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Phenomenology of Sivers Effect with TMD Evolution

Mariaelena Boglione

INFN Torino

UNIVERSITÀ DEGLI STUDI DI TORINO
ALMA UNIVERSITAS TAURINENSIS

In collaboration with M. Anselmino and S. Melis

Introduction

- The exploration of the 3-dimensional structure of the nucleon, both in momentum and in configuration space, is one of the major issues in high energy hadron physics.

- Information on the 3-dimensional structure of the nucleon is embedded in the Transverse Momentum Dependent distribution and fragmentation functions (TMDs).

- The Sivers function, which describes the number density of unpolarized quarks inside a transversely polarized proton, is particularly interesting, as it provides information on the partonic orbital angular momentum.

Beautifully highlighted by W. Vogelsang, P. Mulders, F. Yuan and A. Bacchetta in their talks

- So far, all phenomenological fits have either neglected the QCD scale dependence of TMDs (which was unknown) or limited it to the collinear part of the unpolarized PDFs, according to the DGLAP scheme.

- Here I will present the first extraction of the Sivers function which takes into account the TMD evolution scheme proposed by Aybat, Collins, Qiu and Rogers and show how the new results compare with the previous extractions.

TMD Evolution Formalism

Let $\tilde{F}$ be either an unpolarized distribution or fragmentation function, or the first derivative of the Sivers distribution function, in the **impact parameter space**.
In general terms, its TMD evolution equation can be written as

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Aybat, Collins, Qiu, Rogers

J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.

S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]

S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]

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Aybat, Collins, Qiu, Rogers

$$\widetilde{R}(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_K(\mu) = \alpha_s(\mu)\,\frac{2\,C_F}{\pi}$$

$$\gamma_F(\mu;\frac{Q^2}{\mu^2}) = \alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2} - \ln\frac{Q^2}{\mu^2}\right)$$

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Input function

Aybat, Collins, Qiu, Rogers

Unknown, but universal and scale Independent, input function

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{\ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{d\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_K(\mu) = \alpha_s(\mu) \frac{2\, C_F}{\pi}$$

$$\gamma_F(\mu; \frac{Q^2}{\mu^2}) = \alpha_s(\mu) \frac{C_F}{\pi} \left(\frac{3}{2} - \ln \frac{Q^2}{\mu^2}\right)$$

TMD Evolution Formalism

The appropriate Fourier transform allows us to obtain the distribution and fragmentation functions F̃ in the **momentum space**

$$f_{q/p}(x,k_\perp;Q) = \frac{1}{2\pi}\int_0^\infty db_T\, b_T\, J_0(k_\perp b_T)\, \widetilde{f}_{q/p}(x,b_T;Q)\,,$$

$$D_{h/q}(z,p_\perp;Q) = \frac{1}{2\pi}\int_0^\infty db_T\, b_T\, J_0(k_T b_T)\, \widetilde{D}_{h/q}(z,b_T;Q)\,,$$

$$f_{1T}^{\perp f}(x,k_\perp;Q) = \frac{-1}{2\pi k_\perp}\int_0^\infty db_T\, b_T\, J_1(k_\perp b_T)\, \widetilde{f}_{1T}^{\prime\,\perp q}(x,b_T;Q)\,,$$

$$\begin{aligned} f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{S};Q) &= f_{q/p}(x,k_\perp;Q) - f_{1T}^{\perp q}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{M_p} \\ &= f_{q/p}(x,k_\perp;Q) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{k_\perp} \end{aligned}$$

Parameterization of unknown functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$g_K(b_T) = \frac{1}{2} g_2 b_T^2 \qquad \text{with} \qquad g_2 = 0.68$$

$$\widetilde{f}_{q/p}(x, b_T; Q_0) = f_{q/p}(x, Q_0) \exp\left\{ -\alpha^2 b_T^2 \right\}$$

$$f_{q/p}(x, k_\perp; Q_0) = f_{q/p}(x, Q_0) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

$$\alpha^2 = \langle k_\perp^2 \rangle / 4$$

Parameterization of unknown functions

Then the evolution equations for unpolarized TMDs become simply:

$$\widetilde{f}_{q/p}(x, b_T; Q) = f_{q/p}(x, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-b_T^2\left(\alpha^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\},$$

$$\widetilde{D}_{h/q}(z, b_T; Q) = \frac{1}{z^2} D_{h/q}(z, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-b_T^2\left(\beta^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

