W/Z/DY + jet as a probe of BFKL evolution

Martin Hentschinski

IFT UAM/CSIC Madrid

Based on results obtained with C. Salas Hernandez

Workshop on Deep Inelastic Scattering and related topics 2012

BFKL evolution & High Energy Factorization

- hadronic scattering processes with a hard scale $Q^2 \gg \Lambda^2_{\text{QCD}}$ described within perturbative QCD
- limit of high c.o.m. eneries $s \gg Q^2$ ➡ enter multi scale regime ➡ $\alpha_s(Q^2) \ln s/Q^2 \sim 1$ ➡ require resummation

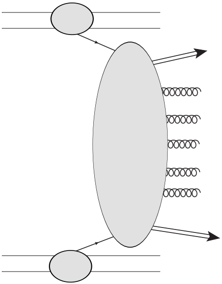
BFKL evolution:
- LL [Fadin, Kuraev, Lipatov (1977)], [Balitsky, Lipatov (1978)]
- NLL [Fadin, Lipatov (1998)], [Ciafaloni, Gamici (1998)]

- Predicts rise of cross-sections with s
- Derived from QCD & QFT
- Hints, but no clear experimental evidence till nowadays

possible reason:
- background of soft & collinear corrections
- BFKL calculation asymptotic

Question: Is it possible to isolate BFKL dynamics at the LHC ?

gold plated process at hadron collider: **forward-backward jets** [Mueller, Navelet (1987)]

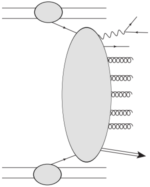


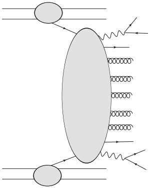
- large relative rapidity opens phase space for n parton emissions ➡ BFKL evolution
- entire process treatable within collinear factorization ➡ no modelling of non-perturbative effects needed
- BFKL description currently known at NLL level [Fadin, Fiore Quartarolo (1994)], [Ciafaloni (1998)], [Ciafaloni, Colferai (1999)], [Bartels, Colferai, Vacca (2002), (2003)], [Colferai, Schwennsen, Szymanowski, Wallon (2010)], [Hentschinski, Sabio Vera (2011)], [Caporale, Ivanov, B. Murdaca, A. Papa (2011)]
- numerical analysis [Colferai, Schwennsen, Szymanowski, Wallon (2010)] reveals strong dependence on impact factors

advisable to study also other processes:

→ isolate effects associated with specific final state
→ test high energy factorization (universality)

here: forward W/Z/γ^*+ backward jet [also forward-backward W/Z/γ^*]



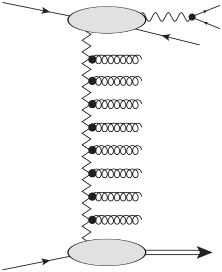


+ color singlet final state → less sensitivity to collinear effects
+ leptonic final state → no hadronization effects

– less phase space for BFKL evolution, ($|\eta_l| < 2.7$) → are other effects sufficiently suppressed?
– complications due to additional quark in final state

- $W/Z+$ jet and WW/ZZ production interesting process by itself: important background process at LHC for Higgs and new physics searches
- provide as a by-product high energy limit of cross-section + high energy resummation

Construction of the high energy resummed $Z+$ jet X-section



- High energy factorization: absorb large high energy logarithms $\alpha_s \ln(s/s_0)$ into gluon (BFKL) Green's function f_{BFKL}
- reggeization scale s_0: analog to factorization scale for high energy factorization
- At first: hybrid study ➡ ϕ's at LO, f_{BFKL} at NLO
  - Complete NLO study possible on a mid-term basis ➡ high energy effective action [Lipatov (1995)]; [MH, Sabio Vera (2011)], [Chachamis, MH, Madrigal, Sabio Vera (2011)]

X-sec. as convolution in transverse momentum space:

$$\frac{d\sigma}{d\boldsymbol{q}dY\,dM_Z^2} = \int \frac{d^2\boldsymbol{k}_1}{\pi} \int \frac{d^2\boldsymbol{k}_2}{\pi} \phi_{Zq}\left(\boldsymbol{k}_1, z, M_Z^2, \boldsymbol{q}\right) f_{\text{BFKL}}\left(\frac{s}{s_0}, \boldsymbol{k}_1, \boldsymbol{k}_1\right) \phi_J\left(\boldsymbol{k}_2, \boldsymbol{p}^2\right)$$

