LIDER

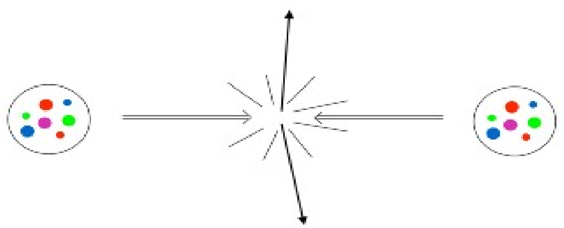
Resummed form of BK equation and its extension towards exclusive final states

Krzysztof Kutak

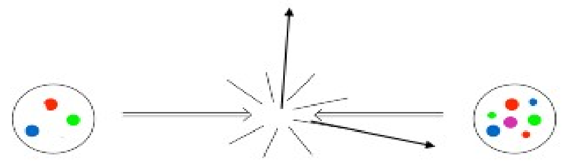
Instytut Fizyki Jądrowej
im. Henryka Niewodniczańskiego
Polskiej Akademii Nauk

Narodowe Centrum
Badań i Rozwoju

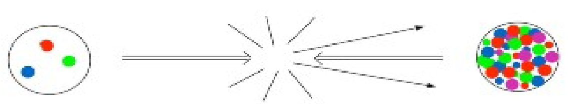
LHC as a scaner of gluon



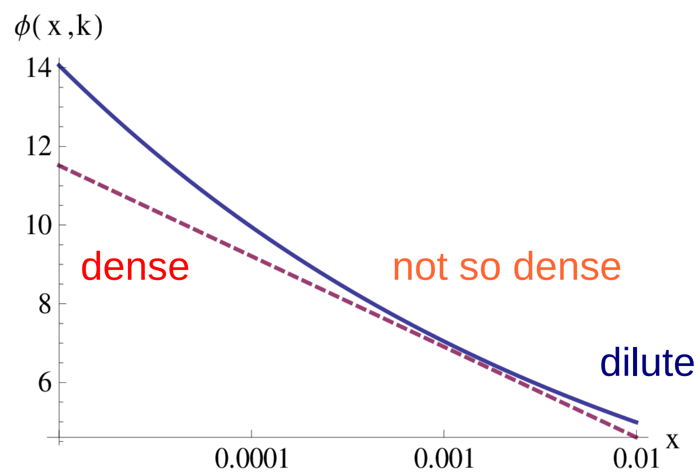
central-central i.e.
not so dense-not so dense



forward-central i.e.
dilute – not so dense



forward-forward i.e.
dilute -dense



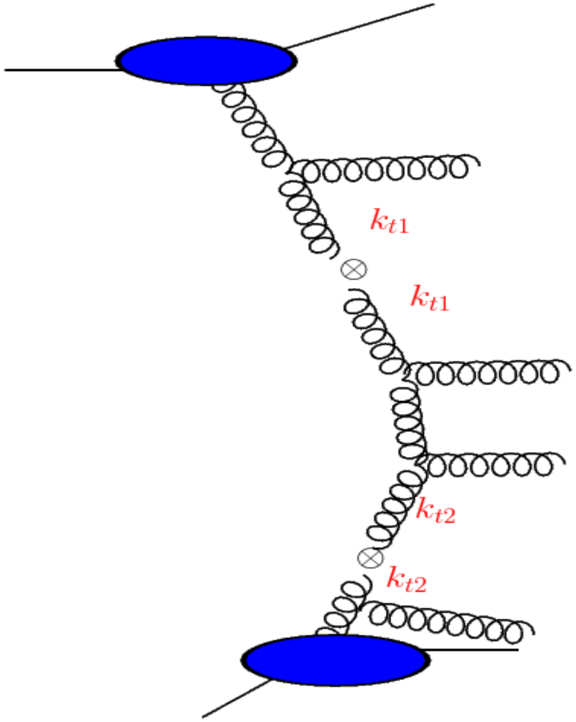
From C. Marquet

High energy limit of QCD

$$\sigma = \int dx_1 dx_2 d^2 k_{1T} d^2 k_{2T} f(x_1, k_{1T}) \hat{\sigma} f(x_2, k_{2T})$$

Ciafaloni, Catani, Hautman '93

Implemented in Monte Carlo generator CASCADE
(*H. Jung*)



