zeta_2 = \left(\frac{p_1}{s} + z_{2\perp}^2 p_2 + z_{2\perp}\right)$$

Here $x^2 = -x_\perp^2$, $x_* \equiv x_\mu p_2^\mu$ (similarly for y); $\mathcal{R} = \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$

$$I_{\mu\nu}^{\rm LO}(z_1, z_2) = \frac{\mathcal{R}^2}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)} \frac{\partial^2}{\partial x^\mu \partial y^\nu}\left[(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) - \frac{1}{2}\kappa^2(\zeta_1\cdot\zeta_2)\right]$$





$$\mathcal{Z}_3 \equiv \frac{(x-z_3)^2_\perp}{x^+} - \frac{(y-z_3)^2_\perp}{y^+}$$

$$I^{\rm NLO}_{\mu\nu}(x,y;z_1,z_2,z_3;\eta) \;=\; -\,I^{\rm LO}_{\mu\nu} \times \frac{\alpha_s}{2\pi}\frac{z^2_{13}}{z^2_{12}z^2_{23}}\ln\frac{\sigma s}{4}\mathcal{Z}_3 \;+\; {\rm conf.}$$

The NLO impact factor is not Möbius invariant $\Rightarrow$ the color dipole with the cutoff $\eta = \ln\sigma$ is not invariant.

However, if we define a composite operator (a - analog of μ^{-2} for usual OPE)

$$[{\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf} \;=\; {\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

$$+\;\frac{\alpha_s}{4\pi}\int d^2z_3\;\frac{z^2_{12}}{z^2_{13}z^2_{23}}[\frac{1}{N_c}{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - {\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]\ln\frac{az^2_{12}}{z^2_{13}z^2_{23}} \;+\; O(\alpha_s^2)$$

the impact factor becomes conformal in the NLO.

Operator expansion in conformal dipoles

Analogy:

When the UV cutoff does not respect the symmetry of a local operator, the composite local renormalized operator must be corrected by finite counter-terms order by order in perturbation theory.

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\, I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm tr}[\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf}$$

$$+ \int d^2z_1 d^2z_2 d^2z_3\, I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[\frac{1}{N_c}{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - {\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

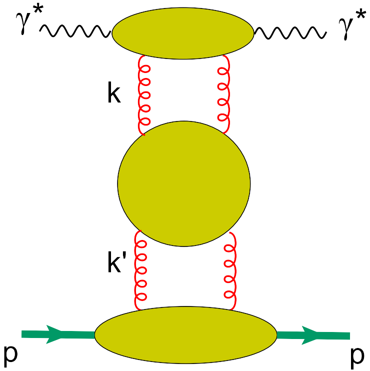
$$I^{\rm NLO}_{\mu\nu} = -I^{\rm LO}_{\mu\nu}\,\frac{\alpha_s N_c}{4\pi}\int dz_3 \frac{z^2_{12}}{z^2_{13}z^2_{23}}\ln\frac{z^2_{12}e^{2\eta}as^2}{z^2_{13}z^2_{23}}\mathcal{Z}^2_3 + {\rm conf.}$$

The new NLO impact factor is conformally invariant.

In conformal $\mathcal{N}=4$ SYM theory one can construct the composite conformal dipole operator order by order in perturbation theory.

Fourier transformation for the impact factors in the forward case corresponding to deep inelastic scattering at $x_B = \frac{Q^2}{2p\cdot q}$

$$\int d^4x\, e^{iqx} \int d^4z\, \delta(z_\bullet) T\{\hat{j}_\mu(x+z)\hat{j}_\nu(z)\}$$



