

Low x physics as a critical phenomenon?

O. Nachtmann, Univ. Heidelberg
together with C. Ewerz and A. von Manteuffel

How can we understand the behaviour of

$F_2(x, Q^2)$ for $x \rightarrow 0$?

- Our guess (Hebecker, Meggiolaro, O.N., NP B571, 26 (2000); O.N., EPJC 26, 579 (2003)):

Low x physics can be viewed as a critical phenomenon.

Starting point: virtual forward Compton scattering amplitude

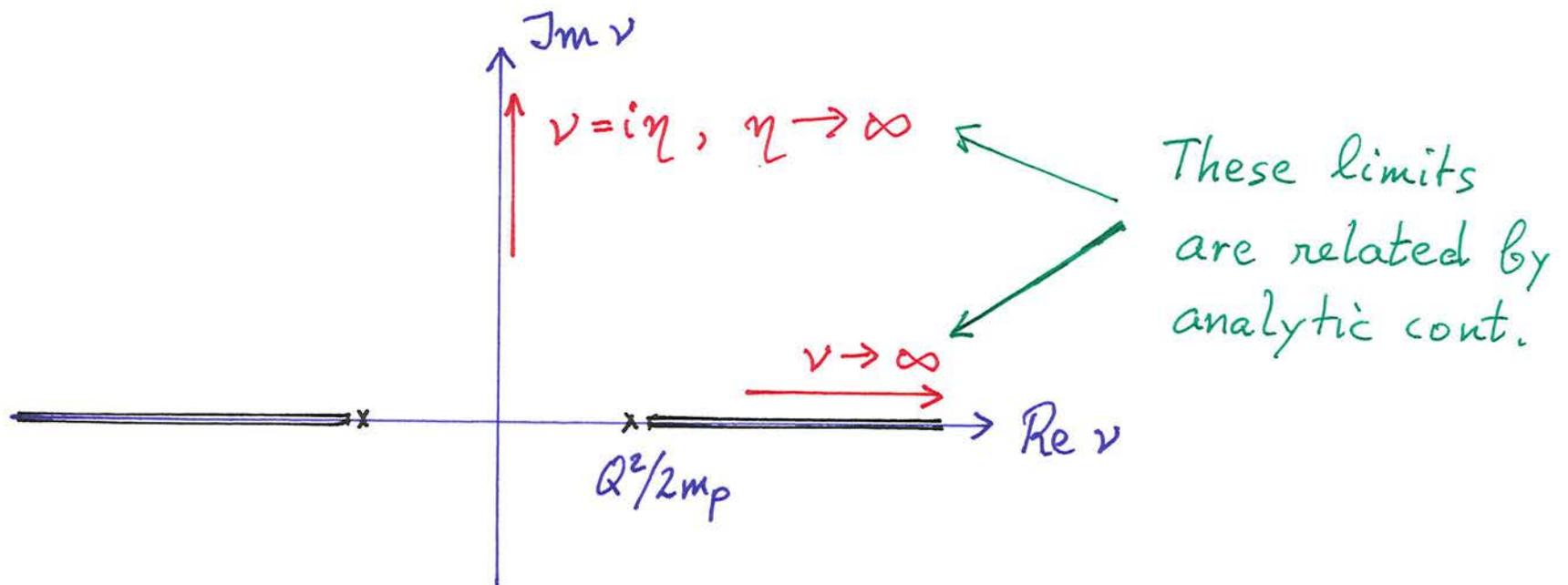
$$\text{Amplitude } (\gamma^* p \rightarrow \gamma^* p) = T_{\mu\nu}(p, q) =$$

$$\frac{i}{2\pi m_p} \int d^4x e^{iq \cdot x} \theta(x^0) \langle p(p) | [J_\mu(x), J_\nu(0)] | p(p) \rangle$$

We want to study the limit Q^2 fixed, $\nu \rightarrow \infty$

$$T_{\mu\nu}(p, q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{m_p^2} \left(p_\mu - \frac{(p \cdot q) q_\mu}{q^2} \right) \left(p_\nu - \frac{(p \cdot q) q_\nu}{q^2} \right) T_2(\nu, Q^2)$$

Analyticity properties of $T_j(\nu, Q^2)$ for fixed Q^2 :



We use the Phragmén - Lindelöf theorem!

