On the NNLO QCD corrections to DIS heavy-quark production

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Workshop DIS 2012 *Heavy flavours/structure functions Working Group*, Bonn, Mar 28, 2012

Plan

- Talk based on results on . . .
  - . . . improved heavy-quark coefficient functions at NNLO
    H. Kawamura , S.M., A. lo Presti and A. Vogt to appear
  - . . . a determination of the running charm-quark mass
    S. Alekhin , K. Daum , K. Lipka and S.M. to appear
  - . . . a spin-off on top-quark hadro-production
    S.M., P. Uwer and A. Vogt to appear

Heavy-quark production in DIS

QCD factorization

- Structure functions for DIS heavy-quark production
- Perturbative expansion at NLO, e.g., neutral current

$$F_2^c(x,Q^2,m^2) = \frac{\alpha_s\, e_q^2}{\pi^2} \sum_{i=q,\bar{q},g} f_i\left(z,\mu^2\right) \otimes \left\{c_{i,k}^{(0)}\,(\eta,\xi) + (4\pi\alpha_s)\, c_{i,k}^{(1)}\left(\eta,\xi,\mu^2/m^2\right)\right\}$$

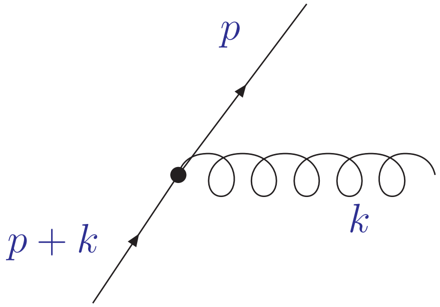
- Kinematic variables
  - distance from threshold $\eta = s/(4m^2) - 1$
  - virtuality $\xi = Q^2/m^2$

NLO

- NLO for coefficient functions $c_{i,k}$
  - neutral current Laenen, Riemersma, Smith, van Neerven ‘93
  - charged current Gottschalk ‘81; Glück, Kretzer, Reya ‘96

Strategy beyond NLO

- Use universal features of soft/collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: $E_g = 0$, soft θ_{qg} = 0 collinear



$$\frac{1}{(p+k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g \, d\sin\theta_{qg} \, \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

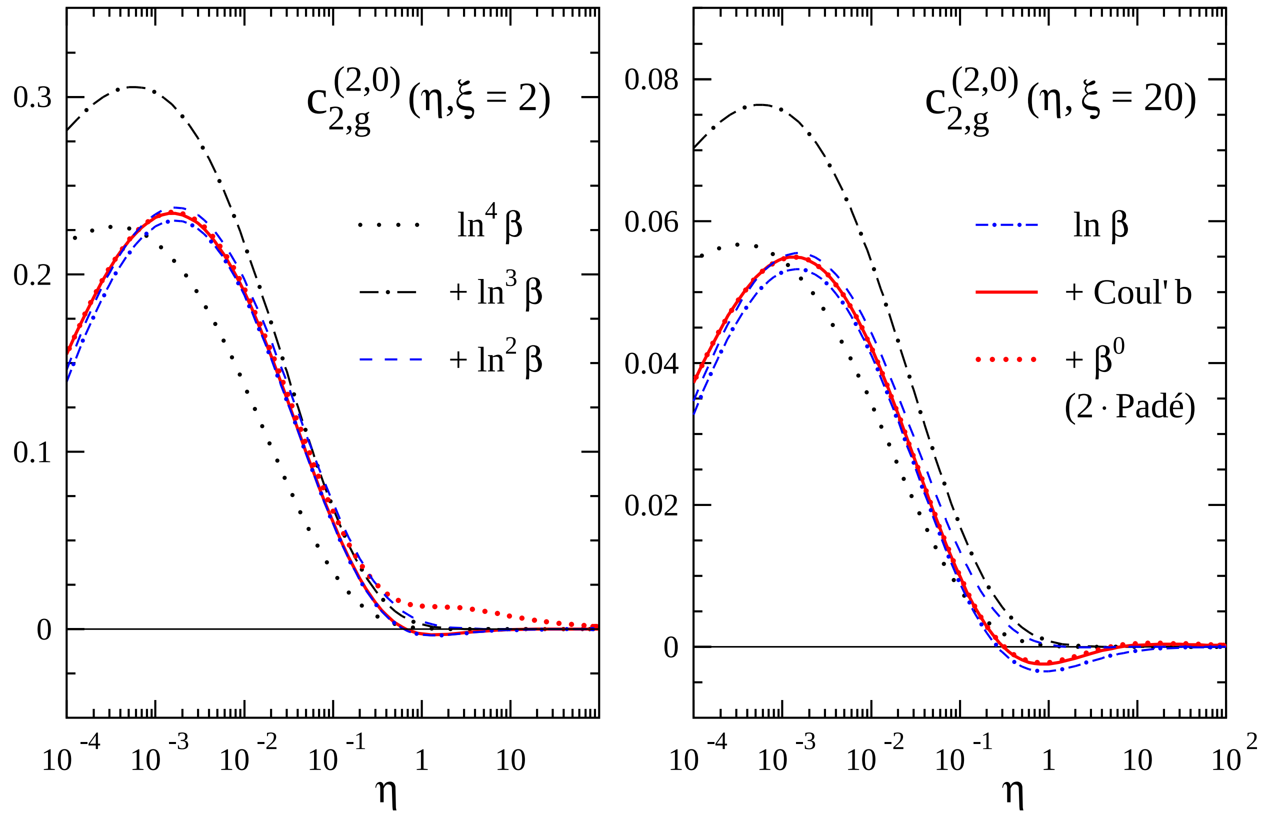
$$\longrightarrow \alpha_s \ln^2(\dots)$$

Threshold improvements beyond NLO

- Parton cross section close to threshold $s \simeq 4m^2$
  - Sudakov logarithms $\ln(\beta)$ with velocity of heavy quark $\beta = \sqrt{1-4m^2/s}$ at $n^{\rm th}$-order: $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
- Resummation in Mellin space (renormalization group equation) predicts fixed orders in perturbation theory
  - approximate expressions to NNLO