I. Balitsky and G. A. C. 2010

$$\Delta \equiv (x-y), \qquad x_* = x^+\sqrt{s/2}, \qquad y_* = y^+\sqrt{s/2}, \qquad R \equiv \frac{\Delta^2 z_{12\perp}^2}{x_* y_* \mathcal{Z}_1 \mathcal{Z}_2}$$

$$I^{NLO}_{\mu\nu}(x,y) = \frac{\alpha_s}{4\pi^7\Delta^4}\frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\int\frac{dz_1 dz_2}{z_{12}^4}\,\mathcal{U}(z_1,z_2)R^2\Big\{-\frac{2}{\kappa^2}\Big(g^{\alpha\beta}-2\frac{\kappa^\alpha\kappa^\beta}{\kappa^2}\Big)$$

$$+\frac{\zeta_1^\alpha\zeta_2^\beta+\zeta_1\leftrightarrow\zeta_2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[4\mathrm{Li}_2(1-R)-\frac{2\pi^2}{3}+\frac{2\ln R}{1-R}+\frac{\ln R}{R}-4\ln R+\frac{1}{2R}-2-4C-\frac{2C}{R}$$

$$+2\big(\ln\frac{1}{R}+\frac{1}{R}-2\big)\big(\ln\frac{1}{R}+2C\big)\Big]+\Big(\frac{\zeta_1^\alpha\zeta_1^\beta}{(\kappa\cdot\zeta_1)^2}+\zeta_1\leftrightarrow\zeta_2\Big)\Big[\frac{\ln R}{R}-\frac{2C}{R}+2\frac{\ln R}{1-R}-\frac{1}{2R}\Big]$$

$$+\Big[-2\frac{\ln R}{1-R}-\frac{\ln R}{R}+\ln R-\frac{3}{2R}+\frac{5}{2}+2C+\frac{2C}{R}\Big]\Big[\frac{\zeta_1^\alpha\kappa^\beta+\zeta_1^\beta\kappa^\alpha}{(\kappa\cdot\zeta_1)\kappa^2}+\zeta_1\leftrightarrow\zeta_2\Big]$$

$$+\frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2)}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[\frac{2\pi^2}{3}-4\mathrm{Li}_2(1-R)-2\big(\ln\frac{1}{R}+\frac{1}{R}+\frac{1}{2R^2}-3\big)\big(\ln\frac{1}{R}+2C\big)$$

$$+6\ln R-\frac{2}{R}+2+\frac{3}{2R^2}\Big]\Big\}$$

Photon Impact Factor at NLO

Conformal vectors

$$\kappa^\mu = \frac{\sqrt{s}}{2x_*}\left(\frac{p_1^\mu}{s} - x^2 p_2^\mu + x_\perp^\mu\right) - \frac{\sqrt{s}}{2y_*}\left(\frac{p_1^\mu}{s} - y^2 p_2^\mu + y_\perp^\mu\right)$$

$$\zeta_1^\mu = \left(\frac{p_1^\mu}{s} + z_{1\perp}^2 p_2^\mu + z_{1\perp}^\mu\right), \qquad \zeta_2^\mu = \left(\frac{p_1^\mu}{s} + z_{2\perp}^2 p_2^\mu + z_{2\perp}^\mu\right)$$

DIS photon impact factor is a linear combination of the following tensor basis

$$\mathcal{I}_1^{\mu\nu} = g^{\mu\nu} \qquad \mathcal{I}_2^{\mu\nu} = \frac{\kappa^\mu \kappa^\nu}{\kappa^2}$$

$$\mathcal{I}_3^{\mu\nu} = \frac{\kappa^\mu \zeta_1^\nu + \kappa^\nu \zeta_1^\mu}{\kappa \cdot \zeta_1} + \frac{\kappa^\mu \zeta_2^\nu + \kappa^\nu \zeta_2^\mu}{\kappa \cdot \zeta_2}$$

$$\mathcal{I}_4^{\mu\nu} = \frac{\kappa^2 \zeta_1^\mu \zeta_1^\nu}{(\kappa \cdot \zeta_1)^2} + \frac{\kappa^2 \zeta_2^\mu \zeta_2^\nu}{(\kappa \cdot \zeta_2)^2} \qquad \mathcal{I}_5^{\mu\nu} = \frac{\zeta_1^\mu \zeta_2^\nu + \zeta_2^\mu \zeta_1^\nu}{\zeta_1 \cdot \zeta_2}$$