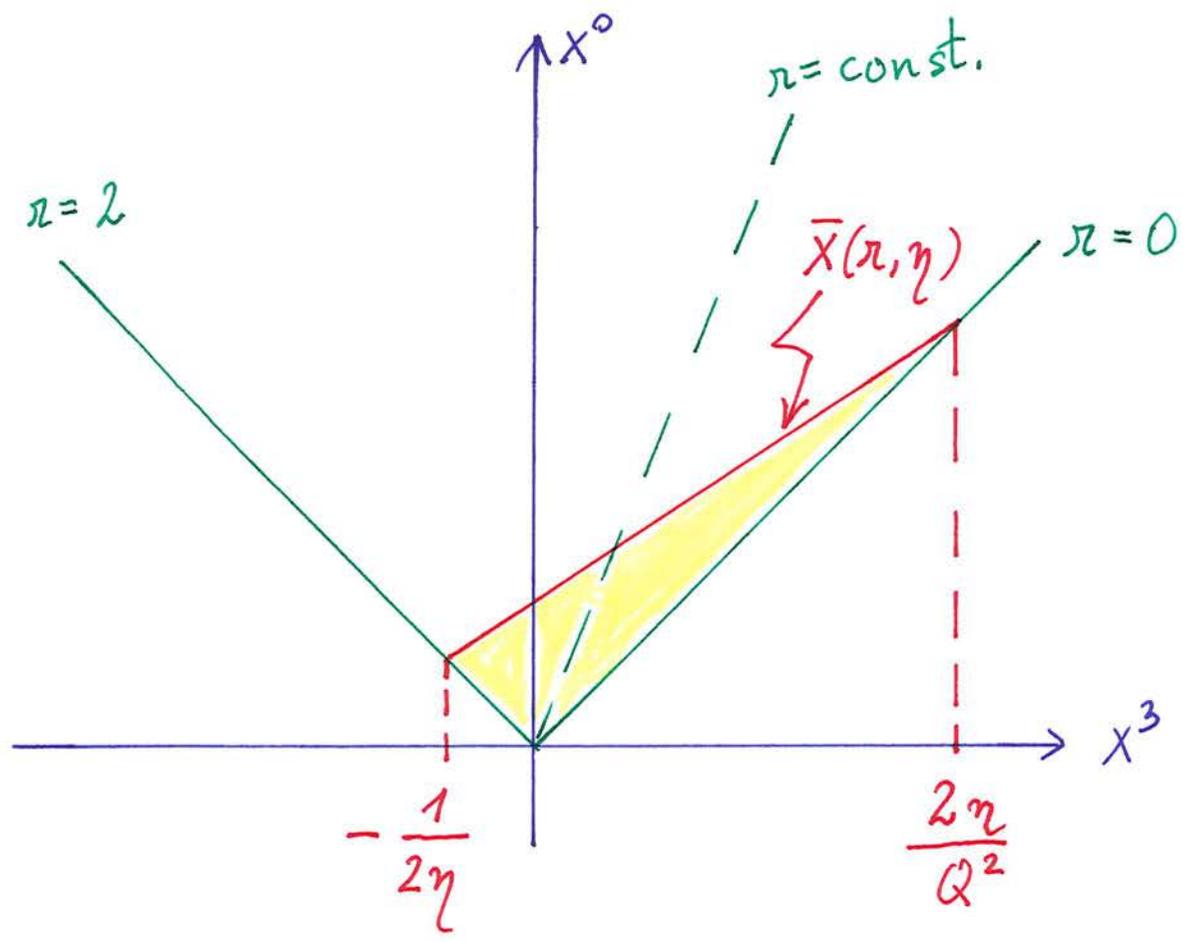
We work in the rest system of the proton and continue in v to the imaginary axis.

$$p = \begin{pmatrix} m_p \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad v = i\eta, \quad \eta > 0, \quad -iq = \begin{pmatrix} \eta \\ 0 \\ 0 \\ \sqrt{\eta^2 - Q^2} \end{pmatrix}$$

$$\exp(iq \cdot x) = \exp \left[-x^0 \left(\eta - (1-\kappa) \sqrt{\eta^2 - Q^2} \right) \right] = \exp \left(-\frac{x^0}{\bar{x}(\kappa, \eta)} \right)$$