While for the Sivers function we have

$$\widetilde{f}'^{\perp}_{1T}(x, b_T; Q) = -2\,\gamma^2\, f^{\perp}_{1T}(x; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, b_T\, \exp\left\{-b_T^2\left(\gamma^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

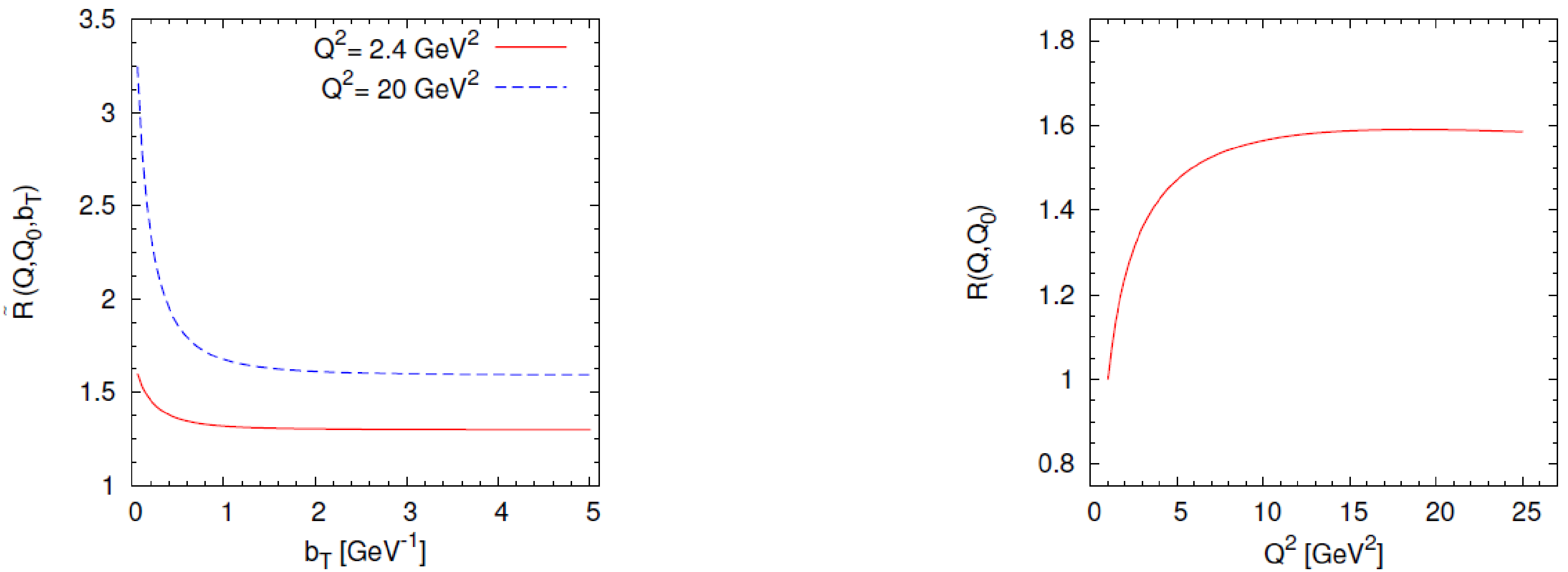
$$4\,\gamma^2 \equiv \langle k_\perp^2 \rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2 + \langle k_\perp^2\rangle}$$

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right)\right\}$$

Analytical solution of the TMD evolution equations

$R(Q,Q_0,b_T)$ shows a weak dependence on (small) b_T, which correspond to large $k_\perp$;