Cornalba, Costa, Penedones (2010)

$$\int \frac{d^2z_1 d^2z_2}{z_{12}^2} I_{LO}^{\mu\nu}(z_1,z_2)\Big(\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\Big)^\gamma = \frac{1}{\Delta^2 x_* y_*} B(1-\gamma)\Gamma(\gamma+2)\Gamma(3-\gamma)$$
$$\times\Big\{\frac{\gamma(1-\gamma)D_1^{\mu\nu}}{12(1+\gamma)(2-\gamma)} + \frac{D_2^{\mu\nu}}{2(1+\gamma)(2-\gamma)} - \frac{D_3^{\mu\nu}}{8(1+\gamma)(2-\gamma)}$$
$$-\frac{\gamma(1-\gamma)D_4}{16(1+2\gamma)(3-2\gamma)(1+\gamma)(2-\gamma)} - \frac{D_1^{\mu\nu}+D_2^{\mu\nu}}{8}\Big\}_{\mu\nu}\Big(\frac{\kappa^2}{(\kappa\cdot\zeta_0)^2}\Big)^\gamma$$

where

$$(D_1+D_2)^{\mu\nu} = -2\Delta^2 x_* y_* \kappa^{-2}\partial_x^\mu\partial_y^\nu\kappa^2$$
$$D_2^{\mu\nu} = -\Delta^2 x_* y_* \partial_x^\mu(\ln\kappa^2)\partial_y^\nu \ln\kappa^2$$
$$D_3^{\mu\nu} = 4\gamma\Delta^2 x_* y_*\big[(\partial_x^\mu \ln\kappa^2)\partial_\nu^y \ln(\kappa\cdot\zeta_0) + (\partial_y^\nu \ln\kappa^2)\partial_\mu^x \ln(\kappa\cdot\zeta_0) - (\partial_x^\mu \ln\kappa^2)\partial_y^\nu \ln\kappa^2\big]$$
$$D_4^{\mu\nu} = 4\gamma(1+2\gamma)\Delta^2 x_* y_*\big[-\frac{1}{3}\partial_x^\mu\partial_y^\nu \ln\kappa^2 - \partial_x^\mu(\ln\kappa^2)\partial_y^\nu\ln\kappa^2$$
$$+(\partial_x^\mu \ln\kappa^2)\partial_\nu^y \ln(\kappa\cdot\zeta_0) + (\partial_y^\nu\ln\kappa^2)\partial_\mu^x\ln(\kappa\cdot\zeta_0) - 2\partial_\mu^x\ln(\kappa\cdot\zeta_0)\partial_\nu^y\ln(\kappa\cdot\zeta_0)\big]$$

Mellin representation of the NLO photon impact factor

Conformal spin 0: NLO impact factor for the unpolarized forward structure functions

$$\int \frac{d^2z_1 d^2z_2}{z_{12}^4} I^{\mu\nu}_{NLO}(z_1,z_2)\left(\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\right)^{\gamma} = \alpha_s \frac{B(1-\gamma)\Gamma(3-\gamma)\Gamma(2+\gamma)}{(2-\gamma)(1+\gamma)} \times$$

$$\left\{ \frac{D_1^{\mu\nu}}{3\sin^2(\gamma\pi)}\left[(1-\cos(2\gamma\pi))\left(\chi - 1 - \gamma(1-\gamma)\left(C\chi - \frac{1}{2}\right)\right) - \gamma(1-\gamma)\frac{\pi^2}{3}(5+\cos(2\gamma\pi))\right]\right.$$

$$+D_2^{\mu\nu}\left[-\frac{3}{\gamma(1-\gamma)} + 2\chi\left(\frac{1}{\gamma(1-\gamma)} - 2C + 1\right) + \frac{4}{3}\pi^2(1 - \frac{3}{\sin^2(\gamma\pi)})\right]$$

$$+D_3^{\mu\nu}\left[C\chi - \frac{1}{2} - \frac{1}{\gamma(1-\gamma)} - \frac{\chi}{4}\left(1 + \frac{2}{\gamma(1-\gamma)}\right) - \frac{\pi^2}{3}(1 - \frac{3}{\sin^2(\gamma\pi)})\right]$$

