α_s and Higher Twist in an Upolarized Valence Quark Analysis and $g_2^{\rm tw3}(x,Q^2)$ & $g_1^{\rm tw3}(x,Q^2)$

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DESY

- Non-Singlet Analysis in the Valence Region
- $\alpha_s(M_Z^2)$
- Higher Twist Contributions: $l^\pm p, l^\pm d$
- $g_2^{\rm tw3}(x,Q^2)$ and $g_1^{\rm tw3}(x,Q^2)$ from the g_2 World Data

Unpolarized NS-Analysis in the Valence Region

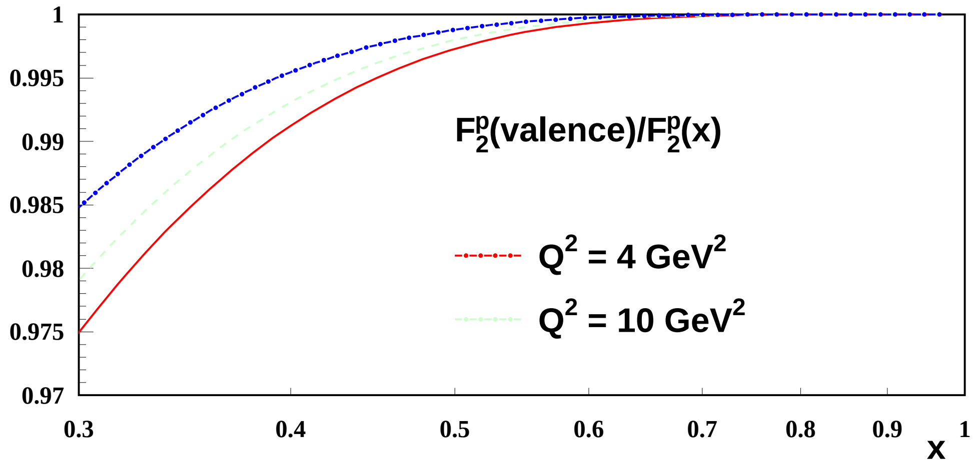
Data:

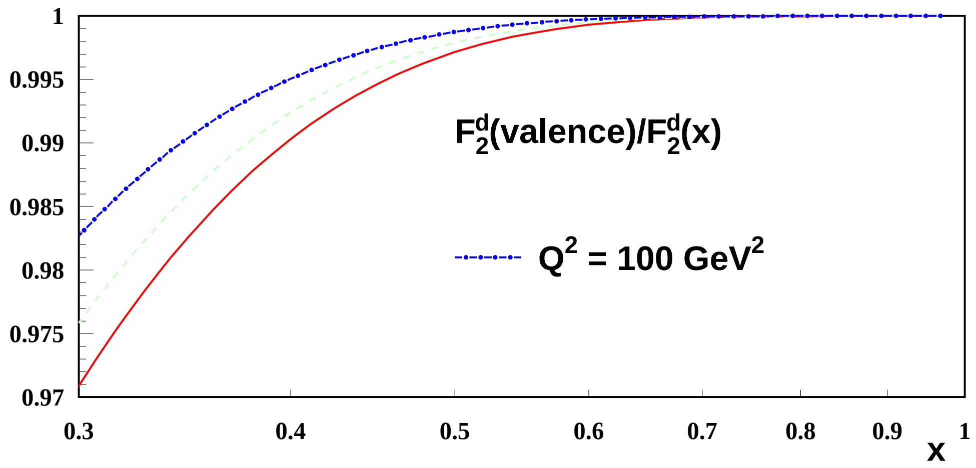
Experiment	x	Q^2, GeV^2	F_2^p	$F_2^p HT$	Norm
BCDMS (100)	0.35 – 0.75	21.50 – 75.00	29	0	1.005
BCDMS (120)	0.35 – 0.75	24.50 – 99.00	32	0	0.998
BCDMS (200)	0.35 – 0.75	43.00 – 137.50	28	0	0.998
BCDMS (280)	0.35 – 0.75	57.00 – 230.00	26	0	0.998
NMC (90)	0.35 – 0.35	8.73 – 8.73	1	1	1.000
NMC (120)	0.35 – 0.47	8.86 – 14.39	6	0	1.000
NMC (200)	0.34 – 0.48	15.02 – 34.80	7	0	1.000
NMC (280)	0.33 – 0.48	20.74 – 62.34	9	0	1.000
SLAC (comb)	0.30 – 0.62	7.30 – 21.39	57	259	1.013
H1 (hQ2)	0.40 – 0.65	200 – 30000	26	0	1.020
H1 (eQ2)	0.40 – 0.65	250 – 30000	21	0	0.974
ZEUS (hQ2)	0.40 – 0.65	650 – 30000	15	0	1.007
proton			257	260	
Experiment	x	Q^2, GeV^2	F_2^d	$F_2^d HT$	Norm
BCDMS (120)	0.35 – 0.75	24.50 – 99.00	32	0	1.001
BCDMS (200)	0.35 – 0.75	43.00 – 137.50	28	0	0.998
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	26	0	1.003
NMC (90)	0.34 – 0.34	8.73 – 8.73	1	1	1.000
NMC (120)	0.35 – 0.48	8.86 – 14.38	5	1	1.000
NMC (200)	0.34 – 0.47	15.01 – 34.79	7	0	1.000
NMC (280)	0.33 – 0.48	20.73 – 62.24	9	0	1.000
SLAC (comb)	0.30 – 0.62	5.00 – 21.40	59	268	0.990
CLAS (comb)	0.46 – 0.61	4.07 – 5.13	0	149	0.994
deuteron			167	419	
Experiment	x	Q^2, GeV^2	F_2^{NS}	$F_2^{NS} HT$	Norm
BCDMS (120)	0.070 – 0.275	8.75 – 43.00	30	0	0.983
BCDMS (200)	0.070 – 0.275	17.00 – 75.00	28	0	0.999
BCDMS (280)	0.100 – 0.275	32.50 – 115.50	26	0	0.997
NMC (120)	0.090 – 0.090	8.77 – 8.77	1	0	1.000
NMC (200)	0.049 – 0.276	8.96 – 26.49	11	0	1.000
NMC (280)	0.340 – 0.480	8.88 – 15.07	9	0	1.000
SLAC (comb)	0.153 – 0.293	4.18 – 5.50	28	1	0.994
non − singlet			133	1	
total			557	680	