    Laenen, S.M. '98; Alekhin, S.M. '08; Lo Presti, Kawamura, S.M., Vogt '10

Approximate coefficient functions at NNLO



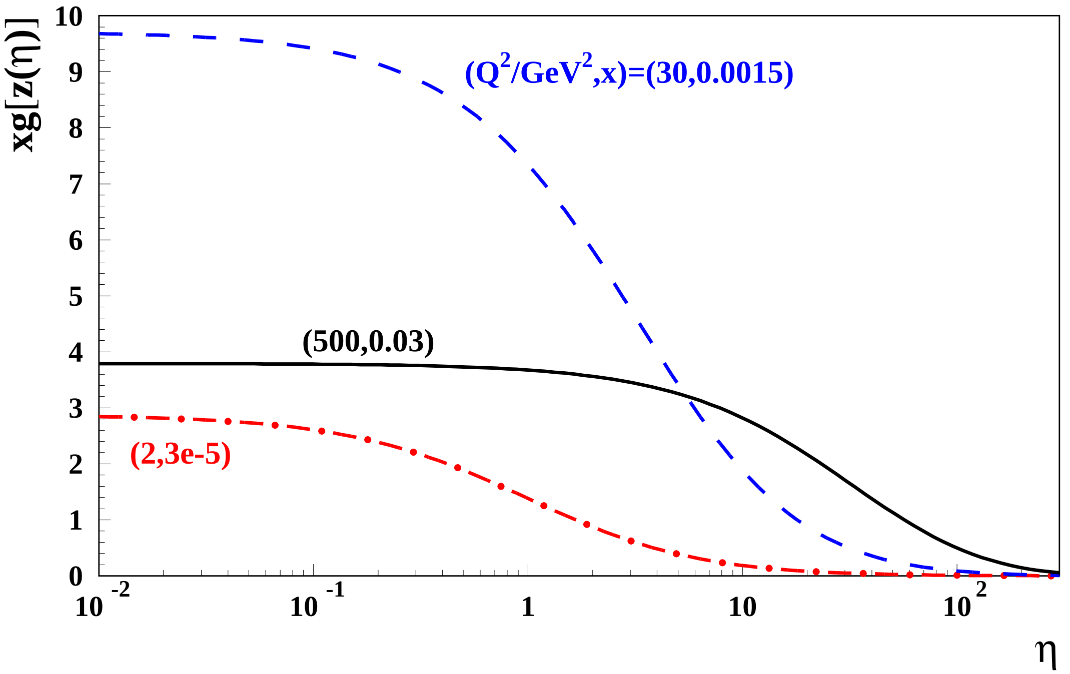
- Distance from threshold $\eta = s/(4m^2) - 1$
  - Sudakov logarithms dominant for $\eta \ll 1$ now known to NNLL
    Lo Presti, Kawamura, S.M., Vogt '10
- Small impact of subleading terms $\mathcal{O}(\beta^0)$ (Padè estimate)

Convolution with gluon PDF

- Recall QCD factorization

$$x^{-1} F_2^c(x, Q^2, m^2) = \frac{\alpha_s\, e_q^2}{\pi^2} \sum_{i=q,\bar{q},g} \int_0^{\eta_{max}} d\eta\; f_i\left(z(\eta), \mu^2\right)\; c_{i,k}\left(\eta, \xi, \mu^2\right)$$

  - gluon PDF gives large weight to parton dynamics near threshold for $Q^2 \lesssim 10 - 30$ GeV^2



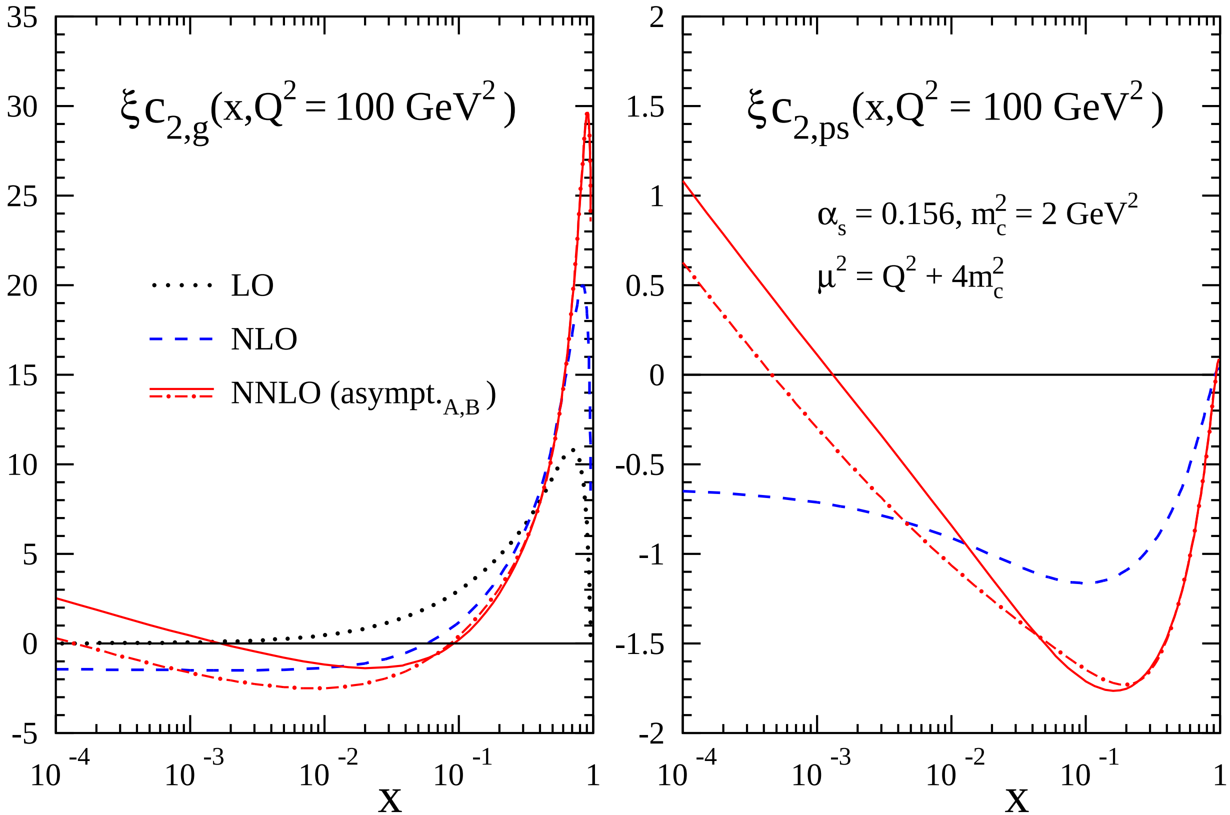
Asymptotics beyond NLO

- Exact factorization at large $Q^2 \gg m^2$ ($\xi \to \infty$)
  Buza, Matiounine, Smith, Migneron, van Neerven '95
  - decompose heavy-quark $c_{k,i}$ into light-quark $C_{k,j}^{\text{light}}$ and operator matrix elements (OMEs) A_{ji}

$$c_{k,i}(\eta, \xi, \mu^2) \rightarrow \left[A_{ji} \left(\frac{m^2}{\mu^2} \right) \otimes C_{k,j}^{\text{light}} \left(\frac{Q^2}{\mu^2} \right) \right] (x)$$