The BFKL gluon Green's function obtained as solution to BFKL evolution equation – formulated in moment pace

$$f\left(\frac{s}{s_0}, \boldsymbol{k}_1, \boldsymbol{k}_2\right) = \int \frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} f_\omega(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

Integral equation: $$\omega f_\omega(\boldsymbol{k}_1, \boldsymbol{k}_2) = \delta^2(\boldsymbol{k}_1 - \boldsymbol{k}_2) + \int \frac{d^2\boldsymbol{k}}{\pi} K(\boldsymbol{k}_1, \boldsymbol{k}) f_\omega(\boldsymbol{k}, \boldsymbol{k}_2)$$

two ways to solve:

- Monte Carlo event generator
- analytic solution

LO: $E_{n,\gamma}(\boldsymbol{q}) = (\boldsymbol{q}^2)^{\gamma-1} e^{in\phi}$ conformal eigenfunctions of the LO BFKL kernel

$$K_{LO} \otimes E_{n,\gamma} = \bar{\alpha}_s \chi_0(\gamma, n) E_{n,\gamma}$$

- $\gamma = i\nu + \frac{1}{2}$: moments of transverse momentum ➡ anomalous dimension
- n: conformal spin ➡ Fourier modes

analytic solution for LO Green's function

$$f_{LO}\left(\frac{s}{s_0}, \boldsymbol{k}_1, \boldsymbol{k}_2\right) = \int \frac{d\gamma}{2\pi i}\left(\frac{s}{s_0}\right)^{\bar{\alpha}_s \chi_0(\gamma, n)} \frac{1}{\boldsymbol{k}_1^2}\left(\frac{\boldsymbol{k}_1^2}{\boldsymbol{k}_2^2}\right)^{\gamma}$$

NLO: running coupling corrections induced non-diagonal terms

→ conformal LO eigenfunctions are not NLO eigenfunctions

→ can include these effects in conformal LO basis to NLO accuracy

construct following NLO BFKL eigenvalue:

$$\omega_{\text{BFKL}}(\gamma, n) = \bar{\alpha}_s \chi_0(\gamma, n) + \bar{\alpha}_s^2 \big(\chi_1(\gamma, n) + \chi_{\text{R.C.}}(\gamma, n)\big) + \bar{\alpha}_s^3 \chi_{\text{R.G.}}(\gamma, n)\big)$$

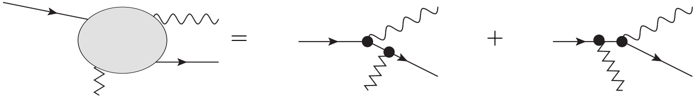
→ allows to use the LO solution

- χ_0, χ_1: conformal LO & NLO BFKL kernel [Fadin, Lipatov (1998)]; [Kotikov, Lipatov (2001)]
- $\chi_{\text{R.C.}}$ running coupling corrections: exact to NLO accuracy,
  - essentially derivative w.r.t. γ of logarithm of impact factors in (γ, n) representation → Here: rather complicated expression
  - correct to NLO accuracy, but care is needed → Monte-Carlo solution might be able to provide improved treatment
  - scale of running coupling determined by external particles

Resummation:

- $\chi_{\text{R.G.}}$: NLO kernel leads to numerical instability – ∃ (anti-)collinear double poles inside the kernel – soft & collinear origin
  - resummation: RG improved NLO BFKL kernel – stabilizes evolution
  - present study uses 'all-pole-approximation' [Sabio Vera (2005)]; advantage: kernel exponentiates – consistent with reggeization

the W/Z/γ^* impact factor (from now on: the Z-boson alone)



- Matrix element $\mathcal{M}_{qg^* \to Zq}$ vanishes for gluon virtuality $\boldsymbol{k}^2 \to 0$ ➡ color singlet final state
- soft singularity for vanishing quark momentum
- obtain differential momentum space impact factor as

$$\Phi_{Zq}(\boldsymbol{k}, z, \boldsymbol{q}) dz d^2\boldsymbol{q} = \frac{1}{8\pi x \boldsymbol{k}^2} d\Phi^{(2)} \int dx_g |\mathcal{M}|^2_{qg^* \to qZ}$$