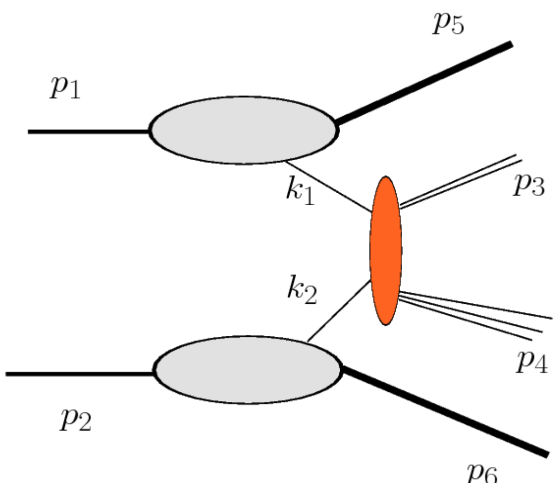
- Parton density depends on kt
  - Off shell initial state partons off shellness ~kt
    - In collinear limit reduces to collinear factorization

High energy factorization and forward jets

Deak, Jung, Hautmann Kutak '09

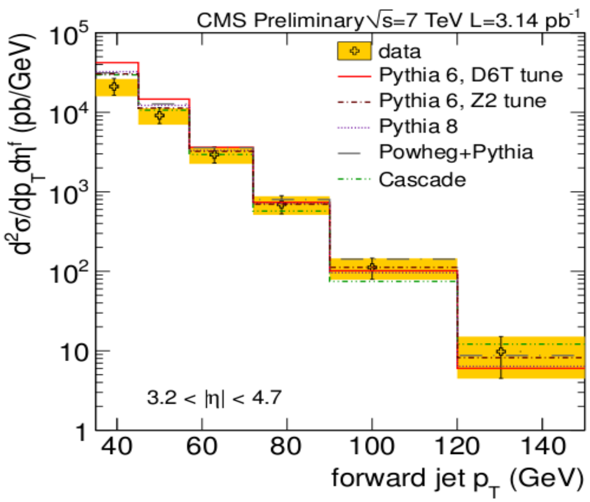
$$\frac{d\sigma}{d^2p_{t1}d^2p_{t2}dy_1dy_2} = \sum_a \int d^2k_t \phi_{a/A}(x_1,\mu) \otimes |M|^2 \otimes \phi_{g^*/B}(x_2,k_t^2,\mu)$$

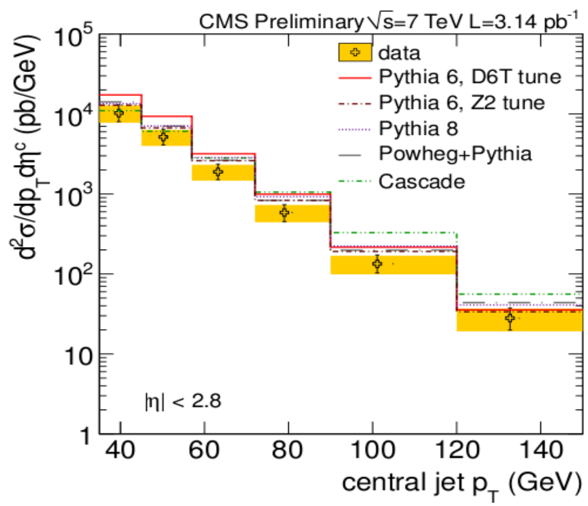
Consistent resumation both logs of rapidity
and
logs of hard scale

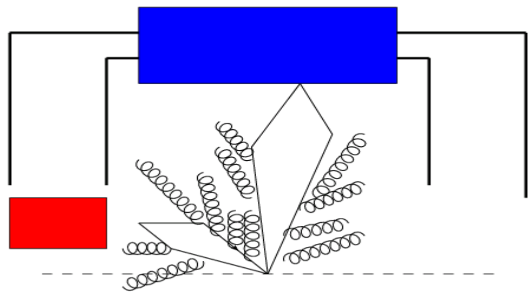


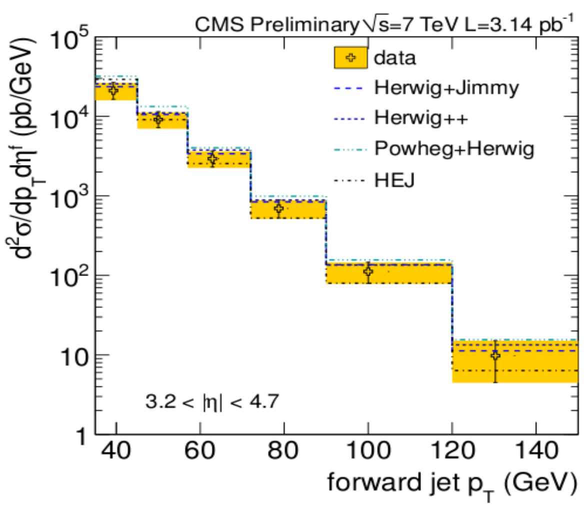
Knowing well parton densities at largr x one can
get information about low x physics