- NNLO corrections require three-loop results
  - massless coefficient functions Vermaseren, S.M., Vogt '05
  - heavy-quark OMEs; currently only fixed Mellin moments available
    Bierenbaum, Blümlein, Klein '09

Approximate coefficient functions at NNLO



- Asymptotically large $\xi = Q^2/m^2$
  - complete functional dependence on x now known to NNLO

    Lo Presti, Kawamura, S.M., Vogt to appear
- Sizable impact of parametrization uncertainties in OMEs at small x
  - significant improvements with more Mellin moments

High-energy limit beyond NLO

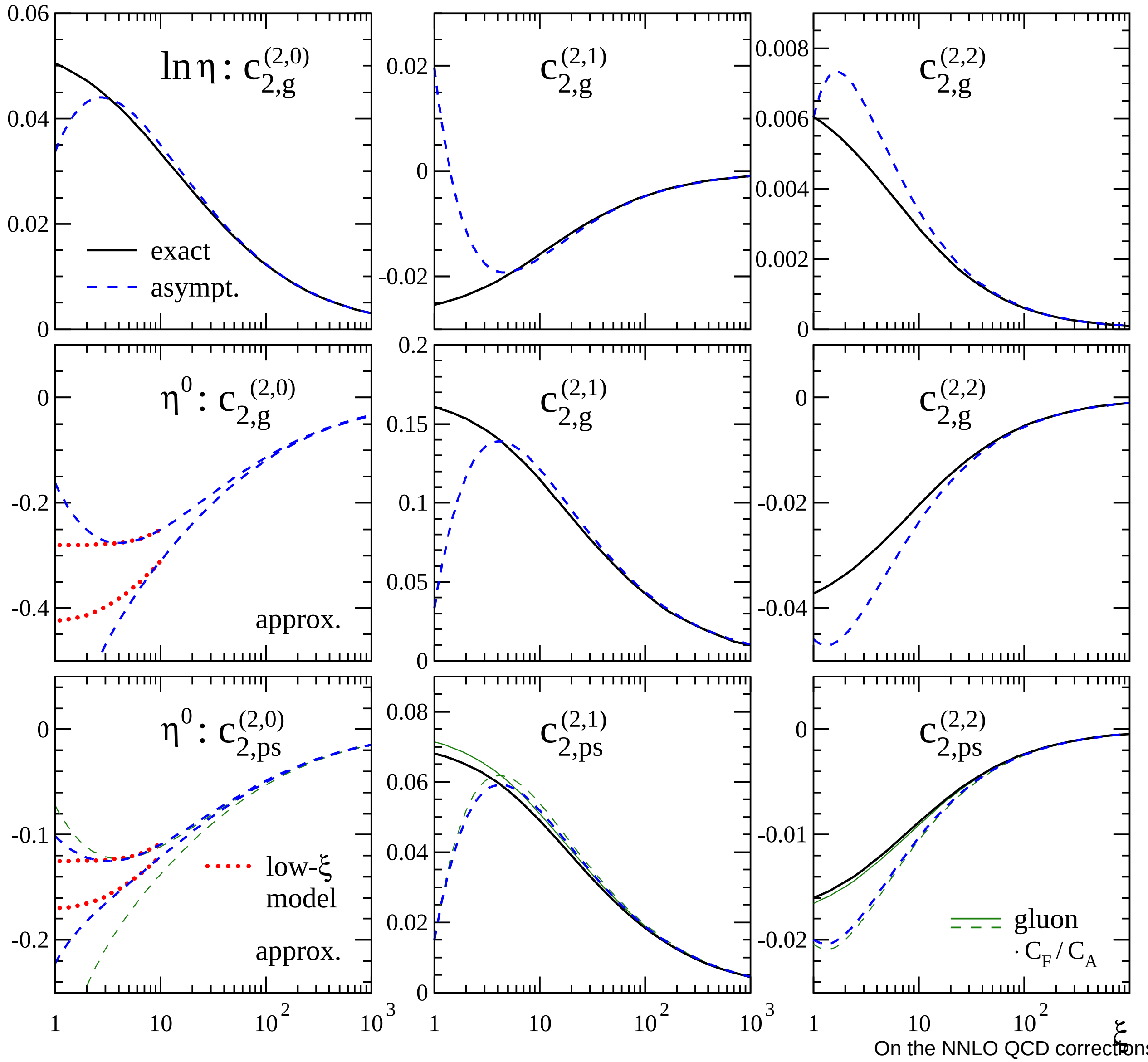
- Well-known k_t-factorization at large η (small x)
  Catani, Ciafaloni, Hautmann '91
  - decompose heavy-quark structure function $F_2^c(x, Q^2, m^2)$ into gluon PDF un-integrated in k_t and heavy-quark impact factor
- Schematic expansion up to NNLO

$$
\begin{aligned}
c^{(1)}_{k,i}(\eta, \xi, \mu^2) &\approx c^{(1)}_{\mathrm{LL}_x}\, \mathrm{const}_\eta \\
c^{(2)}_{k,i}(\eta, \xi, \mu^2) &\approx c^{(2)}_{\mathrm{LL}_x} \ln \eta + c^{(2)}_{\mathrm{NLL}_x}\, \mathrm{const}_\eta
\end{aligned}
$$

- Small-x dependence governed by BFKL anomalous dimension
  - prediction of $c^{(l)}_{\mathrm{LL}_x}$
- Result valid for all ξ;
  - for $c^{(2)}_{\mathrm{LL}_x}$ known exactly Catani, Ciafaloni, Hautmann '91
  - for $c^{(2)}_{\mathrm{NLL}_x}$ at large ξ from asymptotic factorization

