High energy exclusive leptoproduction of the $\rho-$meson: theory and phenomenology

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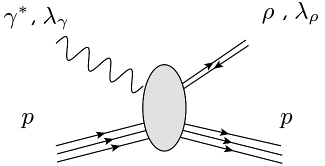
Introduction

Helicity amplitudes of the diffractive leptoproduction of the ρ meson

- Helicity Amplitudes $T_{\lambda_\rho \lambda_\gamma}$

$T_{\lambda_\rho \lambda_\gamma} =$



Examples :

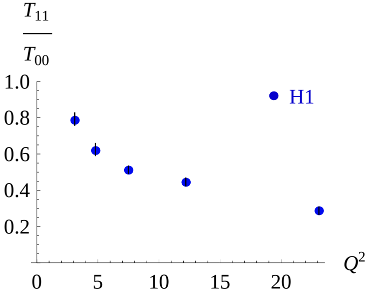
$$T_{00} \iff \gamma^*_L\ p \to \rho_L\ p$$
$$T_{11} \iff \gamma^*_T\ p \to \rho_T\ p$$

- Perturbative Regge Limit :
  - Regge Limit : $s = W^2 >> Q^2, |t|\, , M^2_{\text{hadron}}$
  - Hard scale : $Q >> \Lambda_{QCD}$

Introduction

Experimental data of helicity amplitudes at high energy

- Helicity amplitudes $T_{\lambda_\rho \lambda_\gamma}$: $\gamma^*_{\lambda_\gamma} + p \to \rho_{\lambda_\rho} + p$
- H1 and ZEUS data for Helicity Amplitudes at HERA:



Power counting:

$$\frac{T_{11}}{T_{00}} \propto \frac{1}{Q}$$

$$\frac{T_{01}}{T_{00}} \propto \frac{\sqrt{|t|}}{Q}$$

$$\frac{T_{10}}{T_{00}} \propto \frac{\sqrt{|t|}}{Q^2}$$

S. Chekanov et al. (2007), F.D Aaron et al. (2010)

- Kinematics
  - High energy in the center of mass $30\,\text{GeV} < W < 180\,\text{GeV}$
  - Photon Virtuality $2.5\,\text{GeV}^2 < Q^2 < 60\,\text{GeV}^2$
  - $|t| < 1\,\text{GeV}^2$

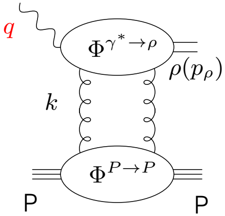
$$\Rightarrow s_{\gamma^* p} = W^2 >> Q^2 >> \Lambda^2_{QCD}$$

Introduction

A Theoretical approach within k_T factorisation

k_T factorisation

- Amplitudes with gluons exchange in $t-$channel dominate at large s ($s = W^2$)



Born order: 2 t-channel gluons

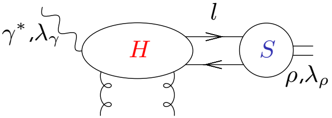
- $T_{\lambda_\rho \lambda_\gamma} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \to \rho(\lambda_\rho)}(\underline{k}) \; \Phi^{P \to P}(-\underline{k})$

Introduction

A theoretical approach of the $\Phi^{\gamma^* \to \rho}$ impact factor up to twist 3

Impact factors $\Phi^{\gamma^* \to \rho}$ and $\Phi^{P \to P}$

$Q^2 >> \Lambda^2_{QCD}$



- Twist t : Impact factor behaves as $1/Q^{t-1}$
- $T_{00} \equiv \gamma^*_L \to \rho_L$ impact factor : Dominant term at twist 2 $\equiv 1/Q$
- $T_{11} \equiv \gamma^*_T \to \rho_T$ impact factor : Dominant term at twist 3 $\equiv 1/Q^2$

Recently computed at $t = t_{min} \approx 0$

Nucl. Phys. B **828** (2010) 1-68. Anikin, Ivanov, Pire, Szymanowski, Wallon

Introduction

Construction of phenomenological models

$$T_{\lambda_\rho \lambda_\gamma} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \to \rho(\lambda_\rho)}(\underline{k}) \; \Phi^{P \to P}(-\underline{k})$$

- Phenomenological model to compare to H1 and ZEUS data (PhysRevD.84.054004 in collaboration with I. V. Anikin, D .Yu. Ivanov, B. Pire, L. Szymanowski, S. Wallon):

$$\Phi^{P \to P}(\underline{k}; M^2) \propto \left[\frac{1}{M^2} - \frac{1}{M^2 + \underline{k}^2} \right]$$ J.F Gunion, D.E Soper