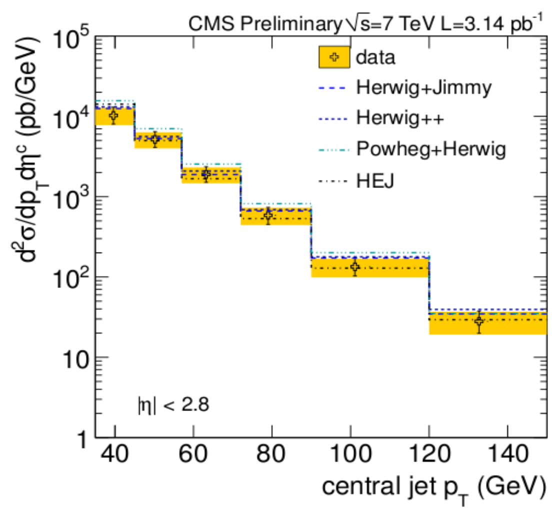
Forward central – jet production









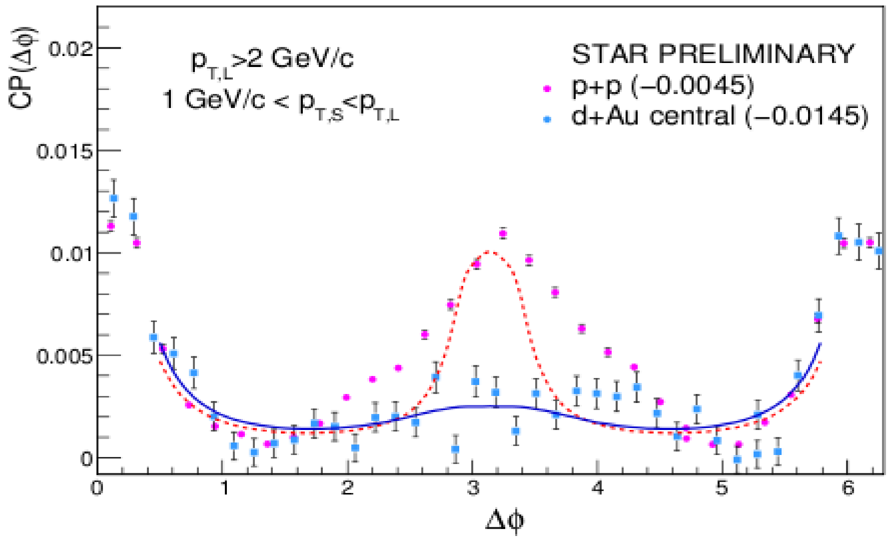


Why differences?

- HEJ and Cascade based on unordered in kt emissions but use different parton densities

- Herwig and PYTHIA use kt odered shower but differ in approximations in ME and ordering conditions in shower

Saturation and production of forward dijets in d Au at RHIC



$$CP(\Delta\phi) = \frac{N_{pair}(\Delta\phi)}{N_{trig}}$$

$$N_{pair}(\Delta\phi) = \int_{y_i,|p_{i\perp}|} \frac{dN^{pA\to h_1 h_2 X}}{d^3p_1 d^3p_2}$$

$$N_{trig} = \int_{y,\ p_\perp} \frac{dN^{pA\to hX}}{d^3p}$$

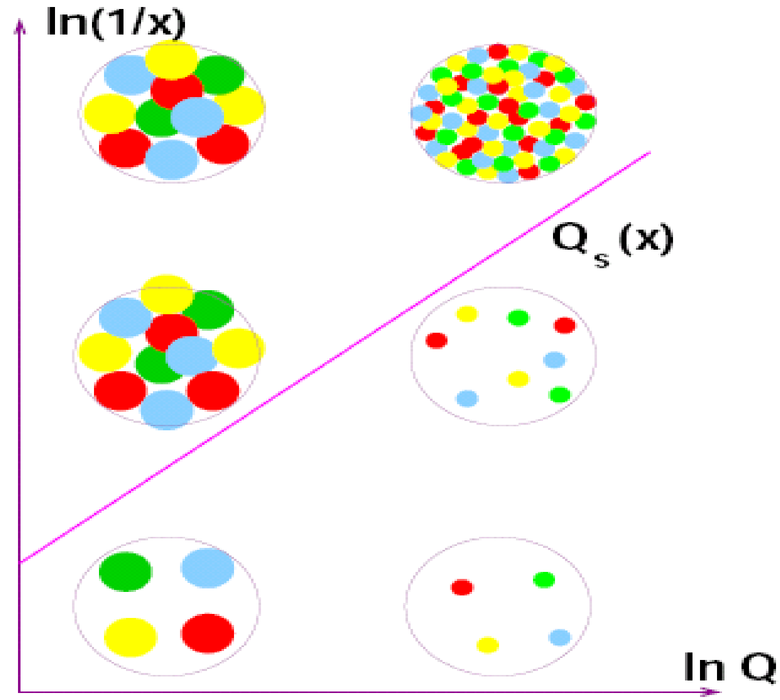
Albacete, Marquet '10; Tuchin 10

Huan,Stasto,Xiao'11

Features: allow for direct studies of
saturation effects of the correlation function.
d-Au no smearing due to collecive flow as in A A

