A GLOBAL ANALYSIS OF DIFFRACTIVE EVENTS AT HERA

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A GLOBAL ANALYSIS OF DIFFRACTIVE EVENTS AT HERA

• Motivation
• DIS & DDIS phenomena
• Introduction of different diffractive data sets
• Global fit procedure
• Results and conclusion
• Structure functions are a measure of the partonic structure of hadrons, which is important for any process which involves colliding hadrons.

• They are key ingredient for deriving PDFs in nucleons.

• These PDFs allow us to predict scattering cross sections at particle colliders. A good knowledge of PDFs is of prime importance for the success of the physics program.


The H1 and ZEUS collaborations presented their results on inclusive and various exclusive reactions, which is being actively studied by theorists and give access to a broader understanding of proton structure.

**Although data-taking there has been stopped, new results continue to appear.**

Approximately 10% of DIS phenomena are of diffractive nature
DIS kinematics

Before the proper analysis we need to define the usual kinematic variables. The main variables used for the description of DDIS are similar to those of DIS variable.

\[ Q^2 = \text{virtuality of photon} \]
\[ = (4\text{-momentum exchanged at e vertex})^2 \]
\[ W = \text{invariant mass of photon-proton system} \]
\[ x = \text{Bjorken’s variable for the Proton} \]
\[ = \text{fraction of Proton’s momentum carried by struck quark} \]
\[ y = \text{inelasticity} \]

\[
\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{xQ^4} \left[ 1 - y + \frac{y^2}{2[1 + R(x, Q^2)]} \right] F_2(x, Q^2)
\]

DIS probes the partonic structure of the proton
The variables related to DDIS bear a close resemblance to those of DIS.

\[ x_{IP} = \text{fraction of proton’s momentum taken by Pomeron} \]

\[ \beta = \text{Bjorken’s variable for the Pomeron} \]
\[ = \text{fraction of Pomeron’s momentum carried by struck quark} \]
\[ = x/x_{IP} \]

\[ t = (4\text{-momentum exchanged at p vertex})^2 \]
\[ \text{typically: } |t| < 1\text{GeV}^2 \]

\[ M_X = \text{invariant mass of photon-Pomeron system} \]

\[
\frac{d^4\sigma}{d\beta dQ^2 dx_{IP} dt} = \frac{4\pi\alpha^2}{\beta Q^4} \left\{ 1 - y + \frac{y^2}{2(1 + R_{D(4)})} \right\} F_2^{D(4)}(\beta, Q^2, x_{IP}, t)
\]
Theoretical basis of our analysis
There are 4 different theoretical approaches to analyze diffractive data:

1) The Pomeron structure function (PSF) model formulated in the framework of Regge phenomenology
2) The Bartels-Ellis-Kowalski-Wusthoff (BEKW) two gluon exchange dipole model
3) The Bialas-Peschanski (BP) model based on the BFKL Pomeron approach
4) The saturation model of Golec-Biernat and Wusthoff (GBW)

These four frameworks are based on completely different theoretical concepts. The best description of all available measurements can be achieved with either the PSF based model or the BEKW approach.
Diffractive cross sections

The data are often presented in the form of a t-integrated cross section

\[
\frac{d^3 \sigma_{e^+e^- \rightarrow X}}{dx_{IP} \, d\beta \, dQ^2} = \frac{4\pi\alpha^2}{xQ^4} \left(1 - y + \frac{y^2}{2}\right) \sigma_r^{D(3)}(x_{IP}, \beta, Q^2),
\]

or in terms of a diffractive structure function

\[
\sigma_r^{D(3)} = F_2^{D(3)} - \frac{y^2}{Y_+} F_L^{D(3)}
\]

\[
Y_+ = 1 + (1 - y)^2
\]

can be neglected anywhere but at very large \( y \)

\( sxy = Q^2 \) Longitudinal structure function plays an important role in low-x physics
It implies that the cross section for the diffractive process can be written in terms of convolution of DPDFs with universal partonic cross sections.

\[ d\sigma_{ep \to eXY}^{ep} (x, Q^2, x_P, t) = \sum_i f_i^D(x, Q^2, x_P, t) \otimes d\hat{\sigma}^{ei}(x, Q^2) \]
The proton vertex factorization framework, in which the DPDFs are factorized into a term depending only on \( X_{IP} \) and \( t \) and a term depending only on \( \beta \) and \( Q^2 \), is often assumed:

\[
f_i^D (x, Q^2, x_{IP}, t) = f_{IP/p} (x_{IP}, t) \cdot f_i (\beta, Q^2)
\]

Flux factor

The $x_I^P$ dependence is parametrised using a flux factor motivated by Regge theory.