$$+\frac{D_4^{\mu\nu}}{4[3+4\gamma(1-\gamma)]}\left[\frac{15}{\gamma(1-\gamma)} + 10 + \gamma(1-\gamma) - \chi - 2\gamma(1-\gamma)\left(C\chi - \frac{\pi^2}{3} + \frac{\pi^2}{\sin^2(\gamma\pi)}\right)\right]$$

$$+\frac{D_1^{\mu\nu} + D_2^{\mu\nu}}{[2+\gamma(1-\gamma)]^{-1}}\left[-\frac{1}{2} - \frac{\pi^2}{3} + \frac{\pi^2}{\sin^2\pi\gamma} + \frac{4\gamma(1-\gamma)+3}{2\gamma(1-\gamma)(1+\gamma)(2-\gamma)}\right.$$

$$\left.\left. + C\chi(\gamma) - \frac{1+2\gamma(1-\gamma)}{\gamma(1-\gamma)(1+\gamma)(2-\gamma)}\chi(\gamma)\right]\right\}$$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

C is the Euler constant. and $\gamma = \frac{1}{2} + i\nu$ and $\bar\gamma = 1 - \gamma$

Mellin representation of the NLO photon impact factor

Conformal spin 2: NLO impact factor for the polarized forward structure functions

$$\frac{N_c}{\pi^6(x-y)^4}\int\frac{d^2z_1d^2z_2}{z_{12}^4}\left[I^{\mu\nu}_{LO}(z_1,z_2)+I^{\mu\nu}_{NLO}(z_1,z_2)\right]\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^{\gamma}\left(\frac{\tilde z_{12}}{\tilde z_{10}\tilde z_{20}}\right)\left(\frac{\bar z_{12}}{\bar z_{10}\bar z_{20}}\right)^{-1}$$

$$= \ -\frac{N_c}{\pi^4\Delta^4}\frac{B(2-\gamma)}{2\Delta^2}\Gamma(3-\gamma)\Gamma(\gamma+2)\left(\frac{\Delta^2}{x_*y_*\mathcal{Z}_0^2}\right)^{\gamma}$$

$$\times\left[g^{\mu1}-ig^{\mu2}+2x^{\mu}\frac{\bar x y_*-\bar y x_*}{x_*y_*\mathcal{Z}_0}+p_2^{\mu}\frac{x^2\bar y-y^2\bar x}{x_*y_*\mathcal{Z}_0}\right]\left[g^{\nu1}-ig^{\nu2}+2y^{\nu}\frac{\bar x y_*-\bar y x_*}{x_*y_*\mathcal{Z}_0}+p_2^{\nu}\frac{x^2\bar y-y^2\bar x}{x_*y_*\mathcal{Z}_0}\right]$$

$$\times\ \left[1+\frac{\alpha_sN_c}{4\pi}\left\{\frac{4\pi^2}{\sin^2\pi\gamma}-\frac{4\pi^2}{3}+4C\chi(2,\gamma)-\frac{4}{\gamma^2}-\frac{4}{\bar\gamma^2}-2-6\frac{1+\chi(2,\gamma)}{2+\bar\gamma\gamma}\right\}\right]$$

$\chi(2,\gamma)=\chi(\gamma)-\frac{1}{\gamma(1-\gamma)}$ $\qquad \bar x = x^1 - ix^2$

Fourier transform of the Photon Impact Factor for BFKL pomeron

"energy scale" $a_0 = -\kappa^{-2}$ for color dipoles depends on x and y

$\hat{\mathcal{U}}_{a_0}$ in terms of $\hat{\mathcal{U}}_{a_m}$ with a_m independent of coordinates x and y. We choose $a_m = 1/x_B$. $\Rightarrow$ Impact Factor does not scale with energy