High energy factorization and saturation

Saturation – state where number of gluons stops growing due to high ocupation number.

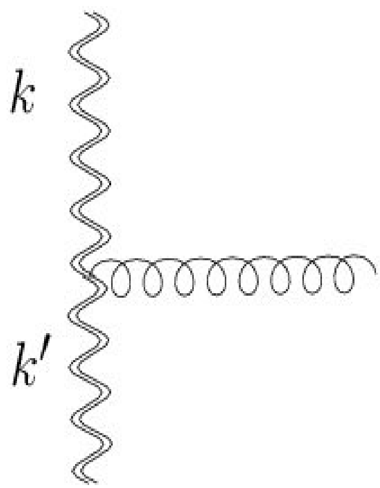


Cross sections change their behaviour from power like to logarithmic like.

CGC: BK, JIMWLK, GBW

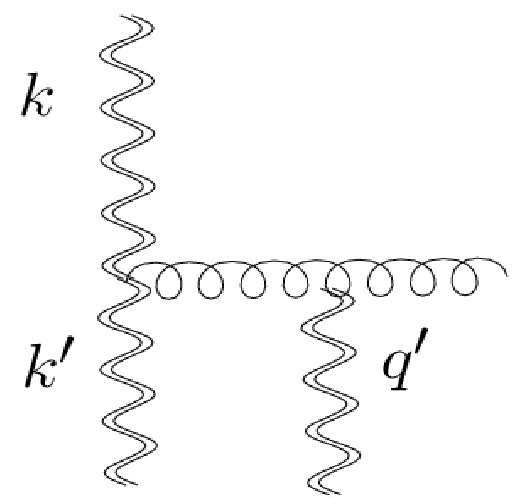
On microscopic level it means that gluon apart from

splitting



Linear evolution equation

recombination

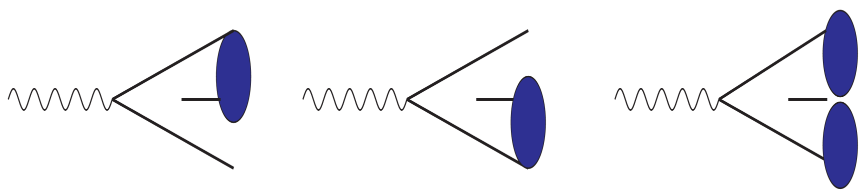


Nonlinear evolution equation BK

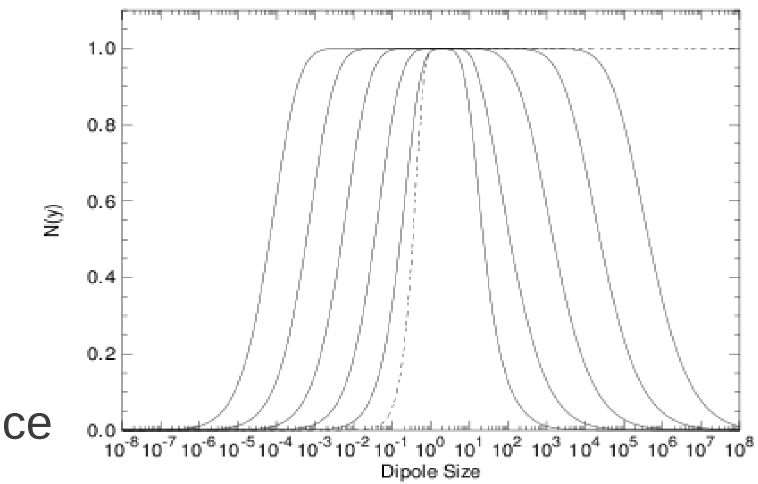
Simple evolution equation with nonlinearities

Kovchegov '99

$$\frac{\partial \mathcal{N}(r,x)}{\partial \ln(x_0/x)} = \int d^2\underline{r}_1 \; K(\underline{r},\underline{r}_1,\underline{r}_2)\,[\mathcal{N}(r_1,x) + \mathcal{N}(r_2,x) - \mathcal{N}(r,x) - \mathcal{N}(r_1,x)\mathcal{N}(r_2,x)]$$