$R(Q,Q_0,b_T)$ becomes constant for $b_T > 1$ GeV $^{-1}$



We can therefore neglect the $\tilde{R}$ dependence on b_T and define

$$R(Q,Q_0) \equiv \widetilde{R}(Q,Q_0,b_T \to \infty)$$

Analytical solution of the TMD evolution equations

Finally, for the unpolarized TMD parton distribution function, we have

$$\widetilde{f}_{q/p}(x, \boldsymbol{b}_T; Q) = f_{q/p}(x, Q_0)\ R(Q, Q_0)\ \exp\left\{ -b_T^2 \left(\alpha^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right) \right\}$$

Which is Gaussian in b_T, and will then Fourier-transform into a Gaussian in $k_\perp$

$$f_{q/p}(x, k_\perp, Q) = f_{q/p}(x, Q_0)\ R(Q, Q_0)\ \frac{e^{-k_\perp^2/w^2}}{\pi w^2}$$

$$w^2(Q, Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}$$

Analytical solution of the TMD evolution equations

For the unpolarized TMD parton fragmentation function, we have

$$D_{h/q}(z,p_\perp,Q) = D_{h/q}(z,Q_0)\; R(Q,Q_0)\; \frac{e^{-p_\perp^2/w_F^2}}{\pi w_F^2}$$

$$w_F^2 \equiv w_F^2(Q,Q_0) = \langle p_\perp^2 \rangle + 2z^2 g_2 \ln \frac{Q}{Q_0}$$

Analytical solution of the TMD evolution equations

For the Sivers distribution function, we find

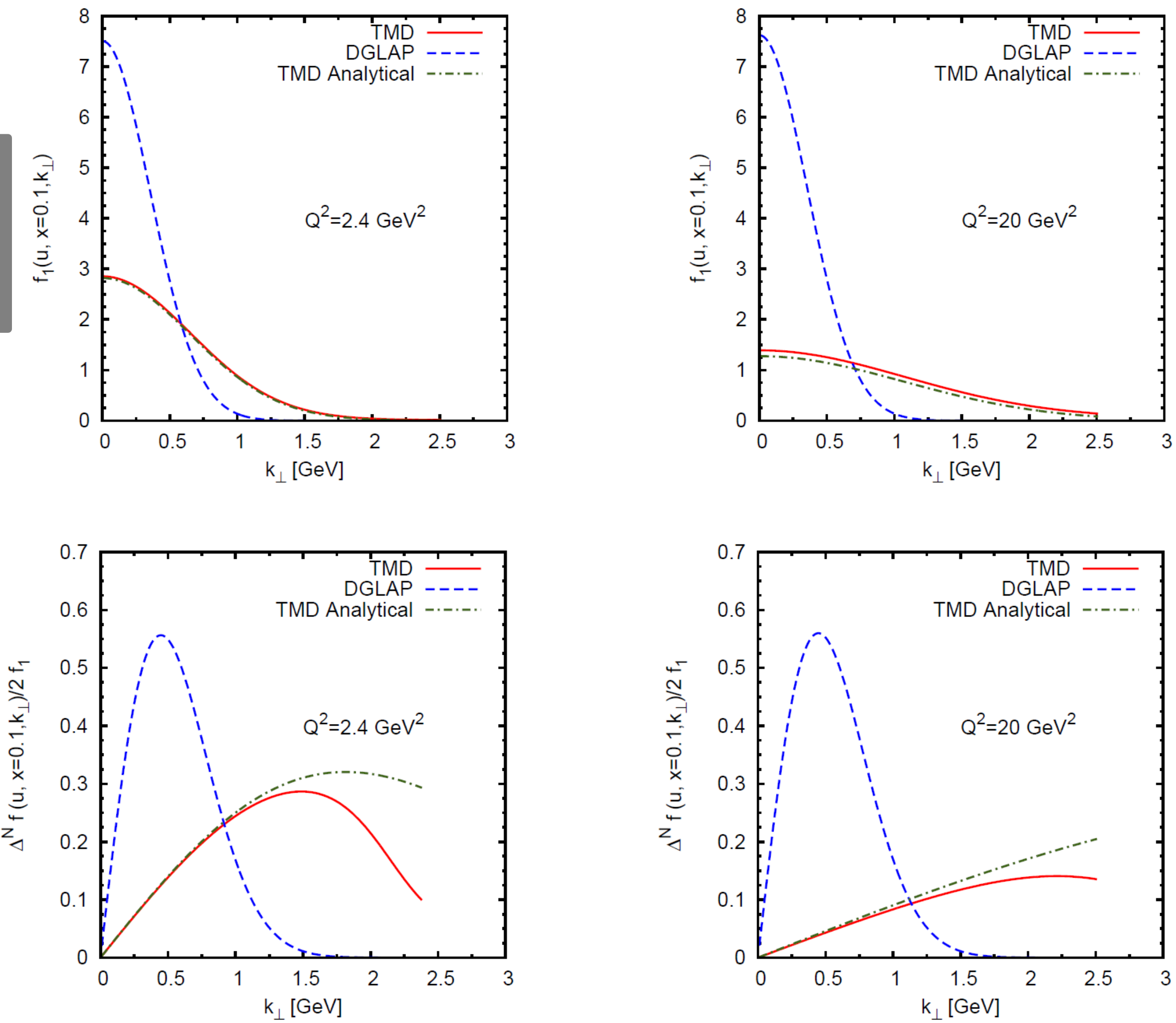
$$\Delta^N f_{q/p^\uparrow}(x,k_\perp;Q) = \frac{k_\perp}{M_1}\sqrt{2e}\,\frac{\langle k_\perp^2\rangle_S^2}{\langle k_\perp^2\rangle}\,\Delta^N f_{q/p^\uparrow}(x,Q_0)\,R(Q,Q_0)\,\frac{e^{-k_\perp^2/w_S^2}}{\pi w_S^4}$$

