NNPDF determination of polarized PDFs at NLO

XX International Workshop on Deep-Inelastic Scattering and Related Subjects

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Outline

1. Introduction
   - Why do we need polarized PDFs?
   - Issues in Standard PDF determination
2. NNDPF fitting approach
   - A general overview
   - Monte Carlo sampling and Neural Networks
3. Towards NNPDFpol1.0
   - Experimental dataset and PDF parametrization
   - Results: polarized PDFs and the spin content of the proton
4. Conclusions
   - Summary and outlook

1. Introduction

Why do we need polarized PDFs?

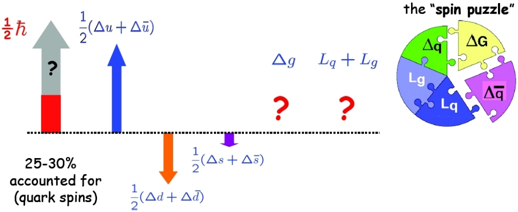
1. How do quarks and gluons carry proton's spin?

$$a_0 = \Delta\Sigma \equiv \int_0^1 \Delta\Sigma(x)dx = 2\langle S_z^{\text{quarks}}\rangle \sim 1 \qquad \Delta\Sigma(x) = \sum_{i=u,d,s} (\Delta q_i + \Delta\bar{q}_i)$$

EMC experiment (1988): $a_0 = 0.114 \pm 0.012 \pm 0.026$ "SPIN CRISIS"

$$\langle S_z\rangle = \frac{1}{2}\langle\Delta\Sigma\rangle \longrightarrow \langle S_z\rangle = \frac{1}{2}\langle\Delta\Sigma\rangle + \langle\Delta g\rangle + L_q + L_g$$

2. Focus on quark and gluon pieces of the "spin puzzle"



3. Rich phenomenology, explore QCD beyond helicity-averaged case

Issues in standard PDF determination

- Extraction of a set of functions with error bands from a set of data points.
- We need an error band, i.e. a probability density $\mathcal{P}[\Delta q(x)]$ in the space of PDFs:

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\Delta q \, \mathcal{P}[\Delta q] \mathcal{O}[\Delta q]$$

$$\sigma_{\mathcal{O}}^2 = \int \mathcal{D}\Delta q \, \mathcal{P}[\Delta q] (\mathcal{O}[\Delta q] - \langle \mathcal{O} \rangle)^2$$

Standard approach

1. Choose a fixed functional form like

   $$\Delta q_i(x, Q_0^2) = A_i x^{b_i} (1-x)^{c_i} (1+\dots)$$

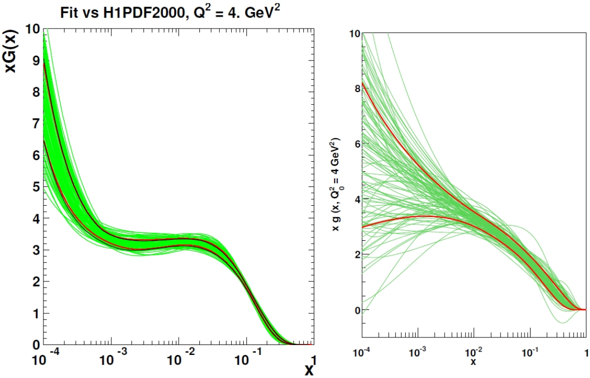
2. Determine best-fit parameters
3. Errors determined via Gaussian linear error propagation

But...

1. Is the parametrization flexible enough?
2. What is the error associated to any particular choice?
3. Need to rely on linear error propagation

Simple functional forms vs Neural Networks

HERA-LHC 2009 PDF benchmarks



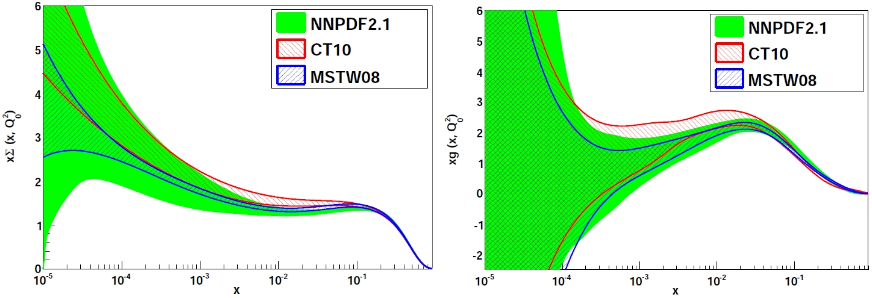
simple functional forms — Neural Networks

- Simple functional forms $\Delta q(x) = Ax^b(1-x)^c P(x)$
  $\longrightarrow$ systematic underestimation of uncertainties $\Rightarrow$ tolerance
- Artificial Neural Networks as universal interpolants
  $\longrightarrow$ reduce theoretical bias from choice of PDF functional form

PDF fitting: a new approach

NNPDF: a new approach to PDF fitting based on
Monte Carlo sampling and Neural Networks

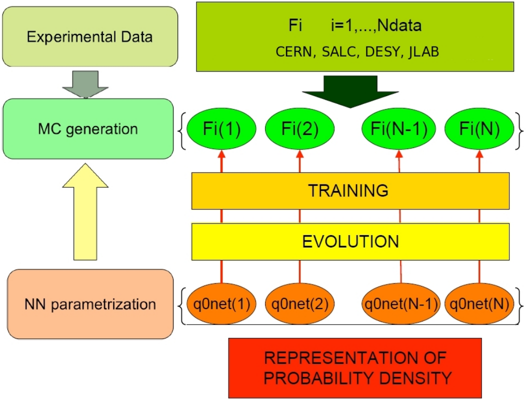
The NNPDF Collaboration, Nucl.Phys. B849 (2011) 296, 1101.1300