The Analysis:

- Consider the extended data set for $x > 0.3$
- Include target mass corrections
- Correct for NS-tails
- Fit the data for $W^2 > 12.5$ GeV^2, $Q^2 > 4$GeV^2
- Extrapolate to lower values of $W^2 > 4$GeV^2 to extract Higher Twist contributions
- Determine $\alpha_s(M_Z^2)$

Correcting for non NS-tails





Using ABKM09 we corrected for non NS-tails in $F_2(x, Q^2)$.

The fit parameters:

		NLO	$NNLO$	N^3LO
u_v	a	0.246 ± 0.006	0.260 ± 0.007	0.264 ± 0.008
	b	3.807 ± 0.040	3.910 ± 0.037	3.916 ± 0.037
	ρ	6.369	6.338	6.699
	γ	34.321	34.975	35.540
d_v	a	0.316 ± 0.028	0.357 ± 0.031	0.372 ± 0.032
	b	4.793 ± 0.220	5.037 ± 0.231	5.098 ± 0.234
	ρ	2.268	2.322	1.891
	γ	16.602	16.369	16.107
$\Lambda_{QCD}^{(4)}, MeV$		264 ± 28	225 ± 26	231 ± 27
χ^2/ndf		$519/552 = 0.94$	$501/552 = 0.91$	$495/552 = 0.90$