The Pomeron trajectory is assumed to be linear.

$$f_i^D(x, Q^2, x_I^P, t) = f_{IP/p}(x_I^P, t) \cdot f_i(\beta, Q^2)$$

$$f_{IP/p}(x_I^P, t) = A_I^P \cdot \frac{e^{B_I^P t}}{x_I^P^{2\alpha_I^P(t)-1}}$$

$$\alpha_I^P(t) = \alpha_I^P(0) + \alpha_I' p t$$

$$f_i^D(x, Q^2, x_I^P, t) = f_{IP/p}(x_I^P, t) \cdot f_i(\beta, Q^2) + n_{IR} \cdot f_{IR/p}(x_I^P, t) \cdot f_i^{IR}(\beta, Q^2)$$

We step into the process of this project by performing a QCD fit under the same conditions and conventions as in H12006. We obtained almost identical result. Then we tried to vary $\Sigma$ distribution functional form to improve our fitting procedure.

H1 parameterization form

$$z \Sigma(z, Q_0^2) = A_\Sigma z^{B_\Sigma} (1 - z)^{C_\Sigma} e^{a_{-0.01}/1-z}$$

Fit A

$$zg(z, Q_0^2) = A_g (1 - z)^{C_g} e^{a_{-0.01}/1-z}$$

Fit B

$$zg(z, Q_0^2) = A_g e^{a_{-0.01}/1-z}$$

Our form

$$z \Sigma(z, Q_0^2) = A_\Sigma z^{B_\Sigma} (1 - z)^{C_\Sigma} (1 + D_\Sigma z + E_\Sigma z^{F_\Sigma})$$

Provides flexibility to obtain a good description of the data.

It ensures that the distribution vanish at $z=1$, as required for evolution equation to be solvable.

Jet data is really urgent to constrain the gluon inside the DPDF.
The tool which experimental groups use for predictions is NLOJET++ adapted for diffraction. This Monte Carlo program however cannot be directly interfaced in a fit since it is very slowly. A possible way out is to use pre-calculated table obtained with FastNLO but unfortunately such a code/table doesn't exist at the moment. As soon as the FastNLO for diffractive jet data gets prepared, we include them in our analysis.
Data Analysis
There is no unique definition of a cross section for DDIS. Different methods exist to select diffractive events. These methods select samples which contain different fractions of proton dissociative events. Cross sections are usually given without corrections for proton dissociation.

Three distinct methods have been employed by the HERA experiments, which select inclusive diffractive events.
Purpose: Direct measurement of the scattered proton
   Measurement of $t$

**Advantage:** No p-diss background ($M_Y = m_p$)

**Disadvantage:** Low statistics due to the Roman Pot detector acceptance

Require a large rapidity gap adjacent to the outgoing proton.

Escaping scattered proton cross section integrated over $t$

**Advantage:** Large statistics, large range in $Q^2$, $x_{IP}$, $\beta$

**Disadvantage:** Contamination of p-diss. ($M_Y < 1.6 \text{ GeV}$) & non diff. background

$$\eta = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right]$$

Require a large rapidity gap adjacent to the outgoing proton.

$t$ measurement is not possible

**Advantage:** Large statistics, No non diff. background

**Disadvantage:** Contamination of p-diss ($M_Y < 2.3 \text{ GeV}$)

$$\frac{dN}{d \ln M_x^2} = D + ce^{b \ln M_x^2}$$
The full HERA data sample analysis is a powerful technique to achieve the best precision possible in extracting DPDFs. First steps are taken towards the combination of the H1 and ZEUS results. However, already at the present level, much can be done with existing data for the understanding of diffraction at HERA. For all data and fit comparisons, all data are transported to the H1-LRG-06 measurement range $M_\gamma < 1.6\text{GeV}$.

- The first challenge is that $M_X$, used by ZEUS to separate diffractive from non diffractive events, and LRG methods use the same data and thus they are strongly correlated.

- Although the leading-proton data previously suffered from low statistics and hence were unlikely to have much influence on the fit results, the high statistics of the present data make them competitive in precision with the result of the LRG method.