$$\hat{\mathcal{U}}_{a_0}(\nu, z_0) = \hat{\mathcal{U}}_{a_m}(\nu, z_0)\big(a_0 x_B\big)^{\frac{\omega(\nu)}{2}}$$
$$\hat{\mathcal{U}}^{(2)}_{a_0}(\nu, z_0) = \hat{\mathcal{U}}^{(2)}_{a_m}(\nu, z_0)\big(a_0 x_B\big)^{\frac{\omega(2,\nu)}{2}}$$

$$\frac{1}{N_c}\int d^4x d^4y\ \delta(y_\bullet) e^{iq\cdot(x-y)} T\{\bar\psi\gamma_\mu\psi(x)\bar\psi\gamma_\nu\psi(y)\}$$
$$= \int \frac{d\nu}{\pi^3}\int d^2z_0 \Big\{ \frac{\Gamma\big(\bar\gamma + \frac{\omega(\nu)}{2}\big)\Gamma^2\big(2-\gamma+\frac{\omega(\nu)}{2}\big)}{\Gamma(4-2\gamma+\omega(\nu))\Gamma\big(2+\gamma+\frac{\omega(\nu)}{2}\big)}\frac{2\gamma-1}{2\gamma+1}\Gamma(2-\gamma)$$
$$\Big[(\gamma\bar\gamma+2)P_1^{\mu\nu}\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\Phi_1(\nu)\Big) + (3\gamma\bar\gamma+2)P_2^{\mu\nu}\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\Phi_2(\nu)\Big)\Big]\mathcal{U}_{a_m}(z_0,\nu)$$
$$-\frac{\bar\gamma\Gamma(3-\gamma)\Gamma\big(\bar\gamma+\frac{\omega(2,\nu)}{2}\big)\Gamma^2\big(2-\gamma+\frac{\omega(2,\nu)}{2}\big)}{2\Gamma(4-2\gamma+\omega(2,\nu))\Gamma\big(2+\gamma+\frac{\omega(2,\nu)}{2}\big)}\Big[\bar{\mathcal{U}}^{(2)}_{a_m}(\nu,z_0)\bar P^{\mu\nu}+\tilde{\mathcal{U}}^{(2)}_{a_m}(\nu,z_0)\tilde P^{\mu\nu}\Big]$$
$$\times\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_6(\nu)\Big)\Big\}\frac{\Gamma^2(\bar\gamma)}{\Gamma(2\bar\gamma)}\frac{(Q^2)^{\gamma-1}\Gamma(2+\gamma)}{4^{\gamma+1}}$$

Fourier transform of the Photon Impact Factor for BFKL pomeron

"energy scale" $a_0 = -\kappa^{-2}$ for color dipoles depends on x and y

$\hat{\mathcal{U}}_{a_0}$ in terms of $\hat{\mathcal{U}}_{a_m}$ with a_m independent of coordinates x and y. We choose $a_m = 1/x_B$. $\Rightarrow$ Impact Factor does not scale with energy

$$\hat{\mathcal{U}}_{a_0}(\nu, z_0) \;=\; \hat{\mathcal{U}}_{a_m}(\nu, z_0)\big(a_0 x_B\big)^{\frac{\omega(\nu)}{2}}$$

$$\hat{\mathcal{U}}^{(2)}_{a_0}(\nu, z_0) \;=\; \hat{\mathcal{U}}^{(2)}_{a_m}(\nu, z_0)\big(a_0 x_B\big)^{\frac{\omega(2,\nu)}{2}}$$

$k_{\rm T}$-factorization form of the high-energy OPE

$$\int d^4x\, e^{iqx} \int d^4z\,\delta(z_\bullet)\langle p_B|T\{\hat{j}_\mu(x+z)\hat{j}_\nu(z)\}|p_B\rangle = \int d^2k_\perp I_{\mu\nu}(q, k_\perp)\langle\langle p_B|\mathcal{U}(k_\perp)|p_B\rangle\rangle$$