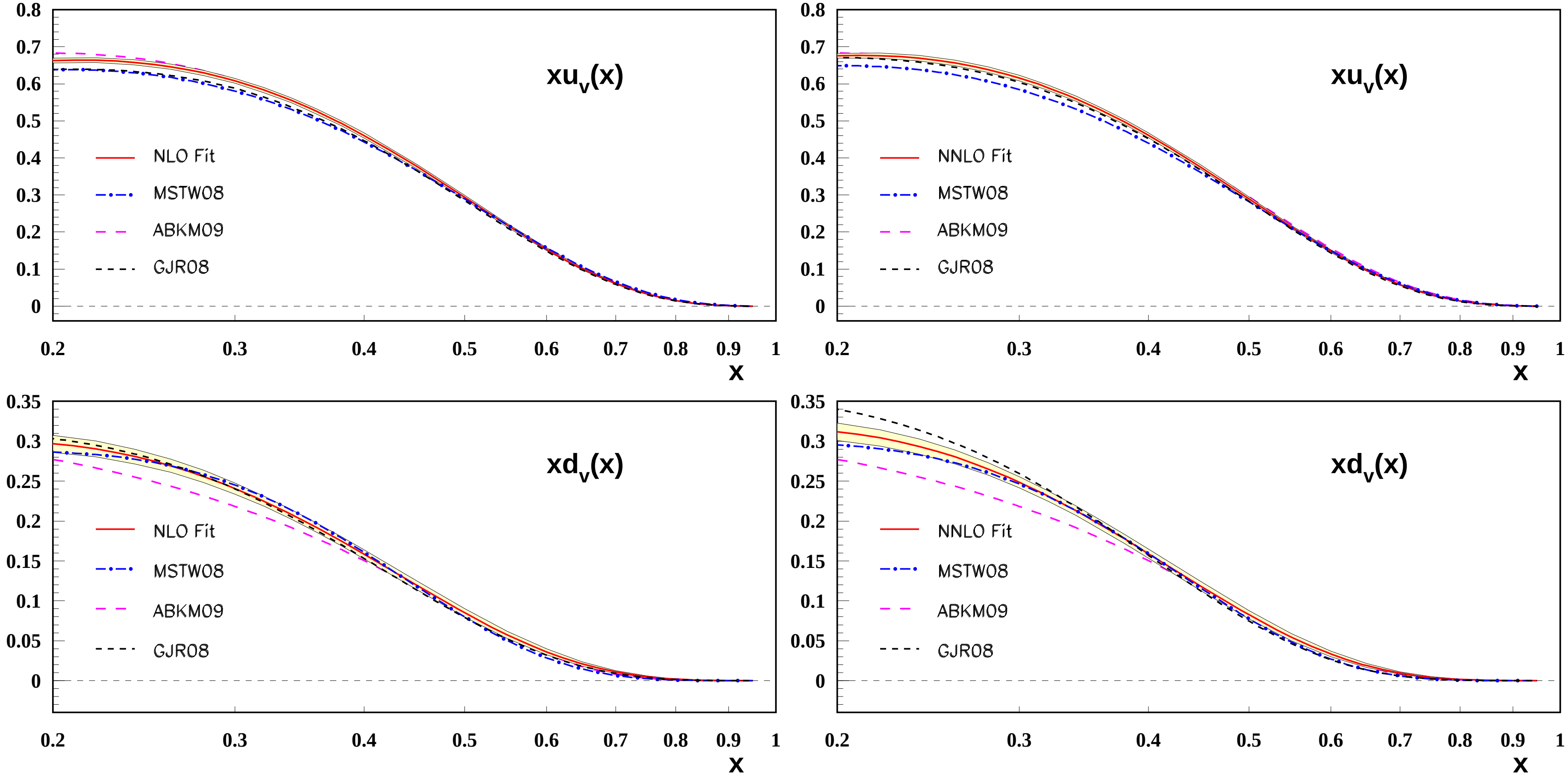
Table 2: Parameter values of the NLO, $NNLO$ and N^3LO non–singlet QCD fit at $Q_0^2 = 4\ GeV^2$. The values without error have been fixed after the first minimization since the data do not constrain these parameters well enough (see text).

The covariance matrix

NLO	$\Lambda_{QCD}^{N_f=4}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}
$\Lambda_{QCD}^{(4)}$	**8.02E-4**				
a_{u_v}	7.32E-5	**4.14E-5**			
b_{u_v}	-5.57E-4	1.24E-4	**1.57E-3**		
a_{d_v}	3.53E-5	-3.94E-4	-3.89E-4	**7.76E-4**	
b_{d_v}	-1.17E-4	-1.93E-4	-3.07E-3	5.53E-3	**4.83E-2**
NNLO	$\Lambda_{QCD}^{N_f=4}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}
$\Lambda_{QCD}^{(4)}$	**6.90E-4**				
a_{u_v}	9.37E-5	**5.11E-5**			
b_{u_v}	-3.28E-4	1.44E-4	**1.40E-3**		
a_{d_v}	8.16E-5	-3.65E-5	-4.50E-4	**9.64E-4**	
b_{d_v}	2.74E-4	-1.45E-4	-3.42E-3	6.52E-3	**5.33E-2**
N3LO	$\Lambda_{QCD}^{N_f=4}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}
$\Lambda_{QCD}^{(4)}$	**7.33E-4**				
a_{u_v}	1.19E-4	**5.98E-5**			
b_{u_v}	-3.17E-4	1.44E-4	**1.40E-3**		
a_{d_v}	1.17E-4	-3.01E-5	-4.72E-4	**1.03E-3**	
b_{d_v}	3.69E-4	-1.21E-4	-3.46E-3	6.82E-3	**5.48E-2**

Table 3: The covariance matrix of the NLO, NNLO, and N3LO Non–Singlet QCD fit at $Q_0^2 = 4\ GeV^2$.

