Conformal symmetry and the relations between perturbative contributions to the Bjorken and Ellis-Jaffe sum rule of the polarized DIS

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Plan of Presentation

Introduction – Reminding of the concept of Conformal Symmetry

Existing perturbative Quenched U(1) identity $O(E^3)$ result: Coefficient functions of Ellis-Jaffe and Bjorken sum rules consider! Q1 Accident or Not?

A1
- Not! Statement: true in all orders of PT
Explanation due to Kataev- 2010 derivation of new Crewther-type relation from triangle diagram of singlet A- V-V currents for the Green function of AA -currents and EJ SI structure function and comparing with basic non-singlet Crewther relation for A-V-V triangle Crewther 1972
Positive conclusion of discussion between Kataev 1996 and Crewther 1997

A2
The conformal-invariant result for DIS sum rules at $O(A_s^3)$-level the case of $SU(N_c)$ Kataev and Mikhailov 2010, Teor.Mat.Fiz. 2012

Conclusions
1) Possible applications- tests of future higher-order ’ EJ SR analytical calculations
2) Possible relations between polarized parton distributions in the CI limit.
Conformal Invariance

is valid in the quark-parton model limit, and perturbative quenched QED. It is the symmetry under the following transformations of coordinates:

1. Translations \( x'{}^\mu = x{}^\mu + \alpha{}^\mu \) with 4 parameters \( \alpha{}^\mu \),
2. Scale (or dilaton) transformation \( x'{}^\mu = \rho x{}^\mu \) with 1 parameter \( \rho > 0 \),
3. Special conformal transformations \( x'{}^\mu = \frac{x{}^\mu + \beta{}^\mu x^2}{1 + 2\beta x + \beta^2 x^2} \) with 4 parameters \( \beta{}^\mu \) and
4. Homogeneous Lorentz transformations \( x'{}^\mu = \Lambda{}_{\nu}{}^{\mu} x{}^\nu \), which also contain 4 parameters.
5. Consequences are widely studied at present, though in renormalized QFT models the CI is violated- appearance of \( \beta \)-function and the effects of running of the coupling constants- QCD, QED
Bjorken polarized sum rule:

\[
B_{jp}(Q^2) = \int_0^1 \left( g_1^{lp}(x, Q^2) - g_1^{ln}(x, Q^2) \right) dx = \frac{1}{6} g_a C_{B_{jp}}^{ns}(A_s)
\]

(1)

Depends from \( Q^2 \) through the running of \( A_s(Q^2) = \alpha_s(Q^2)/(4\pi) \),
Consider pQED limit- \( C_F = 1 \), \( N_F = 0 \), \( A = \alpha/(4\pi) \)

\[
C_{B_{jp}}^{ns} = 1 - 3A + \frac{21}{2} A^2 - \frac{3}{2} A^3 - \left( \frac{4823}{8} + 96\zeta(3) \right) A^4 + O(A^5)
\]

(2)

\( O(A^3) \) Larin and Vermaseren (1991); \( O(A^4) \) Baikov, Chetyrkin, Kuhn (2010)
Ellis-Jaffe sum rule:

\[
E_{jp}^{(n)}(Q^2) = \int_0^1 g_1^{lp(n)}(x, Q^2) dx = C_{B_{jp}}^{ns}(A_s(Q^2)) \left( \pm \frac{1}{12} a_3 + \frac{1}{36} a_8 \right) + C_{E_{jp}}^{s}(Q^2) \frac{1}{9} \Delta \Sigma(Q^2)
\]

(3)

where \( a_3 = \Delta u - \Delta d \), \( a_8 = \Delta u + \Delta d - 2\Delta s \), \( \Delta u, \Delta d \) and \( \Delta s \) are the polarized distributions and \( \Delta \Sigma \) depends from the scheme choice. In the \( \overline{MS} \)-scheme it is defined as \( \Delta \Sigma = \Delta u + \Delta d + \Delta s \).
Note, that in the \( \overline{\text{MS}} \)-scheme the definition of the singlet coefficient function reads (Larin (1994), Larin, van Ritbergen, Vermaseren (1997)).

\[
C^s_{EJp} = \overline{C}^s_{EJp}/Z^s_5
\]  \hspace{1cm} (4)

In the perturbative quenched QED the result is

\[
\overline{C}^s_{EJ} = 1 - 7A + \frac{89}{2}A^2 - \left( \frac{1397}{6} - 96\zeta(3) \right)A^3 + O(A^4)
\]  \hspace{1cm} (5)