$$w_S^2(Q,Q_0) = \langle k_\perp^2\rangle_S + 2g_2 \ln\frac{Q}{Q_0}$$

Comparative analysis of TMD evolution equations

DGLAP evolution is extremely slow in this Q^2 range

TMD evolution Very rapidly widens and dilutes the functions



For the unpol. function, the analytical approx. holds until large $k_\perp$

For the Sivers function, the analytical approx. breaks down at large $k_\perp$

The Sivers function from HERMES and COMPASS SIDIS data

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S \, d\phi_h \, d^2k_\perp \, \Delta^N f_{q/p^\uparrow}(x,k_\perp,Q) \sin(\varphi-\phi_S) \, \frac{d\hat\sigma^{\ell q\to \ell q}}{dQ^2} \, D_q^h(z,p_\perp,Q) \sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S \, d\phi_h \, d^2k_\perp \, f_{q/p}(x,k_\perp,Q) \, \frac{d\hat\sigma^{\ell q\to \ell q}}{dQ^2} \, D_q^h(z,p_\perp,Q)}$$

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp,Q_0) = \Delta^N f_{q/p^\uparrow}(x,Q_0)\, h(k_\perp) = 2\mathcal{N}_q(x)\, h(k_\perp)\, f_{q/p}(x,k_\perp,Q_0)$$

$$\mathcal{N}_q(x) = N_q \, x^{\alpha_q}(1-x)^{\beta_q} \, \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}} \, ,$$