Valence Quark Distributions



α_s vs Exp. and Order of the Fit

Experiment	$\alpha_s(M_Z)$			
	NLO_{exp}	NLO	NNLO	N^3LO^*
BCDMS	0.1111 ± 0.0018	0.1138 ± 0.0007	0.1126 ± 0.0007	0.1128 ± 0.0006
NMC	$0.117 \ {}^{+\,0.011}_{-\,0.016}$	0.1166 ± 0.0039	0.1153 ± 0.0039	0.1153 ± 0.0035
SLAC		0.1147 ± 0.0029	0.1158 ± 0.0033	0.1152 ± 0.0027
BBG		0.1148 ± 0.0019	0.1134 ± 0.0020	0.1141 ± 0.0021
BB		0.1147 ± 0.0021	0.1132 ± 0.0022	0.1137 ± 0.0022

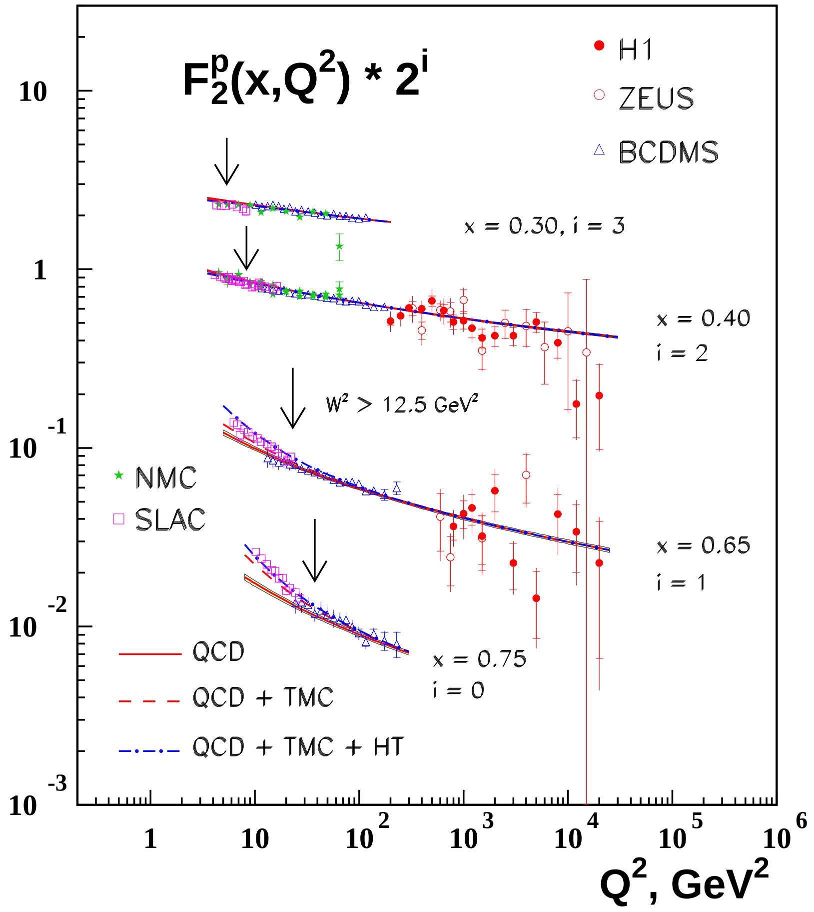
Table 4.6: Comparison of the values of $\alpha_s(M_Z)$ obtained by BCDMS [83] and NMC [103] at NLO with the results of the flavor non-singlet fits BBG [60] and BB [110] of the DIS flavor non-singlet world data, at NLO, NNLO, and N^3LO^* with the response of the individual data sets, combined for the experiments BCDMS [83, 118, 119], NMC [84], and SLAC [120].

