High energy effective theory and NLO BFKL jet phenomenology

Martin Hentschinski

IFT UAM/CSIC Madrid

Based on results obtained with B. Murdaca & A. Sabio Vera

Workshop on Deep Inelastic Scattering and related topics 2012

BFKL evolution & High Energy Factorization

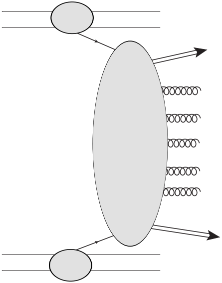
- hadronic scattering processes with a hard scale $Q^2 \gg \Lambda^2_{\rm QCD}$ described within perturbative QCD
- limit of high c.o.m. eneries $s \gg Q^2$ ➡ enter multi scale regime ➡ $\alpha_s(Q^2) \ln s/Q^2 \sim 1$ ➡ require resummation

BFKL evolution:
- LL [Fadin, Kuraev, Lipatov (1977)], [Balitsky, Lipatov (1978)]
- NLL [Fadin, Lipatov (1998)], [Ciafaloni, Gamici (1998)]

- Predicts rise of cross-sections with s
- Derived from QCD & QFT
- Hints, but no clear experimental evidence till nowadays

- study forward & forward-backward observables ➡ probe directly BFKL evolution

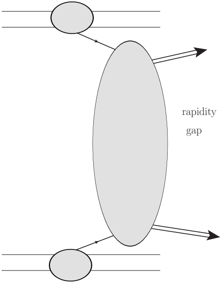
- Develop further the theoretical formulation of high energy factorization ➡ arrive at more precise and stable predictions gold plated process at hadron collider: **forward-backward jets** [Mueller, Navelet (1987)]



- large relative rapidity opens phase space for n parton emissions ➡ BFKL evolution
- entire process treatable within collinear factorization ➡ no modelling of non-perturbative effects needed
- BFKL description currently known at NLL level [Fadin, Fiore Quartarolo (1994)], [Ciafaloni (1998)], [Ciafaloni, Colferai (1999)], [Bartels, Colferai, Vacca (2002), (2003)], [Colferai, Schwennsen, Szymanowski, Wallon (2010)], [Caporale, Ivanov, B. Murdaca, A. Papa (2011)]
- numerical analysis [Colferai, Schwennsen, Szymanowski, Wallon (2010)] reveals strong dependence on impact factors

another interesting jet observable: **forward-backward jets with rapidity gap**

[Mueller, Tang (1987)], [Bartels, Forshaw, Lotter, Lipatov, Ryskin, Wüsthoff (1995)], [Enberg, Ingelman, Motyka (2001)], [Chevallier, Kepka, Marquet, Royon (2009)]



- absence of QCD radiation signals color singlet exchange ➡ can be described by non-forward BFKL Green's function
- entire process treatable within collinear factorization ➡ no modelling of non-perturbative effects neededc
- $\equiv$ diffraction at partonic level
- LO: use conformal symmetry to obtain solution to non-forward BFKL equation
- NLO non-forward BFKL kernel known, but complicated ➡ require Monte-Carlo solution, see also tomorrow's talk by Clara Salas
- impact factors only known at leading order – limits accuracy of the prediction

- current precision calculations reach at least NLO level or go even beyond
- resummations often available at NNLL accuracy
- BFKL impact factors restricted to LO accuracy with few exceptions → limited number of complete NLL cross-sections
- reggeization & renormalization scale dependence + NLL correction often found to be sizeable

Reason for limitation:

- need to extract from exact NLO n-point amplitudes
- difficult, sometimes not even possible

Here: re-activate Lipatov's high energy effective action [Lipatov (1995)]

- tool which should allow to calculate impact factors and evolution kernels directly
- in this talk: jet impact factors, see also tomorrow's talk by Jose Daniel Madrigal

High energy factorization & the effective action

observation: QCD amplitudes factorize in the limit of large center-of-mass energies

QCD: no naive high energy factorization

- cannot simply combine off-shell QCD amplitudes etc. (gauge invariance!)
- different region of phase space (rapidity!) are connected

