Latest theoretical developments in top physics at hadron colliders

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Based on works with:
Michal Czakon '11 and (in progress)
Cacciari, Czakon, Mangano and Nason '11
George Sterman (in progress)
Plan for the talk

① The focus is on the total inclusive cross-section. Will comment on differential too.

① What is well established

① Open questions

① Recent results

① Brand new results (game changer; ~25-years in the waiting)
What is well established

Top-pair production is completely known within NLO/NNLL QCD

Main features:

✓ Very large NLO corrections ~50%
✓ Theory uncertainty at ~10%
✓ Important for Higgs and bSM physics
✓ Experimental improvements down to 5%
✓ Current LHC data agrees well with SM theory
✓ Tevatron data generally agrees too.
   The notable exception: Forward-backward asymmetry from CDF.

Important shift in gears: currently we have fully differential NLO calculations including top decays. This is the direction of the future.
Open questions

✓ Reliably increase the precision of the theoretical predictions? [more next]

✓ What to make of the forward – backward asymmetry?

✓ Can missing theory corrections be large? Yes! (almost universally assumed small)

✓ Is this enough to explain $A_{FB}$? Perhaps not!

This will only be settled once the SM predictions have been exhausted (currently $A_{FB}(tT+X)$ is at LO!)
Recent results

✓ How to improve theory?

All improvements in the last 15-20 years are based on soft gluon resummation.

✓ This has recently been extended to full NNLL level

Czakon, Mitov, Sterman '09
Beneke, Falgari, Schwinn '09

✓ ... and analyzed by a number of groups:

Ahrens, Ferroglia, Neubert, Pecjak, Yang
Kidonakis
Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann
Beneke, Falgari, Klein, Schwinn
Cacciari, Czakon, Mangano, Mitov, Nason

✓ Conclusion: resummation is

- very important at LL,
- relatively important at NLL,
- modest improvement at NNLL(+NLO) (compared to other missing contributions)

at NNLL level total uncertainty is reduced: 10% → 9%

Cacciari, Czakon, Mangano, Mitov, Nason '11

✓ Lots of activity recently.

✓ Note of caution: resummation (or its truncation NNLO_approx) are not substitutes for NNLO.
Recent results

Textbook example: by changing the collider energy go into (out of) the threshold region

Cacciari, Czakon, Mangano, Mitov, Nason `11

- Resummed results are better when close to threshold (as expected)
- One can quantify the question: when are we close to threshold? (below 1 TeV or so)
- Approx_NNLO is a subset of the resumed result. Overused recently. Has accidentally small scale dependence.
Recent results

✓ We have also prepared the tools for top physics:

*Top++*: a C++ program for the calculation of the total cross-section:

✓ Includes:

- fixed order (NLO and NNLO_approx at present)
- and resummation (full NNLL already there)

✓ It is meant to incorporate the full NNLO once available *(to appear)*

✓ Very user friendly.

Czakon, Mitov `11
Brand new results
Motivating question: can $A_{FB}$ be generated (or enhanced) by $tT$ final state interactions?

Work with George Sterman, to appear

Prompted, in turn, by older work in QED

- We have devised an all-order proof of the cancellations of such interactions
- The subtle point is: What is the remainder? All depends on observables’ definition.
- For inclusive observables (with conventional factorization) the remainder is small.
- For observables with rapidity gaps: large corrections are possible.

See, for example, Brodsky, Gillespie `68
Brand new results

NNLO result for $qq \rightarrow tT$ at Tevatron and LHC

Work with M. Czakon, P. Barnreuther

✓ First ever hadron collider calculation at NNLO with more than 2 colored partons.
✓ First ever NNLO hadron collider calculation with massive fermions.

Recall: NLO corrections derived almost 25 years ago
Nason, Dawson, Ellis `88 ,89
Beenakker, Kuijf, van Neerven, Smith `89
✓ What is computed?

\[ \sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\text{max}}} L_{ij}(\beta) \hat{\sigma}(\beta) \]

\[ \rho = \frac{4m_t^2}{s} \quad \beta = \sqrt{1 - \rho} \]

✓ The partonic cross-section computed numerically in 80 points. Then fitted.