Successfully applied in the unpolarized case: most recent global fit NNPDF2.1
Routinely used in LHC data analysis and theory prediction

2. NNPDF fitting approach

A general overview on the recipe



Ingredients:
Monte Carlo sampling and Neural Networks

Ingredient 1: Monte Carlo sampling of experimental data

MONTE CARLO SAMPLING

- Sample the probability density $\mathcal{P}[\Delta q]$ in the space of functions assuming multi-Gaussian data probability distribution

$$g_{1,p}^{(\mathrm{art}),k}(x,Q^2) = (1 + r_{k,N}\sigma_N)\left[g_{1,p}^{(\mathrm{exp})}(x,Q^2) + r_{k,t}\sigma_t(x,Q^2)\right]$$

r_k: Gaussian random numbers σ_N quadratic sum of normalization errors
σ_t: total error (summing in quadrature statistical and systematic errors)

- Generate MC ensemble of N_{rep} replicas with the data probability distribution

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- Generate MC ensemble of N_{rep} replicas with the data probability distribution

MAIN FEATURES

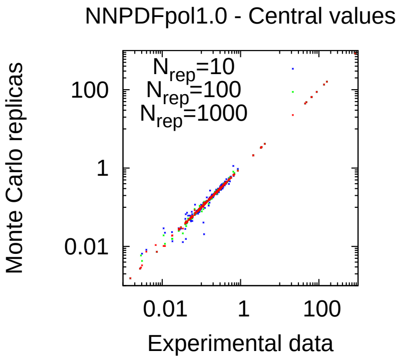
- Expectation values for observables are Monte Carlo integrals

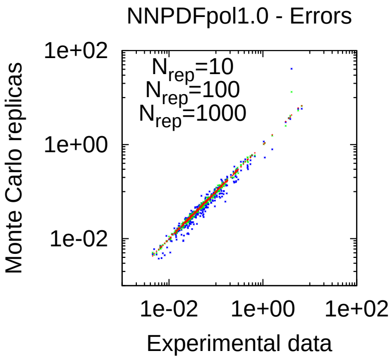
$$\langle \mathcal{O}[\Delta q] \rangle = \frac{1}{N_{\mathrm{rep}}} \sum_{k=1}^{N_{\mathrm{rep}}} \mathcal{O}[\Delta q_k]$$

. . . and the same is true for errors, correlations etc.

- No need to rely on linear propagation of errors
- Possibility to test for non-Gaussian behaviour in fitted PDFs

Ingredient 1: Monte Carlo sampling of experimental data

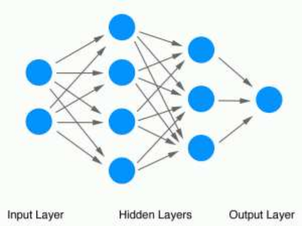




- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy $\longrightarrow$ determine size of the sample
- Accuracy of few % requires $\sim$ 1000 replicas

Ingredient 2: Neural Networks

A convenient functional form
providing redundant and flexible parametrization
used as a generator of random functions in the PDF space



$$\xi_i^{(l)} = g\left(\sum_j^{n_l-1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

$$g(x) = \frac{1}{1+e^{-x}}$$

- made of neurons grouped into layers (define the architecture)
- each neuron receives input from neurons in preceding layer (feed-forward NN)
- activation determined by parameters (weights and thresholds)
- activation determined according to a non-linear function

Ingredient 2: Neural Networks

NEURAL NETWORKS

- Parametrize each polarized PDF replica with flexible Neural Network

DSSV, AAC, LSS, BB	NNPDFpol
$\mathcal{O}(10-20)$ parameters	$\mathcal{O}(200)$ parameters

- Train NN to determine the best fit for each replica
- Compute an ensemble of observables and compare to experimental data

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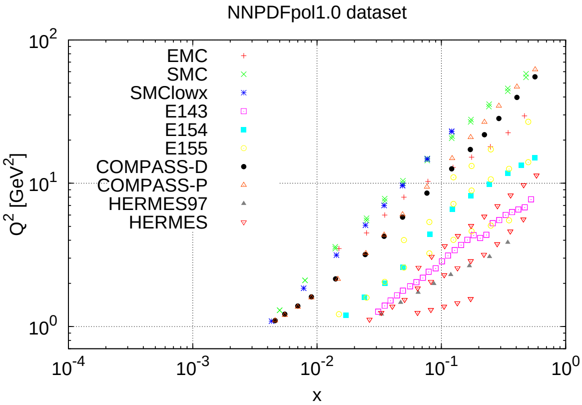
MAIN FEATURES

- Only require smoothness of the fitted function
- Do not require any other prejudice on *a priori* functional form
- Reduce the bias associated to the choice of some functional form
- Given smoothness, the algorithm provided by NN is efficient and can be easily implemented with other algorithms (e.g. genetic algorithms)