High energy effective action: tool to generate high energy expansion of QCD amplitudes

- effective action factorizatizes QCD amplitudes in high energy limit
- interaction mediated through (multiple!) exchanges of new auxiliary scalar field A_+, A_-
- charged under $SU(N_c)$, but invariant under local gauge transformations ➡ gauge invariant factorization
- satisfies $\partial_+ A_- = \partial_- A_+$ – reflects high energy expansion & strong ordering in rapidity of factorized amplitudes
- BFKL insider: identify this field with the reggeized gluon

effective action given as sum of QCD action and induced term:

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind.}}$$

induced term: coupling of auxiliary field $A_\pm = -it^a A_\pm$ ($\equiv$ reggeized gluon) to conventional QCD gluon field $v_\mu = -it^a v_\mu^a$

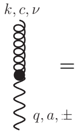
$$S_{\text{ind.}}[v_\mu, A_\pm] = \int \mathrm{d}^4x \, \mathrm{tr}\left[\left(W_+[v(x)] - A_+(x)\right)\partial_\perp^2 A_-(x)\right] + \int \mathrm{d}^4x \, \mathrm{tr}\left[\left(W_-[v(x)] - A_-(x)\right)\partial_\perp^2 A_+(x)\right].$$

Coupling of $A_\pm$ through non-local operator

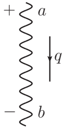
$$W_\pm[v] = v_\pm \frac{1}{D_\pm}\partial_\pm \qquad \text{where} \qquad D_\pm = \partial_\pm + g v_\pm .$$

- no effective action on conventional sense: do not integrate out d.o.f. but add new d.o.f. instead!
- takes into account all necessary contributions ➡ gauge invariance
- to avoid over counting: need either explicit cut-off on (longitudinal) phase space or subtraction mechanism
- subtraction mechanism: subtract from a given matrix element contributions which factorize *i.e.* which can be described through the exchange of an auxiliary field $A_\pm$ (reggeized gluon)

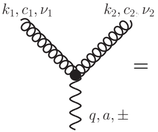
- have in mind: partonic scattering process $p_a + p_b \to p_1 + p_2 + \ldots$. center of mass energy $s = 2p_1 \cdot p_2$, $p_1^2 = 0 = p_2^2$
- define light-cone vectors as $n^+ = 2p_b/\sqrt{s}$ and $n^- = 2p_a/\sqrt{s}$.
- Feynman rules: QCD Feynman rules + infinite many induced vertices [Antonov, Lipatov, Kuraev, Cherednikov (2005)]



$$= -i\boldsymbol{q}^2\delta^{ac}(n^\pm)^\nu, \qquad k^\pm = 0.$$

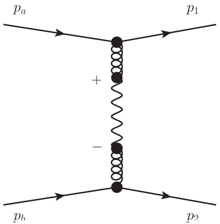


$$= \delta^{ab}\frac{i/2}{\boldsymbol{q}^2}$$



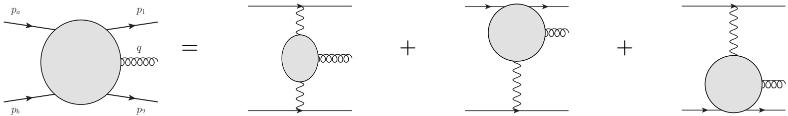
$$= g f^{c_1 c_2 a}\frac{\boldsymbol{q}^2}{k_1^\pm}(n^\pm)^{\nu_1}(n^\pm)^{\nu_2}, \qquad k_1^\pm + k_2^\pm = 0$$

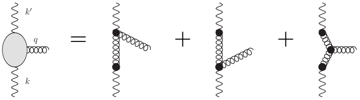
quark-quark scattering in the high energy limit:



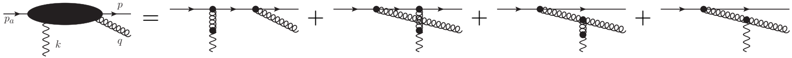
- tree level diagram straightforward
- effective action result coincides with well known QCD result
- Note: action contains also standard QCD diagram ➡ describes effects suppressed in the high energy limit