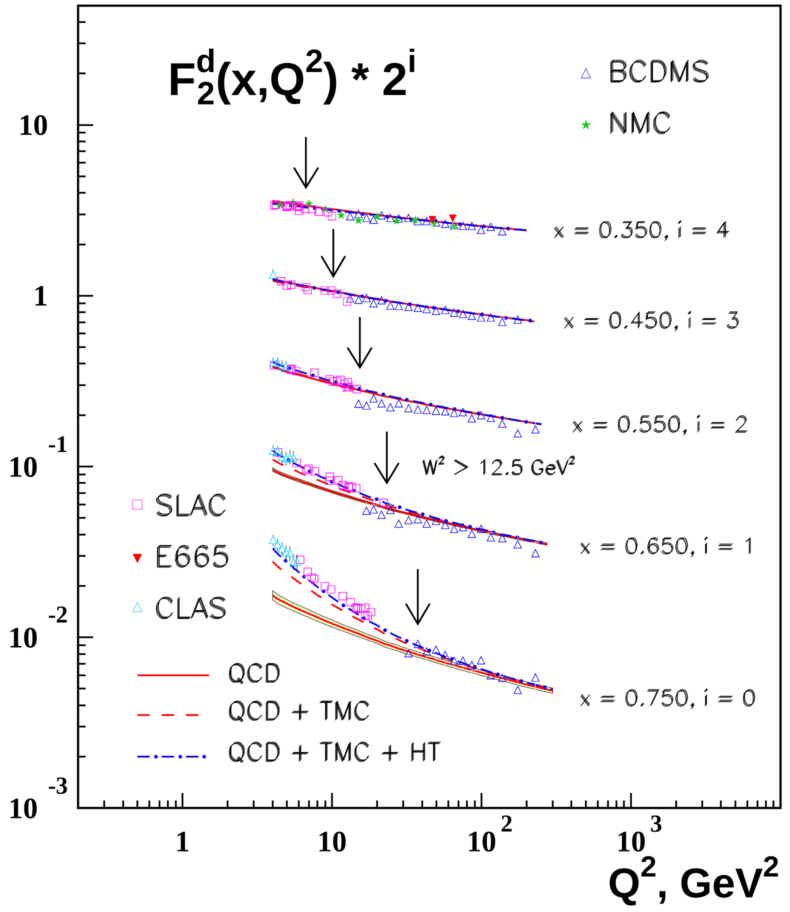
Comparison of α_s from Different Fits

Data Set	ABM11	BBG	NN21	MSTW
BCDMS	0.1048 ± 0.0013	0.1126 ± 0.0007	0.1158 ± 0.0015	0.1101 ± 0.0094
NMC	0.1152 ± 0.0007	0.1153 ± 0.0039	0.1150 ± 0.0020	0.1216 ± 0.0074
SLAC	0.1128 ± 0.0003	0.1158 ± 0.0034	> 0.124	$\{$ 0.1140 ± 0.0060 ep 0.1220 ± 0.0060 ed
HERA	0.1126 ± 0.0002		$\{$ 0.1199 ± 0.0019 0.1231 ± 0.0030	0.1208 ± 0.0058
DY	0.101 ± 0.025	–	–	0.1136 ± 0.0100
	0.1134 ± 0.0011	0.1134 ± 0.0020	0.1173 ± 0.0007	0.1171 ± 0.0014

