Theory overview of TMDs

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Abstract

Transverse momentum dependent (TMD) distribution and fragmentation functions are described as Fourier transforms of matrix elements containing non-local combinations of quark and gluon fields. While the collinear functions are light-cone correlators in which the non-locality is restricted along the light-cone, the transverse momentum dependent functions are light-front correlators including a transverse (space-like) separation away from the light-cone. In the matrix elements the time-ordering is superfluous and they are parts of the full (squared) amplitudes that account for the connections to the hadrons (soft parts).

The collinear (x-dependent) parton (quark or gluon) distribution functions (PDF’s) that appear in the parameterization of collinear leading-twist correlators are interpreted as momentum densities including polarized parton densities in polarized hadrons. They involve only spin-spin densities and they do not allow for a description of single-spin asymmetries in high-energy scattering processes at leading 1/Q order in the hard scale Q.

TMD (x and $p_T$-dependent) PDF’s that appear in the parameterization of TMD correlators include spin-spin as well as momentum-spin correlations and they are able to describe single-spin and azimuthal asymmetries, such as Sivers and Collins effects in semi-inclusive deep inelastic scattering (SIDIS), but there are many open issues on $p_T$-factorization. Upon taking moments in $p_T$ (or taking Bessel weights) the correlators involve higher-twist operators, but evaluated at zero-momentum (gluonic pole matrix elements). They can be incorporated in a ‘generalized’ factorization scheme with specific gluonic pole factors such as the sign in SIDIS versus Drell-Yan, which can be traced back to having TMD’s with non-trivial process-dependent past- or future-pointing gauge links appearing in the light-front separated, non-local operator combinations.

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**Hadron correlators**

- Hadronic correlators establish the diagrammatic link between hadrons and partonic hard scattering amplitude.
- Quark, quark + gluon, gluon, ...

\[
\langle 0 | \psi_i (\xi) | p, s \rangle = u_i (p, s) e^{-ip.\xi}
\]

\[
\langle X | \psi_i (\xi) | P \rangle = e^{+ip.\xi}
\]

- Disentangling a hard process into parts involving hadrons, hard scattering amplitude and soft part is non-trivial.

\[
\langle X | \psi_i (\xi) A^\mu (\eta) | P \rangle e^{i(p-p_1).\xi + ip_1.\eta}
\]

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J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011
**Hadron correlators**

- Basically at high energies soft parts are combined into forward matrix elements of parton fields to account for distributions and fragmentation

\[
\Phi_{ij}(p; P) = \Phi_{ij}(p | p) = \int \frac{d^4 \xi}{(2\pi)^4} e^{ip.\xi} \langle P | \bar{\psi}_j(0) \psi_i(\xi) | P \rangle
\]

- Also needed are multi-parton correlators

\[
\Phi^\alpha_{A:ij}(p - p_1, p_1 | p) = \int \frac{d^4 \xi}{(2\pi)^4} e^{i(p-p_1).\xi + ip_1.\eta} \langle P | \bar{\psi}_j(0) A^\alpha(\eta) \psi_i(\xi) | P \rangle
\]

- Correlators usually just will be parametrized (nonperturbative physics)
In high-energy processes other momenta available, such that $P.P' \sim s$ with a hard scale $s = Q^2 \gg M^2$

Employ light-like vectors $P$ and $n$, such that $P.n = 1$ (e.g. $n = P'/P.P'$) to make a Sudakov expansion of parton momentum

\[
p = xP^\mu + p_T^\mu + \sigma n^\mu \quad \quad x = p^+ = p.n \sim 1
\]

\[
\sim Q \quad \sim M \quad \sim M^2/Q
\]

\[
\sigma = p.P - xM^2 \sim M^2
\]

Enables importance sampling (twist analysis) for integrated correlators,

\[
\Phi(p) = \Phi(x, p_T, p.P) \quad \Rightarrow \quad \Phi(x, p_T) \quad \Rightarrow \quad \Phi(x) \quad \Rightarrow \quad \Phi
\]
(Un)integrated correlators

\[ \Phi(x, p_T, p.P) = \int \frac{d^4 \xi}{(2\pi)^4} e^{ip.\xi} \left\langle P \left| \bar{\psi}(0) \psi(\xi) \right| P \right\rangle \]