- The LRG results from H1 and ZEUS are compatible in most of the kinematic region covered by measurements.

Our decision

1. No $M_X$ data
2. FPS/LPS + LRG

## Published data points

<table>
<thead>
<tr>
<th>Reference</th>
<th>Lable</th>
<th>Data set</th>
<th>$\beta$-range</th>
<th>$x_F$-range</th>
<th>$Q^2$-range</th>
<th>$N_{\text{Data}}$</th>
<th>$N_i$</th>
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<tbody>
<tr>
<td>1.23</td>
<td>H1-LRG-06</td>
<td>$\sigma_r^{D(3)}$</td>
<td>0.004-0.8</td>
<td>0.001-0.03</td>
<td>8.5-1600</td>
<td>190</td>
<td>0.9958</td>
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<td>1.20</td>
<td>H1-FPS-06</td>
<td>$\sigma_r^{D(3)}$</td>
<td>0.02-0.7</td>
<td>0.0011-0.08</td>
<td>10.7-24</td>
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<td>1.33</td>
<td>H1-FPS-10</td>
<td>$\sigma_r^{D(3)}$</td>
<td>0.006-0.562</td>
<td>0.0025-0.075</td>
<td>8.8-200</td>
<td>100</td>
<td>1.0002</td>
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<tr>
<td>1.33</td>
<td>ZEUS-LPS-04</td>
<td>$F_2^{D(3)}$</td>
<td>0.007-0.48</td>
<td>0.0005-0.06</td>
<td>13.5-39</td>
<td>27</td>
<td>1.0005</td>
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<tr>
<td>1.05</td>
<td>ZEUS-LPS-09</td>
<td>$\sigma_r^{D(3)}$</td>
<td>0.013-0.609</td>
<td>0.0009-0.09</td>
<td>14-40</td>
<td>42</td>
<td>0.9845</td>
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<td>0.97</td>
<td>ZEUS-LRG-09</td>
<td>$\sigma_r^{D(3)}$</td>
<td>0.025-0.795</td>
<td>0.0005-0.014</td>
<td>8.5-225</td>
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<td>1.0094</td>
</tr>
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<td>0.99</td>
<td>H1-LRG-11</td>
<td>$F_{2,L}^{D(3)}$</td>
<td>0.089-0.699</td>
<td>0.0005-0.003</td>
<td>11.5-44</td>
<td>20</td>
<td>0.9739</td>
</tr>
<tr>
<td>0.97</td>
<td>H1-LRG-11</td>
<td>$\sigma_r^{D(3)}$</td>
<td>0.089-0.699</td>
<td>0.0005-0.003</td>
<td>11.5-44</td>
<td>25</td>
<td>0.9739</td>
</tr>
<tr>
<td>0.97</td>
<td>H1-LRG-11</td>
<td>$\sigma_{r,(\sqrt{s}=225,252)}^{D(3)}$</td>
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<td>0.0005-0.003</td>
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<td>0.0005-0.003</td>
<td>11.5-44</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>621</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1:** Overview of the published data points for $\beta < 0.8$, $M_x > 2\text{GeV}$ and $Q^2 > 8.5\text{ GeV}^2$ together with the fitted normalization shifts $N_i$.

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Results
Our final results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>TKT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_{\Sigma})</td>
<td>0.17 ± 0.009</td>
</tr>
<tr>
<td>(B_{\Sigma})</td>
<td>0.08 ± 0.031</td>
</tr>
<tr>
<td>(C_{\Omega})</td>
<td>0.53 ± 0.025</td>
</tr>
<tr>
<td>(D_{\Sigma})</td>
<td>4.88 ± 0.14</td>
</tr>
<tr>
<td>(E_{\Sigma})</td>
<td>-2.36 ± 0.064</td>
</tr>
<tr>
<td>(F_{\Sigma})</td>
<td>0.30 ± 0.012</td>
</tr>
<tr>
<td>(A_g)</td>
<td>0.44 ± 0.020</td>
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<tr>
<td>(\alpha_{\beta}(0))</td>
<td>1.108 ± 0.0035</td>
</tr>
<tr>
<td>(N_{R}(H1-LRG-06))</td>
<td>((1.22 ± 0.18) \times 10^{-3})</td>
</tr>
<tr>
<td>(N_{R}(H1-FPS-06))</td>
<td>((1.21 ± 0.23) \times 10^{-3})</td>
</tr>
<tr>
<td>(N_{R}(H1-FPS-10))</td>
<td>((1.36 ± 0.09) \times 10^{-3})</td>
</tr>
<tr>
<td>(N_{R}(ZEUS-LPS-04))</td>
<td>((1.64 ± 0.26) \times 10^{-3})</td>
</tr>
<tr>
<td>(N_{R}(ZEUS-LPS-09))</td>
<td>((2.25 ± 0.12) \times 10^{-3})</td>
</tr>
<tr>
<td>(N_{R}(ZEUS-LRG-09))</td>
<td>((2.19 ± 0.29) \times 10^{-3})</td>
</tr>
<tr>
<td>(\chi^2/\text{dof})</td>
<td>601.92/607 = 0.99</td>
</tr>
</tbody>
</table>