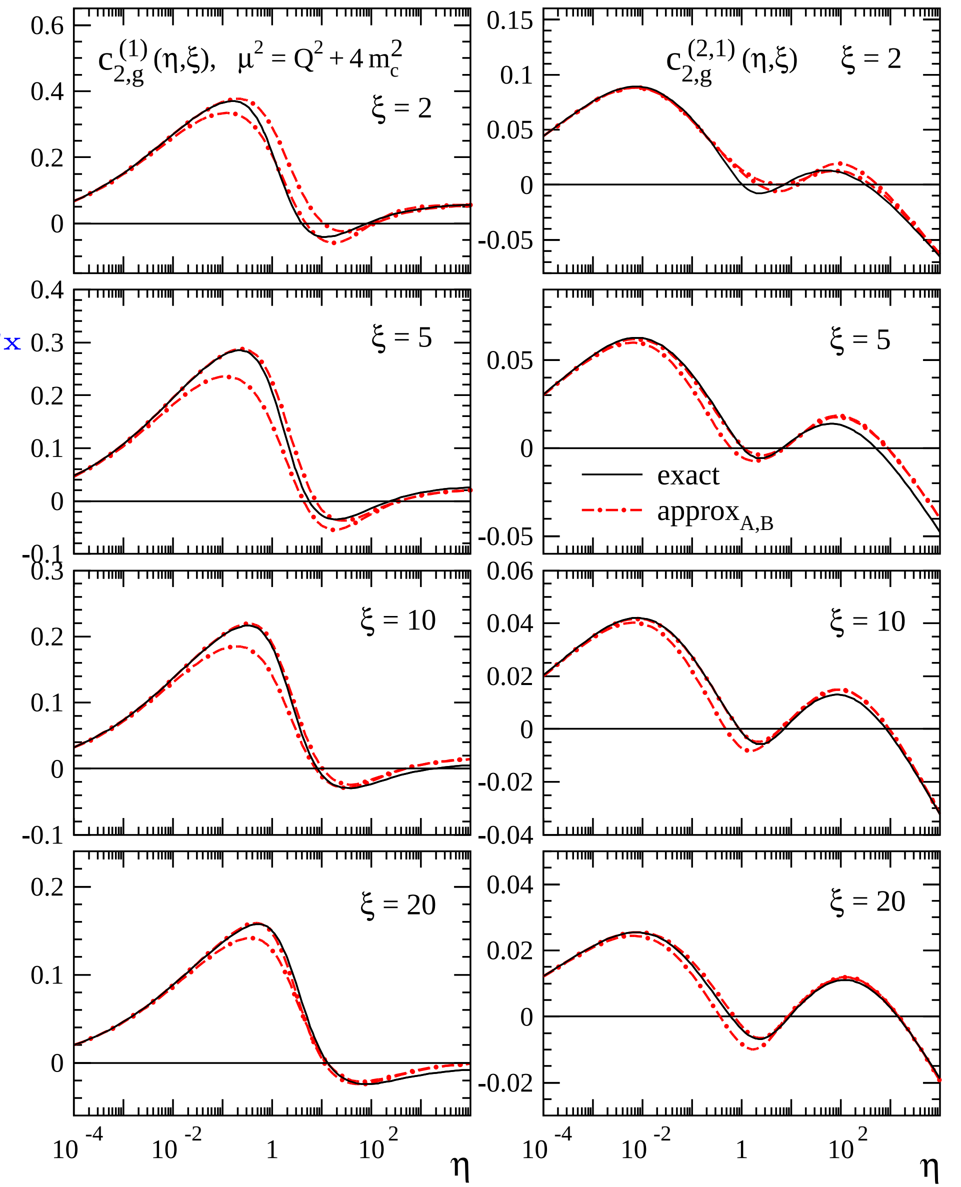
High-energy limit

- Coefficients of large η (small x) expansion at NNLO



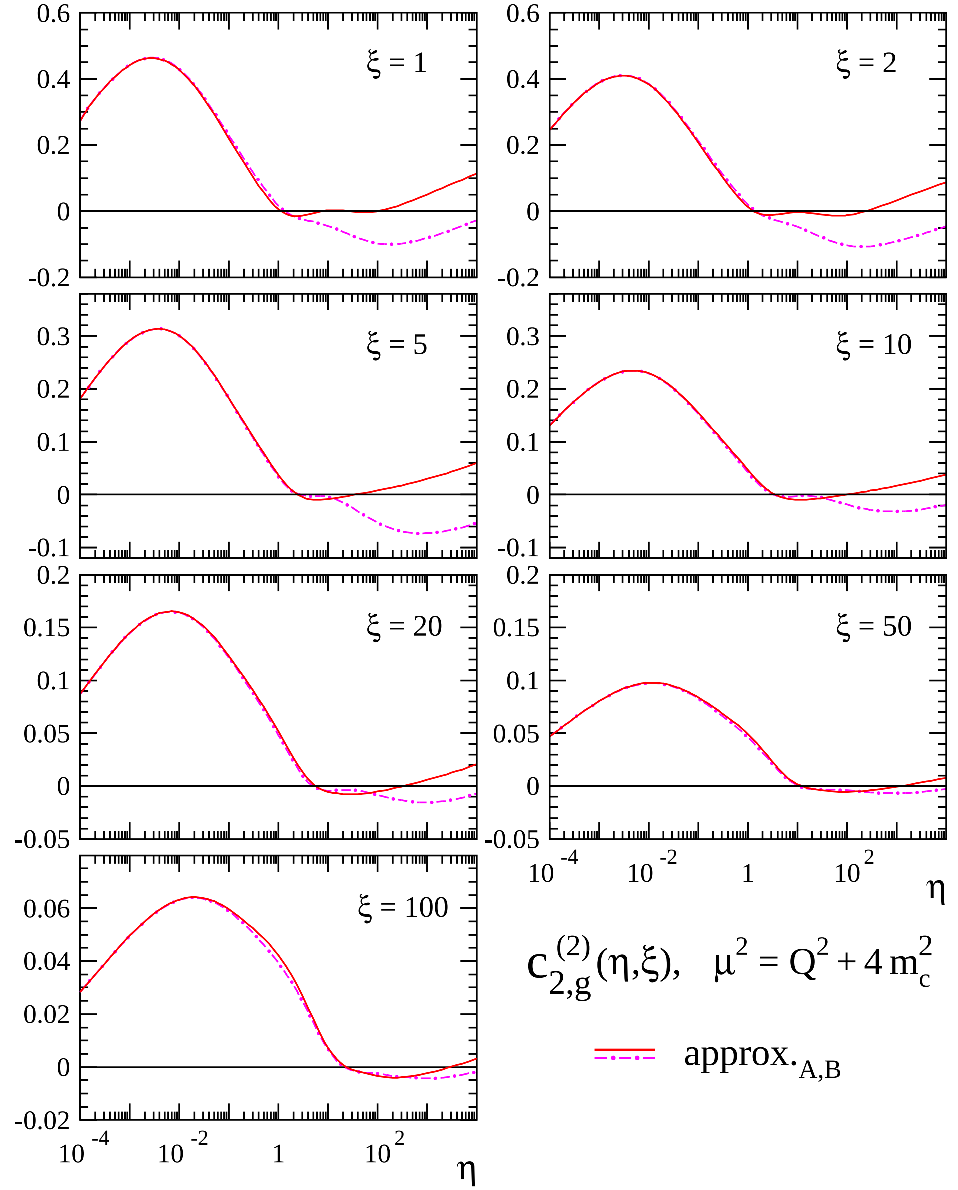
Quality check

- Combine all limits
- Construct approximate result $c_{2,k}^{\rm approx}$ for full kinematic range
  - merge $c_{2,k}^{\rm approx} =$ $c_{2,k}^{\rm thresh} + c_{2,k}^{\rm LL_x} + \eta^\gamma/(C+\eta^\gamma) c_{2,k}^{\rm NLL_x}$
- Tests
  - at NLO for $c_{2,g}^{(1)}$