- unintegrated

\[ \Phi(x, p_T; n) = \int \frac{d(\xi.P)d^2 \xi_T}{(2\pi)^3} e^{ip.\xi} \left\langle P \left| \bar{\psi}(0) \psi(\xi) \right| P \right\rangle \bigg|_{\xi.n=0} \]

- Time-ordering automatic, allowing interpretation as forward anti-parton – target scattering amplitude
- Involves operators of twists starting at a lowest value (which is usually called the ‘twist’ of a TMD)

\[ \Phi(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \left\langle P \left| \bar{\psi}(0) \psi(\xi) \right| P \right\rangle \bigg|_{\xi.n=\xi_T=0 \text{ or } \xi^2=0} \]

- collinear (light-cone)
- Involves operators of a definite twist. Evolution via splitting functions (moments are anomalous dimensions)

\[ \Phi = \left\langle P \left| \bar{\psi}(0) \psi(\xi) \right| P \right\rangle \bigg|_{\xi=0} \]

- local
- Local operators with calculable anomalous dimension
Large $p_T$

- $p_T$-dependence of TMDs

\[
\Phi(x, p_T) \quad \rightarrow \quad \frac{1}{\pi p_T^2} \int_x^1 dy \frac{\alpha_s(p_T^2)}{2\pi} P\left(\frac{x}{y}\right) \Phi(y; p_T^2)
\]

- Fictitious measurement
- Large $\mu^2$ dependence governed by anomalous dim (i.e. splitting functions)

- Consistent matching to collinear situation: CSS formalism

JC Collins, DE Soper and GF Sterman, NP B 250 (1985) 199
Twist analysis

- Dimensional analysis to determine importance in an expansion in inverse hard scale
- Maximize contractions with $n$

  $$\dim[\bar{\psi}(0)\gamma \psi(\xi)] = 2$$
  $$\dim[F^{n\alpha}(0)F^{n\beta}(\xi)] = 2$$
  $$\dim[\bar{\psi}(0)\gamma A^\alpha_T(\eta)\psi(\xi)] = 3$$

- ... or maximize # of P’s in parametrization of $\Phi$

  $$\Phi^q(x) = f^q_1(x) \frac{P}{2} \quad \Leftrightarrow \quad f^q_1(x) = \int \frac{d\lambda}{(2\pi)} e^{ix\lambda} \langle P | \bar{\psi}(0)\gamma \psi(\lambda n) | P \rangle$$

- In addition any number of collinear $n.A(\xi) = A^n(x)$ fields (dimension zero!), but of course in color gauge invariant combinations

  $$\dim 0: \quad i\partial^n \rightarrow iD^n = i\partial^n + gA^n$$
  $$\dim 1: \quad i\partial^\alpha_T \rightarrow iD^\alpha_T = i\partial^\alpha_T + gA^\alpha_T$$
Gauge invariance in a nonlocal situation requires a gauge link $U(0, \xi)$

$$\bar{\psi}(0) \psi(\xi) = \sum_n \frac{1}{n!} \xi^{\mu_1} \ldots \xi^{\mu_N} \bar{\psi}(0) \partial_{\mu_1} \ldots \partial_{\mu_N} \psi(0)$$

$$U(0, \xi) = \mathcal{P} \exp \left( -ig \int_0^\xi ds^\mu A_\mu \right)$$

$$\bar{\psi}(0) U(0, \xi) \psi(\xi) = \sum_n \frac{1}{n!} \xi^{\mu_1} \ldots \xi^{\mu_N} \bar{\psi}(0) D_{\mu_1} \ldots D_{\mu_N} \psi(0)$$

Introduces path dependence for $\Phi(x, p_T)$

$$\Phi^U(x, p_T) \Rightarrow \Phi(x)$$
Which gauge links?