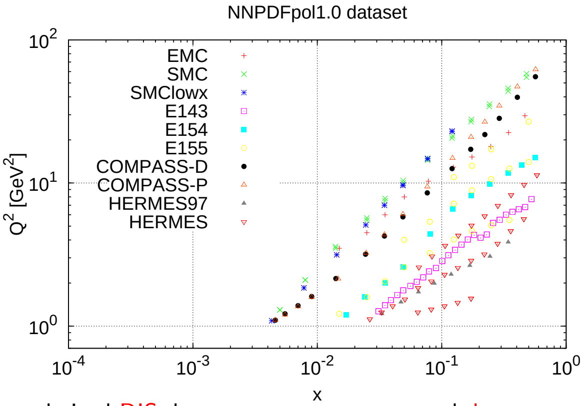
3. Towards NNPDFpol1.0

Experimental dataset



$$g_1(x,Q^2) = A_1(x,Q^2)\frac{F_2(x,Q^2)}{2x(1+R(x,Q^2))}(1+\gamma^2) \qquad \gamma^2 = \frac{4M_N^2x^2}{Q^2}$$

Experimental dataset



1. All relevant polarized DIS data on proton, neutron and deuteron targets
2. Kinematical cuts to remove the sensitivity to dynamical higher-twist
   - $Q^2 > 1 Gev^2$
   - $W^2 = Q^2(1-x)/x \geq 6.25 GeV^2$ (C. Simolo, Ph.D. Thesis. arXiv:0807.1501)

Input polarized PDF basis

1. Four polarized PDFs (gluon + linear combinations of light quarks)
   - singlet $\Delta\Sigma(x) \equiv \Sigma_{i=1}^{n_f}(\Delta q_i(x) + \Delta\bar{q}_i(x))$
   - gluon $\Delta g(x)$
   - triplet $\Delta T_3(x) \equiv (\Delta u(x) + \Delta\bar{u}(x)) - (\Delta d(x) + \Delta\bar{d}(x))$
   - octet $\Delta T_8(x) \equiv (\Delta u(x) + \Delta\bar{u}(x)) + (\Delta d(x) + \Delta\bar{d}(x) - 2(\Delta s(x) + \Delta\bar{s}(x)))$
2. At initial scale $Q_0^2 = 1 GeV^2$ and using $\alpha_s(M_Z^2) = 0.119$
3. Assume all heavy quarks are generated radiatively
4. Must satisfy theoretical constraints:
   - Sum rules

$$[\Delta T_3(Q_0^2)] \equiv \int_0^1 dx \Delta T_3(x, Q_0^2) = a_3 \qquad [\Delta T_8(Q_0^2)] \equiv \int_0^1 dx \Delta T_8(x, Q_0^2) = a_8$$

   - Positivity bound for all proton, neutron and deuteron targets

$$|g_1(x, Q^2)| \leq F_1(x, Q^2)$$

Preprocessing: basic idea

1. Each polarized PDF parametrized with a multi-layer feed-forward NN. All NN have the same architecture (2-5-3-1).
2. Parametrization supplemented with a preprocessing polynomial: exponents m and n randomly choosen in fixed intervals; the NN only fits the deviation from this function.

$$\Delta\Sigma(x, Q_0^2) = (1-x)^{m_{\Delta\Sigma}} x^{-n_{\Delta\Sigma}} NN_{\Delta\Sigma}(x)$$

$$\Delta g(x, Q_0^2) = (1-x)^{m_{\Delta g}} x^{-n_{\Delta g}} NN_{\Delta g}(x)$$

$$\Delta T_3(x, Q_0^2) = A_{\Delta T_3}(1-x)^{m_{\Delta T_3}} x^{-n_{\Delta T_3}} NN_{\Delta T_3}(x)$$

$$\Delta T_8(X, Q_0^2) = A_{\Delta T_8}(1-x)^{m_{\Delta T_8}} x^{-n_{\Delta T_8}} NN_{\Delta T_8}(x)$$

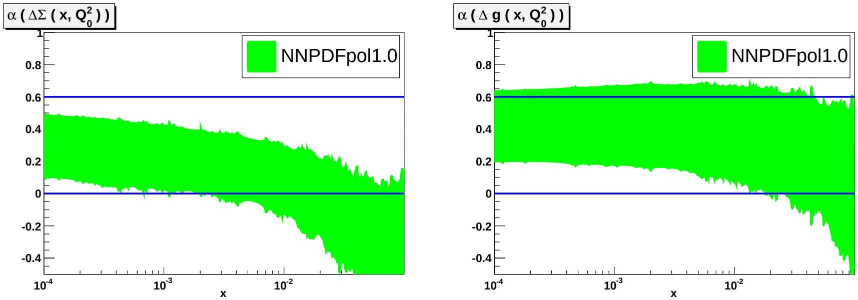
3. Overall normalization constant factored out for triplet and octet. Determined by imposing the sum rules.

$$A_{\Delta T_3} = \frac{a_3}{\int_0^1 dx[(1-x)^{m_{\Delta T_3}} x^{-n_{\Delta T_3}} NN_{\Delta T_3}(x)]}$$

$$A_{\Delta T_8} = \frac{a_8}{\int_0^1 dx[(1-x)^{m_{\Delta T_8}} x^{-n_{\Delta T_8}} NN_{\Delta T_8}(x)]}$$

Preprocessing: effective asymptotic exponents

$0.0 \leq n_{\Delta\Sigma} \leq 0.6$ $\qquad$ $0.0 \leq n_{\Delta g} \leq 0.6$

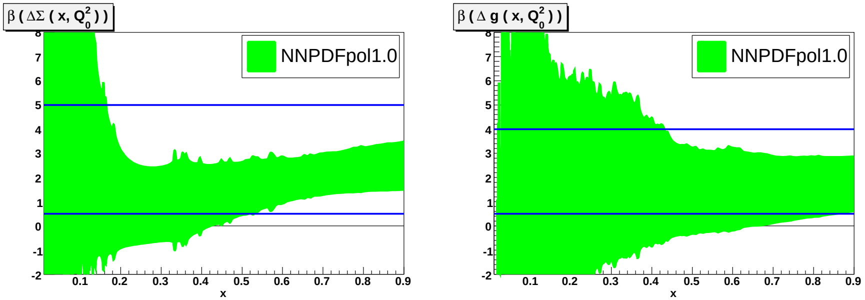


$$\alpha_{\text{eff}}(x, Q^2) \equiv \ln(\Delta q(x, Q^2))/\ln(1/x) \quad \text{at } Q^2 = Q_0^2 = 1GeV^2$$