Real corrections: divided into three groups: central and 2 x quasi-elastic

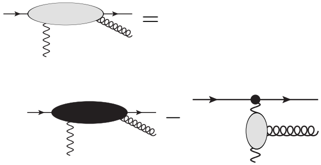




- central production
- well known Lipatov vertex



- reduces in the limit $s_{qg} \to \infty$ into factorized result
- cannot simply add both contributions
- need cut-off or subtraction factorized result
- final result for 5 point amplitude agrees with literature [Ciafaloni (1998)], [Ciafaloni, Colferai (1999)], [Bartels, Colferai, Vacca (2002)]

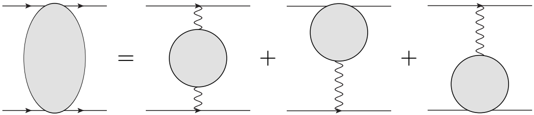


Virtual corrections: factorized results divergent ➡ tilt light cone vectors

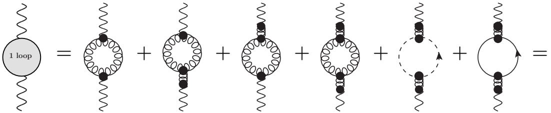
$$n^+ \to n^+ + e^{-\rho} n^-$$
$$n^- \to n^- + e^{-\rho} n^+$$

- evaluate in limit $\rho \to \infty$;
- can interpret ρ as $\ln s$; here treat as parameter

$$i\mathcal{M}^{(1)}_{q_a q_b \to q_1 q_2} =$$



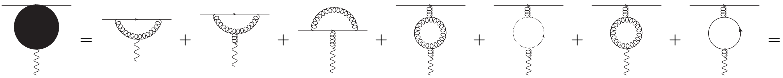
central correction: reggeized gluon self-energy



$$\frac{\alpha_s N_c(-2i\boldsymbol{k}^2)}{2\pi}\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^{\epsilon}\left(\frac{i\pi - 2\rho}{2\epsilon} - \frac{\beta_0}{2N_c\epsilon} + \frac{67}{18} - \frac{10 n_f}{18 N_c}\right) + \mathcal{O}(\epsilon)$$

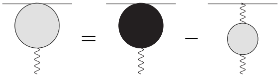
- term linear in ρ: LO gluon Regge trajectory
- resummation yields 'Regge factor' $s^{\omega}(t)$ – justifies identifaction $A_{\pm}$ field with reggeized gluon

quasi-elastic vertex correction:



$$\frac{i\mathcal{M}^{(0)}_{qr^*\to q}}{(-i2k_\perp^2)}\frac{\lambda}{\epsilon}\left\{-\left(\ln\frac{-p_a^+}{\sqrt{k_\perp^2}}+\ln\frac{p_a^+}{\sqrt{k_\perp^2}}+\rho\right)-\frac{1}{(1+2\epsilon)}\left[-\frac{1}{N_c^2}\left(\frac{1}{\epsilon}+\frac{1+2\epsilon}{2}\right)+\frac{11+7\epsilon}{3+2\epsilon}-2\frac{n_f}{N_c}\frac{1+\epsilon}{3+2\epsilon}\right.\right.$$

$$\left.\left.-\frac{2+7\epsilon}{2\epsilon}+(1+2\epsilon)\Big(\psi(1-\epsilon)-2\psi(\epsilon)+\psi(1)\Big)\right]\right\}, \qquad \lambda=\frac{(-2i\boldsymbol{k}^2)g^2N_c}{(4\pi)^{2+\epsilon}}\left(\frac{k_\perp^2}{\mu^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)\Gamma(1+\epsilon)^2}{\Gamma(1+2\epsilon)}$$



- vertex itself contains ρ-dependent terms proportional to gluon Regge trajectory
- need to subtract self-energy correction from vertex correction