\( Z^s_5 \) is the finite renormalization constants of \( \overline{\Psi}\gamma_\mu\gamma_5\Psi \) current, \( Z^{ns}_5 \) is the renormalization constant of \( \overline{\Psi}\gamma_5(\lambda^a/2)\Psi \)-current. They were evaluated in Larin (1994) and Larin and Vermaseren (1991). In the limit of pQED we have

\[
Z^s_5(pqQED) = Z^{ns}_5(pqQED) = 1 - 4A + 22A^2 + \left( -\frac{370}{3} + 96\zeta_3 \right)A^3 + O(A^4)
\]  \hspace{1cm} (6)

Nontrivial scheme-independent pQED consequence of QCD results for SI and NS coefficient functions of EJ sum rule Kataev (2010):

\[
C^s_{EJp}(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 + O(A^4) = C^{ns}_{Bjp}(A)
\]  \hspace{1cm} (7)

Q1: What is the theoretical explanation? Q2: Is this result true in all orders of PT? Q3: Is there any \( Q^2 \)-dependence?
A1: Follow from Crewther-type relations for AVV diagrams in CI limit
A2: Valid in all orders of PT A3: No $Q^2$ dependence- fixed coupling constant

Proof of A1: Using OPE for

$$T^{ab}_{\mu\alpha\beta}(p, q) = i \int <0| TA_\mu(y) V^a_\alpha(x) V^b_\beta(0)|0 > e^{ipx+iqy} dx dy = \delta^{ab} \Delta^{(1-loop)}_{\mu\alpha\beta}(p, q)$$

(8)

where $A_\mu = \bar{\psi} \gamma_\mu \gamma_5 \psi$ and

$$T^{abc}_{\mu\alpha\beta}(p, q) = i \int <0| TA^a_\mu(y) V^b_\alpha(x) V^c_\beta(0)|0 > e^{ipx+iqy} dx dy = d^{abc} \Delta^{(1-loop)}_{\mu\alpha\beta}(p, q)$$

(9)

The consideration of the first and second triangle graphs+ the concept of conformal invariance give the following relations: Kataev 1996 and Crewther 1972

$$C_{EJP}^s(A) \times C_D^A(A) = 1 \ (1996) \quad C_{Bjp}^{ns}(A) \times C_D^{ns}(A) = 1 \ (1972)$$

(10)

where $C_D^{A(ns)}$ are defined from taking $q^2 \frac{d}{dq^2}$ of

$$i \int <0| TA_\mu(x) A_\nu(0)|0 > e^{iqx} dx = \Pi^{ax}_{\mu\nu}(q^2)$$

(11)

$$i \int <0| TA^a_\mu(x) A^b_\nu(0)|0 > e^{iqx} dx = \delta^{ab} \Pi^{ns}_{\mu\nu}(q^2).$$

(12)

Since in the massless limit chiral symmetry is exact, one has:

$$C_D^A(A) = C_D^{ns}(A)$$

(13)
Thus in the CI limit in all orders of PT

\[ C_{EJp}^s(A) = C_{Bjp}^{ns}(A) \quad \text{Kataev 2010} \quad (14) \]

In pQED with \( A = \alpha/(4\pi) \) CI limit we have

\[ C_{EJp}^s(A) = 1 - 3A + \frac{21}{2} A^2 - \frac{3}{2} A^3 = C_{Bjp}^{ns}(A) \quad (15) \]

In the case of the \( SU(N_c) \) with \( A_s = \alpha_s/(4\pi) \) the similar relation is:

\[ C_{Bjp}^{ns} = 1 - 3C_F A_s + \left( \frac{21}{2} C_F^2 - C_F C_A \right) A_s^2 \]

\[ + \left[ \left( -\frac{3}{2} C_F^3 - 65 C_F^2 C_A - \left( \frac{523}{12} - 216 \zeta_3 \right) C_F C_A^2 \right) A_s^3 + O(A_s^4) \right] = C_{EJ}^s(A_s) \quad (16) \]

Numbers obtained from Crewther relation in Kataev and Mikhailov (2010-2012).

This is the CI predictions for \( O(A_s^4) \)-term in \( C_{EJp}^s(A_s) \).
Conclusion
In the CI limit in $SU(N_c)$
1. It is possible to try to get $O(A_s^4)$ for $C_{Bjp}$ from Kataev, Mikhailov (2012)- strong test for analytical evaluation of Ellis-Jaffe sum rule at $A_s^4$ (hope that $SU(N_c)$ results will be obtained soon).
2. In this CI limit we get the following constraints on polarized PDFs:

$$ \frac{EJp^{(n)}}{Bjp} = \pm \frac{1}{2} + \frac{a_8}{6a_3} + \frac{2\Delta\Sigma}{3a_3} $$

(17)

where $a_3 = \Delta u - \Delta d$, $a_8 = \Delta u + \Delta d - 2\Delta s$, and $\Delta\Sigma = \Delta u + \Delta d + \Delta s$.

Question: Is it possible to use this relation in practice?