Effective exponents always contained in the preprocessing exponents range
The polarized PDF is driven only by experimental data

Preprocessing: effective asymptotic exponents

$0.5 \leq m_{\Delta\Sigma} \leq 5.0$ $\qquad$ $0.5 \leq m_{\Delta g} \leq 4.0$



$$\beta_{\text{eff}}(x, Q^2) \equiv \ln(\Delta q(x, Q^2)) / \ln(1-x) \quad \text{at } Q^2 = Q_0^2 = 1 GeV^2$$

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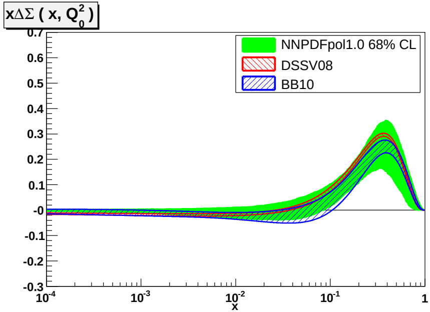
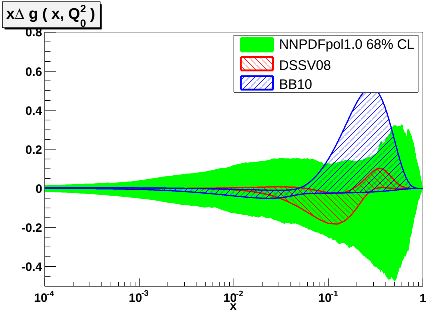
NNPDFpol1.0: results

Comparison with DSSV08 and BB10





Much larger error bands for Singlet and Gluon

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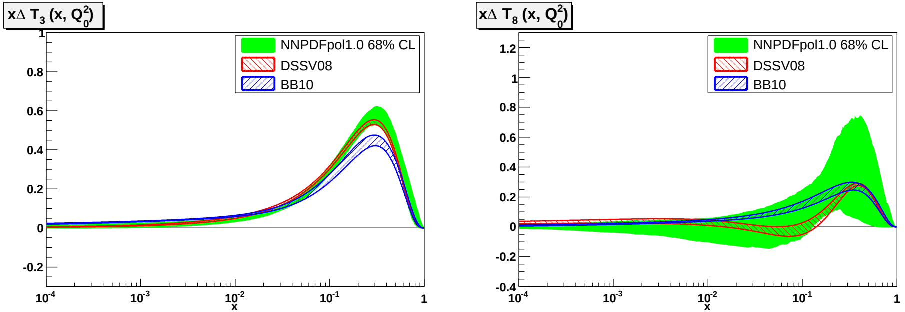

Triplet agrees with DSSV08 fit, but does not with BB10 fit
Much larger error band for Octet

NNPDFpol1.0: results

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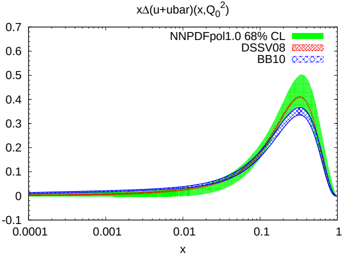

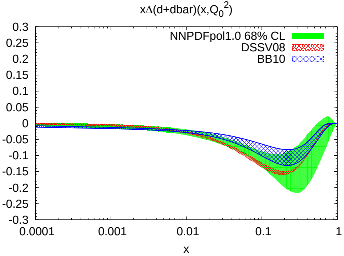

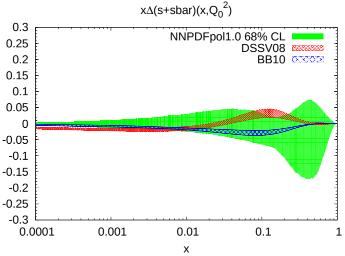

Much larger error bands for all light quark combinations

NNPDFpol1.0 does not agree with BB10 for the $x\Delta(u+\bar{u})$ combination

The spin content of the proton

Singlet and Gluon first moments in $\overline{\mathrm{MS}}$ scheme at $Q_0^2 = 1$ GeV^2

	NNPDFpol1.0	DSSV08	AAC08
$[\Delta\Sigma]$	0.32 ± 0.11	0.26 ± 0.03	0.26 ± 0.06
$[\Delta g]$	-0.2 $\pm$ 1.1	-0.12 $\pm$ 0.12	0.40 ± 0.28

Notice the large uncertainty on the first moments:
Singlet between two and four times
Gluon almost one order of magnitude

$$\langle S_z \rangle = \frac{1}{2}\langle \Delta\Sigma \rangle + \langle \Delta g \rangle + L_q + L_g$$

$$\frac{1}{2} = (-0.1 \pm 1.1) + L_q + L_g$$

Gluon uncertainty dominates the contribution to proton's spin

4. Conclusions

Final remarks

Summary

1. The NNPDF technology provides a statistically sound procedure for PDF fitting
2. NNPDFpol1.0 is the first polarized parton determination using NNPDF approach
3. The analysis from inclusive DIS data leads to
   - able to discriminate Triplet (agreement with DSSV08, not with BB10)
   - large uncertainties on Singlet and Octet and very large on Gluon
   - uncertainty on Singlet first moment between two and four times bigger
   - uncertainty on Gluon first moment almost one order of magnitude bigger

Outlook

1. Include data sets from other processes (open charm and jet production with fixed target, inclusive jet production, W boson production at RHIC, ...)
2. Determine the strong-coupling constant from polarized DIS data
3. Investigate the sensitivity of polarized data to a_3 and a_8 axial constants

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Thank you for your attention!