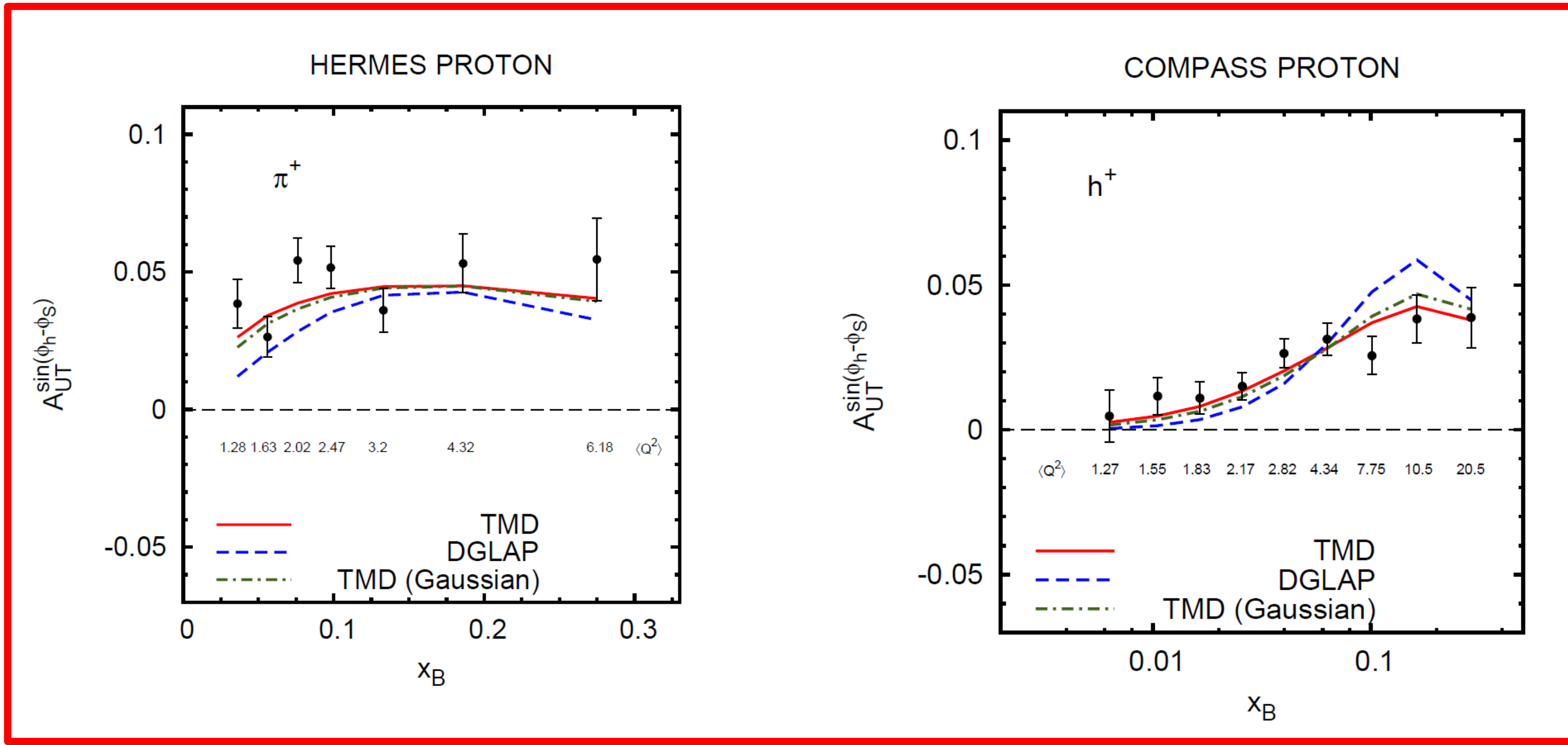
$$h(k_\perp) = \sqrt{2e}\, \frac{k_\perp}{M_1}\, e^{-k_\perp^2/M_1^2} \, ,$$

N_{u_v}	N_{d_v}	N_s
$N_{\bar u}$	$N_{\bar d}$	$N_{\bar s}$
α_{u_v}	α_{d_v}	α_{sea}
β	M_1 (GeV/c) .	

The Sivers function from HERMES and COMPASS SIDIS data

We perform 3 different fits:

- TMD-fit (computing TMD evolution equations numerically)
- TMD-analytical fit (solving TMD evolution equations in the analytical approx.)
- DGLAP fit (using DGLAP evolution equation for the collinear part of the TMD)