✓ Many contributing partonic channels:

\[ q\bar{q} \rightarrow t\bar{t} \]
\[ q\bar{q} \rightarrow t\bar{t}g \]
\[ q\bar{q} \rightarrow t\bar{t}gg \]
\[ q\bar{q} \rightarrow t\bar{t}q'\bar{q}' , \quad q \neq q' \]
\[ gg \rightarrow t\bar{t} \]
\[ gg \rightarrow t\bar{t}g \]
\[ gg \rightarrow t\bar{t}gg \]
\[ gg \rightarrow t\bar{t}q\bar{q} \]
\[ qg \rightarrow t\bar{t}q \]
\[ qg \rightarrow t\bar{t}qq \]
\[ qg \rightarrow t\bar{t}qq' , \quad q \neq \bar{q}' \]
\[ q\bar{q} \rightarrow t\bar{t}q\bar{q} \]

Computed. Dominant at Tevatron (~85%)

All of the same complexity. No more conceptual challenges expected (just lots of CPU)
Results: partonic cross-section through NNLO

\[
\hat{\sigma}_{qq\rightarrow t\bar{t}}(\beta) = \frac{\alpha_S^4(m_t)}{m_t^2} \left\{ \text{LO} + \text{NLO} + \text{NNLO} \right\}
\]

Notable features:

- Small numerical errors
- Agrees with limits

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\[
\beta = \sqrt{1 - \rho} \\
\rho = \frac{4m_t^2}{s}
\]
What happens once we add the flux?

- The difference increases!

\[ \sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\text{max}}} L_{ij}(\beta) \hat{s}(\beta) \]

Approx\_NNLO - blue
Exact NNLO – red
Here are the numbers for the Tevatron:

**NNLO:**

\[ \sigma_{\text{tot}} = 7.004 + 2.9\% - 4.4\% \text{ [pb].} \]

**NNLO+NNLL:**

\[ \sigma_{\text{tot}} = 7.048 + 1.9\% - 3.2\% \text{ [pb].} \]

- Very weak dependence on unknown parameters (sub 1%): gg NNLO, A, etc.
- 50% scales reduction compared to the NLO+NNLL analysis of Cacciari, Czakon, Mangano, Mitov, Nason ‘11

\[ 6.72 + 3.6\% - 6.1\% \]
Summary and Conclusions

- Long (~15 years) and turbulent chapter in top physics is closing
  - It saw uses of soft gluon resummation to a number of approximations at NNLO
  - It was theoretically very fruitful: engine for theoretical developments

- We have derived the full NNLO result $qq \rightarrow tt$ (numeric – very good precision)
  - at Tevatron it cuts scale uncertainty in half compared to NLO+NNLL
  - Words like approx_NNLO, etc., belong in the past.

- Methods are very general and applicable to differential distributions

- Applications for dijets and $W+$jet, $H+$jet, etc @ NNLO

- Only restriction – availability of two-loop amplitudes and computing speed

- We are on the verge of the NNLO revolution (NLO wish-list already exhausted 😊)
Backup slides
What’s needed for NNLO?

There are 3 principle contributions:

- ✔ 2-loop virtual corrections (V-V)
- ✔ 1-loop virtual with one extra parton (R-V)
- ✔ 2 extra emitted partons at tree level (R-R)

And 2 secondary contributions:

- ✔ Collinear subtraction for the initial state
  Known, in principle. Done numerically.
- ✔ One-loop squared amplitudes (analytic)
  Korner, Merebashvili, Rogal `07
What’s needed for NNLO? V-V

Required are the two loop amplitudes: \(qq \rightarrow QQ\) and \(gg \rightarrow QQ\).

- Their high energy limits and their poles are known analytically
  
  Czakon, Mitov, Moch ’07
  Czakon, Mitov, Sterman ’09
  Ferroglia, Neubert, Pecjak, Yang ’09

- Directly used here: The \(qq \rightarrow QQ\) amplitude is known numerically
  
  Czakon ‘07

- Numerical work underway for the \(gg \rightarrow QQ\)
  
  Czakon, Bärnreuther, to appear

What’s the future here?

- Right now this is the biggest (and perhaps only) obstacle for NNLO phenomenology on a mass scale
A wonderful result By M. Czakon

The method is general (also to other processes, differential kinematics, etc).

Explicit contribution to the total cross-section given.

Just been verified in an extremely non-trivial problem.
What’s needed for NNLO? R-V

✔ Counterterms all known (i.e. all singular limits)

The finite piece of the one loop amplitude computed with a private code of Stefan Dittmaier.

Extremely fast code!

A great help!

Many thanks!

Bern, Del Duca, Kilgore, Schmidt '98-99
Catani, Grazzini ’00
Bierenbaum, Czakon, Mitov ’11