• $\kappa = 1 - x^3/x^0$, *important variable in the following!*

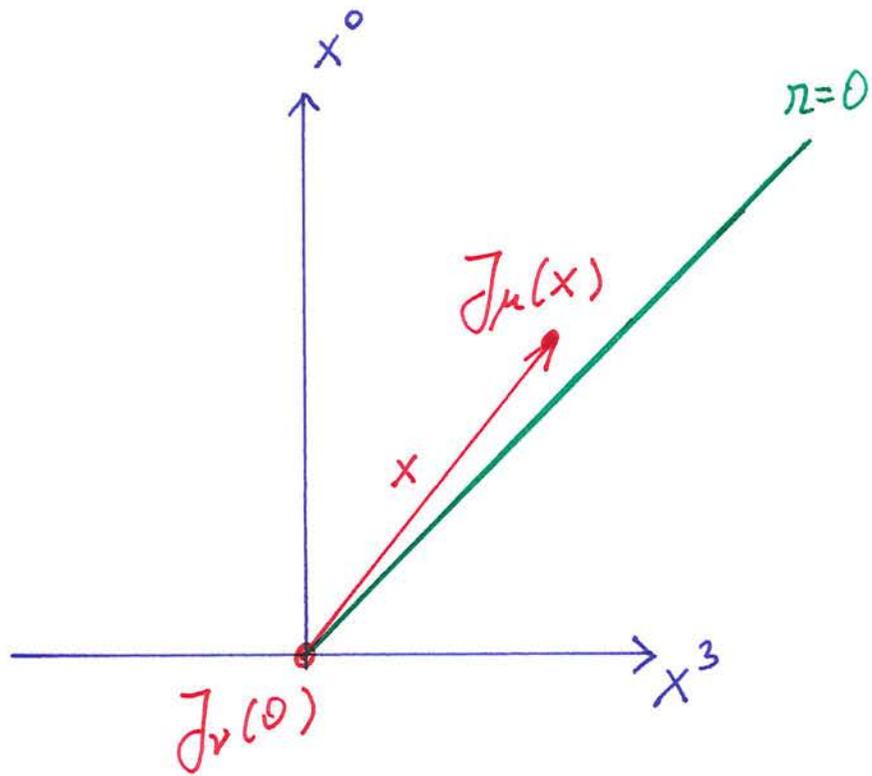
$$\bar{x}(\kappa, \eta) \cong \frac{1}{\eta \left(\kappa + \frac{Q^2}{2\eta^2} \right)} \quad \text{for large } \eta$$



As $\eta \rightarrow \infty$ the integration region shrinks towards the line $r=0$ on the forward light cone.

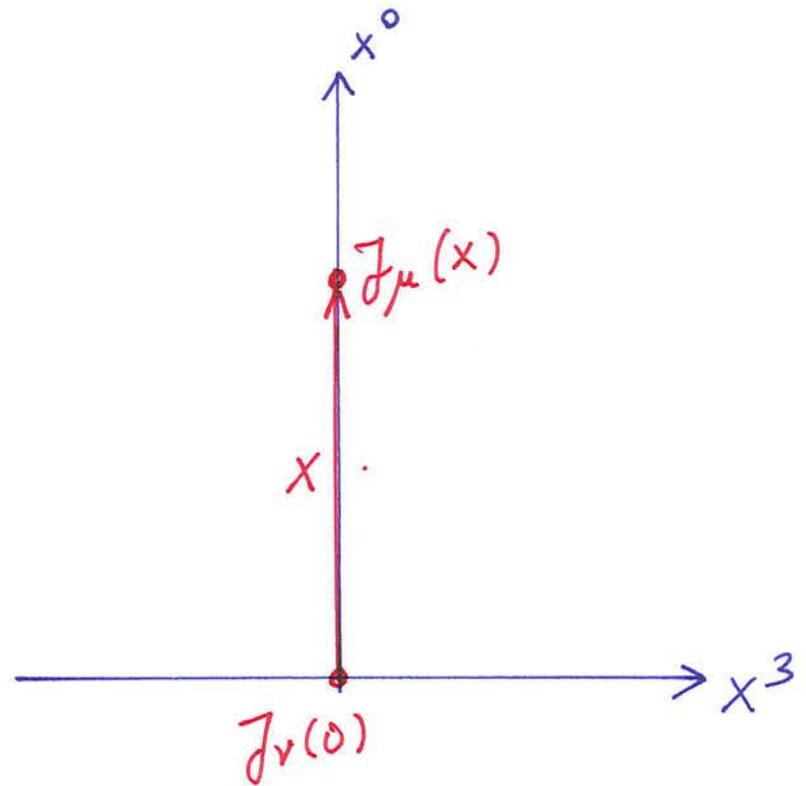
In the original theory we need the behaviour of the current correlation function in the proton near the light cone.