k_T-factorization form

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$$I^{\mu\nu}(q,k_\perp) = \frac{N_c}{32}\int\frac{d\nu}{\pi\nu}\frac{\sinh\pi\nu}{(1+\nu^2)\cosh^2\pi\nu}\left(\frac{k_\perp^2}{Q^2}\right)^{\frac{1}{2}-i\nu}$$
$$\Big\{\Big[\Big(\frac{9}{4}+\nu^2\Big)\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_1(\nu)\Big)P_1^{\mu\nu}+\Big(\frac{11}{4}+3\nu^2\Big)\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_2(\nu)\Big)P_2^{\mu\nu}\Big]$$
$$+\frac{\frac{1}{4}+\nu^2}{2k_\perp^2}\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_3(\nu)\Big)\left[\tilde{P}^{\mu\nu}\bar{k}^2+\bar{P}^{\mu\nu}\tilde{k}^2\right]\Big\}$$

$$P_1^{\mu\nu} = g^{\mu\nu}-\frac{q_\mu q_\nu}{q^2}$$
$$P_2^{\mu\nu} = \frac{1}{q^2}\Big(q^\mu-\frac{p_2^\mu q^2}{q\cdot p_2}\Big)\Big(q^\nu-\frac{p_2^\nu q^2}{q\cdot p_2}\Big)$$
$$\bar{P}^{\mu\nu} = \Big(g^{\mu1}-ig^{\mu2}-p_2^\mu\frac{\bar{q}}{q\cdot p_2}\Big)\Big(g^{\nu1}-ig^{\nu2}-p_2^\nu\frac{\bar{q}}{q\cdot p_2}\Big)$$
$$\tilde{P}^{\mu\nu} = \Big(g^{\mu1}+ig^{\mu2}-p_2^\mu\frac{\tilde{q}}{q\cdot p_2}\Big)\Big(g^{\nu1}+ig^{\nu2}-p_2^\nu\frac{\tilde{q}}{q\cdot p_2}\Big)$$

Photon Impact Factor for BFKL pomeron in momentum space

$$
\begin{aligned}
\mathcal{F}_{1(2)}(\nu) &= \Phi_{1(2)}(\nu) + \chi_\gamma \Psi(\nu), \quad \mathcal{F}_3(\nu) = F_6(\nu) + \Big(\chi_\gamma - \frac{1}{\bar\gamma\gamma}\Big)\Psi(\nu),\\
\Psi(\nu) &\equiv \psi(\bar\gamma) + 2\psi(2-\gamma) - 2\psi(4-2\gamma) - \psi(2+\gamma),\\
F_6(\gamma) &= F(\gamma) - \frac{2C}{\bar\gamma\gamma} - 1 - \frac{2}{\gamma^2} - \frac{2}{\bar\gamma^2} - 3\frac{1+\chi_\gamma - \frac{1}{\gamma\bar\gamma}}{2+\bar\gamma\gamma},\\
\Phi_1(\nu) &= F(\gamma) + \frac{3\chi_\gamma}{2+\bar\gamma\gamma} + 1 + \frac{25}{18(2-\gamma)} + \frac{1}{2\bar\gamma} - \frac{1}{2\gamma} - \frac{7}{18(1+\gamma)} + \frac{10}{3(1+\gamma)^2}\\
\Phi_2(\nu) &= F(\gamma) + \frac{3\chi_\gamma}{2+\bar\gamma\gamma} + 1 + \frac{1}{2\bar\gamma\gamma} - \frac{7}{2(2+3\bar\gamma\gamma)} + \frac{\chi_\gamma}{1+\gamma} + \frac{\chi_\gamma(1+3\gamma)}{2+3\bar\gamma\gamma},\\
F(\gamma) &= \frac{2\pi^2}{3} - \frac{2\pi^2}{\sin^2\pi\gamma} - 2C\chi_\gamma + \frac{\chi_\gamma - 2}{\bar\gamma\gamma}
\end{aligned}
$$

Non linear evolution equation: BK equation

$$U_z^{ab} = \mathrm{Tr}\{t^a U_z t^b U_z^\dagger\} \quad \Rightarrow (U_x U_y^\dagger)^{\eta_1} \to (U_x U_y^\dagger)^{\eta_2} + \alpha_s(\eta_1 - \eta_2)(U_x U_z^\dagger U_z U_y^\dagger)^{\eta_2}$$