  - at NNLO for $c_{2,g}^{(2,1)}$ (scale dependent part)



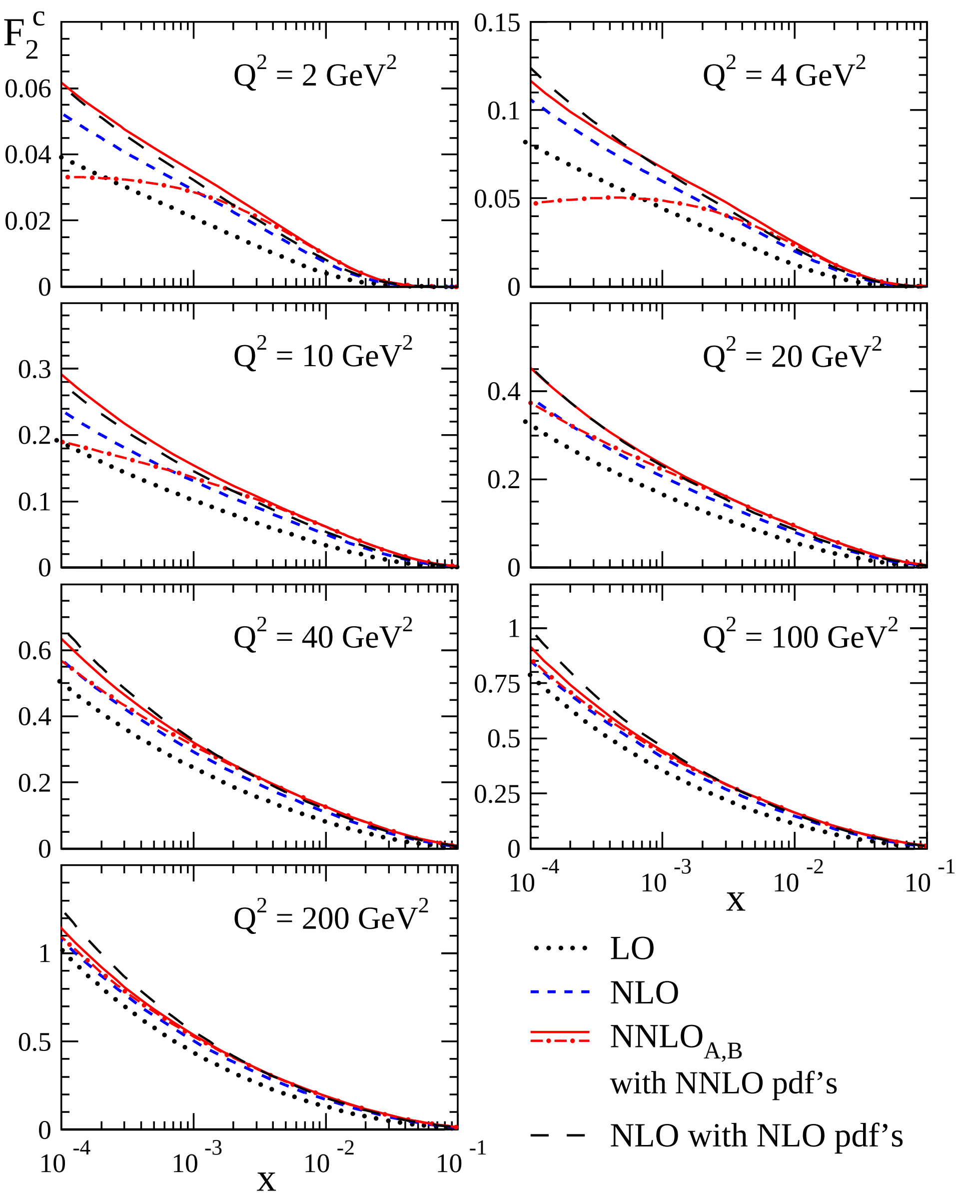
Approximate coefficient functions at NNLO

- New approximate NNLO result
  - very well constrained at large ξ
  - sizable uncertainties for $\xi \lesssim 10$
- Realistic estimate of uncertainty through combination of all limits
- Progress would require
  - more Mellin moments for A_{ji}
  - computation of $c_{2,k}^{\mathrm{NLL_x}}$ (NLO impact factor for k_t-factorization)