Our idea: transform the theory such that only the correlation function on the time axis is needed.



original theory :

Hamiltonian = H ,
 3rd comp. of
 momentum op. = P^3



effective theory :

Hamiltonian =
 $H_{\text{eff}}(x) = H - (1-x)P^3$

Example of an κ -theory: scalar field theory

- Original Lagrangian

$$\mathcal{L}(x) = \frac{1}{2} \partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{1}{2} m^2 \phi^2(x) - \frac{\lambda}{4!} \phi^4(x)$$

- κ -theory in Minkowski space

$$\mathcal{L}_{M,\kappa}(x) = \frac{1}{2} (\partial_0 \phi - (1-\kappa) \partial_3 \phi)^2 - \frac{1}{2} (\vec{\nabla} \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4$$

- κ -theory in Euclidean space

$$\mathcal{L}_{E,\kappa} = \frac{1}{2} (\partial_0 \phi + \underline{i(1-\kappa) \partial_3 \phi})^2 + \frac{1}{2} (\vec{\nabla} \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4$$

anisotropic propagator.

Correlation length in time direction

$$\xi(\kappa) = \frac{1}{m \sqrt{\kappa}}$$

$$\underline{\xi(\kappa) \rightarrow \infty \text{ for } \kappa \rightarrow 0}$$

$$\langle p(p) | \mathcal{J}_0(x^3, x^0) \mathcal{J}_0(0) | p(p) \rangle =$$

$$\langle p(p) | \mathcal{J}_0(0, x^0) \mathcal{J}_0(0) | p(p) \rangle \Big|_{\pi\text{-theory}}$$

$$\equiv \mathcal{M}_{00}(x^0, \pi)$$

- The parameter π appears now only in the Lagrangian.

- From a study of free field theories and of the energy gap we expect that in the effective κ -theory there will be a large correlation length $\xi(\kappa)$ in time-like direction for $\kappa \rightarrow 0$.

In free field theory:

$$\xi(\kappa) \propto 1/\sqrt{\kappa}$$

- This indicates a critical phenomenon, with κ playing the role of $(T - T_c)/T_c$.
- We expect then scaling behaviour of $\mathcal{M}_{00}(x^0, \kappa)$ for $0 < x^0 < \xi(\kappa)$

- Assumption: simple power law behaviour of the matrix elements in the scaling region

$$M_{00}(x^0, r) \propto (x^0 m_p)^{a-3} (-r)^{2-\frac{1}{2}\epsilon_0},$$

a, ϵ_0 : critical indices

- This leads to

$$F_2(x, Q^2) \propto (Q^2)^{\frac{1}{2}\epsilon_0} (x)^{-(a+\epsilon_0)}$$

for $x \rightarrow 0$, $Q^2 \gg m_p^2$

- Test of this scaling relation:

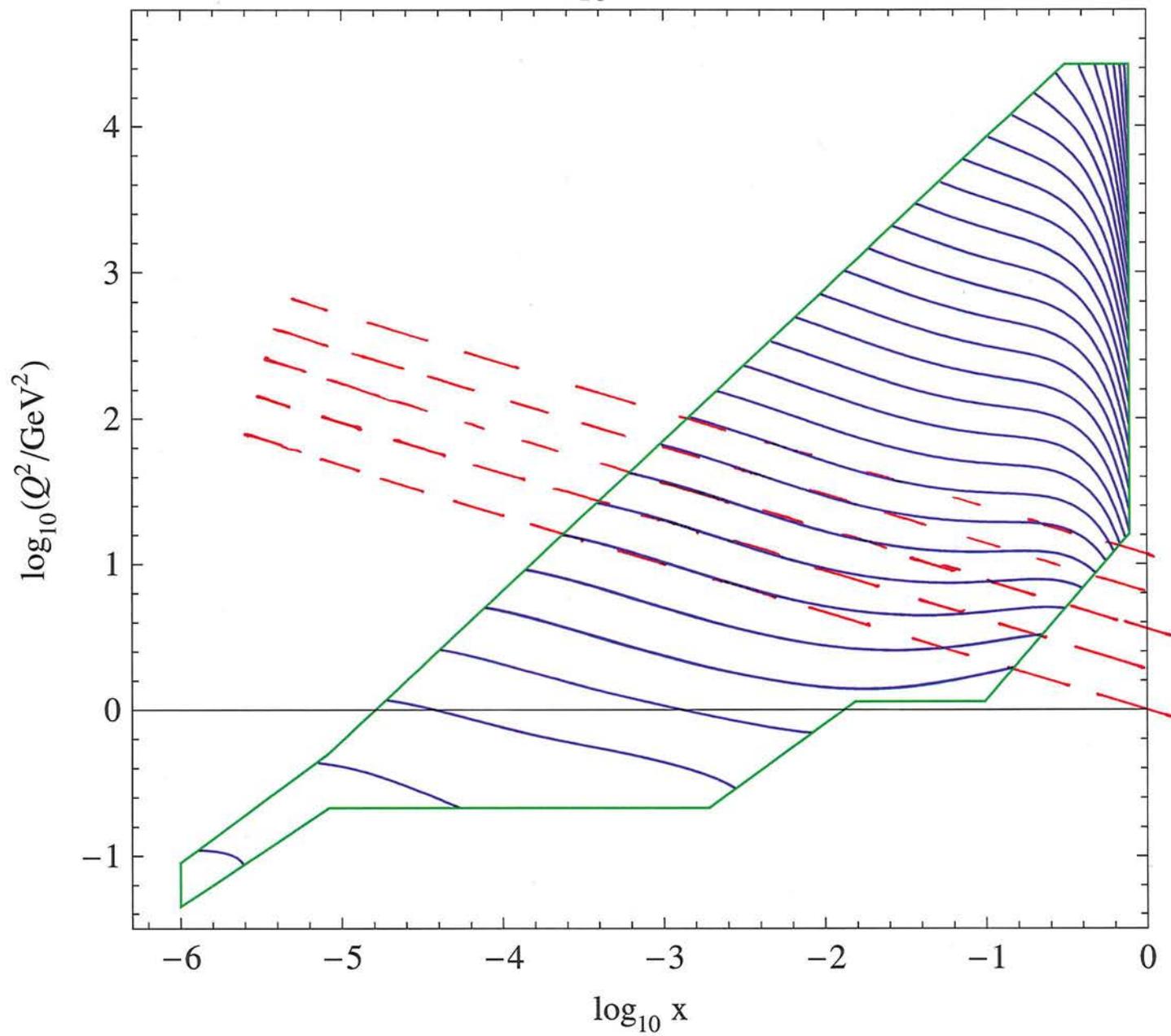
$$\begin{aligned}\sigma_{\gamma^*p}(x, Q^2) &= \frac{4\pi^2\alpha}{Q^2} F_2(x, Q^2) \\ &= \sigma_0 (Q^2)^{-(1-\frac{1}{2}\epsilon_0)} x^{-(a+\epsilon_0)}\end{aligned}$$