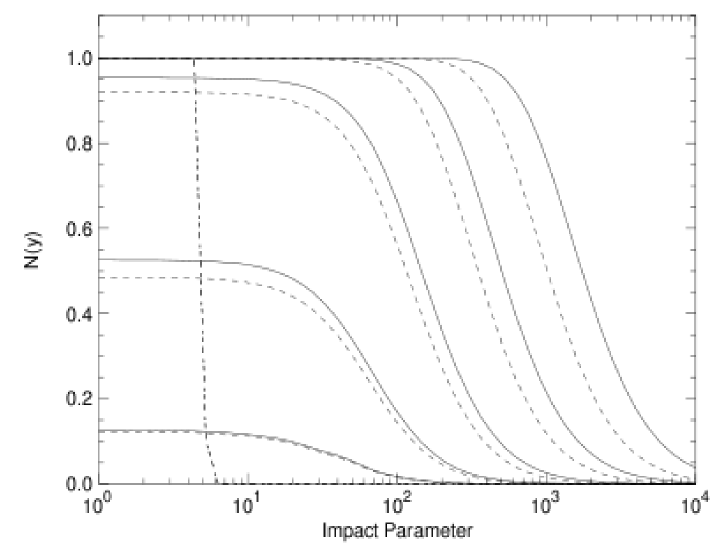


Nonlinear term allows for saturation

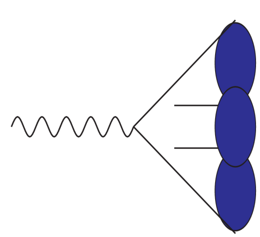


Recently solved with full impact parameter dependence

Berger, Stasto '11



BK is at present known up to NLO
where such transitions are possible



Forward physics way to constrain gluon both at large and small pt

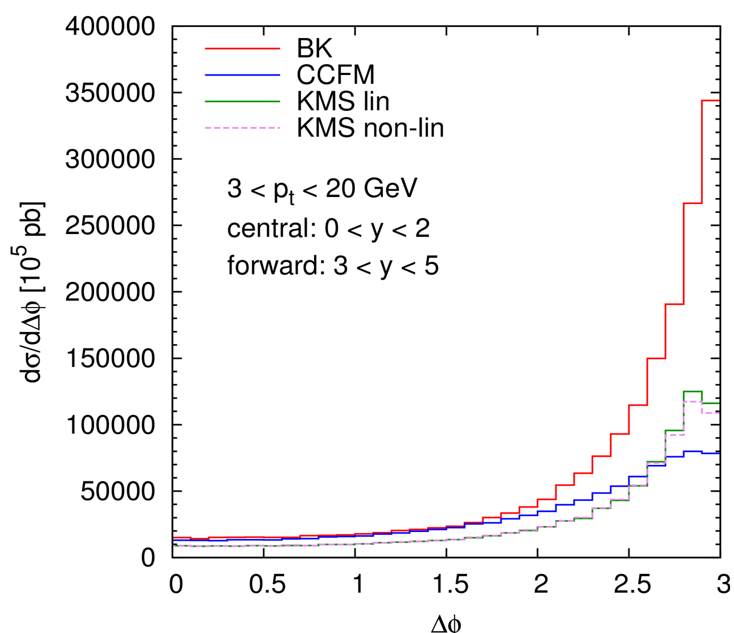
Kutak, Sapeta in preparation

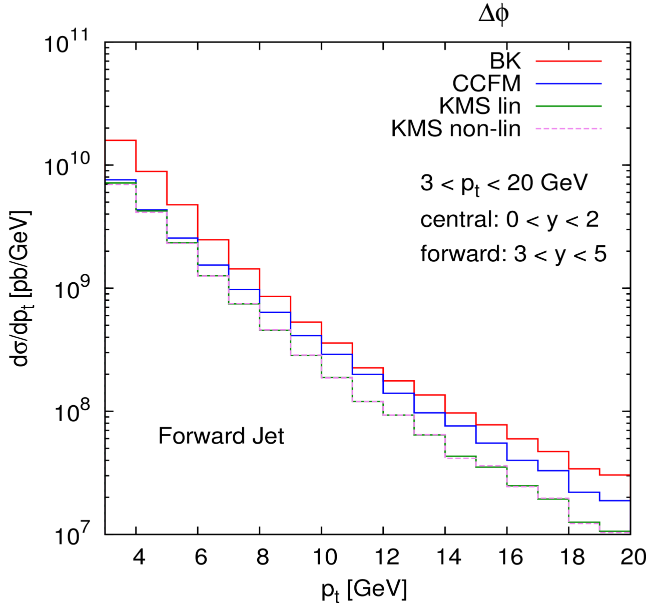
KMS
- BFKL + nonsingular pieces of splitting function + kinematical constraint + quarks