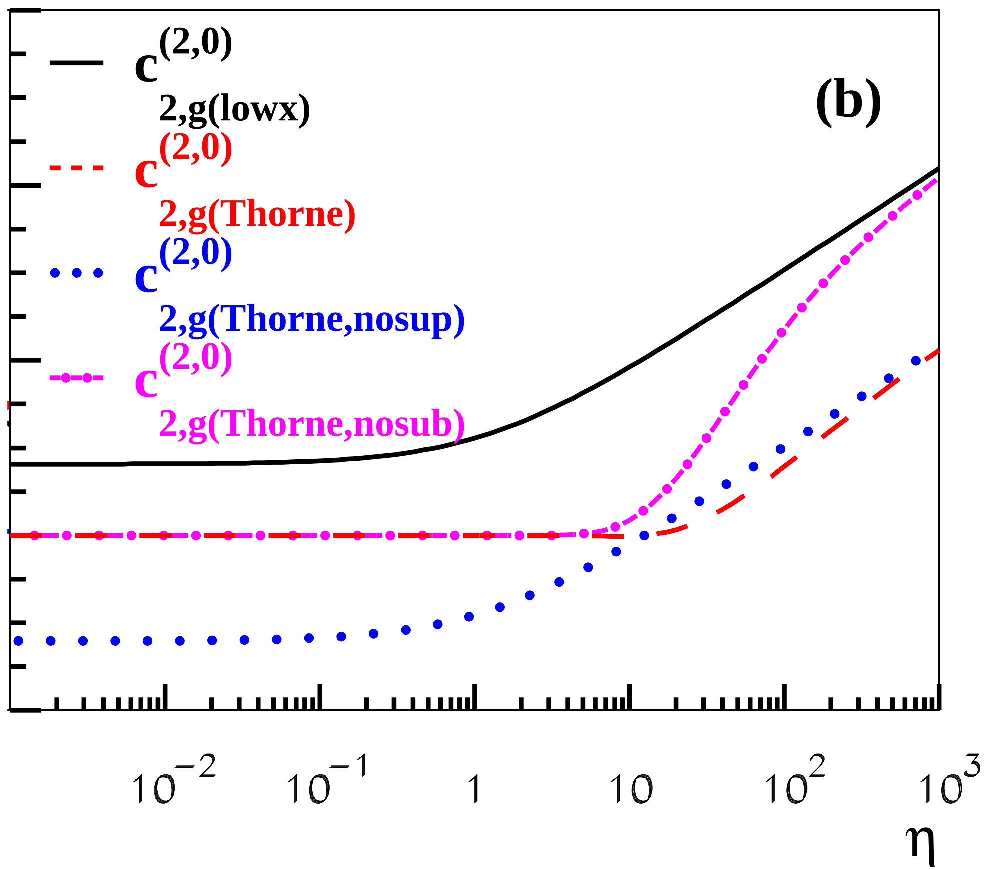


Approximate charm structure functions at NNLO

- New approximate NNLO result
  - convolution with ABM11 PDF set
- Uncertainty on NNLO dominates kinematic region in Q^2 relevant for HERA
  - very well constrained at large Q^2
  - sizable uncertainties for $Q^2 \lesssim 20\,\mathrm{GeV}^2$ (even at not so small x)
  - gluon PDF does not fall fast enough as η grows larger



Charm coefficient functions at NNLO in TR' scheme



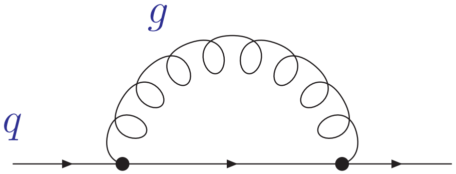
- Coefficient function $c^{(2)}_{2,g}$ at NNLO only modelled at high-x scale
  Thorne '06
  - no valid description in threshold region
  - no uncertainty at high energies attached

Mass measurement

Pole mass

- Based on (unphysical) concept of heavy-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$



  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$

Running quark masses

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
    $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
  - conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory: $m = m(\mu_R)\left(1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)}\right)$

Quark masses in the PDF fits

		m_c/m_b (GeV)	
MSTW	DIS+DY+jets	1.40 / 4.75 (GMVFN)	**EPJC 63, 189 (2009)**
CTEQ	DIS+DY+jets	1.3 / 4.5 (GMVFN)	**PRD 82, 074024 (2010)**
NNPDF	DIS+DY+jets	√2 / 4.75 (GMVFN)	**NPB 849, 296 (2011)**
JR	DIS+DY	1.3 / 4.2 (FFN)	**PRD 70, 074023 (2009)**
HERAPDF	DIS	1.4 / 4.75 (GMVFN)	**JHEP 1001, 109 (2010)**
PDG:		1.66 / 4.79	

PDF fits assume pole mass scheme

The values of masses are systematically lower than those from PDG

In the GMVFN scheme the mass is used as a matching parameter as well

Additional uncertainty in the c.s. predictions

Alekhin

NNLO value of m_c from DIS (theory)

Alekhin

The NNLO* theoretical accuracy:

– the NNLO PDF evolution
– approximate NNLO Wilson coefficients with a linear interpolation between two margins A and B and interpolation coefficient d_N is a parameter of fit

$$A\,(1-d_N) + d_N\,B$$

The PDFs are fixed at the ABM11 NNLO central values with m_c-dependence included

– variation of PDFs within the ABM11 uncertainties makes little effect on the value of m_c

The experimental data are confronted to:

– the structure function F_2^{cc} (m_c) (charm production c.s. σ^{cc}_{full})

– the charm-production c.s with account of the visible phase space cuts σ^{cc}_{full} $(m_c)\varepsilon(m_c)$. The correction factor

$$\varepsilon(m_c)=\sigma^{cc}_{vis}(m_c)/\sigma^{cc}_{full}(m_c)$$

is calculated with the fully exclusive code HVQDIS; the NNLO contribution partially cancel in the ratio. The correction is sensitive to m_c $\rightarrow$ recalculated iteratively in the fit

Harris, Simth NPB 452, 109 (1995)

Fitting m_c (data)