- $\log_{10} \sigma_{\gamma^*p}(x, Q^2) = \log_{10} \sigma_0 - (1-\frac{1}{2}\epsilon_0) \log_{10} Q^2 - (a+\epsilon_0) \log_{10} x$

$$\sigma_{\gamma^*p} \text{ in } \mu\text{b}, \quad Q^2 \text{ in } \text{GeV}^2$$

- Contour lines of $\log_{10} \sigma_{\gamma^*p}$ should be straight, parallel, lines in the $\log_{10} x$ versus $\log_{10} Q^2$ plane for $Q^2 \gg m_p^2$ and small x .

const $\log_{10}(\sigma_{\gamma p}/\mu b)$ lines



- Extraction of critical indices:

$$1 - \frac{1}{2} \varepsilon_0 \cong 0.8 \qquad a + \varepsilon_0 \cong 0.26$$

$1 + a + \varepsilon_0 \cong 1.26$ is the "hard pomeron" intercept

(see C. Ewerz, A.v. Manteuffel, O.N., to be published)

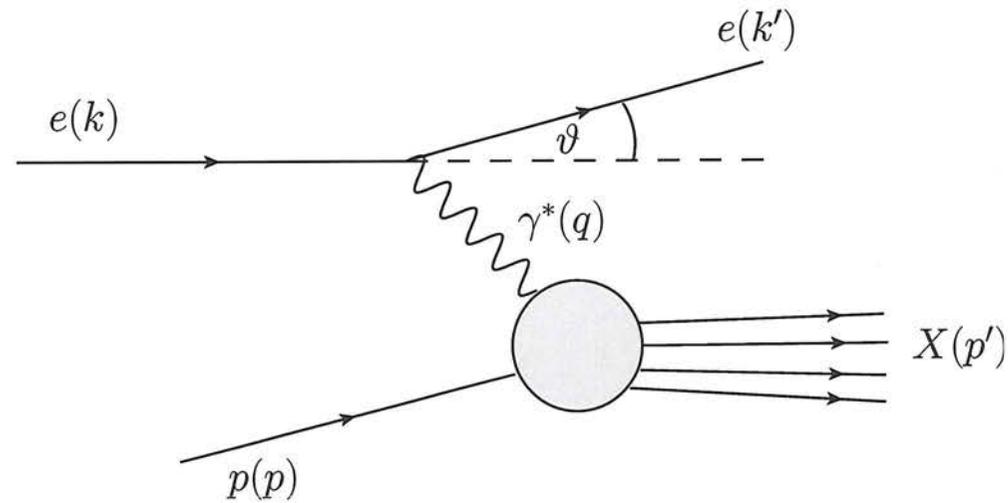
- Our scaling assumption leads to conclusions similar to those found in the phenomenological 2 pomeron model of Donnachie and Landshoff, P.L. B 518, 63 (2001). Their fit has