- both self-energy correction and (subtracted) vertex correction carry ρ-dependence
- cancels for complete scattering amplitude if subtracted elements are used → important check of consistency
- complete amplitude yields expected results [Fadin, Fiore, Quartarolo (1994)], [Del Duca, Schmidt (1998)]

$$i\mathcal{M}^{(1)}_{q_aq_b\to q_1q_2}=i\mathcal{M}^{(0)}_{q_aq_b\to q_1q_2}\left(\frac{1}{2}\left(\ln\frac{s}{\boldsymbol{k}^2}+\ln\frac{-s}{\boldsymbol{k}^2}\right)\omega(-k_\perp^2)+\Gamma_a^{(1)}(\boldsymbol{k}^2)+\Gamma_b^{(1)}(\boldsymbol{k}^2)\right).$$

combination of real & virtual correction can then be used for study of forward & forward/backward jets

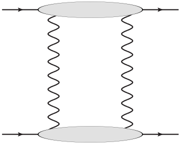
General procedure:

1. calculate QCD correction to propagators, vertices etc. with external reggeized gluon fields
2. subtract corresponding factorized expressions
3. use subtracted expression in combination with reggeized gluon propagators to construct high energy limit of QCD scattering amplitudes

currently checked and confirmed for 2-loop gluon trajectory (quark part) → see tomorrow's talk by Jose Daniel Madrigal

Mueller Tang jets – forward backward jets with rapidity gap

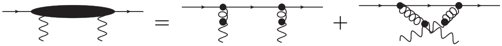
color singlet exchange → at high center of mass energies: 2 reggeized gluon exchange "Pomeron"



effective action:

- kinematic constraint $\partial_+ A_- = 0 = \partial_- A_+$ → loop integral factorizes $dl^- d^{d-2}\boldsymbol{l} d^+$
- associate dl^- (dl^+) with upper (lower) quark - 2 reggeized gluon coupling → impact factor
- final result as convolution in transverse momentum space

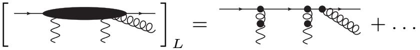
LO: $\phi(\boldsymbol{l}, \boldsymbol{k} - \boldsymbol{l}) = \int \frac{dl^-}{4\pi}$ 



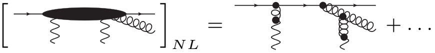
Here: individual diagrams convergent, not the case for color octet → subtraction

NLO: virtual corrections known from conventional calculation [Fadin, Fiore, Kotsky, Papa (2000)]
use effective action for determination of real corrections

organize diagrams into groups



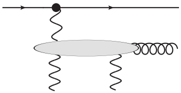
Pomeron couples like particle to quark



composite nature of 2 reggeized gluon state

- individual diagrams divergent w.r.t. l^- integration
- sum converges

- subtraction diagram: reggeized gluon exchange inside the 5 point amplitude
- requires Reggeized gluon - gluon - 2 Reggeized gluon vertex; known from conventional approach Bartels (1980) and effective action Braun, Vyazovsky (2006)
- contributions with $1 \to 2$ reggeized gluon splitting do not contribute [Bartels, MH, (in preparation)]



exact amplitude reduces to factorized expression for $s_{qg} \to \infty$ ➡ cancelation for the subtracted expression ➡ further consistency check of effective action

for actual definition of jet impact factor, cut-off might be more suitable

result currently checked against exact NLO calculation for $qq \to qqg$ amplityde [Kunszt, Signer, Trocsanyi (1994)]

Conclusions

- jets widely separated in rapidity allow promise observables which allow to search for signs for BFKL evolution at a hadron collider
- solid predictions provide improvements on the theory side → NLL accuracy etc.
- a 17 year old new tool: high energy effective action [Lipatov (1995)]; requires
  - regularization
  - subtraction mechanism
- re-calculated amplitudes underlying NLO forward quark jet impact factor (real & virtual) directly from effective action. Result in agreement with the literature [MH, Sabio Vera (2011)].
- new result: real NLO corrections to Mueller-Tang impact factor (jets with rapidity gap) [MH, Murdaca, Sabio Vera (in preparation)] – currently checked against exact calculation; to be combined with known virtual corrections.

Outlook:

Hope, to use effective action for a systematic study of high energy factorization, BFKL evolution and its extension