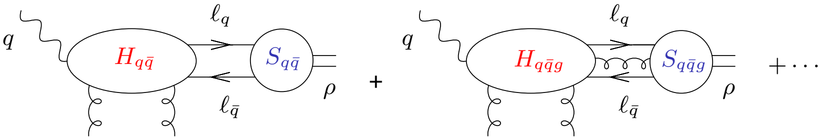
- Link $\Phi^{\gamma^*(\lambda_\gamma) \to \rho(\lambda_\rho)}$ with the dipole picture.
  $\Rightarrow$ Phenomenological model from the scattering amplitude of colour dipoles and nucleon.
  (Work in collaboration with L. Szymanowski, S. Wallon)

Collinear factorization
Light-Cone Collinear approach

- The impact factor $\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}$ can be written as

$$\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)} = \int d^4\ell \cdots \text{tr}[H^{(\lambda_\gamma)}(\ell\cdots) \quad S^{(\lambda_\rho)}(\ell\cdots)]$$

hard part   soft part



- At the 2-body level:

$$S_{q\bar{q}}(\ell) = \int d^4z\, e^{-i\ell\cdot z} \langle\rho(p)|\psi(0)\,\bar{\psi}(z)|0\rangle,$$

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

Momentum factorization

- Decomposition of partons momenta ℓ:

$$\ell = y\, p_\rho + \ell^\perp + (\ell \cdot p_\rho)\, n$$

- Taylor Expand $H(\ell)$ around the p direction:

$$H(\ell) = H(yp) + \left.\frac{\partial H(\ell)}{\partial \ell_\alpha}\right|_{\ell = yp} \ell_\alpha^\perp + \dots$$

- $\int dy$ performs the longitudinal momentum factorization

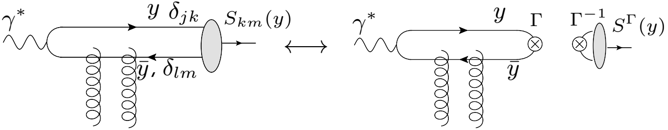
$$\Phi = \int dy\, \{\mathrm{tr}\, [H_{q\bar{q}}(y\, p)\, S_{q\bar{q}}(y)] + \mathrm{tr}\, [\partial_\perp H_{q\bar{q}}(y\, p)\, \partial_\perp S_{q\bar{q}}(y)]\}$$

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

Momentum and spinorial factorization

- Fierz Identity: $\delta_{jk}\delta_{lm} = -\frac{1}{4}(\Gamma)_{jl}(\Gamma^{-1})_{km}$



- Spinor (and color) factorisation:

$$\Phi = \int dy\ \left\{ \text{tr}\left[H_{q\bar{q}}(y\,p)\,\Gamma\right]\ S^{\Gamma}_{q\bar{q}}(y) + \text{tr}\left[\partial_\perp H_{q\bar{q}}(y\,p)\,\Gamma\right]\ \partial_\perp S^{\Gamma}_{q\bar{q}}(y) \right\}$$

$$S^{\Gamma}_{q\bar{q}}(y) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\,\psi(0)|0\rangle$$

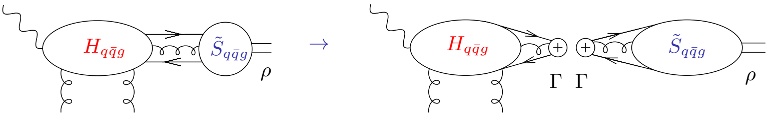
$$\partial_\perp S^{\Gamma}_{q\bar{q}}(y) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\, i\ \overleftrightarrow{\partial_\perp}\ \psi(0)|0\rangle$$

- choose axial gauge condition for gluons, *i.e.* $n \cdot A = 0 \Rightarrow$ no Wilson line

Collinear factorization
Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  $\Rightarrow$ no need for Taylor expansion
- Momentum factorization goes in the same way as for 2-body case
- Spinorial (and color) factorization is similar:

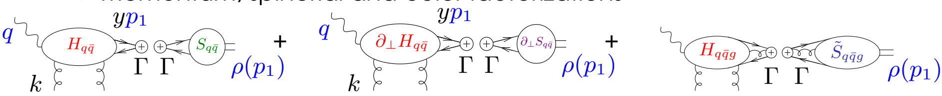


Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements (DAs): 2-body correlators

Collinear factorization of 2-body and 3-body contributions

- Momentum, spinorial and color factorizations



- Parametrization of Soft parts $S_{q\bar{q}}, \partial_\perp S_{q\bar{q}}, S_{q\bar{q}g}$
  - $\Rightarrow$ 5 2-body DAs $\{\varphi_1, \varphi_A, \varphi_3, \varphi_{1T}, \varphi_{AT}\}$
  - $\Rightarrow$ 2 3-body DAs $\{B(y_1, y_2), D(y_1, y_2)\}$
  - Relations between DAs : Equation of motion and n-independence $\Rightarrow$