$$1 - \frac{1}{2} \varepsilon_0 = 0.8 \qquad a + \varepsilon_0 = 0.44$$

Conclusions

- In our view low x physics is describable as a critical phenomenon.
- We have presented the extraction of critical indices from a contour plot of $\log_{10} \sigma_{\gamma^* p}$.
- These critical indices should be calculable from the Euclidean x theory using lattice methods.

► Kinematics



$$q = k - k' = p' - p, \quad Q^2 = -q^2$$

$$E = pk/m_p, \quad E' = pk'/m_p, \quad \nu = pq/m_p$$

$$W^2 = (p + q)^2 = 2m_p\nu - Q^2 + m_p^2, \quad x = \frac{Q^2}{2m_p\nu}, \quad y = \frac{\nu}{E}$$

► Structure functions

$$\frac{\partial^2 \sigma}{\partial E' \partial \Omega'} = \frac{4\alpha^2 E'^2}{Q^4} \left\{ 2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} + W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} \right\}$$

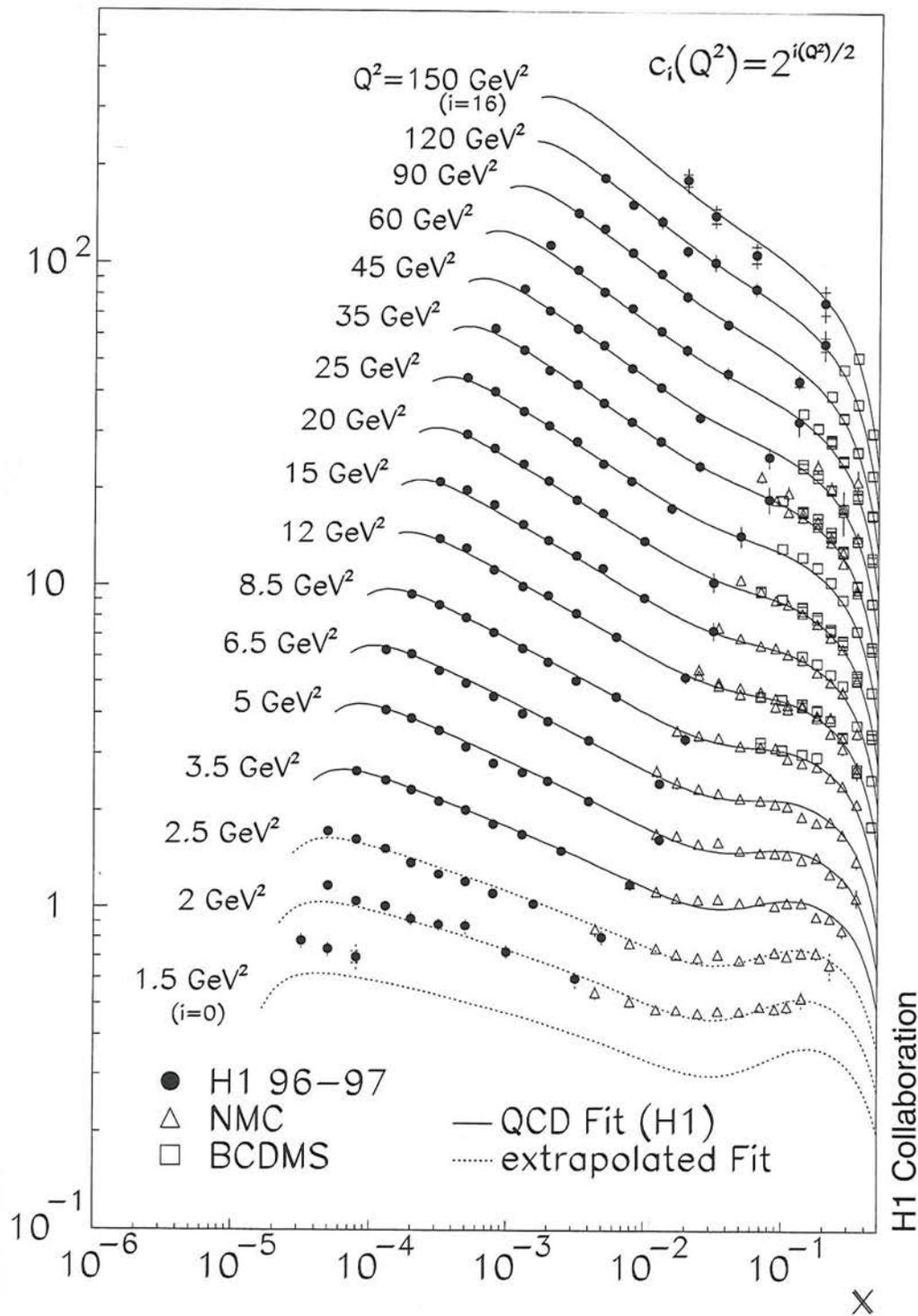
$$F_1(x, Q^2) = 2m_p W_1(\nu, Q^2)$$