5. Backup

PDF fitting: state of the art

1. First stage: first moments of polarized PDFs and polarized sum rules (last 25 years)
   $\longrightarrow$ "historical" experimental collaborations (at CERN, SLAC, DESY, JLAB):
   EMC, SMC, E142, E143, E154, E155, COMPASS, HERMES, CLAS, ...
2. Second stage: polarized PDF fits from global NLO QCD analysis (last ~15 years)
   $\longrightarrow$ different choice of datasets, parton parametrization, treatement of higher twists, ...
   ABFR (arXiv:hep-ph/9803237, 1998), BB (arXiv:1005.3113, 2010) (DIS only);
   AAC (arXiv:0808.0413, 2008), LSS (arXiv:1010.0574, 2010) (DIS+SIDIS);
   DSSV (arXiv:0904.3821, 2009) (DIS+SIDIS+pp)
3. Third stage: provide uncertainties on polarized PDFs (last ~10 years)
   $\longrightarrow$ Gaussian error propagation, Lagrange multiplier + Hessian method; fit with orthogonal polynomials (arXiv:1011.4873, 2010)

Monte Carlo sampling: more detail

- The k-th MC replica is generated assuming a multi-Gaussian distribution

$$g_{1,i}^{(\mathrm{art}),k}(x, Q^2) = (1 + r_{k,N}\sigma_N)\left[g_{1,p}^{(\mathrm{exp})}(x, Q^2) + r_{k,t}\sigma_t(x, Q^2)\right]$$

r_k: Gaussian random numbers σ_N quadratic sum of normalization errors
σ_t: total error (summing in quadrature statistical and systematic errors)

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy $\longrightarrow$ determine size of the sample

$r\,[g_1]$	1.00
$\left\langle \sigma^{(\mathrm{exp})} \right\rangle_{\mathrm{dat}}$ (%)	0.13E+03
$\left\langle \sigma^{(\mathrm{gen})} \right\rangle_{\mathrm{dat}}$	0.11E+03
$r\left[\sigma^{(\mathrm{gen})}\right]$ (%)	0.10E+01
$\left\langle \rho^{(\mathrm{exp})} \right\rangle_{\mathrm{dat}}$	0.64E-01
$\left\langle \rho^{(\mathrm{gen})} \right\rangle_{\mathrm{dat}}$	0.64E-01
$r\left[\rho^{(\mathrm{gen})}\right]$	0.98E+00
$\left\langle \mathrm{cov}^{(\mathrm{exp})} \right\rangle_{\mathrm{dat}}$	0.25E-01
$\left\langle \mathrm{cov}^{(\mathrm{gen})} \right\rangle_{\mathrm{dat}}$	0.25E-01
$r\left[\mathrm{cov}^{(\mathrm{gen})}\right]$	0.10E+01

- Accuracy of few % requires $\sim$ 1000 replicas

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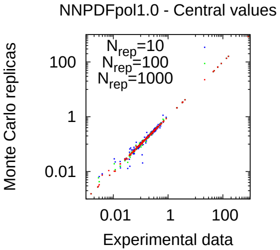

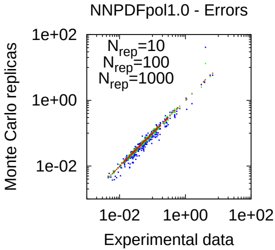

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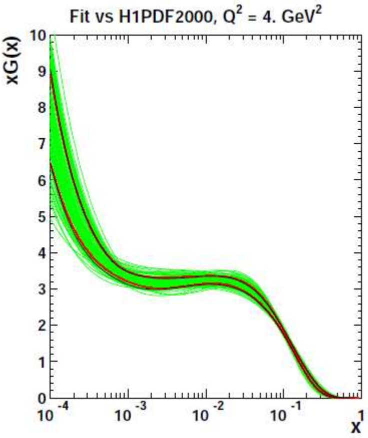
Monte Carlo vs Hessian PDF uncertainties



HERA-LHC 2009 PDf benchmarks

- H1PDF2000 fit done with Hessian method and with Monte Carlo method
- The standard deviation of the 100 PDF replicas (MC method) is in perfect agreement with Hessian errors with $\Delta\chi^2 = 1$
- The MC method to estimate PDF uncertainties reproduces Hessian result when global χ^2 is quadratic