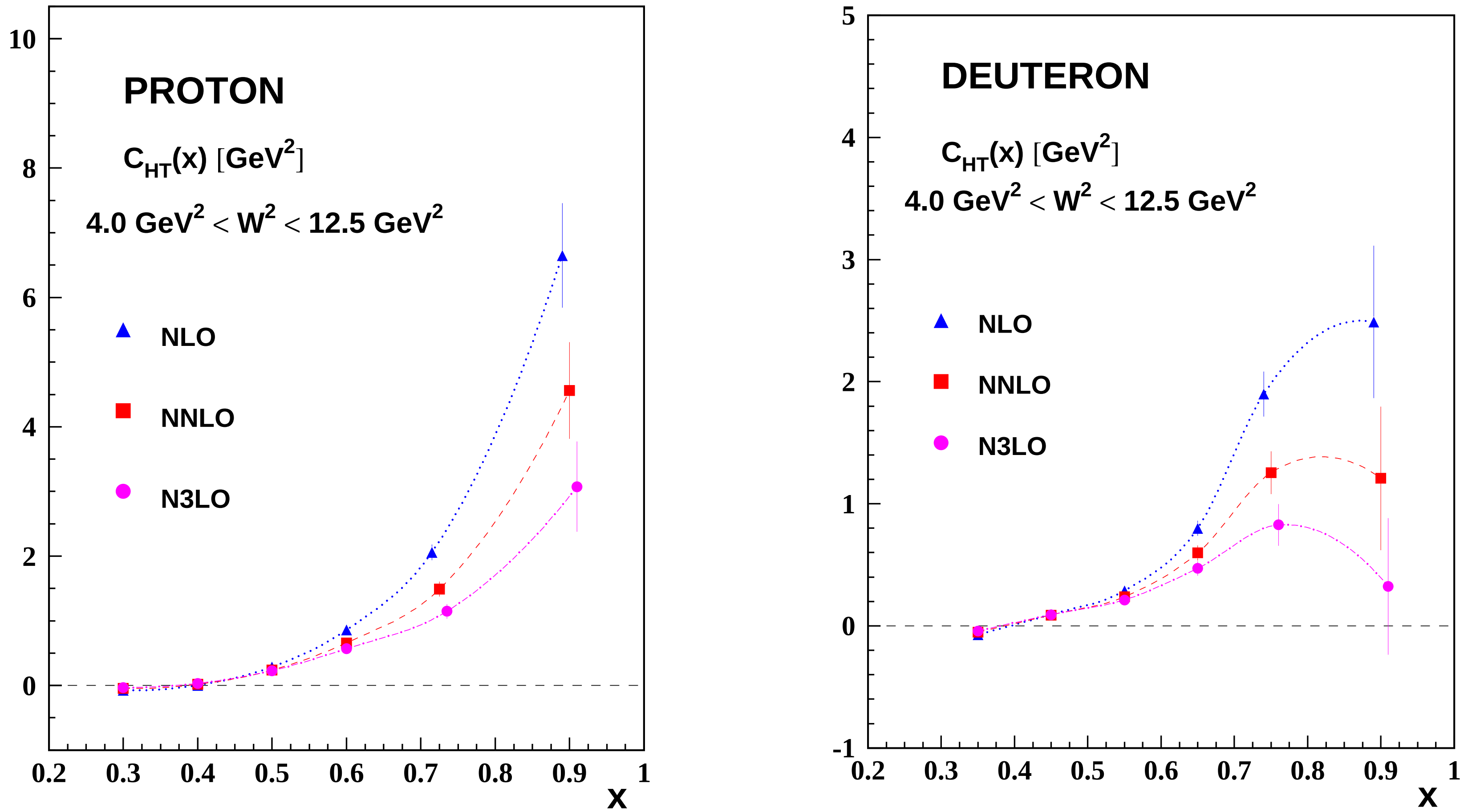
Table 4.9: Comparison of the pulls in $\alpha_s(M_Z)$ per data set between the ABM11, BBG [60], NN21 [124] and MSTW [158] analyses at NNLO.

F_2 NNLO


$F_2^p(x,Q^2) * 2^i$
H1
ZEUS
BCDMS
NMC
SLAC
QCD
QCD + TMC
QCD + TMC + HT
x = 0.30, i = 3
x = 0.40
i = 2
W^2 > 12.5 GeV^2
x = 0.65
i = 1
x = 0.75
i = 0
Q^2, GeV^2


$F_2^d(x,Q^2) * 2^i$
BCDMS
NMC
SLAC
E665
CLAS
QCD
QCD + TMC
QCD + TMC + HT
x = 0.350, i = 4
x = 0.450, i = 3
x = 0.550, i = 2
W^2 > 12.5 GeV^2
x = 0.650, i = 1
x = 0.750, i = 0
Q^2, GeV^2

Updated Higher Twist Contributions



$g_2^{\mathrm{tw3}}(x, Q^2)$

- Usually $g_1(x, Q^2)$ is thought to not receive twist-3 contributions.
- However, [Blümlein and Tkabladze, 1998 Nucl.Phys.B] traget masses yield a twist-3 contribution.
- Twist $\tau = 2$: [Wandzura and Wilczek, 1977 Phys.Lett.B]

$$g_2^{\tau=2}(x, Q^2) = -g_1^{\tau=2}(x, Q^2) + \int_x^1 \frac{dy}{y} g_1^{\tau=2}(y, Q^2)$$

- Twist $\tau = 3$: [Blümlein and Tkabladze, 1998 Nucl.Phys.B]

$$g_1^{\tau=3}(x, Q^2) = \frac{4x^2 M^2}{Q^2} \left[g_2^{\tau=3}(x, Q^2) - 2 \int_x^1 \frac{dy}{y} g_1^{\tau=3}(y, Q^2) \right]$$

- Try to extract $g_2^{\tau=3}(x, Q^2)$ and $g_1^{\tau=3}(x, Q^2)$ from data.