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

BK equation: Ian Balitsky (1996), Yu. Kovchegov (1999)

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) \;=\; \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\;(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z) + \hat{\mathcal{U}}(z,y) - \hat{\mathcal{U}}(x,y) - \hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

Alternative approach: JIMWLK (1997-2000)

LLA for DIS in pQCD $\Rightarrow$ BFKL (LLA: $\alpha_s \ll 1,\ \alpha_s\eta \sim 1$)

LLA for DIS in sQCD $\Rightarrow$ BK eqn (LLA: $\alpha_s \ll 1,\ \alpha_s\eta \sim 1,\ \alpha_s^2 A^{1/3} \sim 1$)

(s for semi-classical)

NLO evolution of composite "conformal" dipoles in QCD

$$\frac{d}{d\eta}[\mathrm{tr}\{\hat U_{z_1} U^\dagger_{z_2}\}]^{\rm conf} = \frac{\alpha_s}{2\pi^2}\int d^2z_3 \Big([\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_2}\} - N_c\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\rm conf}$$

$$\times \frac{z_{12}^2}{z_{13}^2 z_{23}^2}\Big[1 + \frac{\alpha_s N_c}{4\pi}\Big(b\ln z_{12}^2\mu^2 + b\frac{z_{13}^2 - z_{23}^2}{z_{13}^2 z_{23}^2}\ln\frac{z_{13}^2}{z_{23}^2} + \frac{67}{9} - \frac{\pi^2}{3}\Big)\Big]$$

$$+ \frac{\alpha_s}{4\pi^2}\int\frac{d^2z_4}{z_{34}^4}\Big\{\Big[-2 + \frac{z_{14}{}^2 z_{23}^2 + z_{24}{}^2 z_{13}^2 - 4z_{12}^2 z_{34}^2}{2(z_{14}{}^2 z_{23}^2 - z_{24}{}^2 z_{13}^2)}\ln\frac{z_{14}{}^2 z_{23}^2}{z_{24}{}^2 z_{13}^2}\Big]$$

$$\times [\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_4}\}\{\hat U_{z_4}\hat U^\dagger_{z_2}\} - \mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\hat U_{z_4}\hat U^\dagger_{z_2}\hat U_{z_3}\hat U^\dagger_{z_4}\} - (z_4 \to z_3)]$$

$$+ \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}{}^2}\Big[2\ln\frac{z_{12}^2 z_{34}^2}{z_{14}^2 z_{23}^2} + \Big(1 + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\Big)\ln\frac{z_{13}^2 z_{24}{}^2}{z_{14}{}^2 z_{23}^2}\Big]$$

$$\times [\mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}\mathrm{tr}\{\hat U_{z_3}\hat U^\dagger_{z_4}\}\mathrm{tr}\{\hat U_{z_4}\hat U^\dagger_{z_2}\} - \mathrm{tr}\{\hat U_{z_1}\hat U^\dagger_{z_4}\hat U_{z_3}\hat U^\dagger_{z_2}\hat U_{z_4}\hat U^\dagger_{z_3}\} - (z_4 \to z_3)]\Big\}$$

$b = \frac{11}{3}N_c - \frac{2}{3}n_f$ I. Balitsky and G.A.C – PRD**77**, NPB**822**

$K_{\rm NLO\ BK}$ = Running coupling part + Conformal "non-analytic" (in j) part + Conformal analytic ($\mathcal{N} = 4$) part

Linear $K_{\rm NLO\ BK}$ reproduces forward NLO BFKL kernel of Fadin and Lipatov (1998).