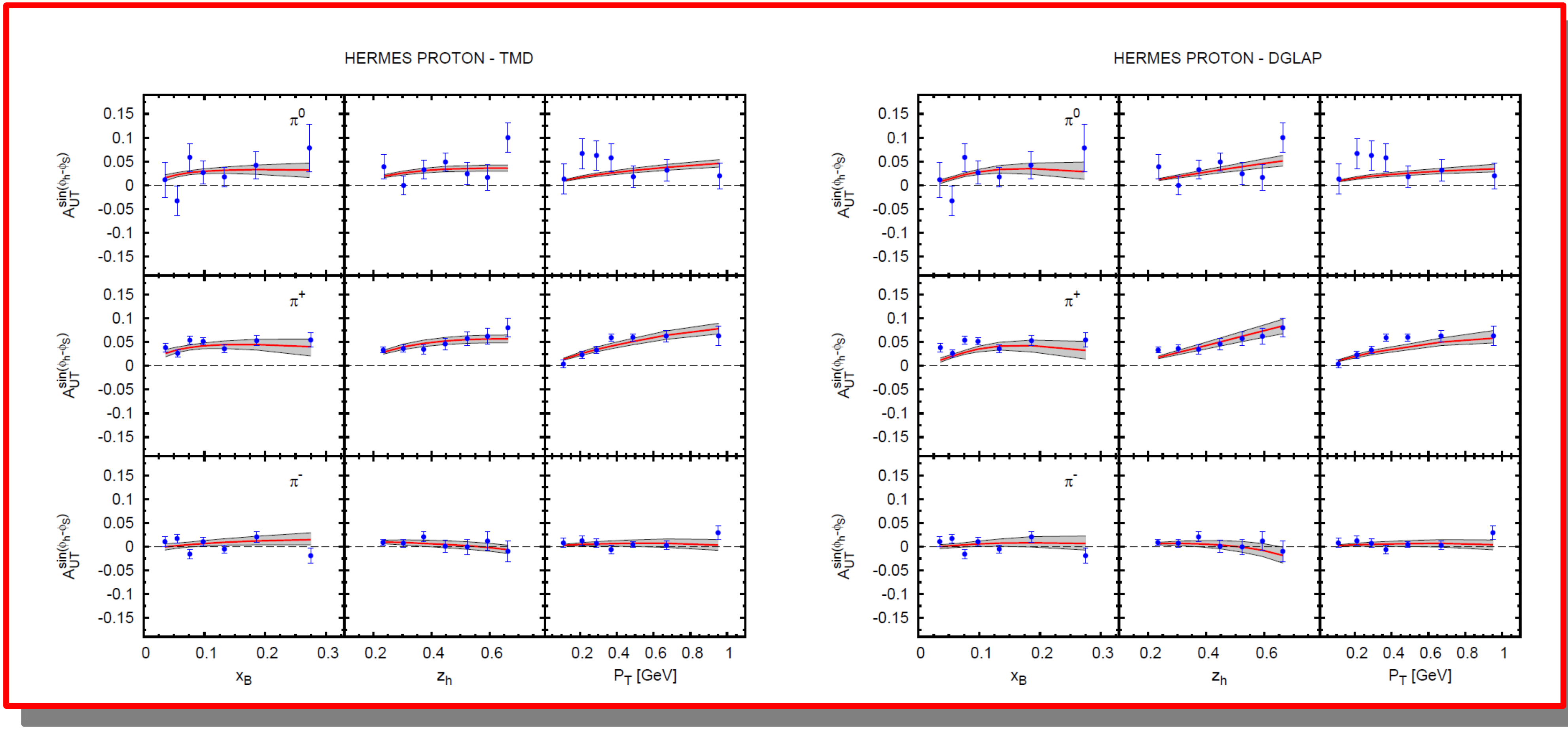
The Sivers function from HERMES and COMPASS SIDIS data

χ^2 tables

	TMD Evolution (Exact)	TMD Evolution (Analytical)	DGLAP Evolution
	$\chi^2_{tot} = 255.8$ $\chi^2_{d.o.f} = 1.02$	$\chi^2_{tot} = 275.7$ $\chi^2_{d.o.f} = 1.10$	$\chi^2_{tot} = 315.6$ $\chi^2_{d.o.f} = 1.26$
HERMES π^+	$\chi^2_x = 10.7$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 9.1$	$\chi^2_x = 12.9$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 10.5$	$\chi^2_x = 27.5$ $\chi^2_z = 8.6$ $\chi^2_{P_T} = 22.5$
COMPASS h^+	$\chi^2_x = 6.7$ $\chi^2_z = 17.8$ $\chi^2_{P_T} = 12.4$	$\chi^2_x = 11.2$ $\chi^2_z = 18.5$ $\chi^2_{P_T} = 24.2$	$\chi^2_x = 29.2$ $\chi^2_z = 16.6$ $\chi^2_{P_T} = 11.8$