Polarized PDF evolution

In Mellin space the DGLAP equations

$$\begin{aligned}
\mu^2 \frac{\partial}{\partial \mu^2} \Delta q_{NS}^{\pm,\nu}(N,\mu^2) &= \Delta\gamma_{NS}^{\pm,\nu} q_{NS}^{\pm,\nu}(N,\mu^2) \\
\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} (N,\mu^2) &= \begin{pmatrix} \Delta\gamma_{qq}(N,\alpha_s(Q^2)) & \Delta\gamma_{qg}(N,\alpha_s(Q^2)) \\ \Delta\gamma_{gq}(N,\alpha_s(Q^2)) & \Delta\gamma_{gg}(N,\alpha_s(Q^2)) \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}
\end{aligned}$$

can be solved analitically

$$\Delta q_{NS}^{\pm,\nu}(N,Q^2) = \Gamma_{NS}^{\pm,\nu}(N,a_s,a_0)\Delta q_{NS}^{\pm,\nu}(N,Q_0^2) \;, a_s \equiv \alpha_s/2\pi$$

where, at NLO,

$$\Gamma_{NS,NLO}^{\pm,\nu}(N,a_s,a_0) = \exp\left\{ \frac{U_1^{\pm,\nu}}{b_1} \ln\left(\frac{1+b_1 a_s}{1+b_1 a_0}\right)\right\} \left(\frac{a_s}{a_0}\right)^{-R_0^{NS}}$$

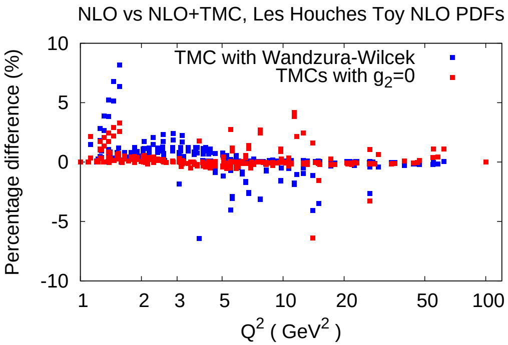
Polarized PDF evolution

NNPDF NLO polarized PDF evolution (**Fast Kernel method**) benchmarked with the Les Houches PDF benchmarks (G. Salam and a. Vogt, hep-ph/0511119)

x	$\epsilon_{\rm rel}\left(\Delta u_V\right)$	$\epsilon_{\rm rel}\left(\Delta d_V\right)$	$\epsilon_{\rm rel}\left(\Delta\Sigma\right)$	$\epsilon_{\rm rel}\left(\Delta g\right)$
10^{-3}	$1.1\,10^{-4}$	$9.2\,10^{-5}$	$9.9\,10^{-5}$	$1.1\,10^{-4}$
10^{-2}	$1.4\,10^{-4}$	$1.9\,10^{-4}$	$3.5\,10^{-4}$	$9.3\,10^{-5}$
0.1	$1.2\,10^{-4}$	$1.6\,10^{-4}$	$5.4\,10^{-6}$	$1.7\,10^{-4}$
0.3	$2.3\,10^{-6}$	$1.1\,10^{-5}$	$7.5\,10^{-6}$	$1.7\,10^{-5}$
0.5	$5.6\,10^{-6}$	$9.6\,10^{-6}$	$1.6\,10^{-5}$	$2.5\,10^{-5}$
0.7	$1.2\,10^{-4}$	$9.2\,10^{-7}$	$1.6\,10^{-4}$	$7.8\,10^{-5}$
0.9	$3.5\,10^{-3}$	$1.1\,10^{-2}$	$4.1\,10^{-3}$	$7.8\,10^{-3}$

Very accurate evolution!

Target mass corrections



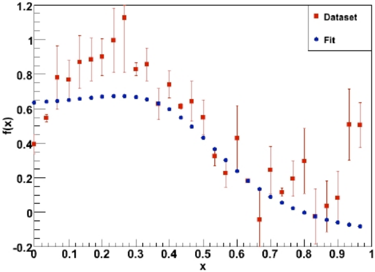
Kinematical cuts exclude the largest-x and smallest-Q^2 data region, where the TMC effects are most important

Moderate impact of TMC corrections (small percent at small Q^2)

One more ingredient: Stopping

1. NN are flexible tools
   - can learn fluctuations
2. Cross-validation method
   - divide data into two subsets (training & validation)
   - train the NN on training subset
   - compute χ^2 for each subset
   - stop when χ^2 of validation subset no longer decreases (NN are learning fluctuations!)

UNDERLEARNING

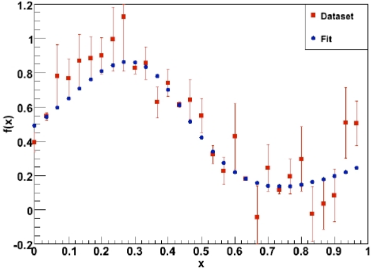


The best fit does not coincide with the χ^2 absolute minimum

One more ingredient: Stopping

1. NN are flexible tools
   - can learn fluctuations
2. Cross-validation method
   - divide data into two subsets (training & validation)
   - train the NN on training subset
   - compute χ^2 for each subset
   - stop when χ^2 of validation subset no longer decreases (NN are learning fluctuations!)

PROPER LEARNING



The best fit does not coincide with the χ^2 absolute minimum

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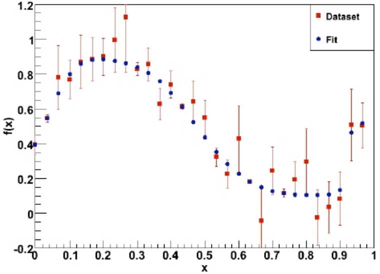
OVERLEARNING



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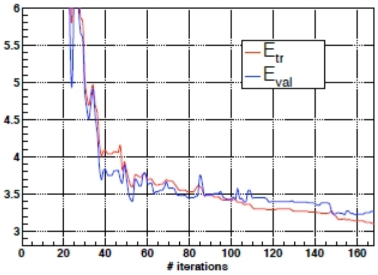