\[ \Phi^{q[C]}_{ij}(x, p_T; n) = \int \frac{d(\xi.P)d^2 \xi_T}{(2\pi)^3} e^{i p \cdot \xi} \langle P | \bar{\psi}_j(0) U^{[C]}_{[0, \xi]} \psi_i(\xi) | P \rangle_{\xi, n=0} \]

\[ \Phi^q_{ij}(x; n) = \int \frac{d(\xi.P)}{2\pi} e^{i p \cdot \xi} \langle P | \bar{\psi}_j(0) U^{[n]}_{[0, \xi]} \psi_i(\xi) | P \rangle_{\xi, n=\xi_T=0} \]

◆ Gauge links for TMD correlators process-dependent with simplest cases

AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJ Mulders and F Pijlman, NP B 667 (2003) 201
The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves $C = C'$

$$F_{\alpha\beta}^{\alpha\beta}(\xi) \rightarrow U_{[\eta,\xi]}^{[C]} F_{\alpha\beta}^{\alpha\beta}(\xi) U_{[\xi,\eta]}^{[C]}$$

Basic (simplest) gauge links for gluon TMD correlators:
Which gauge links?

- With more (initial state) hadrons color gets entangled, e.g. in pp

- Outgoing color contributes future pointing gauge link to $\Phi(p_2)$ and future pointing part of a loop in the gauge link for $\Phi(p_1)$

- Can be color-detangled if only $p_T$ of one correlator is relevant (using polarization, …) but include Wilson loops in final $U$

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T.C. Rogers, PJM, PR D81 (2010) 094006

M.G.A. Buffing, PJM, JHEP 07 (2011) 065
Symmetry constraints

\[ \Phi^{T*}(p; P, S) = \gamma_0 \Phi(p; P, S) \gamma_0 \]

\[ \Phi(p; P, S) = \gamma_0 \Phi(\bar{p}; \bar{P}, -\bar{S}) \gamma_0 \]

\[ \Phi^{[U]}(p; P, S) = (-i\gamma_5 C) \Phi^{[-U]}(\bar{p}; \bar{P}, \bar{S})(-i\gamma_5 C) \]

\[ \Phi^c(p; P, S) = C \Phi^T(-p; P, S) C \]

- Hermiticity
- Parity
- Time reversal
- Charge conjugation (giving antiquark corr)

Parametrization of TMD correlator for unpolarized hadron:

\[ \Phi^{[\pm]q}(x, p_T) = \left( f_1^q(x, p_T^2) \pm i h_1^{\perp q}(x, p_T^2) \frac{p_T}{M} \right) \frac{P}{2} \]

(unpolarized and transversely polarized quarks)

- T-even
- T-odd

Mulders, Tangerman, Boer; Bacchetta, Diehl, Goeke, Metz, M, Schlegel, JHEP02 (2007) 093
unpolarized quark distribution

with \( p_T \)

T-odd

helicity or chirality distribution

with \( p_T \)

T-odd

transverse spin distr. or transversity

with \( p_T \)
Experimental consequences

- Even if (elaborated on below) transverse moments involve twist-3, they may show up at leading order in azimuthal asymmetries, cf DY

\[ \sigma(q_T) = \int \int d^2 p_T d^2 k_T \delta^2 (p_T + k_T - q_T) \Phi_1(p_T) \Phi_2(k_T) \ldots \]

- Integrated:

\[
\int d^2 q_T \sigma(q_T) = \int d^2 p_T \Phi_1(p_T) \int d^2 k_T \Phi_2(k_T) \ldots
\]

- Weighted:

\[
\int d^2 q_T q_T^\alpha \sigma(q_T) = \int d^2 p_T \Phi_1(p_T) \int d^2 k_T k_T^\alpha \Phi_2(k_T) \ldots
\]

\[+ \int d^2 p_T p_T^\alpha \Phi_1(p_T) \int d^2 k_T \Phi_2(k_T) \ldots \]