$g_2^{\rm tw3}(x, Q^2)$

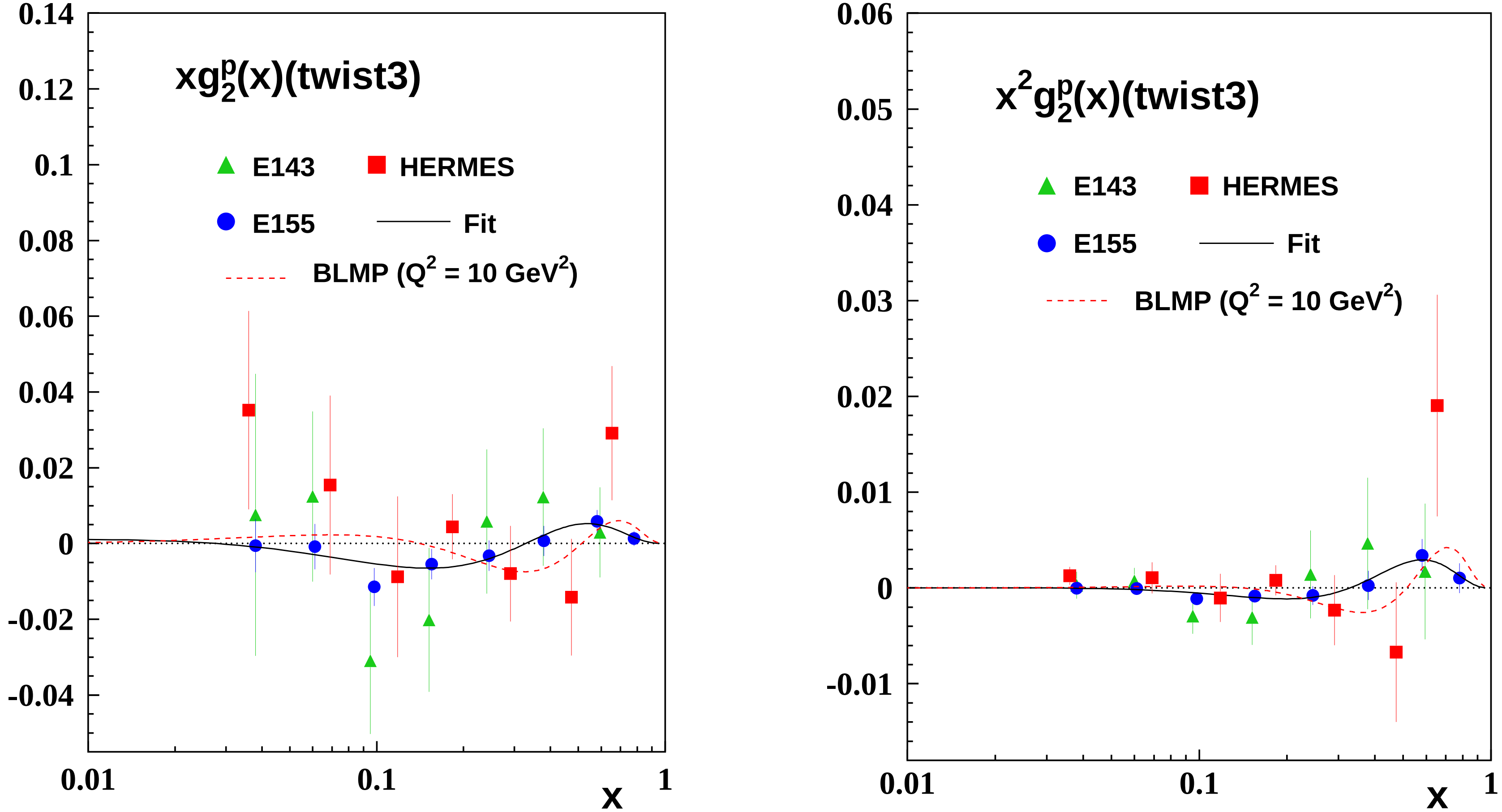
- Errors of the current g_2 data are still rather large.
- E143, E155, Hermes; (NMC)
- Model calculations by: [V. Braun et al., 2011 Phys.Rev.D]
- Are higher moments helpful? – Lattice Simulations ?
- Bukhard-Cottingham :

$$\int_0^1 dx g_2(x, Q^2) = 0$$

- Wandzura-Wilczek :