    3 independent DAs : $\{\varphi_1, B(y_1, y_2), D(y_1, y_2)\}$

Collinear factorization

Wandzura-Wilczek and Genuine contributions

- Solution in the Wandzura-Wilczek Approximation (WW) ≡ Only 2-body contributions

$$\varphi_1 \Rightarrow \{\varphi_3^{WW}(y), \varphi_A^{WW}(y), \varphi_{1T}^{WW}(y), \varphi_{AT}^{WW}(y)\}$$

- Genuine solutions

$$\{B(y_1,y_2), D(y_1,y_2)\} \Rightarrow \{\varphi_3^{gen}(y), \varphi_A^{gen}(y), \varphi_{1T}^{gen}(y), \varphi_{AT}^{gen}(y)\}$$

- Evolution of the DAs   P. Ball, V.M Braun, Y. Koike, K. Tanaka

$$\begin{aligned}
\varphi_1(y,\mu_F^2) &= 6y\bar{y}(1 + a_2(\mu_F^2)\frac{3}{2}(5(y-\bar{y})^2 - 1)) \stackrel{\mu_F^2\to\infty}{\longrightarrow} 6y\bar{y} \\
B(y_1,y_2;\mu_F^2) &= -5040 y_1\bar{y_2}(y_1 - \bar{y_2})(y_2 - y_1) \\
D(y_1,y_2;\mu_F^2) &= -360 y_1\bar{y_2}(y_2 - y_1)(1 + \frac{\omega^A_{\{1,0\}}(\mu_F^2)}{2}(7(y_2 - y_1) - 3))
\end{aligned}$$

with $\mu_F^2 \approx Q^2$ the collinear factorisation scale

Ratios of Helicity Amplitudes

A model for the proton impact factor

- $T_{\lambda_\rho \lambda_\gamma}(Q,M) = is \int \frac{d^2\underline{k}}{(2\pi)^2} \frac{1}{(\underline{k}^2)^2} \Phi^{P\to P}(\underline{k}; M^2) \Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\underline{k}; Q^2)$

- Phenomenological Model for $\Phi^{P\to P}$

$$\Phi^{P\to P}(\underline{k}; M^2) \propto \left[\frac{1}{M^2} - \frac{1}{M^2+\underline{k}^2}\right]$$

J.F Gunion, D.E Soper

- $\gamma^*_L \to \rho_L$ helicity amplitude:

$$T_{00} \propto \frac{is}{(2\pi)} \int_{\lambda^2}^{\infty} d\underline{k}^2 \frac{1}{(\underline{k}^2)^2} \left(\frac{1}{M^2} - \frac{1}{\underline{k}^2+M^2}\right) \frac{1}{Q} \int_0^1 dy\, \varphi_1(y,\mu_F^2) \frac{\underline{k}^2}{\underline{k}^2 + y\bar{y}Q^2}$$

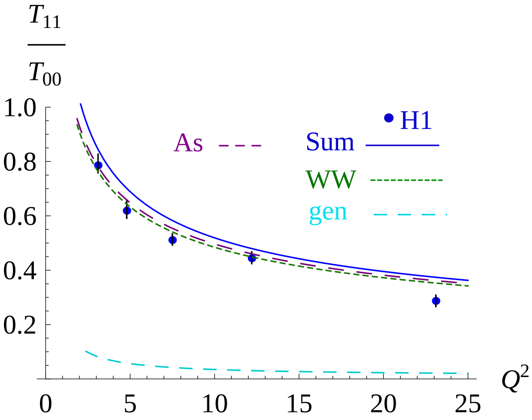
- $\gamma^*_T \to \rho_T$, WW contribution:

$$T_{11}^{WW} \propto \frac{is}{2\pi} \int_{\lambda^2}^{\infty} d(\underline{k}^2) \frac{1}{(\underline{k}^2)^2} \left(\frac{1}{M^2} - \frac{1}{\underline{k}^2+M^2}\right) \frac{(\epsilon_\gamma.\epsilon_\rho^*) m_\rho}{Q^2} \int_0^1 du \frac{\varphi_1(u;\mu_F^2)}{u} \int_0^u dy \frac{\underline{k}^2(\underline{k}^2 + 2y\bar{y}Q^2)}{(\underline{k}^2 + y\bar{y}Q^2)^2}$$