- Examples in DY: Ralston & Soper; Kotzinian; M & Tangerman

- Examples in SIDIS: Collins & Sivers asymmetries

- Examples in pp-scattering: deviations from back-to-back situation in 2-jet production: Boer, Vogelsang

- In particular single-spin asymmetries: T-odd observables (!) requiring T-odd correlators (hard T-odd effects are higher order or mass effects)
Operator structure in collinear case

- Collinear functions

\[ \Phi^q(x) = \int \frac{d(\xi \cdot P)}{(2\pi)} e^{ip\cdot \xi} \left\langle P \left| \bar{\psi}(0) U^{[n]}_{[0, \xi]} \psi(\xi) \right| P \right\rangle_{\xi, n = \xi_T = 0} \]

\[ x^{N-1} \Phi^q(x) = \int \frac{d(\xi \cdot P)}{(2\pi)} e^{ip\cdot \xi} \left\langle P \left| \bar{\psi}(0) (\partial^n)^{N-1} U^{[n]}_{[0, \xi]} \psi(\xi) \right| P \right\rangle_{\xi, n = \xi_T = 0} \]

\[ = \int \frac{d(\xi \cdot P)}{(2\pi)} e^{ip\cdot \xi} \left\langle P \left| \bar{\psi}(0) U^{[n]}_{[0, \xi]} (D^n)^{N-1} \psi(\xi) \right| P \right\rangle_{\xi, n = \xi_T = 0} \]

- Moments correspond to local matrix elements with calculable anomalous dimensions, that can be Mellin transformed to splitting functions

\[ \Phi^{(N)} = \left\langle P \left| \bar{\psi}(0) (D^n)^{N-1} \psi(0) \right| P \right\rangle \]

- All operators have same twist since \( \text{dim}(D^n) = 0 \)
For TMD functions one can consider transverse moments

\[ \Phi(x, p_T; n) = \int \frac{d(\xi, P)d^2 \xi}{(2\pi)^3} e^{ip,\xi} \left\langle P \left| \bar{\psi}(0) \psi(\xi) \right| P \right\rangle_{\xi, n=0} \]

\[ p_T^\alpha \Phi^{[\pm]}(x, p_T; n) = \int \frac{d(\xi, P)d^2 \xi}{(2\pi)^3} e^{ip,\xi} \left\langle P \left| \bar{\psi}(0) UD_T^\alpha (\pm \infty) U \psi(\xi) \right| P \right\rangle_{\xi, n=0} \]

Transverse moments involve collinear twist-3 multi-parton correlators \( \Phi_D \) and \( \Phi_F \) built from non-local combination of three parton fields

\[ \Phi^\alpha_F (x - x_1, x_1 | x) = \int \frac{d\xi, P}{(2\pi)} e^{i(p-p_1),\xi + ip_1,\eta} \left\langle P \left| \bar{\psi}(0) F_{n\alpha} (\eta) \psi(\xi) \right| P \right\rangle_{\xi, n=\xi_T=0} \]

\[ \Phi_D^\alpha (x) = \int dx_1 \Phi_D^\alpha (x - x_1, x_1 | x) \]

\[ \Phi_A^\alpha (x) = PV \int dx_1 \frac{1}{x_1} \Phi_F^{n\alpha} (x - x_1, x_1 | x) \]

T-invariant definition
Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators

\[ \int d^2 p_T \ p_T^\alpha \Phi^{[\pm]}(x, p_T; n) = \Phi_D^\alpha(x) \pm \pi \Phi_G^\alpha(x) \]

- Transverse moments connect to local operators; it is sometimes nicer to work in $b_T$-space with Bessel-weighted asymmetries

\[ e^{iMq_b} F(b_T) = \int d^2 p_T e^{ip_T \cdot b_T} e^{iMq_p} \tilde{F}(p_T) \sim e^{iMq_b} \int p_T dp_T J_M(p_T b_T) \tilde{F}(p_T) \]

- Operators remain nonlocal because of gauge link
  - Accounts in natural way for asymptotic behavior
  - Advantages when considering evolution (cancellation of soft factors)
Distributions versus fragmentation