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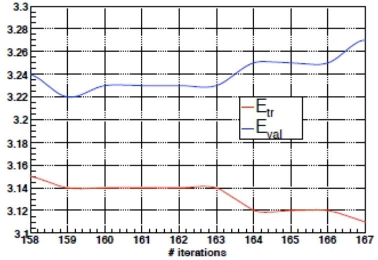


The best fit does not coincide with the χ^2 absolute minimum

Genetic algorithm

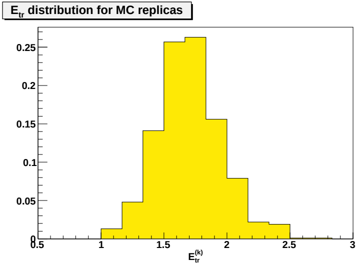
- Set Neural Network parameters randomly
- Make clones of the parameter vector and mutate them
- Define a figure of merit or error function for the k-th replica

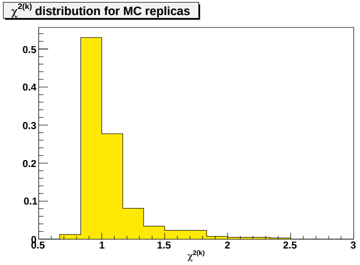
$$E^{(k)} = \frac{1}{N_{\mathrm{rep}}} \sum_{i,j=1}^{N_{\mathrm{rep}}} \left(F_i^{(\mathrm{art})(k)} - F_i^{(\mathrm{net})(k)} \right) \left((\mathrm{cov}_{t_0})^{-1} \right)_{ij} \left(F_j^{(\mathrm{art})(k)} - F_j^{(\mathrm{net})(k)} \right)$$

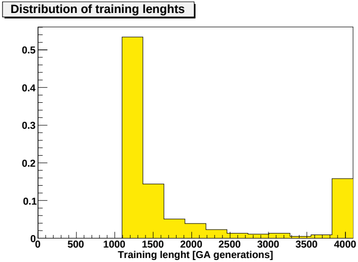
$$(\mathrm{cov}_{t_0})_{ij} = \sigma_{N,i}\sigma_{Nj} g_{1i}^{(0)} g_{1j}^{(0)} + \delta_{ij}\sigma_{i,tot}^2$$

  is the covariance matrix including normalization errors using the t_0 method (arXiv:0912.2276)
- Select the best ones and perform other manipulations (crossing, mutating, ...) until stability is reached

NNPDFpol1.0: global χ^2

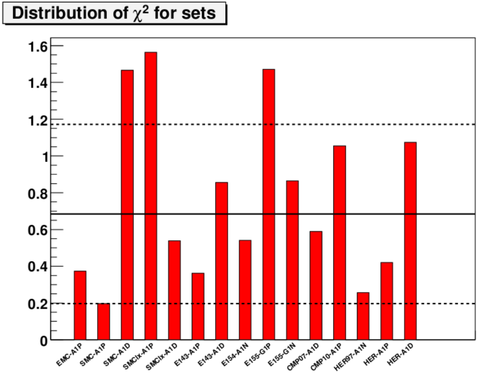






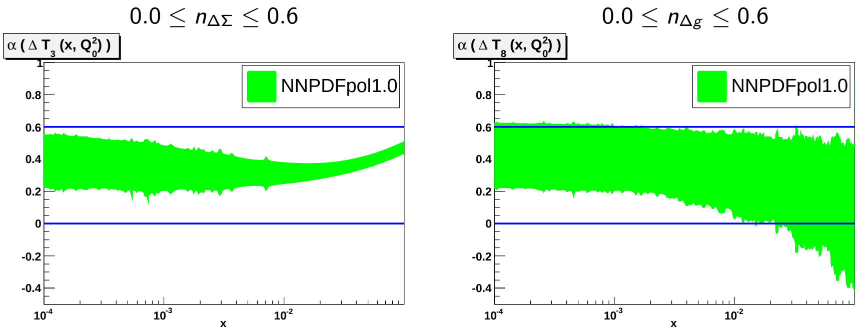
χ^2_{tot}	0.75
$\langle E \rangle \pm \sigma_E$ $\langle E_{\mathrm{tr}} \rangle \pm \sigma_{E_{\mathrm{tr}}}$ $\langle E_{\mathrm{val}} \rangle \pm \sigma_{E_{\mathrm{val}}}$	1.89 ± 0.19 1.64 ± 0.22 2.14 ± 0.33
$\langle \chi^{2(k)} \rangle \pm \sigma_{\chi^2}$	0.96 ± 0.12
$\langle \sigma^{(\mathrm{exp})} \rangle_{\mathrm{dat}} (\%)$ $\langle \sigma^{(\mathrm{net})} \rangle_{\mathrm{dat}} (\%)$	130 48
$\langle TL \rangle$	1825

NNPDFpol1.0: individual experiments χ^2



- No evidence of any specific dataset being inconsistet with each other
- Distribution of individual χ^2 values broadly consistent with statistical expectations

Preprocessing: effective asymptotic exponents

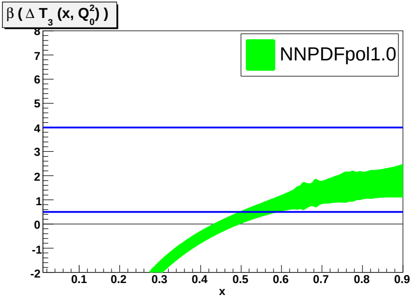


$$\alpha_{\text{eff}}(x, Q^2) \equiv \ln(\Delta q(x, Q^2))/\ln(1/x) \quad \text{at } Q^2 = Q_0^2 = 1 GeV^2$$