$$F_2(x, Q^2) = \nu W_2(\nu, Q^2)$$

$$\sigma_T(W^2, Q^2) = \frac{8\pi^2 m_p \alpha}{W^2 - m_p^2} W_1(\nu, Q^2)$$

$$\sigma_L(W^2, Q^2) = \frac{8\pi^2 m_p \alpha}{W^2 - m_p^2} \left[W_2(\nu, Q^2) \frac{\nu^2 + Q^2}{Q^2} - W_1(\nu, Q^2) \right]$$

$$F_2 \left(1 - \frac{y^2}{1 + (1-y)^2} \frac{\sigma_L}{\sigma_T + \sigma_L} \right)$$



Data :

NMC,
BCDMS,
H1

(from H1 Coll.

EPJC 21, 33 (2001))

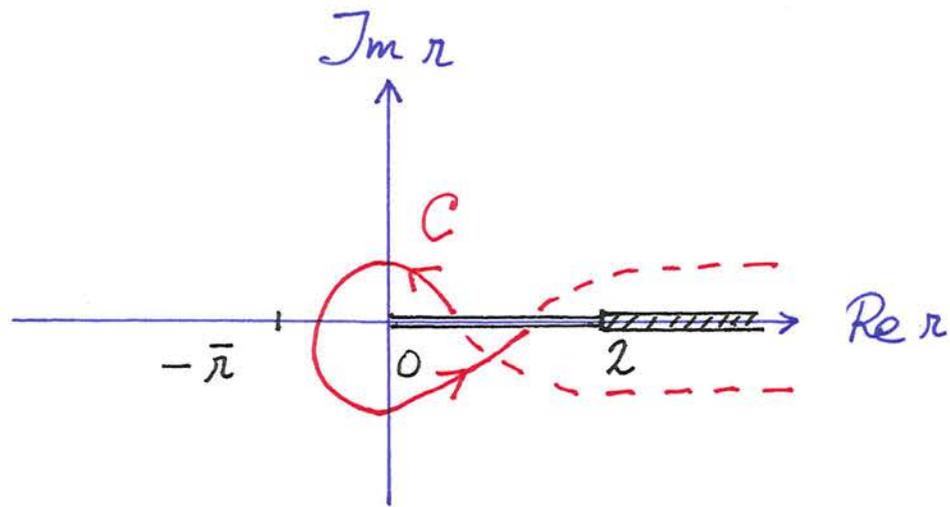
$$M_{\mu\nu}(x, p) = \langle p(p) | J_{\mu}(x) J_{\nu}(0) | p(p) \rangle$$

$$= [g_{\mu\nu} \square - \partial_{\mu} \partial_{\nu}] M_1(x^p, x^2)$$

$$+ [p_{\mu} p_{\nu} \square - (p \partial)(p_{\mu} \partial_{\nu} + p_{\nu} \partial_{\mu}) + g_{\mu\nu} (p \partial)^2] M_2(x^p, x^2)$$

$$\tilde{M}_j(x^0, r) \equiv M_j(x^0 m_p, (x^0)^2 r(2-r))$$

We study the analyticity properties of $\tilde{M}_j(x^p, r)$ for fixed x^0 as fct. of r using the DGS representation.