- z : fraction of the inital quark momentum xp carried on by the Z-boson
  $\boldsymbol{q}$: transverse momentum of the Z-boson
  $\boldsymbol{k}$: transverse momentum of the gluon

use of analytic solution to (N)LL BFKL Green's function ➡ transform ϕ_{Zq} from $\boldsymbol{k}$ representation to representation w.r.t. transverse moments γ and conformal spin n

$$\phi_{Zq}(\boldsymbol{k}) = \frac{1}{\pi}\sum_n \int \frac{d\gamma}{2\pi i}\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^{\gamma} e^{in\theta_{\boldsymbol{k}}} H_{Zq}(n,\gamma)$$

(a) total cross-section: integrate over z & $\boldsymbol{q}$; only conformal spin zero agree with result by [Marzani, Ball (2008)]

$$H_{Zq}(n,\gamma) = \left(4\pi c_f \frac{\sqrt{N_c^2-1}}{N_c}\right)\frac{\alpha_s}{\pi}\frac{\Gamma^2(\gamma)\Gamma^2(1-\gamma)}{(3-\gamma)(2-\gamma)(1-\gamma)}, \qquad \mu^2 = M_Z^2$$

(b) exclusive impact factor: multiple scales

$$H_{Zq}(n,\gamma) = \frac{1}{\boldsymbol{q}^2}\left(\frac{\boldsymbol{q}^2}{M_Z^2}\right)^{n/2} \bar{H}_{Zq}\left(n,\gamma,z,\frac{\boldsymbol{q}^2}{(1-z)M_Z^2}\right), \qquad \mu^2 = \frac{(1-z)M_Z^2}{z^2}$$

non-trivial function

Note: μ^2 sets scale of running coupling inside BFKL Green's function; limit $z \to 1$ delicate

- $\bar{H}$ non-trivial function: $\left(x \equiv \frac{\boldsymbol{q}^2}{(1-z)M_Z^2}\right)$

$$\bar{H}_{Zq}(\gamma, n; z, x) = \frac{\alpha e_f^2 \alpha_s \sqrt{N_c^2 - 1}}{\pi N_c} \left\{ \frac{1 + (1-z)^2}{z} \frac{x}{1+x} \right.$$
$$\frac{\Gamma(\gamma + n/2)\Gamma(1-\gamma+n/2)\Gamma(1-\gamma+n/2)}{\Gamma(1-\gamma+n)\Gamma(1+n)} {}_2F_1\left(1-\gamma+\frac{n}{2}, 1-\gamma+\frac{n}{2}, 1+n; -x\right)$$
$$- 2zx \frac{\Gamma(\gamma-1+n/2)\Gamma(3-\gamma+n/2)\Gamma(2-\gamma+n/2)}{\Gamma(2-\gamma+n)\Gamma(1+n)} {}_2F_1\left(3-\gamma+\frac{n}{2}, 2-\gamma+\frac{n}{2}, 1+n; -x\right)$$
$$\left. + 4\frac{zx}{1+x} \frac{\Gamma(\gamma-1+n/2)\Gamma^2(2-\gamma+n/2)}{\Gamma(2-\gamma+n)\Gamma(1+n)} {}_2F_1\left(2-\gamma+\frac{n}{2}, 2-\gamma+\frac{n}{2}, 1+n; -x\right) \right\}$$

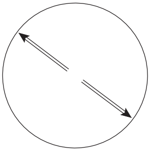
- carries divergence for $z \to 1$

$$\lim_{z\to 1} \bar{H}_{Zq}\left(n, \gamma, z, \frac{\boldsymbol{q}^2}{(1-z)M_Z^2}\right) = \left(c_{Zq}^f \frac{\sqrt{N_c^2-1}}{\pi N_c}\right) \frac{\Gamma(1-\gamma+\frac{n}{2})}{\Gamma(1-\gamma+n)} \ln \frac{\boldsymbol{q}^2}{(1-z)M_Z^2}$$