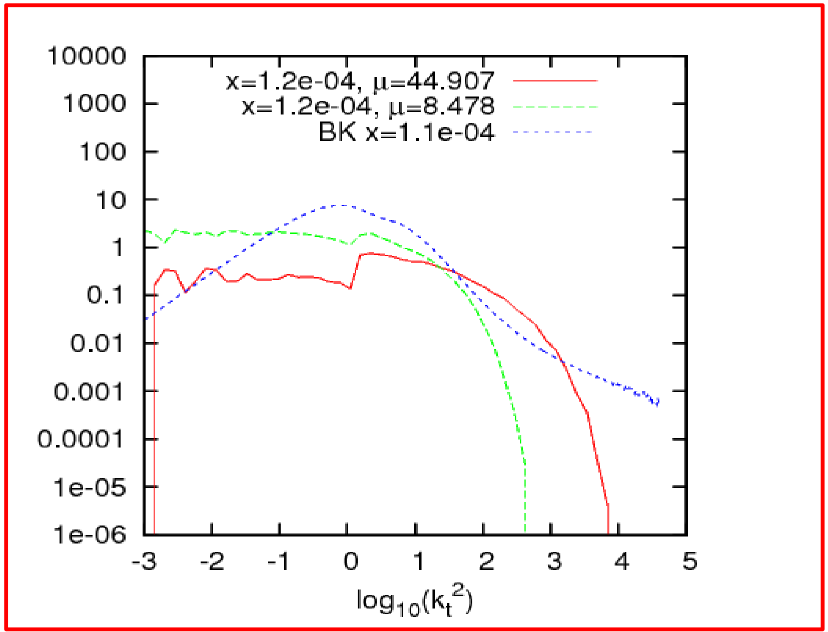
KKS
- BK + nonsingular pieces of splitting function + kinematical constraint + quarks

Importace of NLO corrections.

Not physical behaviour of BK at large kt
Not physical behaviour of CCFM at small pt





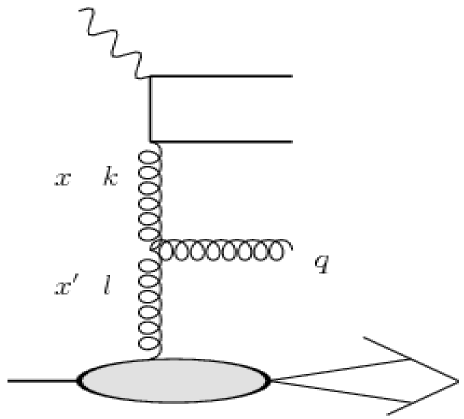


The BK equation and singularities

Go to momentum space

dipol density

$$\Phi(x,k^2) = \int \frac{d^2\mathbf{r}}{2\pi} \exp(-i\mathbf{k}\cdot\mathbf{r}) \frac{N(x,r)}{r^2}$$



gluon density

$$\mathcal{F}_{BK}(x,k^2) = \frac{N_c}{\alpha_s \pi^2} k^2 \nabla_k^2 \Phi(x,k^2)$$

$$\frac{\partial \Phi(x,k^2)}{\partial \ln 1/x} = \overline{\alpha}_s \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2\Phi(x,l^2) - k^2\Phi(x,k^2)}{|k^2-l^2|} + \frac{k^2\Phi(x,k)}{\sqrt{(4l^4+k^4)}} \right] - \overline{\alpha}_s \Phi^2(x,k)$$

Substractions are not easy to deal with in MC. Perform resummation.

Exclusive form of BK

K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek

JHEP 1202 (2012) 117

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z, |\mathbf{k}+\mathbf{q}|^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)$$

Angular dependence restored

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \Phi(x/z, |\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z, |\mathbf{k}+\mathbf{q}|^2)\theta(\mu^2-q^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)\,.$$

Resolution scale introduced

Perform Mellin transform w.r.t x to get rid of z integral

$$\overline{\Phi}(\omega,k^2) = \int_0^1 dx x^{\omega-1}\Phi(x,k^2)$$

$$\Phi(x,k^2) = \int_{c-i\infty}^{c+i\infty} d\omega\, x^{-\omega}\overline{\Phi}(\omega,k^2)$$