Ratios of Helicity Amplitudes

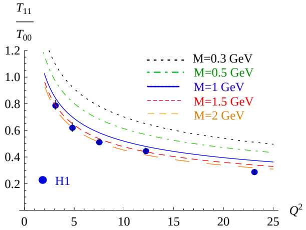
Comparison with H1 data : T_{11}/T_{00}

- Genuine and Wandzura-Wilczek Contributions at $M = 1$ GeV



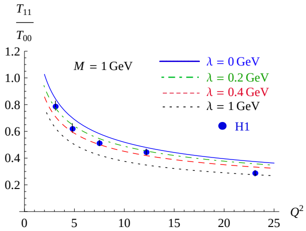
Ratios of Helicity Amplitudes
Dependence on parameters M and λ

- M dependence of the ratio T_{11}/T_{00}



- Result not so sensitive in M arround 0.9-3 GeV
- Result in good agreement with data.

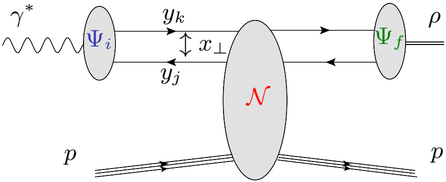
- Soft gluon effect λ IR cut-off on k_T integrals $\int_{\lambda^2} d(\underline{k}^2)$



- $|\underline{k}| < \Lambda_{QCD}$ not important contribution
- $|\underline{k}| < 1$ GeV not negligeable contribution

Dipole Models
Dipole model picture

- Factorization of a high energy scattering amplitude into:

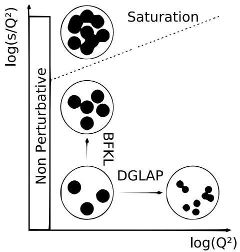


  - Initial Ψ_i and final Ψ_f states wave functions of projectiles
  - Universal scattering amplitude $\mathcal{N} \equiv \mathcal{N}_{\text{dipole-target}}$

- Dipole models are consistent with twist 2 Collinear approximation but are they still consistent with collinear factorization at higher twist order?

Dipole Models
Saturation Effects

- Saturation Effects at high energy:



- Golec-Biernat Wusthoff model:

$$\sigma_{T,L}(x_{Bj}, Q^2) = \int d^2\underline{x} \int_0^1 dy \, |\Psi_{T,L}(y, \underline{x})|^2 \, \hat{\sigma}(x_{Bj}, \underline{x}^2)$$

$$\hat{\sigma}(x_{Bj}, \underline{x}^2) = \sigma_0 \left\{ 1 - \exp\left(-\frac{\underline{x}^2}{4R_0^2(x_{Bj})} \right) \right\}$$

The 2-parton Impact factor

Fourier transform of the $\gamma^* \to \rho$ impact factor

- 2-body impact factor in transverse coordinate space:

$$\Phi^{\gamma^* \to \rho}_{2body} = \int d^4\ell\, Tr(H(\ell)\, S(\ell)) = -\frac{1}{4}\int d^4\ell\, Tr(H\,\Gamma)(\ell)\, S_\Gamma(\ell)$$

$$= -\frac{1}{4}\int dy \int d^2\ell_\perp \int \frac{d^2x_\perp}{2\pi} e^{-i\ell_\perp \cdot x_\perp}\, \tilde{H}^\Gamma(y, x_\perp)$$

$$\int \frac{d^2z_\perp}{2\pi} e^{-i\ell_\perp \cdot z_\perp} \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho | \bar{\psi}(\lambda n + z_\perp)\Gamma\, \psi(0)|0\rangle$$

- Collinear approximation consists in Taylor expanding the transverse fourier transform around $\ell_\perp = 0$:

$$\int \frac{d^2x_\perp}{2\pi} e^{-i\ell_\perp \cdot x_\perp}\, \tilde{H}^\Gamma(y, x_\perp) = \int \frac{d^2x_\perp}{2\pi} (1 - i\ell_\perp \cdot x_\perp + \cdots)\tilde{H}^\Gamma(y, x_\perp)$$

2-parton impact factor
Collinear approximation

Collinear Approximation of the 2-body impact factor up to twist 3

- $e^{-i\ell_\perp \cdot x_\perp} = 1 - i\ell_\perp \cdot x_\perp + \cdots$
- The collinear term "1"(twist 2 and twist 3):

$$\Phi_{2-body}^{\gamma^* \to \rho\,(0)} = -\frac{1}{4}\int dy \int \frac{d^2 x_\perp}{(2\pi)^2}\, 1\, \tilde{H}^{\Gamma,\mu}(y, x_\perp) \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma_\mu^{-1}\,\psi(0)|0\rangle$$