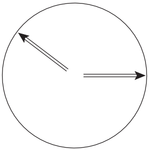
- at first: soft singularity which cannot be avoided for hadronic differential cross-section
  → convolution with initial quark pdf: $z = 1 \equiv$ lower limit for convolution integral

→ develop resummation and/or design observables which are not sensitive to this region

Borrow from the study of forward-backward jets [Stirling (xxx)], [Sabio Vera, Schwennsen (xxx)]



central jets: major QCD correction soft & collinear radiation
➡ transverse momentum of jets basically unchanged
➡ azimuthal angle strongly correlated



fwd.-bkwd jets: phase space opens
➡ additional hard emission between jets
➡ azimuthal angle of jets decorrelated

study angular coefficents

$$\mathcal{C}_m = \int \frac{d\theta}{2\pi} \cos m\theta \frac{d\sigma}{d\theta}$$

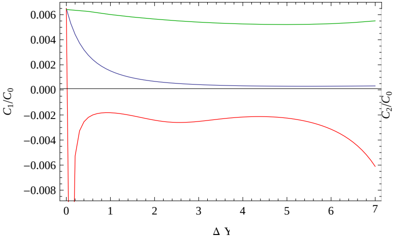
- describe rapidity dependence by BFKL
- ratios $\mathcal{C}_n/\mathcal{C}_m$ less sensitive to soft & collinear logs

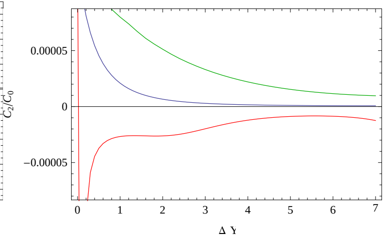
$\boldsymbol{Z+}$ **jet:** picture less illustrative, but ...

- can study rapidity dependence of $\mathcal{C}_n$ with BFKL ➡ azimuthal angle sensitive to multiple BFKL emissions (n-dependence of BFKL eigenvalue!)
- ratios $\mathcal{C}_n/\mathcal{C}_m$ are finite for $z \to 1$ ➡ can start study of $Z+$jet with current knowledge

A first flavor of numerical results

- restrict analysis for the moment to the partonic level ➡ no pdf's included ➡ results not directly applicable to phenomenology
- NLO BFKL kernel: only conformal part; running coupling corrections known but difficult to implement
- use at first fixed running coupling $\bar{\alpha}_s = 0.2$





BLUE: LO RED: NLO BFKL GREEN: RG improved NLO BFKL

- NLO BFKL alone not stable ➡ need RG improvement
- Convergence of ratios $\mathcal{C}_1/\mathcal{C}_0$ and $\mathcal{C}_2/\mathcal{C}_0$ rather poor, even with resummation included

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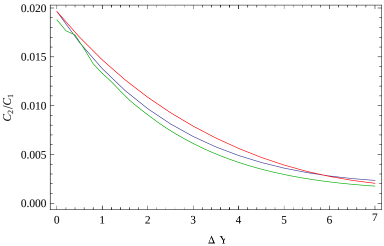

- convergence improves for ratio $\mathcal{C}_2/\mathcal{C}_1$
- promises to be a suitable observable
- similar observation for NLL forward-backward jets [Colferai, Schwennsen, Szymanowski, Wallon (2010)]
- still way to go (running coupling, pdfs, ...)

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Summary

- still miss striking evidence for BFKL evolution - no model, but derived from QCD → should be found
- proposed new final state (for BFKL searches) to be studied at LHC: forward $W/Z/\gamma^*+$ backward jet production
- determined analytic expression for $W/Z/\gamma^*$ impact factor in (γ, n) representation
- plagued by soft divergence → need to develop resummation and/or use 'good' observables
- ratios of angular coefficients: divergence cancels, but problem with running coupling remains
- first numeric results at partonic level with fixed strong coupling for $\mathcal{C}_1/\mathcal{C}_0$, $\mathcal{C}_2/\mathcal{C}_0$, $\mathcal{C}_2/\mathcal{C}_1$

Outlook

- Complete study: full running coupling corrections & parton distribution functions
- mid/long term goal: develop systematic resummation of soft logarithms for exclusive impact factors