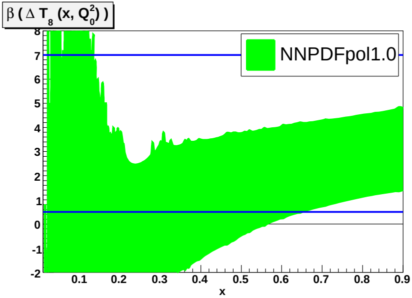
Effective exponents always contained in in the preprocessing exponents range
The polarized PDF is driven only by experimental data

Preprocessing: effective asymptotic exponents

$0.5 \leq m_{\Delta\Sigma} \leq 4.0$



$0.5 \leq m_{\Delta g} \leq 7.0$

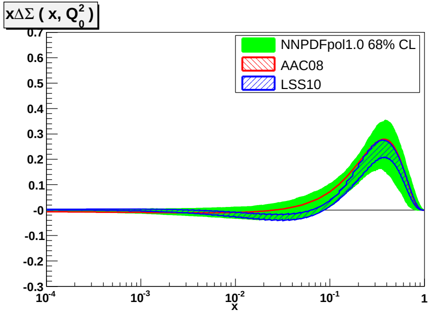


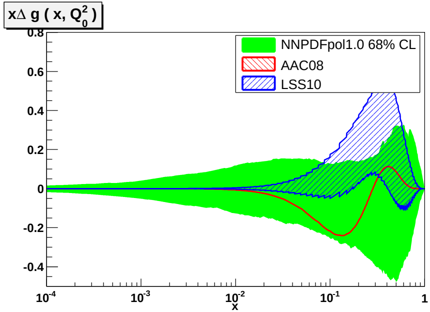
$$\beta_{\rm eff}(x, Q^2) \equiv \ln(\Delta q(x, Q^2))/\ln(1-x) \quad \text{at } Q^2 = Q_0^2 = 1GeV^2$$

Effective exponents always contained in in the preprocessing exponents range
The polarized PDF is driven only by experimental data

NNPDFpol1.0: more results

Comparison with AAC08 and LSS10

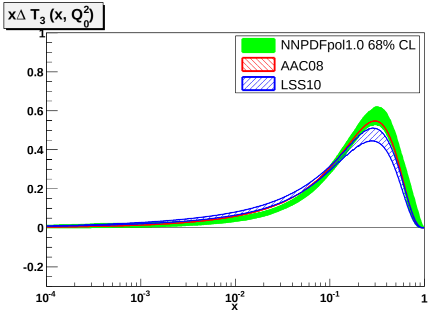


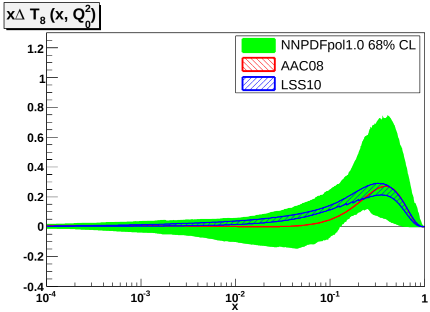


Substantial agreement for Singlet
Gluon essentially unconstrained

NNPDFpol1.0: more results

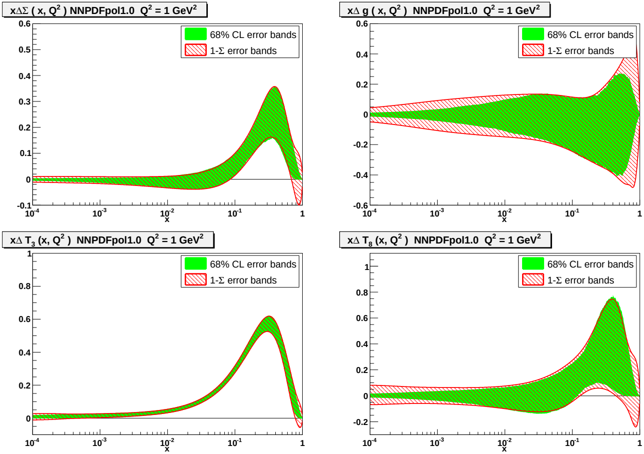
Comparison with AAC08 and LSS10





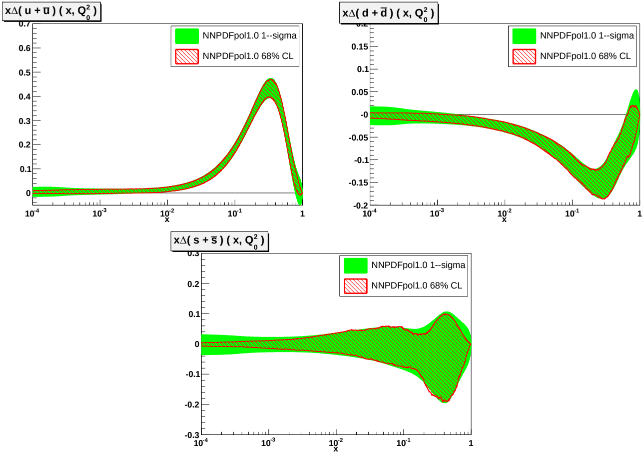
Substantial agreement for Octet, but with larger error band
Triplet slightly bigger

NNPDFpol1.0: 68% confidence levels



comparison between 1σ error bands and 68% confidence level
test for non-Gaussian behaviour
sizeable deviations from Gaussian behaviour in the extrapolation region

NNPDFpol1.0: 68% confidence levels



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