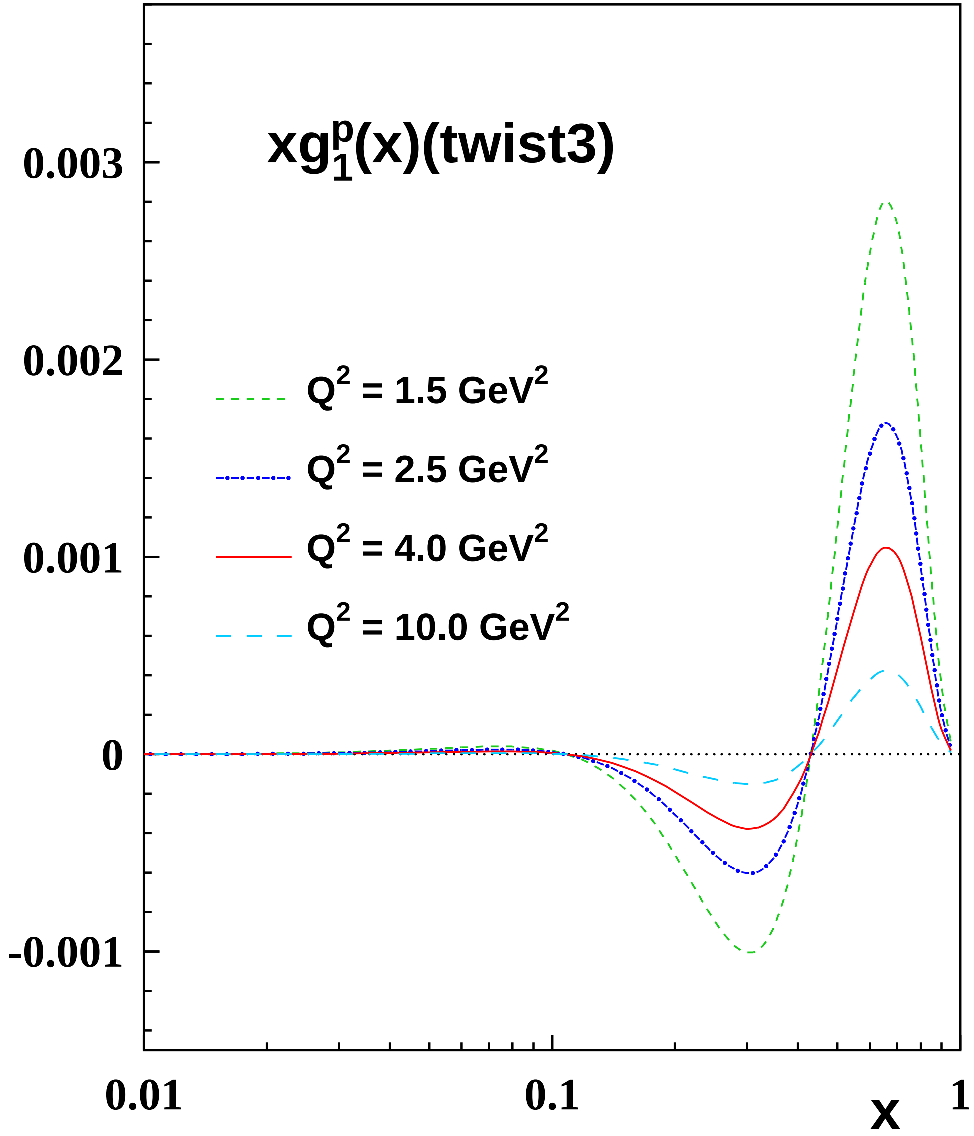
$$\int_0^1 dx g_2^{\tau=2}(x, Q^2) = 0 \quad \Longrightarrow \quad \int_0^1 dx g_2^{\tau=3}(x, Q^2) = 0$$

$g_2^{\mathrm{tw3}}(x, Q^2)$



$$g_2^{\tau=3}(x) = A\left[\ln(x) + (1-x) + \frac{1}{2}(1-x)^2\right] + (1-x)^3\left[B - C(1-x) + D(1-x)^2\right]$$

$g_1^{\mathrm{tw3}}(x, Q^2)$



Conclusions

- We performed a re-analysis of the extended set of deep-inelastic valence data (p,d), accounting for remaining non-valence tails in $F_2(x, Q^2)$, which were calculated using the ABKM09 distributions.
- We obtained slightly lower values of $\alpha_s(M_Z^2)$, far within the 1σ range of the BBG (2006) analysis.
- Very stable predictions are obtained going from NLO to NNLO*.
- We do not confirm the significant differences being reported by MSTW between the SLAC *ep* and *ed* data, and neither their large value for the NMC data. We also disagree with the large value of NNPDF for SLAC, which also contradicts the *ep* value of MSTW.
- An update of the higher twist contributions at large values of x was given.
- First estimates on $g_2^{\tau=3}(x, Q^2)$ and $g_1^{\tau=3}(x, Q^2)$ were derived from data, using general integral relations.