- The first order term "$-i\underline{\ell}\cdot\underline{x}$" (twist 3):

$$\Phi_{2-body}^{\gamma^* \to \rho\,(1)} = -\frac{1}{4}\int dy \int \frac{d^2 x_\perp}{(2\pi)^2}\, x_\perp^\alpha\, \tilde{H}^{\Gamma,\mu}(y, x_\perp) \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p_\rho)|\bar{\psi}(\lambda n)\Gamma_\mu^{-1}\,\overleftrightarrow{\partial_\alpha^\perp}\,\psi(0)|0\rangle$$

2-parton impact factor

Soft part parameterisation

The Soft correlators are parametrized by the DAs, for example $\Gamma \equiv \gamma_\mu$:

$$\int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p) | \bar{\psi}(\lambda n)\, \gamma_\mu\, \psi(0) | 0 \rangle = m_\rho f_\rho (\varphi_1(y) (e_\rho^* \cdot n) p_\mu + \varphi_3(y) e_{\rho\perp\mu}^*)$$

$$\int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p) | \bar{\psi}(\lambda n)\, \gamma_\mu\, \overleftrightarrow{\partial_\alpha^\perp}\, \psi(0) | 0 \rangle = -i m_\rho f_\rho \varphi_{1T}(y) p_\mu e_{\rho\perp\alpha}^*$$

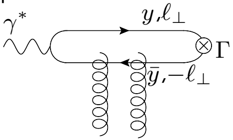
Then the result for the 2-body impact factor up to twist 3

$$\Phi^{\gamma^* \to \rho} = -\frac{1}{4} m_\rho f_\rho \int dy \int \frac{d^2 x_\perp}{(2\pi)} \left\{ \left(\varphi_3(y) e_{\rho\mu}^* + i\varphi_{1T}(y) p_{1\mu} (e_\rho^* \cdot \underline{x})\right) \tilde{H}^{\gamma,\mu}(y, \underline{x}) + \left(i\varphi_A(y) \varepsilon_{\mu e_\rho^* p_1 n} + \varphi_{AT}(y) p_{1\mu} \varepsilon_{x_\perp e_\rho^* p_1 n}\right) \tilde{H}^{\gamma_5 \gamma,\mu}(y, \underline{x}) \right\}$$

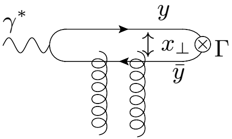
2-parton impact factor

Comutations of the hard part

- Hard parts in transverse coordinate space :

$H^\Gamma(y,\ell_\perp) =$ 

$\longleftrightarrow$

$\tilde{H}^\Gamma(y,x_\perp) =$ 

- For $\Gamma \equiv \gamma^\mu$:

$$\tilde{H}^{\gamma,\mu}(y,\underline{x}) = -4\frac{eg^2}{\sqrt{2}}\frac{\delta^{ab}}{2N_c} y\bar{y} K_0(\mu|\underline{x}|)\, e_\gamma^\mu$$

$$+4i\frac{eg^2}{\sqrt{2}s}\frac{\delta^{ab}}{2N_c}(y-\bar{y})\mu\frac{\underline{e}\cdot\underline{x}}{|\underline{x}|}K_1(\mu|\underline{x}|)\left(\underbrace{(1-e^{i\underline{k}\cdot\underline{x}})(1-e^{-i\underline{k}\cdot\underline{x}})}_{\mathcal{N}(\underline{x},\underline{k})}-1\right)p_2^\mu$$

- For $\Gamma \equiv \gamma^\mu\gamma_5$:

$$\tilde{H}^{\gamma\gamma_5,\mu}(y,\underline{x}) = 4\frac{eg^2}{s\sqrt{2}}\frac{\delta^{ab}}{2N_c}\varepsilon^{\mu\nu\rho\sigma}(e_{\gamma\nu}\frac{x_{\perp\rho}}{|\underline{x}|}p_{2\sigma})\mu K_1(\mu|\underline{x}|)\times(\mathcal{N}(\underline{x},\underline{k})-1)$$

2-parton impact factor

Role of the equation of motion of QCD

- Convolution with soft parts:

$$\Phi^{\gamma^* \to \rho} = -\frac{1}{4} m_\rho f_\rho \int dy \int \frac{d^2 x_\perp}{(2\pi)} \left\{ \left(\varphi_3(y) e^*_{\rho\mu} + i\varphi_{1T}(y) p_{1\mu} (e^*_\rho \cdot \underline{x}) \right) \tilde{H}^{\gamma,\mu}(y,\underline{x}) + \left(i\varphi_A(y) \varepsilon_{\mu e^*_\rho p_1 n} + \varphi_{AT}(y) p_{1\mu} \varepsilon_{x_\perp e^*_\rho p_1 n} \right) \tilde{H}^{\gamma_5 \gamma,\mu}(y,\underline{x}) \right\}$$