The data on D* c.s. **H1 Collaboration EPJC 71, 1769 (2011)**

D* cross section transformed into charm cross section using fragmentation function from **[H1 Collaboration EPJC59:589(2009)]**, with code *hvqdispawutil* **[K. Daum]**

correction for the unmeasured region is necessary

consistent treatment of the F_L contribution

The data on structure function obtained with the shifted vertex

H1 Collaboration EPJC 65, 89 (2010)

inessential cut on p_T → the correction for the unmeasured region is small

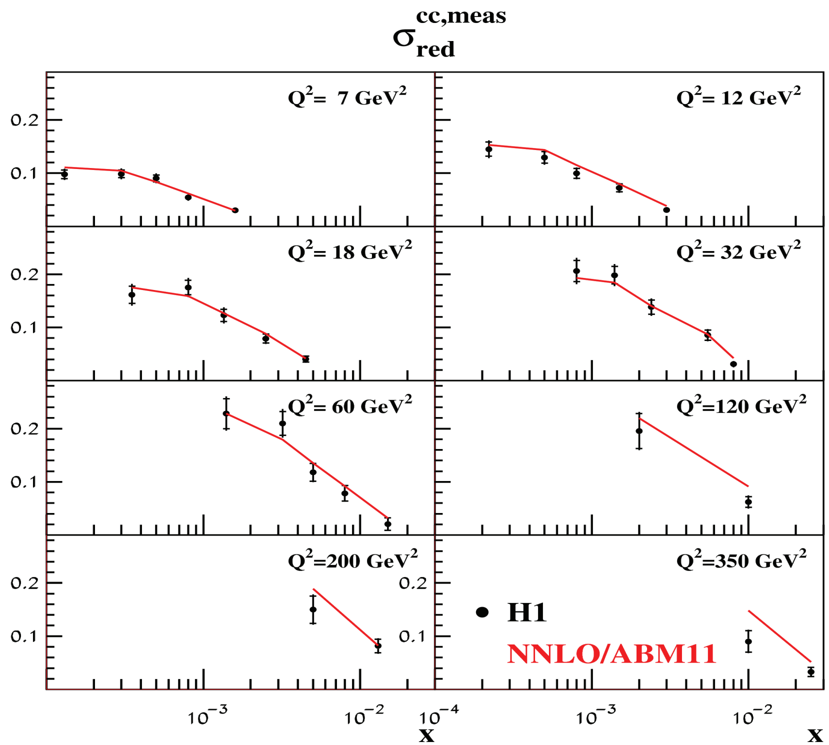
Alekhin

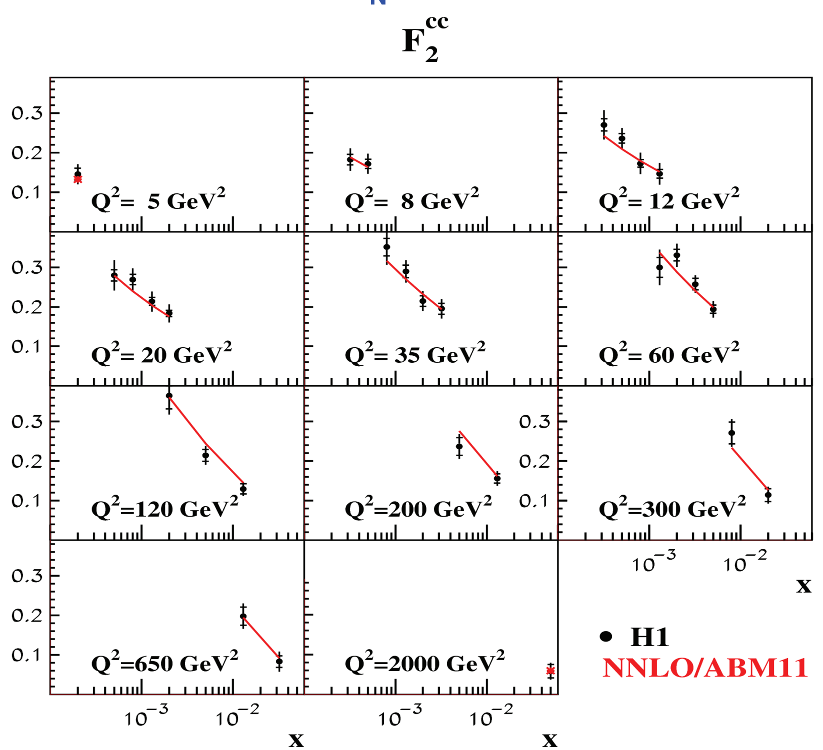
Results

Alekhin

d_N	1.	0.	-0.44±0.13..	
$m_c(m_c)$/GeV	0.98±0.05	1.25±0.03	1.36±0.05	
X^2	160	73	64	NDP=60