Table 2: Pomeron quark and gluon densities parameters and their statistical errors for combined data sets in MS scheme at the input scale \(Q_0^2=3\ \text{GeV}^2\).
Figure 1: Comparison between the total quark singlet and gluon distributions obtained from our model, H1 2006 DPDF Fit B, ZEUS SJ and MRW at four different values of $Q^2$ as a function of $z$. The ZEUS SJ and MRW DPDFs plotted here are normalised to $M_Y < 1.6$ GeV by multiplying by a factor 1.23 relative to $M_Y = m_p$. 
The contribution of the sub-leading Reggeon trajectories should be considered for $x_{IP}$ values substantially larger than 0.01. The contribution from these trajectories is modeled using the pion structure function. The pion PDFs are used in a region of low $\beta$ where they are not directly constrained by data. In order to limit the influence of sub-leading Reggeon trajectory, we exclude data points with $x_{IP} > 0.01$ and use a single $N_{IR}$ for all required data sets. No significant difference exists between both fits showing the weak dependence of the fit on the precise knowledge on the Reggeon structure function.

*Figure 2:* Comparison between the total quark singlet and gluon distributions obtained from our model. Results are presented with no cut on $x_{IP}$ (solid) and $x_{IP} < 0.01$ (dashed).
Individual analysis

To study the degree of compatibility, we perform individual analysis on all data sets. It justifies our approach to combine all data sets.

<table>
<thead>
<tr>
<th>Label</th>
<th>$\chi^2$/dof</th>
<th>Data points</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1-LRG-06</td>
<td>0.79</td>
<td>190</td>
</tr>
<tr>
<td>H1-FPS-06</td>
<td>0.53</td>
<td>40</td>
</tr>
<tr>
<td>H1-FPS-10</td>
<td>0.91</td>
<td>100</td>
</tr>
<tr>
<td>ZEUS-LPS-04</td>
<td>0.23</td>
<td>27</td>
</tr>
<tr>
<td>ZEUS-LPS-09</td>
<td>0.78</td>
<td>42</td>
</tr>
<tr>
<td>ZEUS-LRG-09</td>
<td>1.08</td>
<td>155</td>
</tr>
<tr>
<td>H1-LRG-11</td>
<td>0.89</td>
<td>67</td>
</tr>
</tbody>
</table>

Table 3: $\chi^2$ values per data set for the global QCD fit with statistical and systematic errors added in quadrature.

Figure 3: Singlet and gluon distributions of Pomeron as a function of z derived from QCD fits on H1-FPS-10 data alone, H1-LRG-06 data alone, ZEUS-LRG-09 data alone and all the data sets together.
Cross section is largely flat in the measured $\beta$ range. Keeping in mind the similarity between $\beta$ in diffractive DIS and $x_{Bj}$ in inclusive DIS, this is very different from the behavior of the usual cross section, which strongly decreases for $x_{Bj} > 0.2$.

**Figure 4:** The ZEUS-LPS-09 $x_{IP} \sigma^{D(3)}$ as a function of $\beta$ for different regions of $Q^2$ and $x_{IP}$. The curves show our model reduced by a global factor 1.33 to correct for the contributions of proton dissociation processes.

F. D. Aaron et al. [H1 and ZEUS Collaboration], JHEP 1001, 169 (2010) [arXiv:0911.0884 [hep-ex]].
Figure 5: The H1-LRG-06 $x_{IP} \sigma^{D(3)}$ as a function of $\beta$ for different regions of $Q^2$ and $x_{IP}$.
The contribution of the sub-leading exchange alone is also shown in $x_{IP} = 0.03$ and 0.01.
Positive scaling violations are due to the growth of the sea quark and gluon densities.