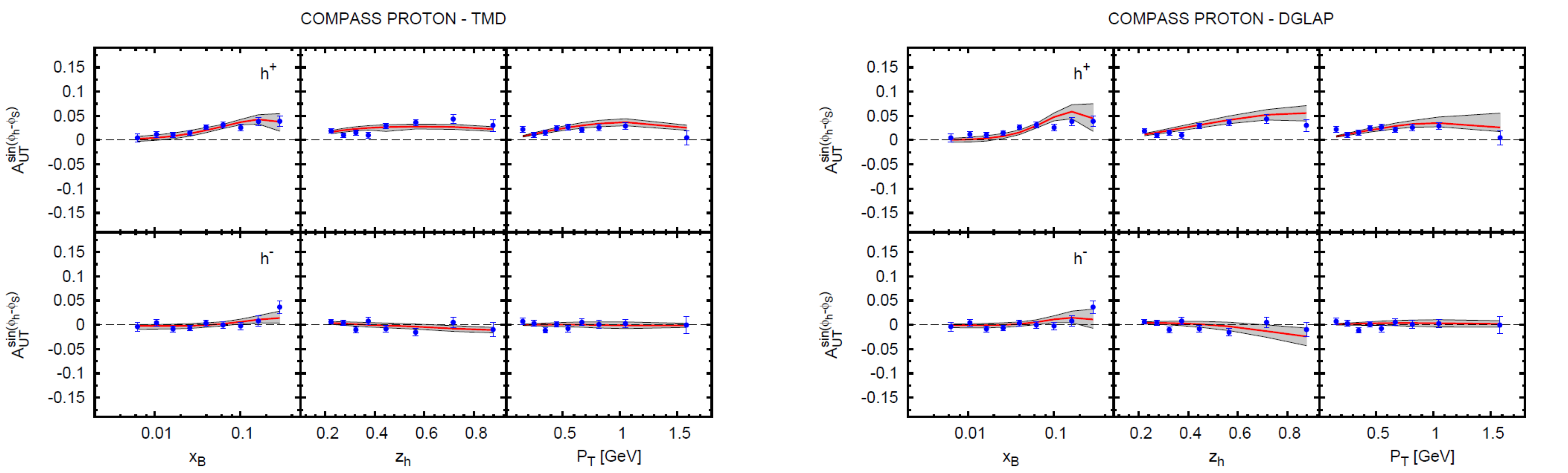
The Sivers function from HERMES and COMPASS SIDIS data

A. Airapetian et al., Phys. Rev. Lett. 103, 152002 (2009), arXiv:0906.3918 [hep-ex]



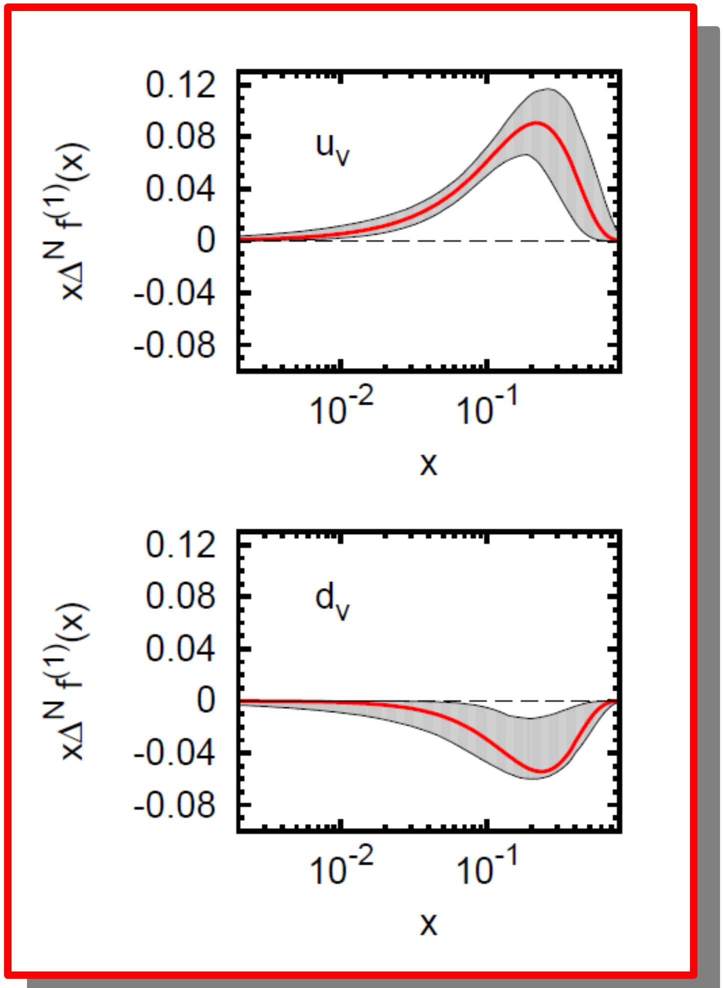
The Sivers function from HERMES and COMPASS SIDIS data