- Equations of motion of QCD:

$$\text{Terms} \times \mathcal{N}(\underline{x},\underline{k}) + \text{Terms} \times \underbrace{\left(2y\bar{y}\varphi_3(y) + (y-\bar{y})\varphi_{1T}(y) + \varphi_{AT}(y)\right)}_{=0 \text{ in the WW approximation}}$$

2-parton impact factor
Results in the WW approximation

Twist 3, 2-body impact factors in WW approximation:

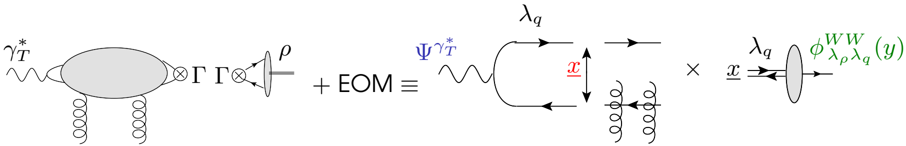
$$\Phi^{\gamma^* \to \rho\, WW}_{2-\mathrm{parton,n.f.}} = \frac{m_\rho f_\rho}{\sqrt{2}} \int dy \int d^2\underline{x}\, g^2\, \delta^{ab}\, \mathcal{N}(\underline{x}, \underline{k}) \sum_{\lambda} \phi^{WW}_{i\lambda}\, \Psi^{\gamma^*_T}_{i\lambda}\,,$$

$$\Phi^{\gamma^* \to \rho\, WW}_{2-parton,f} = \frac{m_\rho f_\rho}{\sqrt{2}} \int dy \int d^2\underline{x}\, g^2\, \delta^{ab}\, \mathcal{N}(\underline{x}, \underline{k}) \sum_{\lambda, i\neq j} \phi^{WW}_{i\lambda}\, \Psi^{\gamma^*_T}_{j\lambda}$$

with the following combinations of WW DAs:

$$\phi^{WW}_{\lambda_\rho \pm} = -i(\underline{e}^{\lambda_\rho *}_{\rho} \cdot \underline{x}) \frac{1}{8N_c} (\varphi^{T\, WW}_{A} \pm \varphi^{T\, WW}_{1})$$

$$\Psi^{\gamma^*_T}_{++}(y, \underline{x}) = i \frac{e}{\pi} y \frac{\underline{e}^{+}_{\gamma} \cdot \underline{x}}{|\underline{x}|} \mu K_1(\mu |\underline{x}|)$$

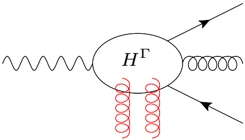


3-parton impact factor
Expression and kinematics

- The 3-body amplitude in transverse coordinate space after collinear approximation

$$\Phi^{\gamma^* \to \rho}_{3-body} = -\frac{i m_\rho f_\rho}{4} \int dy_1 dy_g \int \frac{d^2 x_{1\perp}}{(2\pi)^2} \frac{d^2 x_{g\perp}}{(2\pi)^2}$$
$$\Big(\zeta^V_{3\rho} B(y_1, y_2) p_\mu e_{\rho\perp\alpha} \; \tilde{H}^{\alpha,\gamma^\mu}(y_1, y_g, x_{1\perp}, x_{g\perp})$$
$$+ \zeta^A_{3\rho} i D(y_1, y_2) p_\mu \varepsilon_{\alpha e_{\rho\perp} p n} \; \tilde{H}^{\alpha,\gamma^\mu \gamma_5}(y_1, y_g, x_{1\perp}, x_{g\perp}) \Big)$$

- Kinematics of Hard parts:



$$\ell_1 = y_1 p_1 + \ell_{1\perp} + \frac{\ell_1^2}{y_1 s} p_2$$
$$\ell_g = y_g p_1 + \ell_{g\perp} + \frac{\ell_g^2}{y_g s} p_2$$
$$\ell_2 = \bar{y}_2 p_1 + \ell_{2\perp} + \frac{\ell_2^2}{\bar{y}_2 s} p_2$$

partons are on-shell

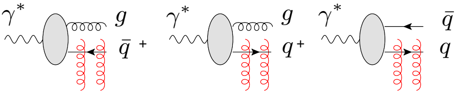
3-parton impact factor
Colour dipole configurations

- Analogy:(as a guideline) Partons kinematics in transverse space $\leftrightarrow$ nonrelativistic 2D mechanics of an equivalent system:

Partons	Masses	Momenta
quark	$m_q \propto y_1 Q$	$\underline{\ell}_1$
anti-quark	$m_{\bar{q}} \propto \bar{y}_2 Q$	$\underline{\ell}_2$
gluon	$m_g \propto y_g Q$	$\underline{\ell}_g$

- The t-channel gluons interact with three 2-body systems :

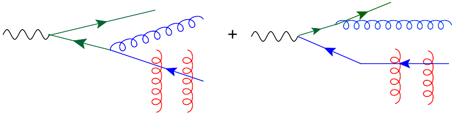
$\{\bar{q}g\}$, $\{qg\}$ and $\{q\bar{q}\}$



3-parton impact factor
Dynamics of the dipoles

Example: $\{\bar{q}g\}$ system

- Diagrams associated:



- Momentum of the center of mass G of $\{\bar{q}g\}$: $\underline{\ell}_G = \underline{\ell}_2 + \underline{\ell}_g$
- Momentum of the reduced particle:

$$m_{\{\bar{q}g\}} = \frac{m_{\bar{q}} m_g}{m_{\bar{q}} + m_g}$$

$$\underline{\ell}_{\bar{q}g} = \frac{m_{\{\bar{q}g\}}}{m_{\bar{q}}} \underline{\ell}_2 - \frac{m_{\{\bar{q}g\}}}{m_g} \underline{\ell}_g$$

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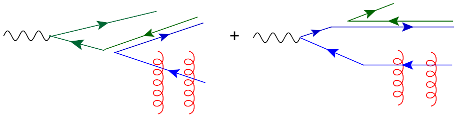

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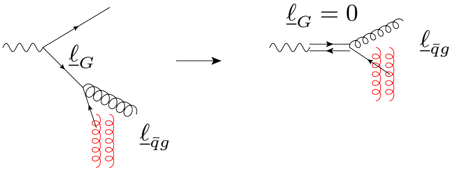
3-parton impact factor

Results form of the 3-parton impact factor

- Collinear approximation and Fourier transforms :

$$\Phi^{\gamma^*\to\rho}_{\bar{q}g} \propto \int dy_1 dy_g \int d^2\underline{x} d^2\underline{x}_G \tilde{H}(\underline{x},\underline{x}_G;\underline{k}) = \int dy_1 dy_g \int d^2\underline{x}\ \tilde{H}'(\underline{x},\underline{\ell}_G = 0;\underline{k})$$



- 3-partons results:

$$\Phi^{\gamma^*\to\rho,nf}_{3-parton} \propto \int d^2\underline{x} S(y_1,y_2)\mu^2 K_0(\mu\,|\underline{x}|) \times \mathcal{N}(\underline{x},\underline{k}) + \int dy_1\, dy_2\, \frac{2S(y_1,y_2)}{\bar{y}_1}$$

$$\Phi^{\gamma^*\to\rho,f}_{3-parton} \propto \int d^2\underline{x} S(y_1,y_2)\mu^2 K_2(\mu\,|\underline{x}|) \times \mathcal{N}(\underline{x},\underline{k})$$

$$S(y_1,y_2) = \zeta^V_\rho(\mu^2) B(y_1,y_2;\mu^2) + \zeta^A_\rho(\mu^2) D(y_1,y_2;\mu^2)$$

Impact factor
2- and 3-parton contributions

- Impact factor 2-parton+3-parton:

$$\Phi^{\gamma^* \to \rho} \propto \int dy_i \int d^2\underline{x}\, \mathcal{N}(\underline{x}, \underline{k})\, \tilde{f}(y_i, \underline{x})$$
$$+ \underbrace{\int \frac{dy}{y\bar{y}} \left(2y\bar{y}\varphi_3(y) + (y-\bar{y})\varphi_1^T(y) + \varphi_A^T(y)\right) + \int dy_1\, dy_2\, \frac{2S(y_1, y_2)}{\bar{y}_1}}_{=0 \text{ due to EOM of QCD}}$$

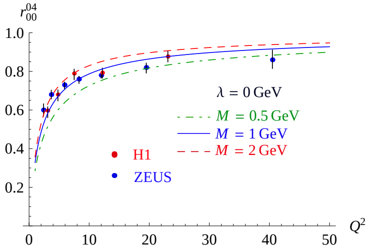
- Final result consistent with dipole picture

$$\Phi^{\gamma^* \to \rho} \propto \int dy_i \int d^2\underline{x}\, \mathcal{N}(\underline{x}, \underline{k})\, \tilde{f}(y_i, \underline{x})$$