Operators:

\[ \Phi^{[\pm]}(p \mid p) \sim \langle P \mid \bar{\psi}(0) U_p \psi(\xi) \mid P \rangle \]

\[ \Phi_\delta^\alpha(x) = \tilde{\Phi}_\delta^\alpha(x) \pm \pi \Phi_G^\alpha(x) \]

T-even \hspace{1cm} T-odd (gluonic pole)

\[ \Phi_G^\alpha(x) = \Phi_F^{n\alpha}(x, 0 \mid x) \neq 0 \]

Operators:

\[ \Delta(k \mid k) \sim \sum_X \langle 0 \mid \psi(\xi) \mid K_h X \rangle \langle K_h X \mid \bar{\psi}(0) \mid 0 \rangle \]

\[ \Delta^\alpha_G(x) = \Delta_F^{n\alpha}(\frac{1}{Z}, 0 \mid \frac{1}{Z}) = 0 \]

\[ \Delta_\delta^\alpha(x) = \tilde{\Delta}_\delta^\alpha(x) \]

T-even operator combination, but no T-constraints!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503
Higher azimuthal asymmetries

Transverse moments can be extended to higher moments, involving twist-4 correlators $\Phi_{FF}$ etc., where each of the gluon fields can be a gluonic pole. This is relevant for $\cos(2\phi)$ and $\sin(2\phi)$ asymmetries. Relevant e.g. in the study of transversely polarized quarks in a proton

$$\Phi^q_T(x, p_T) = \ldots + \left( h^q_{1T}(x, p_T^2) \gamma_5 S - h^{\perp q}_{1T}(x, p_T^2) \frac{p_T \cdot S_T}{M} \gamma_5 \frac{P_T}{M} \right) \frac{P}{2}$$

For gluons one needs operators $<F F>$, $<F [F,F]>$, $<F, \{F,F\}>$, $<[F,F] [F,F]>$, $<\{F,F\} \{F,F\}>$ etc. again with increasing twist and several gluonic poles. Relevant in study of linearly polarized gluons in proton

$$\Phi^{\mu\nu}_{g}(x, p_T) = \frac{1}{2x} \left( -g^{\mu\nu}_{T} f_1^g(x, p_T^2) + \left( \frac{p_T^\mu p_T^\nu + \frac{1}{2} g^{\mu\nu}}{M^2} \right) h^{\perp g}_{1}(x, p_T^2) \right)$$

Both $\Phi^{[+,+]}_{g} + \Phi^{[+,+]}_{g}$ and $\Phi^{[+,+]}_{g} + \Phi^{[+,+]}_{g}$ are T-even with 2nd moments containing $<[F,G] [G,F]>$ and $<\{F,G\} \{G,F\}>$ operator terms respectively

C Bomhof and PJ Mulders, NPB 795 (2008) 409
F Dominguez, J-W Qiu, B-W Xiao and F Yuan, PR D85 (2012) 045003
**k_T-factorization**

- TMD moments involve higher twist operators, with many possibilities, distinguishing T-even/odd, chiral-even/odd, ...
- To study scale dependence one needs to have a full definition that accounts for (transitions to) all regions, requiring renormalization scale, regularization of rapidity divergences, ...
- In a process one needs to consider collinear and soft gluons, ...

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J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011

http://projects.hepforge.org/tmd/

Aybat, Prokudin, Rogers, Qiu, Collins,
Conclusions

- TMD physics is rapidly evolving with opportunities such as incorporating spin-momentum correlations visible in T-odd observables (single spin asymmetries) with many theoretical challenges to be addressed.

- Many links and relations with small-x physics, NLO results in collinear case.

- Opportunities exist both theoretically and experimentally! Low and high energies


Talks C Adolph (COMPASS), F Giordano, T Burton ...
Talk F Yuan (Orbital angular momentum),
Talk W den Dunnen (Higgs coupling)
Talks M Boglione, A Bacchetta (Sivers effect)
Talk A Mukherjee (SSA in J/psi production)

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