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$$\int d^4x\, e^{iqx}\int d^4z\delta(z_\bullet)\langle p_B|T\{\hat{j}_\mu(x+z)\hat{j}_\nu(z)\}|p_B\rangle = \int d^2k_\perp I_{\mu\nu}(q,k_\perp)\langle\langle p_B|\mathcal{U}(k_\perp)|p_B\rangle\rangle$$

$I_{\mu\nu}(q,k_\perp)$ is the Impact Factor at NLO accuracy given in page 20.
The NLO evolution of the matrix element $\langle\langle p_B|\mathcal{U}(k_\perp)|p_B\rangle\rangle$ in momentum spcae is

$$\begin{aligned}
2a\frac{d}{da}\mathcal{U}_a(k_\perp) &= \frac{\alpha_s N_c}{2\pi^2}\int d^2k'_\perp\frac{k'^2_\perp}{k^2_\perp}\Big\{1+\frac{\alpha_s b}{4\pi}\Big[\frac{\ln\mu^2/k^2_\perp}{(k-k')^2_\perp}\Big(2\mathcal{U}(k'_\perp)-\frac{k^2_\perp}{k'^2_\perp}\mathcal{U}_a(k_\perp)\Big)\\
&-\frac{2}{(k-k')^2_\perp}\Big(\mathcal{U}_a(k'_\perp)\ln\frac{(k-k')^2_\perp}{{k'_\perp}^2}-\mathcal{U}_a(k_\perp)\frac{k^2_\perp}{k'^2_\perp}\ln\frac{(k-k')^2_\perp}{k^2_\perp}\Big)\\
&+\mathcal{U}_a(k'_\perp)\frac{4(k',k-k')_\perp}{k'^2_\perp(k-k')^2_\perp}\ln\frac{(k-k')^2_\perp}{k^2_\perp}\Big]\Big\}+\frac{\alpha_s^2N_c^2}{4\pi^3}\int d^2k'_\perp\frac{k'^2_\perp}{k^2_\perp}\Big[-\frac{1}{(k-k')^2_\perp}\ln^2\frac{k^2_\perp}{k'^2_\perp}\\
&+F(k_\perp,k'_\perp)+\Phi(k_\perp,k'_\perp)\Big]\,\mathcal{U}_a(k'_\perp)+3\frac{\alpha_s^2N_c^2}{2\pi^2}\zeta(3)\mathcal{U}_a(k_\perp)
\end{aligned}$$

where

$$F(q,q') = \Big(1+\frac{n_f}{N_c^3}\Big)\frac{3(q,q')^2-2q^2z'^2}{16z^2q'^2}\Big(\frac{2}{q^2}+\frac{2}{q'^2}+\ \frac{q^2-q'^2}{q^2q'^2}\ln\frac{z^2}{q'^2}\Big)-\Big[3+ \Big(1+\frac{n_f}{N_c^3}\Big)\Big(1-\frac{(q^2+q'^2)^2}{8q^2q'^2}+\frac{3q^4+3q'^4-2q^2q'^2}{16q^4q'^4}(q,q')^2\Big)\Big]\int_0^\infty\frac{dt}{q^2+t^2q'^2}\ln\frac{1+t}{|1-t|}$$

and

$$\Phi(q,q') = \frac{(q^2-q'^2)}{(q-q')^2(q+q')^2}\Big[\ln\frac{q^2}{q'^2}\ln\frac{q^2q'^2(q-q')^4}{(q^2+q'^2)^4}+2\mathrm{Li}_2\Big(-\frac{q'^2}{q^2}\Big)-2\mathrm{Li}_2\Big(-\frac{q^2}{q'^2}\Big)\Big] -\Big(1-\frac{(q^2-q'^2)^2}{(q-q')^2(q+q')^2}\Big)\Big[\int_0^1-\int_1^\infty\Big]\frac{du}{(q-q'u)^2}\ln\frac{u^2q'^2}{q^2}$$

Conclusions

- High-energy operator expansion in color (composite) dipoles works at the NLO level.
- An analytic (and compact) expression for the NLO Impact Factor for BFKL pomeron in momentum space has been calculated: k_{T} factorization for DIS at NLO has been proved explicitly.
- Gauge invariance has been checked at each step: in coordinate space, in Mellin space, and momentum space.
- The NLO BK kernel in QCD and $\mathcal{N} = 4$ SYM is in agreement with the NLO BFKL eigenvalues when the NLO BK eq. is linearized.
- The linearized and forward NLO BK kernel in momentum space has been obtained and can be convoluted with the analytic NLO Impact Factor here presented.