The value of $m_c(m_c)$ is anti-correlated with d_N

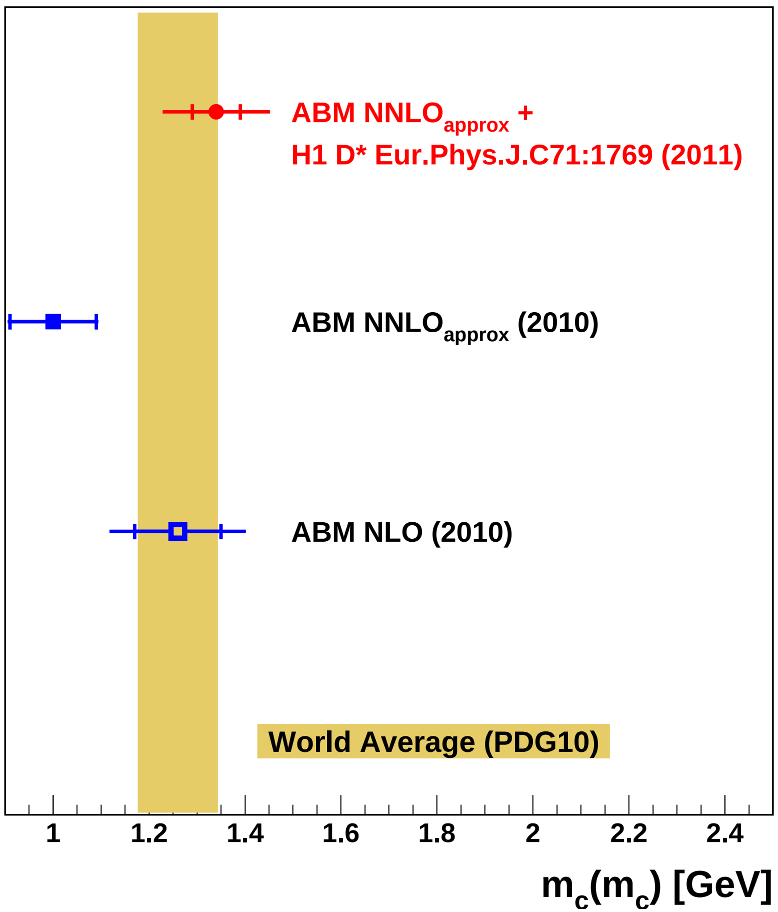


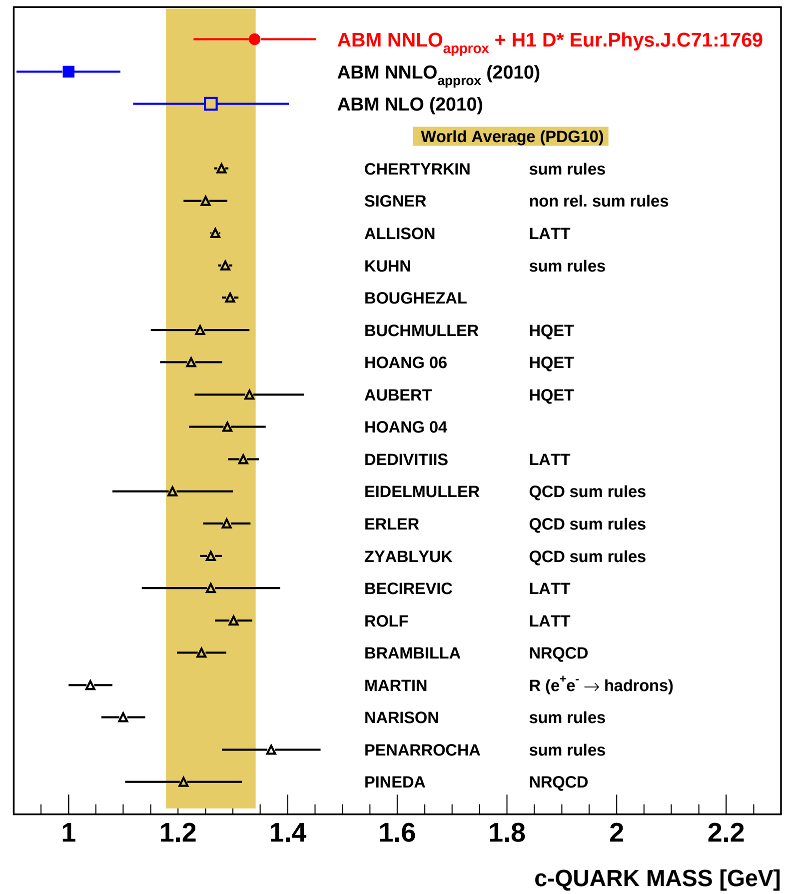


$m_c(m_c)$=1.36 ± 0.05(exp.) ± 0.10(theory/NNLO) (GeV)

Running charm mass from HERA

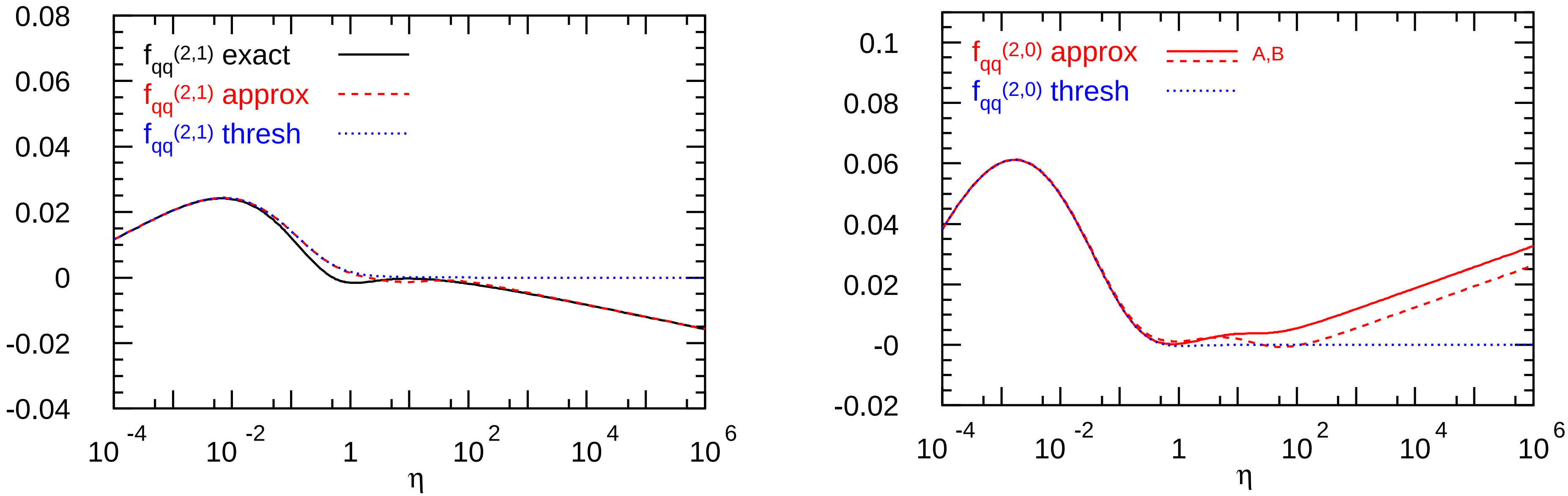
- Determination of $\overline{\mathrm{MS}}$-mass $m_c(\bar{m}_c)$ in DIS
  - significant impact of data on F_2^c from HERA II
  - ongoing work (Alekhin, Daum, Lipka, S.M.)





Top-pair hadro-production

- Spin-Off from work on heavy-quark DIS S.M, Uwer, Vogt to appear
- Threshold at $s \simeq 4m_t^2$ with logarithms $\ln(\beta)$ in velocity of heavy quark $\beta = \sqrt{1 - 4m_t^2/s}$ at $\mathrm{n^{th}}$-order
  S.M, Uwer '08; Beneke, Czakon, Falgari, Mitov, Schwinn '09
- High-energy limit for $\rho = 4m_t^2/s \to 1$
  Catani, Ciafaloni, Hautmann '91; Ball, Ellis '01



Summary

Heavy-quark DIS

- Charm structure function F_2^c
  - progress on approximation at NNLO QCD
  - large uncertainty due to poorly constrained NLL_x term at NNLO

Heavy quark masses

- Determination of running charm-quark mass from DIS data
- Outlook:
  - tuning of ABM11 PDFs with updated Wilson coefficients
  - comparision to $m_c(m_c)$ at NLO scale variation uncertainty
  - impact of the combined HERA charm data

Top-quark hadro-production

- Improved approximate NNLO cross sections with small uncertainty