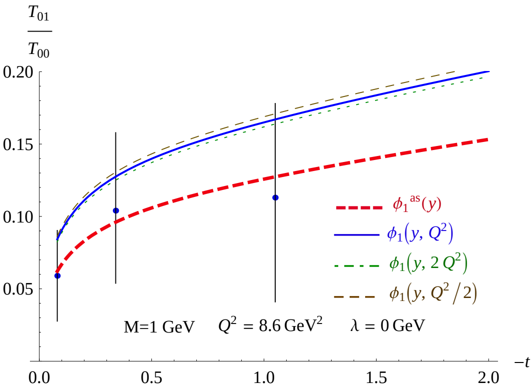
Conclusion

- The dipole/$t-$channel gluons scattering amplitude appears in the factorised form up to twist 3.
- The wave function of the photon factorised out in the WW approximation.
- We have now the starting point for including Saturation effects in the diffractive meson production up to twist 3 accuracy.

Comparison of r_{00}^{04} in the S-Channel Helicity Conservation approximation with ZEUS and H1 data:



- T_{01}/T_{00} at $M = 1$ GeV, Dependence on μ^2 at $M = 1$ GeV :



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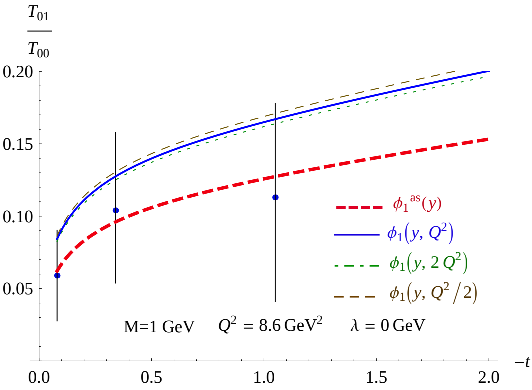
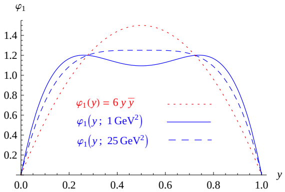


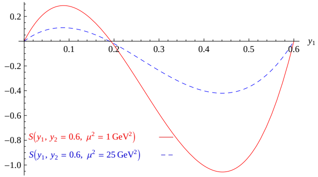
Collinear factorisation
DAs dependence on μ^2

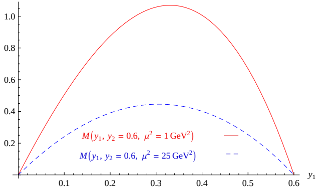
- $\varphi_1(y, \mu^2)$



- $M(y_1, y_2) = \zeta_\rho^V(\mu^2) B(y_1, y_2; \mu^2) - \zeta_\rho^A(\mu^2) D(y_1, y_2; \mu^2) \xrightarrow{\mu^2 \to \infty} 0$

  $S(y_1, y_2) = \zeta_\rho^V(\mu^2) B(y_1, y_2; \mu^2) + \zeta_\rho^A(\mu^2) D(y_1, y_2; \mu^2) \xrightarrow{\mu^2 \to \infty} 0$





- Colour dipole model and saturation effects:

  - Fourier transform $\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\underline{k})$:

$$\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\underline{k}) = \int dy \int d^2\underline{x}\, \Psi_\rho^{*\lambda_\rho}(y,\underline{x})\, \mathcal{N}(\underline{x},\underline{k})\, \Psi_{\gamma^*}^{\lambda_\gamma}(y,\underline{x})$$

$\mathcal{N}(\underline{x},\underline{k})$: interaction of t-channel gluon with color dipole

  - Phenomenological model:

$$T_{\lambda_\rho\lambda_\gamma} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2(\underline{k}^2)^2}\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\underline{k})\; \Phi^{P\to P}(-\underline{k})$$

$$= is \int dy \int d^2\underline{x}\, \Psi_\rho^{*\lambda_\rho}(y,\underline{x})\, \Psi_{\gamma^*}^{\lambda_\gamma}(y,\underline{x}) \int \frac{d^2\,\underline{k}}{(2\pi)^2(\underline{k}^2)^2}\mathcal{N}(\underline{x},\underline{k})\; \Phi^{P\to P}(-\underline{k})$$

$$= is \int dy \int d^2\underline{x}\, \Psi_\rho^{*\lambda_\rho}(y,\underline{x})\, \Psi_{\gamma^*}^{\lambda_\gamma}(y,\underline{x})\mathcal{N}(x_{Bj},\underline{x})$$

- Golec-Biernat Wusthoff model:

$$\hat{\sigma}(x_{Bj},\underline{x}^2) = \sigma_0 \left\{1 - \exp\left(-\frac{\underline{x}^2}{4R_0^2(x_{Bj})}\right)\right\}$$