Towards exclusive and resummed form of BK

Using in unresolved real part $|\mathbf{k}+\mathbf{q}|^2 \approx \mathbf{k}^2 \longleftarrow q^2 < \mu^2$

$$\overline{\Phi}(\omega, k^2) = \overline{\Phi}^0(\omega, k^2) + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{q^2}[\overline{\Phi}(\omega, |\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2)] + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{q^2}\overline{\Phi}(\omega, k^2)[\theta(\mu^2-q^2)-\theta(k^2-q^2)] - \frac{\overline{\alpha}_s}{\omega}\int_0^1 dy y^{\omega-1}\Phi^2(y, k^2)$$

$$\overline{\Phi}(\omega, k^2) = \overline{\Phi}^0(\omega, k^2) + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{\pi q^2}\overline{\Phi}(\omega, |\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) - \frac{\overline{\alpha}_s}{\omega}\overline{\Phi}(\omega, k^2)\ln\frac{k^2}{\mu^2} - \frac{\overline{\alpha}_s}{\omega}\int_0^1 dy y^{\omega-1}\Phi^2(y, k^2)\,.$$

BK equation in resummed exclusive form

K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek

arXiv:1111.6928

$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\,\frac{\Delta_R(z,k,\mu)}{z}\left[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\right]$$

The same resumed piece for linear and nonlinear

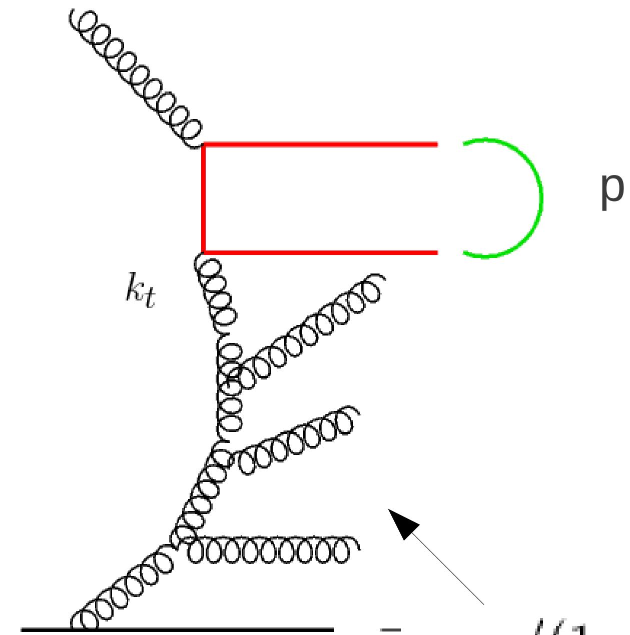
Suggestive form to promote the CCFM equation to nonlinear equation

CCFM evolution equation - evolution with observer

- Linear equation based on strong ordering in angle

$$\bar{\xi} > \xi_i > \xi_{i-1} > \dots > \xi_1 > \xi_0$$

$$\eta_i = \frac{1}{2}\ln(\xi_i) \qquad \bar{\xi} = p^2/(x^2 s)$$



$$\bar{\mathbf{q}} = \mathbf{q}/(1-z)$$

- Interpolates between DGLAP and BFKL
- Gluon density is build by constructive interference of gluons
- Sumes up also logs of hard scale

$$k' = |\boldsymbol{k} + (1-z)\bar{\boldsymbol{q}}|$$

$$\mathcal{A}(x,k^2,p) = \mathcal{A}_0(x,k^2,p) + \bar{\alpha}_s \int_x^1 dz \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2}\, \theta(p - z\bar{q})\Delta_s(p, z\bar{q}) \left(\frac{\Delta_{ns}(z,k,q)}{z} + \frac{1}{1-z} \right) \mathcal{A}\left(\frac{x}{z}, k'^2, \bar{q}\right)$$

Extension of CCFM to non linear equation

- The second argument should be kt motivated by analogy to BK
- The third argument should reflect locally the angular ordering

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$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\frac{\Delta_R(z,k,\mu)}{z}\Big[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\Big]$$

$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p) + \bar{\alpha}_s \int_x^1 dz \int \frac{d^2\bar{\mathbf{q}}}{\pi\bar{q}^2}\,\theta(p-z\bar{q})\Delta_s(p,z\bar{q})\left(\frac{\Delta_{ns}(z,k,q)}{z} + \frac{1}{1-z}\right)\left[\mathcal{E}\left(\frac{x}{z},k'^2,\bar{q}\right) - \delta(\bar{q}^2-k^2)\,\mathcal{E}^2(\frac{x}{z},\bar{q}^2,\bar{q})\right].$$