F. Bradamante, arXiv:1111.0869 [hep-ex]



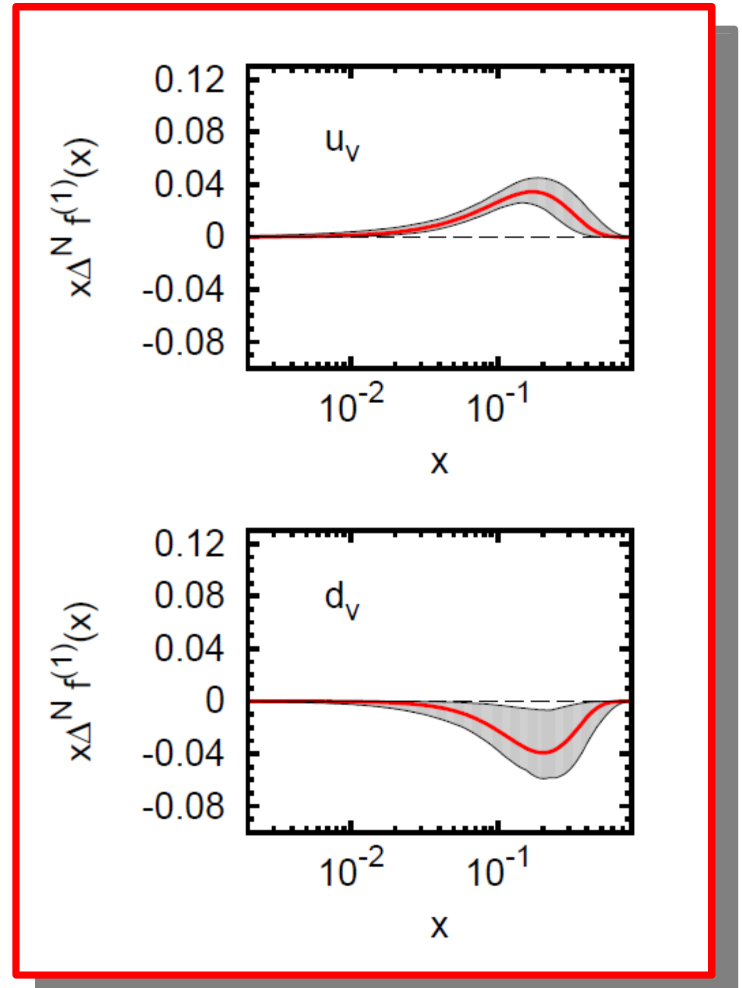
The Sivers function from HERMES and COMPASS SIDIS data

TMD Evolution



Q_0=1 GeV

DGLAP Evolution



Conclusions and further remarks

- We have analyzed the Sivers effect by up-grading old fits with the addiction of the most recent HERMES and COMPASS SIDIS data.

- For the first time, we have applied **TMD evolution equations** to the phenomenological analysis of the Sivers effect.

- We have **compared** the analysis obtained using TMD evolution equations with the results found by considering the DGLAP evolution of the collinear part of the TMDs.

- **SIDIS data support the TMD evolution**, but further experimental data covering a **wider range of Q^2 values** are necessary to confirm these results.