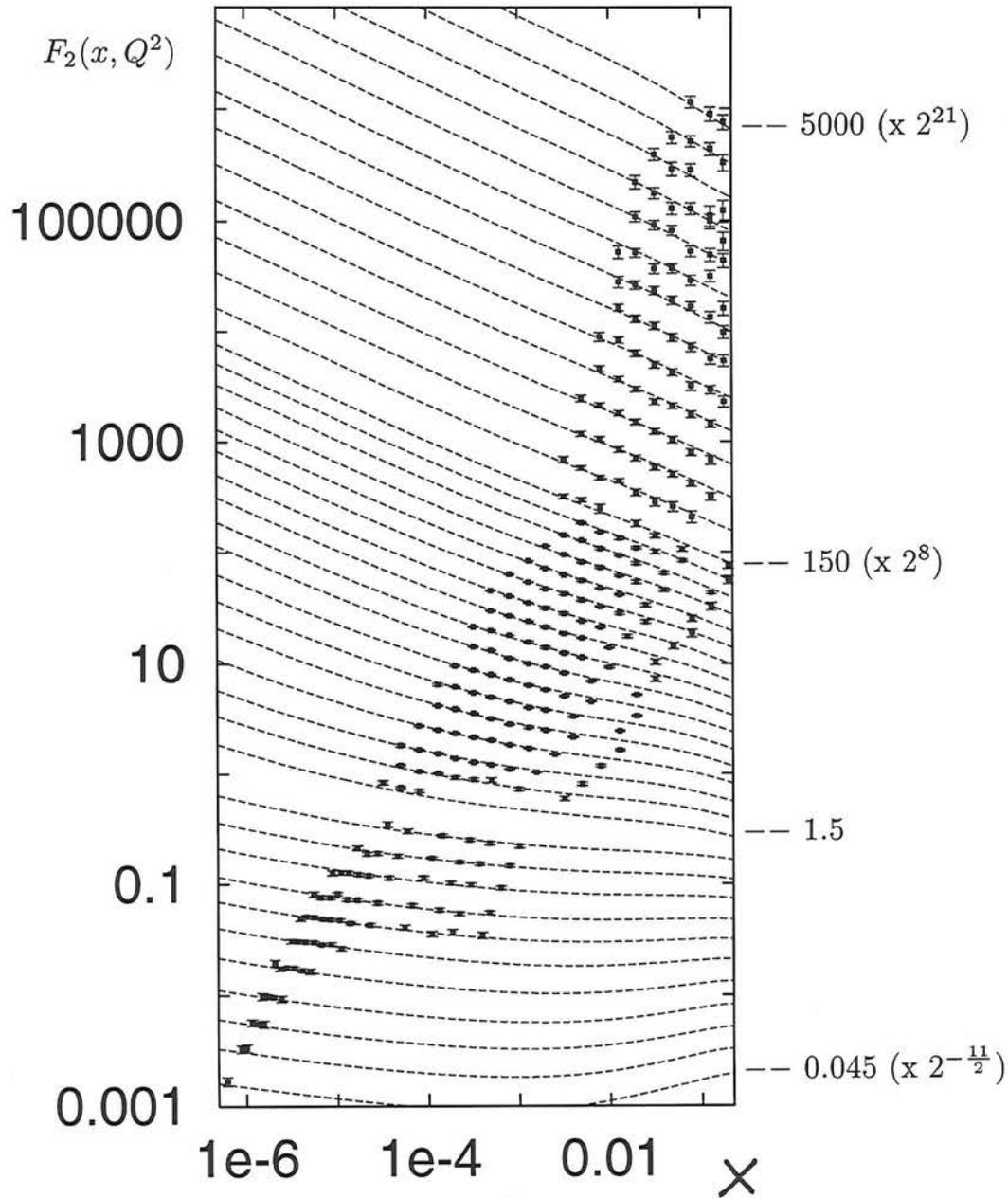
$$\bar{\pi} = \frac{Q^2}{2\eta^2}$$

$$T_2(i\eta, Q^2) = \frac{im_p Q^2}{\sqrt{\eta^2 - Q^2}} \int_C d\pi (1-\pi) \int_0^\infty dx^0 (x^0)^2 \tilde{M}_2(x^0, \pi) \exp\left[-\frac{x^0}{\bar{x}(\eta, \pi)}\right]$$

$$\propto (x^0)^{\alpha-1} (-\pi)^{-\frac{1}{2}} \epsilon_0 \exp\left[-\frac{x^0}{\xi(\pi)}\right]$$

For $Q^2 \gg m_p^2$ we find $\xi(-\bar{\pi}/2) > \bar{x}(\eta, -\bar{\pi}/2)$.

The integral is dominated by the scaling region.



Data: H1 and ZEUS
 2 Pomeron fit by DL

$$F_2(x, Q^2) = f_0(Q^2) x^{-\epsilon_0} + f_1(Q^2) x^{-\epsilon_1} + f_2(Q^2) x^{-\epsilon_2}.$$

Hard pomeron intercept:

$$\underline{1 + \epsilon_0 \approx 1.44}$$

In our theory this should be a critical index calculable from first principles.

Fig. from Donnachie + Landshoff, 2001