Extension of CCFM to nonlinear equation

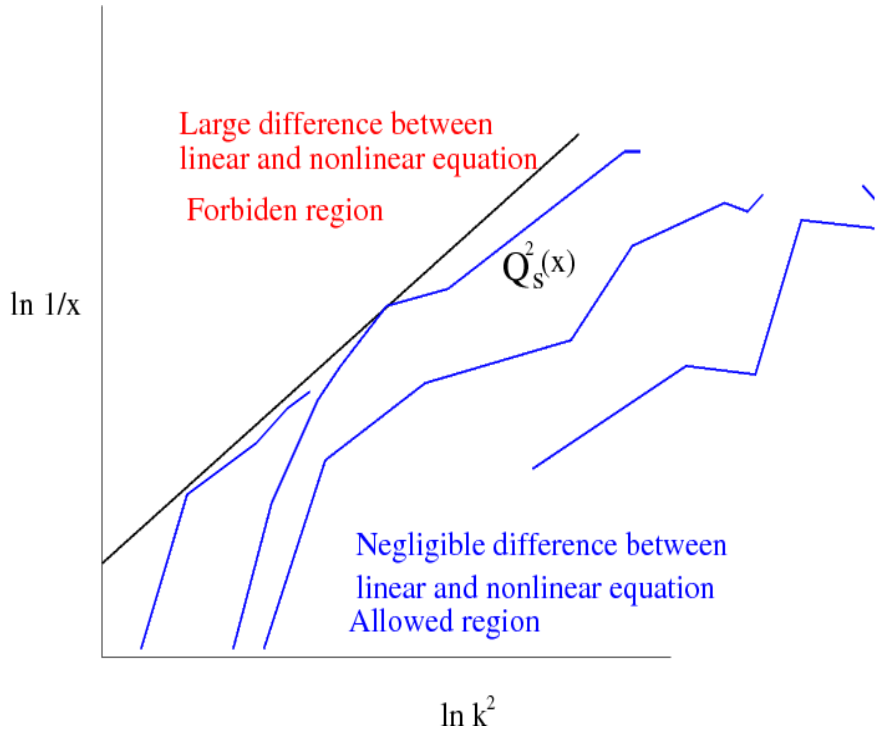
The unintegrated gluon density is obtained from

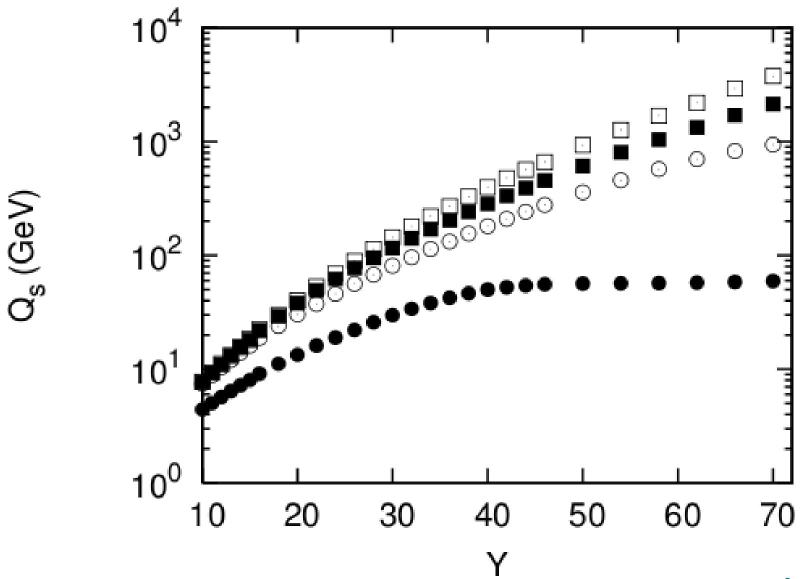
$$\mathcal{A}_{non-linear}(x, k^2, p) = \frac{N_c}{\alpha_s \pi^2} k^2 \nabla_k^2 \mathcal{E}(x, k^2, p)$$

The nonlinear term can be understood as a way to introduce the decoherence in emission of gluons which build unintegrated gluon density.

CCFM with saturation – another approach

Jung, Kutak '09
Avsar, Iancu '09





Avsar, Stasto '10

introduce line which will introduce effectively saturation effects in evolution. trajectories which enter the saturation region are rejected.

saturation scale saturates itself because of limited phase space due to existence of hard scale

Consequences for entropy production

$$S = \frac{6C_F A_\perp}{\pi \alpha_s} Q_s^2(x) + S_0$$

K.Kutak '11
Kiritsis, Tsalios '11

Conclusions and outlook

- There comes oportunity to test parton densities both when the parton density is probed at low x and at high kt.

- Used so far equations did not allow for this

- New representation for BK equation allowed for ansatz for well motivated equation which incorporates both saturation effects and coherence

- In the future it will be interesting to check whether this equation predicts saturation of the saturation scale as in other frameworks