Negative scaling violations for $x_{Bj} > 0.2$ manifest the existence of the valence quarks radiating gluons.

F. D. Aaron et al. [H1 and ZEUS Collaboration], JHEP 1001, 109 (2010) [arXiv:0911.0884 [hep-ex]].
**LRG data versus $Q^2$**

**Figure 6:** Comparison between the H1-LRG-06 and ZEUS-LRG-09 measurements after correcting the latter data set to $M_Y < 1.6$ GeV by applying scale factor of 1.05. The measurements are compared with our model and the results of the H1 2006 DPDF Fit B, which was based on the H1 data shown.


Figure 7: $\chi_{IP} F_2^{D(3)}$, as a function of $Q^2$ for different regions of $\beta$ and $x_{IP}$. Our model reduced by 1.33 to correct for the contributions of proton dissociation processes.

Figure 8: The diffractive cross section multiplied by $x_{IP}$, as a function of $Q^2$ for different regions of $\beta$ and $x_{IP}$. The curves show our model reduced by a global factor 1.20 to correct for the contributions of proton dissociation processes as described previously.

Figure 9: The diffractive cross section multiplied by $x_{IP}$, as a function of $Q^2$ for different regions of $\beta$ and $x_{IP}$. The curves show our model reduced by a global factor 1.23 to correct for the contributions of proton dissociation processes as described previously.

We superimposed our results on $M_X$ data. It shows that the fit can describe most kinematic regions. It leads to the conclusion that there seems to be compatibility between all data sets.

**Figure 10:** The diffractive cross section multiplied by $x_{IP}$, as a function of $Q^2$ for different regions of $\beta$ and $x_{IP}$. The curves show our model reduced by a global factor 0.86 to correct for the contributions of proton dissociation processes as described previously.

Figure 11: $x_{IP}\sigma_{D(3)}$, as a function of $\beta$ at fixed $Q^2$ and $x_{IP}$ for (from left to right) the 460, 575, 820 and 920 GeV data sets. The curves show our model reduced by a global factor 0.97, 0.99 and 0.97 for $E_p=460$, 575 and 920 GeV, respectively.

A measurement of $F_L^D$ provides a very powerful independent tool to verify our understanding of the underlying dynamics of diffraction and to test the DPDFs. This is particularly important at the lowest $\beta$ values, where direct information on the gluon density cannot be obtained via other methods.

**Figure 12:** $x_{IP}\sigma_{D(3)}$, as a function of $\beta$ at fixed $Q^2$ and $x_{IP}$ for (from left to right) the 460, 575, 820 and 920 GeV data sets. The curves show our model reduced by a global factor 0.97, 0.99 and 0.97 for $E_p=460$, 575 and 920 GeV.

Figure 13: $x_{IP}F_{L}^{D}$ measurement as a function of $\beta$ measured at fixed $Q^2$ and $x_{IP}$. The present fit is the solid curve. Also shown are the results of H1 2006 Fit A and and Fit B.
Figure 14: The ratio measurement, $R_D$, as a function of $\beta$ at the indicated values of $x_{IP}$ and $Q^2$. Our result is the solid curve.
Our result on charm cross sections and structure function

Charm production in diffractive DIS has been measured by H1 and ZEUS collaborations. It allows quantitative tests of models due to the sensitivity of charm production to gluon density.

**Figure 15:** Comparison of our result for the contribution of the charm quarks to the diffractive cross section and structure function with H1 2006 DPDF Fit A, Fit B and MRW shown as a function of $\beta$ for different values of $x_{IP}$. The data obtained from the H1 and ZEUS D* production and from H1 displaced track method.


Sara Taheri Monfared (Semnan University and IPM)
Conclusion

IPM Parton Distribution Function

Title: FORTRAN Package for Diffractive Parton Distribution Functions and Structure Functions with their Errors (TKT2012)

A global analysis of diffractive events at HERA

arXiv:1109.0912

Author: S. Taheri Monfared, A. Khorramian, S. Atashbar Tehran


To access the latest Diffractive Parton Distribution Functions (DPDFs) and Structure Functions with their errors, please use the Fortran package dpdf-TKT2012.tar.gz file.

Title: FORTRAN Package for Polarized Parton Distribution Functions and Polarized Structure Functions for Nucleons and Nuclei (KATAOpolinfo)

Author: A. Khorramian, S. Atashbar Tehran, S. Taheri Monfared, F. Arbabifar, F. I. Olness

Package Version: 09 Nov, 2010

To access the latest Polarized Parton Distribution Functions and Polarized Structure Functions for Nucleons and Nuclei please use the Fortran package KATAO2010.tar.gz file.

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In conclusion, this has been a general overview of what really fascinated me through the course of this study.

• We have shown that the diffractive observables measured in the H1 and ZEUS experiments at HERA can be well described by a perturbative QCD analysis which fundamental quark and gluon distributions, evolving according to the NLO DGLAP equations, are assigned to the Pomeron and Reggeon exchanges.

• We introduce new set of quark distribution which provide flexibility to obtain a proper description of the data.

• Although these data obtained by various methods with very different systematic, they are broadly consistent in the shape of the distribution. This is a very important message from HERA that diffractive DPDFs are well compatible for both experiments

• Our next steps are toward calculation of double diffractive phenomena in hadron–hadron colliders ...
Thanks for your paying attention
Diffractive structure function to all order in perturbation theory can be described by

\[ F_i(\beta, Q^2) = \sum_j C_i^j(\beta, \frac{Q^2}{\mu^2}) \otimes f_j(\beta, \mu^2), \]

Generally speaking, structure function can be written like

\[ F_i(\beta, Q^2) = F_i^{\text{light}}(\beta, Q^2) + F_i^{\text{heavy}}(\beta, Q^2, m_h^2). \]

\[ i = 2, L \]

The flavor singlet contribution up to NLO is given by

\[ \frac{1}{x} F_i^{\text{light}}(\beta, Q^2) = \frac{2}{9} \left( C_{i,q} \otimes \Sigma + C_{i,g} \otimes g \right)(\beta, Q^2). \]

NLO coefficient functions


The treatment of heavy quarks is something that nearly every group does slightly and it can lead to surprisingly different results for PDFs extracted.

- **VFNS**: $p \rightarrow (u, d, s, g) + (c, b, t)$
- **FFNS**: $p \rightarrow (u, d, s, g)$

\[ F_i^h(\beta, Q^2) = \sum_k C_{i,k}^{F_n f}(Q^2/m_h^2) \otimes f_{i,k}^{n f}(Q^2). \]

\[ z_{\text{max}} = Q^2/(Q^2 + 4m^2) \]

\[ F_k(\beta, Q^2, m^2) = \frac{Q^2 \alpha_s}{4\pi^2 m^2} \int_{\beta}^{z_{\text{max}}} \frac{dz}{z} \left[ c_{H}(\frac{\beta}{z}, \mu^2)c_{k,g}^{(0)} \right] \]
\[ + \frac{Q^2 \alpha_s}{\pi m^2} \int_{\beta}^{z_{\text{max}}} \frac{dz}{z} \left[ c_{H}(\frac{\beta}{z}, \mu^2)c_{k,g}^{(1)} + \bar{c}_{k,g}^{(1)} \ln \frac{\mu^2}{m^2} \right] \]
\[ + \sum_{i=q,g} \left[ c_{L,i} f_i(\frac{\beta}{z}, \mu^2)(d_{k,i}^{(1)} + \bar{d}_{k,i}^{(1)} \ln \frac{\mu^2}{m^2}) \right], \]


Non diffractive
\[ \frac{dN}{d \ln M_x^2} = c e^{b \ln M_x^2} \]

Diffractive
\[ \frac{dN}{d \ln M_x^2} \approx \text{const.} = D \]
Error calculation

Overall normalization factor: obtained via first fit

\[ \chi^2_{\text{global}}(p) = \sum_{i=1}^{n_{\text{data}}} \left[ \left( \frac{1 - N_i}{\Delta N_i} \right)^2 + \sum_{j=1}^{n_{\text{data}}} \left( \frac{N_j F_{2,i}^{D,\text{data}} - F_{2,i}^{D,\text{theor}}(p)}{N_j \Delta F_{2,i}^{D,\text{data}}} \right)^2 \right] \]

- Number of data points included
- Set of independent parameters
- Data value
- Theoretical value
- Measurement uncertainty

Experimental normalization uncertainty: quoted by the experiments

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